Sudakov Logarithms in Four-Fermion Electroweak Processes at High Energy

Sven-Olaf Moch*

DESY Theorie
Zeuthen

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* in collaboration with **B. Feucht**, **J.K. Kühn**, **A.A. Penin** and **V.A. Smirnov**

Motivation

Electroweak processes at high energy

– Perturbation theory $\longrightarrow$ large logarithmic corrections in calculation of observables

  – typically at one-loop $\alpha \ln^2(s/M^2)$

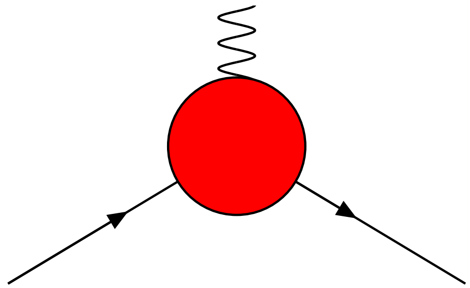
    Ciafaloni, Comelli '98 ; Kühn, Penin, Smirnov '00 ; Fadin, Lipatov, Martin, Melles '00 ; Denner, Pozzorini '00 ; Beenakker, Werthenbach '01 ; Beccaria, Renard, Verzegnassi '01, ...

– Improve perturbative expansion $\longrightarrow$ all order resummation

Resummation

– Why resum beyond logarithmic accuracy ?
  – experiments :
    – both LHC and a next linear collider demand better standard model predictions
  – theory :
    – large K-factors at NLO $\longrightarrow$ higher orders ?
    – error on leading logarithms $\longrightarrow$ assess quality of resummation

Factorization and resummation



– Classic example :
  QED form factor of massless fermion
  Sudakov ‘56 ; Fishbane, Sullivan ‘71 ; Collins ‘89 ; Korchemsky ‘89, ...

  – n-th order : $\alpha_s^n \ln^{2n}(Q/Q_0)$ with Q/Q_0 : large/small scale

– Standard multiplicative renormalization of Green’s function G

$$G_{\mathrm{bare}}\left(p, \frac{\Lambda}{Q}, \alpha_s\right) = \prod_i Z_i^{1/2}\left(\frac{\mu}{\Lambda}, \alpha_s(\mu)\right) G_{\mathrm{ren}}\left(p, \frac{\mu}{Q}, \alpha_s(\mu)\right) .$$

  – factorize UV-cutoff Λ
  – for renormalization scale μ require : $\mu \frac{d}{d\mu} G_{\mathrm{bare}} = 0\,.$

– Solve renormalization group equation :
  – determine anomalous dimensions γ_i from the Z_i
  – integrate $\mu(d/d\mu)G_{\mathrm{bare}} \longrightarrow$ resummation of large logarithms done

Factorization

– Understand resummation from underlying factorization

Operator approach

– Factorization of amplitude $\mathcal{A}$ into various functions Collins, Soper ‘81, Gatheral ‘83, Sterman ‘87, ...

$$\mathcal{A} \quad = \quad H \quad S \quad J$$

- $H \longrightarrow$ hard off-shell momenta at short distances
- $J \longrightarrow$ collinear jet-like particles near the light cone
- $S \longrightarrow$ soft quanta of long wave-length

Alternative approaches

– Apply expansion in various kinematical regions directly to Feynman diagrams
$\longrightarrow$ strategy of regions Beneke, Smirnov ‘98 ; Smirnov ‘99 ; Kühn, Penin, Smirnov ‘00

– Infrared evolution equations Fadin, Lipatov, Martin, Melles ‘00 ; Melles ‘01

The jet-function

– Operator definition for J with path-ordered exponential along light-like ζ

$$J\,(p\cdot\zeta) = \langle 0|\mathrm{Pexp}\left\{\mathrm{i}g\int_0^{\infty} dt\,\zeta\cdot A(t\zeta)\right\}\psi(0)|p\rangle$$

