Massive two-loop Bhabha Scattering — the Factorizable subset

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Outline

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Motivation

Why Bhabha scattering?

- important process for luminosity monitoring
- indirect relevance to all processes measured at Linear Collider
- simple enough Born process to be optimistic that higher order calculation is feasible

Why two-loop calculation?

- experimental precision for next generation of LC $\mathcal{O}(1/1000)$
- theoretical uncertainties need to be controlled

Why massive electrons?

- new theoretical challenge massless results: Bern, Dixon, Ghinculov; Glover, Tausk, van der Bij
- possible foundation for electroweak calculation
- avoid collinear singularities important for matching with existing real calculations

One-loop Contributions

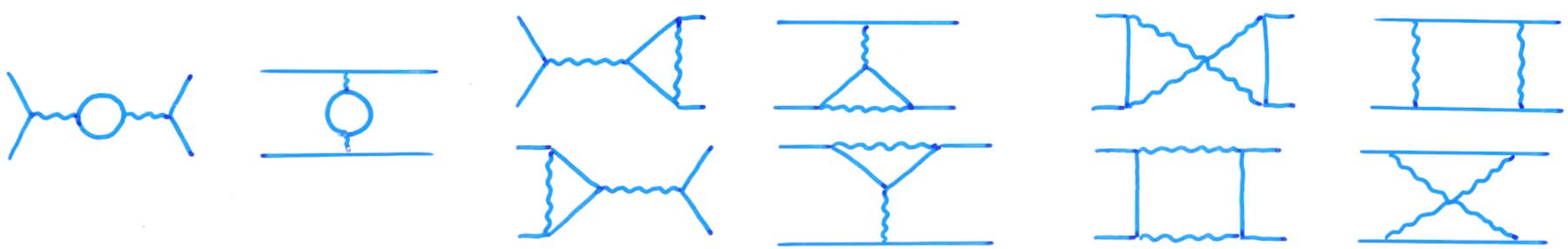
contribution to two-loop calculation as (1-loop)^2

working in Feynman gauge   no tadpoles contribute

- utilize DIagramANAlyser (Fleischer, Tentyukov)

  to generate diagrams (using Qgraf)
  - as a .ps file
  - in symbolic notation for further algebraic manipulation

self-energy diagrams   vertex diagrams   box diagrams



- decompose matrix element into 12 amplitudes

  $\mathcal{M}_1 = [1 \times 1]\, F_1,\ \mathcal{M}_5 = [\gamma_\mu \times \gamma_\mu]\, F_5,\ \mathcal{M}_6 = [\gamma_\mu \not{p}_4 \times \gamma_\mu]\, F_6, \ldots$

- parametrize calculation through corresponding 12 form factors

Symmetry Properties

- Important for box diagrams   self-energies and vertex diagrams are substantially simpler

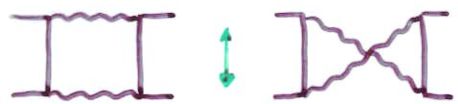
▸ Internal Symmetry

relation of form factors within a given box diagram

→ 6 independent form factors

▸ Conventional Crossing Symmetry S - T

▸ Crossing Symmetry T - U



non-trivial relation between form factors

→ sufficient to calculate one box diagram and obtain form factors of remaining 3 box diagrams via those symmetry relations