– Resummation of all leading logarithms via two evolution equations for J

Sterman '87 ; Korchemsky '89

– renormalization group equation $\longrightarrow \mu\frac{\partial}{\partial\mu}$

– evolution equation for kinematics dependence $\longrightarrow p\cdot\zeta\frac{\partial}{\partial p\cdot\zeta}$

Idea

– Identify kinematics vector $\zeta^{\mu} \longrightarrow p\cdot\zeta$ and gauge vector $n^{\mu} \longrightarrow n\cdot A = 0$
$\longrightarrow$ turn kinematics dependence into gauge dependence Collins, Soper '81

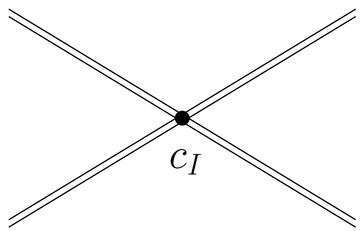
– Use boson propagator in axial gauge to calculate changes in n^{μ}

$$\frac{\partial}{\partial n_a}\left(-g^{\mu\nu} + \frac{n_{\mu}l_{\nu} + n_{\nu}l_{\mu}}{n\cdot l} - n^2\frac{l_{\mu}l_{\nu}}{(n\cdot l)^2}\right)$$

– Explicit gauge dependence compensated by soft function S

The soft function

- Construction of S_I as composite operator
  - coupling to Wilson-lines $\longrightarrow$ particles in eikonal approximation
  - matrix in colour/chiral space



- S obeys renormalization group equation

Contopanagos, Laenen, Sterman ‘97 ; Kidonakis, Sterman ‘97

$$\mu \frac{d}{d\mu} S_I = -\chi_{IA} S_J$$

- UV-divergency of composite operator $\longrightarrow$ soft anomalous dimension χ_{IJ}
- Sensitive to gauge group and chiral structure of scattering process

Four-fermion electroweak processes

Form factor for $SU(2)$ toy model

– Born term $\mathcal{F}_\mathrm{B} = \bar{\psi}(p_2)\gamma_\mu\psi(p_1)$

– Evolution equation

Sen '81 ; Collins '89 ; Korchemsky '89, ...

$$\frac{\partial}{\partial \ln Q^2}\mathcal{F} = \left[\int_{M^2}^{Q^2}\frac{dx}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(M^2))\right]\mathcal{F}$$

– Anomalous dimensions γ, ζ and ξ

– $\gamma^{(1)} = -2C_F$, $\quad \gamma^{(2)} = -2C_F\left[\left(\frac{67}{9} - \frac{\pi^2}{3}\right)C_A - \frac{20}{9}T_F n_f\right]$, ...

Kodaira, Trentadue '81

– $\zeta^{(1)} = 3C_F$, ...

– Upshot :

$$\mathcal{F} = \mathcal{F}_\mathrm{B}\, F_0(\alpha(M^2)) \exp\left\{\int_{M^2}^{Q^2}\frac{dx}{x}\left[\int_{M^2}^{x}\frac{dx'}{x'}\gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(M^2))\right]\right\}$$

Four-fermion amplitude for $SU(2)$ toy model

– Spinor basis for scattering process $f + f' \to f + f'$

$$\mathcal{A}^d = (\bar{\psi}_2 \gamma^\mu \psi_1)\,(\bar{\psi}_4 \gamma_\mu \psi_3)$$
$$\mathcal{A}^\lambda = (\bar{\psi}_2 t^a \gamma^\mu \psi_1)\,(\bar{\psi}_4 t^a \gamma_\mu \psi_3)$$

– Factorization of ampltitude ($\tilde{\mathcal{A}} = \mathcal{A} *$ collinear logs)

$$\mathcal{A} = \frac{ig^2}{s}\ \mathcal{F}^2\ \tilde{\mathcal{A}}$$

– Evolution equation for $\tilde{\mathcal{A}}$ Sen ‘81 ; Contopanagos, Laenen, Sterman ‘97 ; S.M., Kühn, Penin, Smirnov ‘01

$$\frac{\partial}{\partial \ln Q^2} \begin{pmatrix} \tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d \end{pmatrix} = \chi\left(\alpha\left(Q^2\right)\right) \begin{pmatrix} \tilde{\mathcal{A}}^\lambda \\ \tilde{\mathcal{A}}^d \end{pmatrix}$$

– Solution requires diagonalization of matrix χ

$$\tilde{\mathcal{A}} = \sum_i \tilde{\mathcal{A}}_{0\,i}(\alpha(M^2)) \exp\left[\int_{M^2}^{Q^2} \frac{dx}{x} \chi_i(\alpha(x))\right]$$

NNLL matching

– Explicit one-loop calculation (logarithmic accuracy) :

$$\mathcal{A}^{(1)} = \frac{ig^2}{s}\frac{\alpha}{2\pi}\left[\left\{-C_F\left(\ln^2\left(\frac{s}{M^2}\right)-3\ln\left(\frac{s}{M^2}\right)\right)-\left(C_A\ln\left(\frac{-u}{s}\right)\right.\right.\right.$$
$$\left.\left.-2\left(C_F-\frac{T_F}{N}\right)\ln\left(\frac{u}{t}\right)\right)\ln\left(\frac{s}{M^2}\right)\right\}\mathcal{A}^{\lambda}+\left\{2\frac{C_F T_F}{N}\ln\left(\frac{u}{t}\right)\ln\left(\frac{s}{M^2}\right)\right\}\mathcal{A}^{d}+\mathcal{A}_0^{(1)}\Bigg]$$

– Check for all logarithms from NLL resummation

– Determine $\mathcal{A}_0^{(1)}$ for NNLL matching

Results

S.M., Kühn, Penin, Smirnov '01

$$\sigma = \sigma_B + \frac{\alpha}{4\pi}\sigma^{(1)} + \left(\frac{\alpha}{4\pi}\right)^2 \sigma^{(2)} + \dots$$

– $u\bar{u} \to u\bar{u}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{80}{3}\ln\left(\frac{s}{M^2}\right) - \left(\frac{25}{9} + 3\pi^2\right)\right]\sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{449}{6}\ln^3\left(\frac{s}{M^2}\right) + \left(\frac{4855}{18} + \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B$$

– $u\bar{u} \to d\bar{d}$

$$\sigma^{(1)} = \left[-3\ln^2\left(\frac{s}{M^2}\right) + \frac{26}{3}\ln\left(\frac{s}{M^2}\right) + \left(\frac{218}{9} - 3\pi^2\right)\right]\sigma_B$$

$$\sigma^{(2)} = \left[\frac{9}{2}\ln^4\left(\frac{s}{M^2}\right) - \frac{125}{6}\ln^3\left(\frac{s}{M^2}\right) - \left(\frac{799}{9} - \frac{37\pi^2}{3}\right)\ln^2\left(\frac{s}{M^2}\right)\right]\sigma_B$$

– Large coefficients, alternating signs

Standard Model : W, Z, γ

– Examine $f' \overline{f'} \to f \overline{f}$

$$A_B = \frac{ig^2}{s} \sum_{I,J=L,R} \left(T^3_{f'} T^3_f + t^2_W \frac{Y_{f'} Y_f}{4} \right) A^{f'f}_{IJ} \qquad \text{with} \qquad A^{f'f}_{IJ} = \left(\overline{f'_I} \gamma^\mu f'_I \right) \left(\overline{f_J} \gamma_\mu f_J \right)$$

– Account separately for corrections from photon radiation up to cutoff $\omega \ll M_{W,Z}$

Fadin, Lipatov, Martin, Melles '00

– Neglect W, Z mass differences in evolution equations $\longrightarrow M_W = M_Z = M$