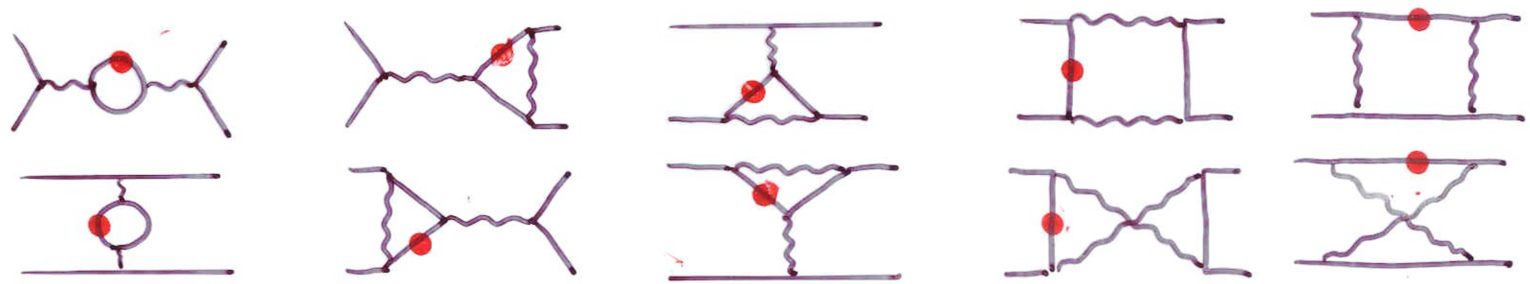
Factorizable two-loop Contributions

two substantial complications when electron is considered massive

→ mass renormalization $$\frac{1}{k^2-(m_e+\delta m_e)^2} \simeq \frac{1}{k^2-m_e^2}\left(1+\frac{2m_e\delta m_e}{k^2-m_e^2}\right)$$

multiplicative 1-loop x 1-loop

need to calculate the following additional diagrams (with dots on lines)



1) electron-propagator appears squared

- New method is needed to reduce this to master integrals

2) mass renormalization terms contain $1/\varepsilon$ UV poles

all one-loop contributions are needed up to $O(\varepsilon)$

- New method is needed to calculate those master integrals

see J. Fleischers talk

Recurrence Relations

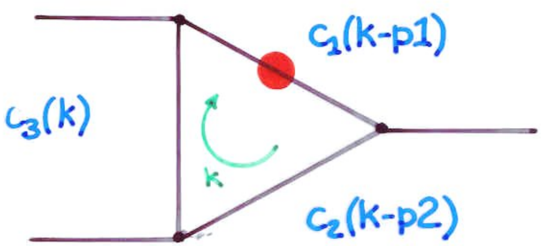
J. Fleischer, F. Jegerlehner and O.V. Tarasov, Nucl. Phys. B566 (2000) p. 423

To illustrate the method choose simple example:

e.g. Three-Point Function

$C_i = (k-p_i)^2 - m_i^2 + i\varepsilon$

$C_3 = k^2 - m_3^2 + i\varepsilon$



Loop-momentum k

reduce additional denominator as well as tensor structure

$$\int \frac{k_\mu[d]}{c_1^2 c_2 c_3} \to -\int \frac{p_{1\mu}[d+2]}{c_1^3 c_2 c_3} - \boxed{\int \frac{p_{2\mu}[d+2]}{c_1^2 c_2^2 c_3}}$$

Def. Cayley-determinant

$$()_n = \begin{vmatrix} 0 & 1 & 1 & \cdots & 1 \\ 1 & Y_{11} & Y_{12} & \cdots & Y_{1n} \\ 1 & Y_{12} & Y_{22} & \cdots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_{1n} & Y_{2n} & \cdots & Y_{nn} \end{vmatrix}$$

with $Y_{ij} = -(p_i - p_j)^2 + m_i^2 + m_j^2$

- Removing dots from lines and changing dimension

$$()_n \, \nu_j \, \mathbf{j}^+ I_n^{(d+2)} = \left[-\begin{pmatrix} j \\ 0 \end{pmatrix}_n + \sum_{k=1}^{n} \begin{pmatrix} j \\ k \end{pmatrix}_n \mathbf{k}^- \right] I_n^{(d)}$$