– Define for $\sqrt{s} = 1$ TeV (2 TeV)

$$L(s) = \frac{g^2}{16\pi^2} \ln^2\left(\frac{s}{M^2}\right) = 0.07 \quad (0.11)$$

$$l(s) = \frac{g^2}{16\pi^2} \ln\left(\frac{s}{M^2}\right) = 0.014 \, (0.017)$$

$$a = \frac{g^2}{16\pi^2} = 0.003$$

Results

S.M., Kühn, Penin, Smirnov '01

– Total cross-sections :

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to Q\bar{Q}) = 1 - 1.66\,L(s) + 5.31\,l(s) - 15.86\,a$$
$$+1.93\,L^2(s) - 9.43\,L(s)l(s) + 28.73\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to q\bar{q}) = 1 - 2.18\,L(s) + 20.58\,l(s) - 36.34\,a$$
$$+2.79\,L^2(s) - 50.06\,L(s)l(s) + 295.12\,l^2(s)$$

$$\sigma/\sigma_{\mathrm{B}}(e^+e^- \to \mu^+\mu^-) = 1 - 1.39\,L(s) + 10.12\,l(s) - 31.33\,a$$
$$+1.42\,L^2(s) - 18.43\,L(s)l(s) + 99.89\,l^2(s)$$

– Asymmetries : corrections similar and even larger for A_{LR}, smaller for A_{FB}

– Leading logarithms confirmed by explicit two-loop calculation   Hori, Kawamura, Kodaira '00

Stability of logarithmic approximation

– Subleading terms in form factor of massive $U(1)$-model

$$\mathcal{F} = \mathcal{F}_{\mathrm{B}}\left[1+\frac{\alpha}{4\pi}\mathcal{F}^{(1)}+\frac{\alpha^2}{16\pi^2}\mathcal{F}^{(2)}+\ldots\right].$$

– Logarithms

S.M., Feucht, Kühn

$$\mathcal{F}^{(1)} = -\ln\left(\frac{s}{M^2}\right)^2+3\ln\left(\frac{s}{M^2}\right)-\frac{7}{2}-\frac{2}{3}\pi^2$$

$\longrightarrow$ all fermionic (n_f-enhanced) contributions at two loops

$$\mathcal{F}^{(2)}\Big|_{n_f} = n_f\left(-\frac{4}{9}\ln\left(\frac{s}{M^2}\right)^3+\frac{38}{9}\ln\left(\frac{s}{M^2}\right)^2-\frac{34}{3}\ln\left(\frac{s}{M^2}\right)+\frac{115}{9}+\frac{16}{27}\pi^2\right)$$