- Removing remaining dots from lines without changing dimension

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}_n \nu_j \, \mathbf{j}^+ I_n^{(d)} = \left\{ \left(1 + \sum_{i=1}^{n} \nu_i - d \right) \begin{pmatrix} 0 \\ j \end{pmatrix}_n - \sum_{k=1}^{n} \begin{pmatrix} 0j \\ 0k \end{pmatrix}_n (\nu_k - 1) \right\} I_n^{(d)} - \sum_{i,k:\, i \neq k}^{n} \begin{pmatrix} 0j \\ 0k \end{pmatrix}_n \nu_i \, \mathbf{k}^- \mathbf{i}^+ I_n^{(d)}$$

- Reducing dimension

$$\left(d - \sum_{i=1}^{n} \nu_i + 1 \right) ()_n \, I_n^{(d+2)} = \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}_n - \sum_{k=1}^{n} \begin{pmatrix} 0 \\ k \end{pmatrix}_n \mathbf{k}^- \right] I_n^{(d)}$$

Three-Point Package

$$(\quad)_3 \, 1^+ \int \frac{d^{d+2}k}{c_1 c_2^2 c_3} = \left[-(\quad)_3 + (\quad)_3 \, c_1 + (\quad)_3 \, c_2 + (\quad)_3 \, c_3 \right] \int \frac{d^d k}{c_1 c_2^2 c_3}$$

$$(\quad)_3 \, 2^+ \int \frac{d^d k}{c_1 c_2 c_3} = \left[(1+3-d) (\quad)_3 \right] \int \frac{d^d k}{c_1 c_2 c_3} - \left[(\quad)_3 \left(\frac{c_1}{c_2} + \frac{c_1}{c_3} \right) + (\quad)_3 \left(\frac{c_2}{c_1} + \frac{c_2}{c_3} \right) + (\quad)_3 \left(\frac{c_3}{c_2} + \frac{c_3}{c_1} \right) \right] \int \frac{d^d k}{c_1 c_2 c_3}$$

needed for higher tensor structures

Two-Point Package

skipped because no [d+]

$$\left(\quad\right)_2 1^+ \int \frac{d^d k}{c_1 c_2} = \left[(1+2-d)\left(\quad\right)_2\right] \int \frac{d^d k}{c_1 c_2} - \left[\left(\quad\right)_2 \left(\frac{c_1}{c_2}\right) + \left(\quad\right)_2 \left(\frac{c_2}{c_1}\right)\right] \int \frac{d^d k}{c_1 c_2}$$

One-Point Package

$$\left(\quad\right)_1 \mathbf{1}^{+} \int \frac{d^d k}{C_1} = \left[(1 + 1 - d) \left(\vert\!-\!\!- \right)_1 \right] \int \frac{d^d k}{C_1}$$

Zero Gram-determinant

- How to proceed if $(\ )_3 = 0$?

  similar mechanism exists for zero Gram-determinants
  and is implemented as well

- Zero Gram-determinant means effectively that scalar n-point integrals can be further simplified into (n-1)-, (n-2)-, ...point integrals

$$C_0(m_e^2, s, m_e^2; 0, m_e^2, m_e^2) = -\frac{(d-2)}{(d-4)}\frac{1}{s-4m_e^2}\frac{A_0(m_e^2)}{m_e^2} + 2\frac{(d-3)}{(d-4)}\frac{1}{s-4m_e^2}B_0(s, m_e^2, m_e^2)$$

$$B_0(m_e^2, m_e^2, 0) = \frac{1}{2}\frac{(d-2)}{(d-3)}\frac{A_0(m_e^2)}{m_e^2}$$

Summary

presented progress on the two-loop process $e^+e^- \to e^+e^-$

starting point: DIANA

1-loop calculation

- selfenergies, vertices and box diagrams
- decomposition into 12 amplitudes → 12 form factors
- exploiting symmetries → 6 independent form factors

2-loop calculation

- mass renormalization → dotted selfenergies, vertices and box diagrams
  - reduce to scalar master integrals making use of recurrence relations
  - expansion of scalar master integrals to $O(\varepsilon)$

▸ Compact expression of all dotted diagrams as function of master integrals A_0, B_0, C_0 and D_0

everything that is needed for factorizable two-loop calculation is well under control