– Pattern of alternating signs and growing coefficients persists

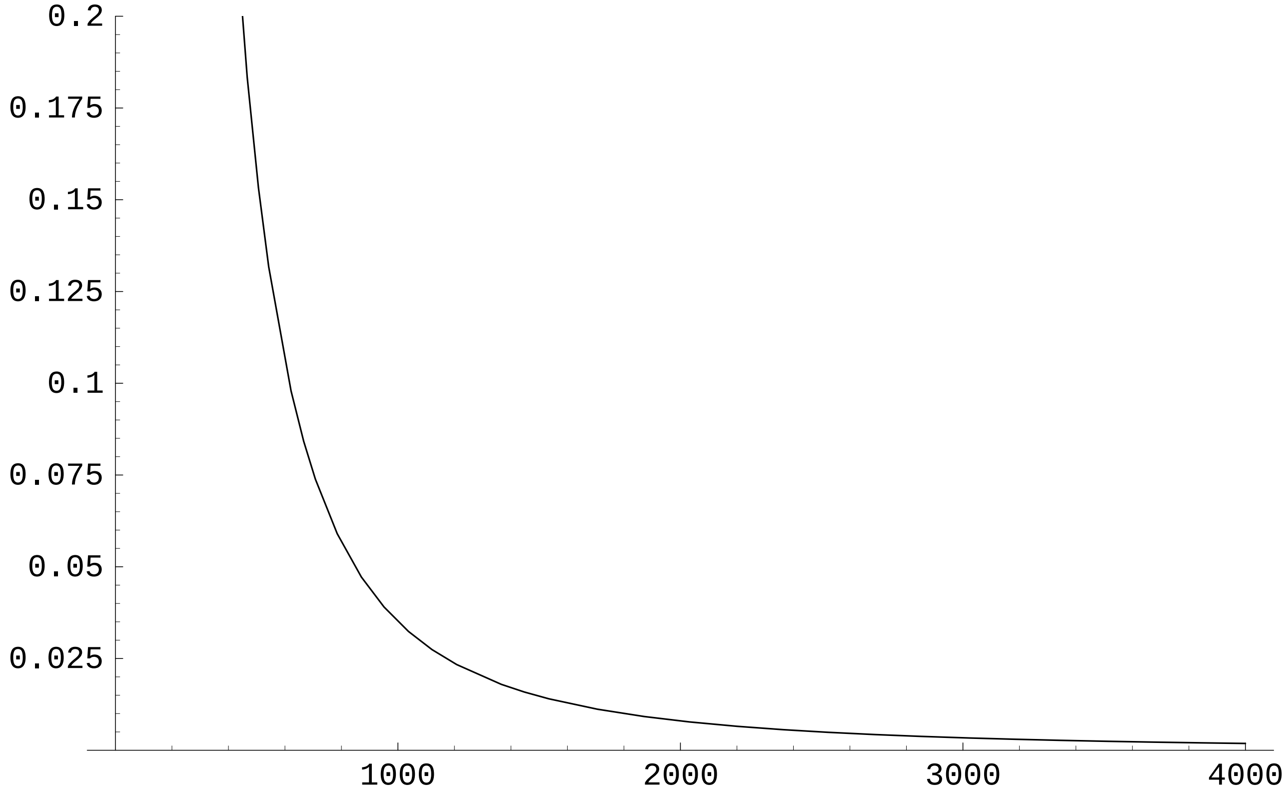
– Exact results $\longrightarrow$ examine M^2/s corrections S.M., Feucht, Kühn

$$\mathcal{F}^{(1)} = 2z - \frac{7}{2} - (1-z)^2\left\{2\mathrm{Li}_2(1-z) + \ln(z)^2 + \frac{\pi^2}{3}\right\} - (3-2z)\ln(z)$$

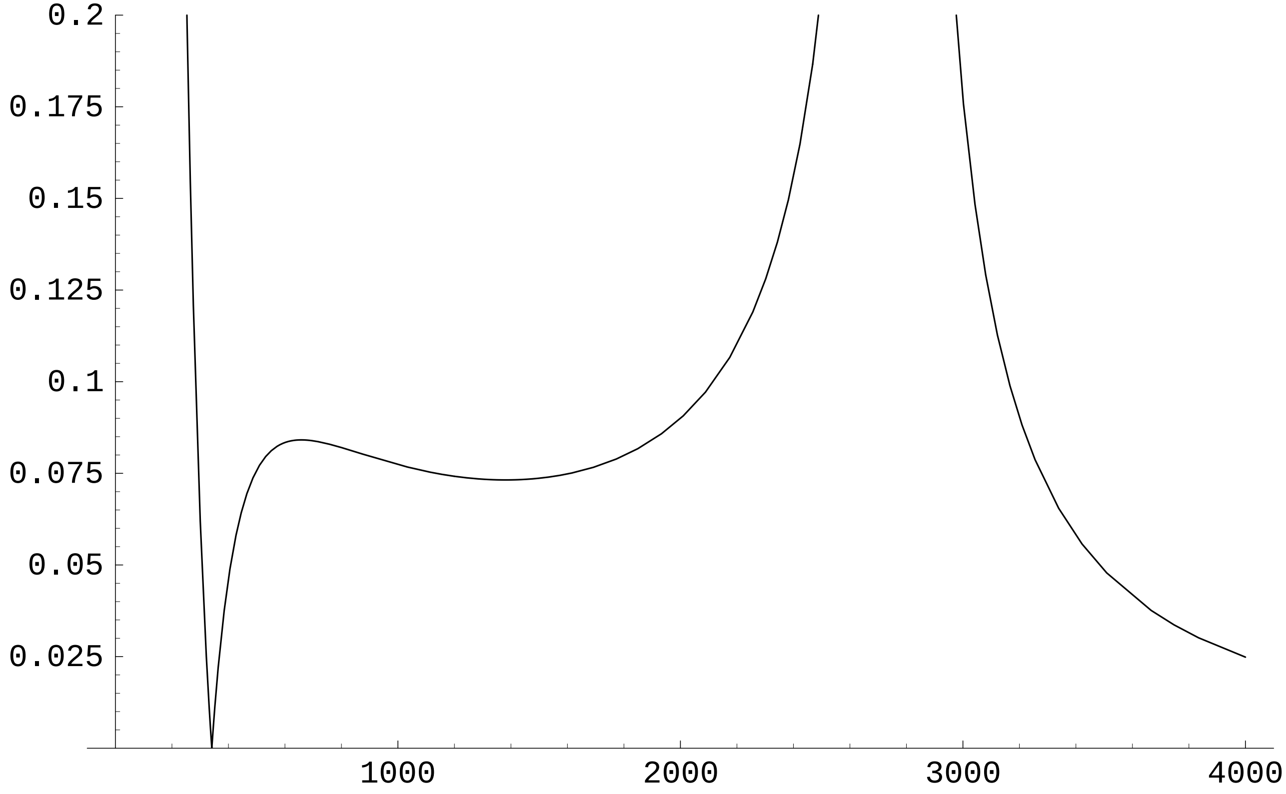
$$\begin{aligned}\mathcal{F}^{(2)}\Big|_{n_f} = \;& n_f\Bigg(\frac{115}{9} - \frac{88}{9}z + (1-z)^2\left\{\frac{16}{9}\mathrm{Li}_2(1-z) + \frac{8}{27}\pi^2\right\} + \ln(z)\left\{\frac{34}{3} - \frac{88}{9}z\right\}\\ & +(1-4z+3z^2)\left\{\frac{4}{9}\ln(z)^3 + \frac{4}{3}\ln(z)^2\ln(1-z) + \frac{8}{3}\mathrm{Li}_3(z) + \frac{8}{3}\ln(z)\mathrm{Li}_2(1-z)\right.\\ & \left. -\frac{4}{9}\ln(z)\pi^2\right\} + \ln(z)^2\left\{\frac{38}{9} - \frac{52}{9}z + \frac{8}{9}z^2\right\}\Bigg].\end{aligned}$$

with $z = M^2/s$

– Plot of $\frac{(\mathcal{F}^{(1)}_{\text{exact}} - \mathcal{F}^{(1)}_{\text{approx.}})}{\mathcal{F}^{(1)}_{\text{exact}}}$ for mass M = 100 MeV against $\sqrt{s} =$ in MeV



– Plot of $\dfrac{(\mathcal{F}^{(2)}\big|_{n_f,\mathrm{exact}} - \mathcal{F}^{(2)}\big|_{n_f,\mathrm{approx.}})}{\mathcal{F}^{(2)}\big|_{n_f,\mathrm{exact}}}$ for mass M = 100 MeV against $\sqrt{s} =$ in MeV



Summary

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- Factorization and resummation very closely connected
- Electroweak processes at high energy
  $\longrightarrow$ resummation of subleading logarithms and complete one-loop matching
  $\longrightarrow$ two-loop approximation up to the NNLL logarithms
- Stability
  $\longrightarrow$ explicit calculation of some subleading logarithms at two loops

Outlook

- Calculate full two-loop form factor in massive $U(1)$ model