Lepton Flavour Violating Z Decays in the MSSM

José I. Illana

in collaboration with: **Manuel Masip**

Universidad de Granada

- **Present status of LFV**
- **SUSY: parametrization of slepton mass matrices**
- $\boxed{Z \to \ell_I \ell_J}$ **at DESY TESLA GigaZ**
- **Bounds from** $\boxed{\ell_J \to \ell_I \gamma}$
- **LFV and** $\boxed{(g_\mu - 2)}$
- **Summary and conclusions**

[hep-ph/0207328]

Status of LFV

BR($\mu \to e\gamma$) $< 1.2 \times 10^{-11}$	[MEGA '99]	BR($\mu \to 3e$) $< 1.0 \times 10^{-12}$	[SINDRUM '88]
BR($\tau \to e\gamma$) $< 2.7 \times 10^{-6}$	[CLEO '99]	BR($\tau \to 3e$) $< 2.9 \times 10^{-6}$	[CLEO '98]
BR($\tau \to \mu\gamma$) $< 1.1 \times 10^{-6}$	[CLEO '00]	BR($\tau \to 3\mu$) $< 1.9 \times 10^{-6}$	[CLEO '98]

R(μTi $\to e$Ti) $< 6.1 \times 10^{-13}$ [SINDRUM '98]

BR($Z \to \mu e$) $< 1.7 \times 10^{-6}$	[OPAL '95]	$\Longrightarrow$	$< 2.0 \times 10^{-9}$	[GigaZ]
BR($Z \to \tau e$) $< 9.8 \times 10^{-6}$	[OPAL '95]	$\Longrightarrow$	$< \kappa \times 6.5 \times 10^{-8}$	($\kappa = 0.2$ – 1.0)
BR($Z \to \tau\mu$) $< 1.2 \times 10^{-5}$	[DELPHI '97]	$\Longrightarrow$	$< \kappa \times 2.2 \times 10^{-8}$	

- Neutrino oscillations (neutral LFV) $\Rightarrow$ νSM predicts tiny charged LFV
- SUSY models (unbroken R–parity) $\Rightarrow$ slepton mass matrix natural source of LFV

Aim: $\boldsymbol{Z \to \ell_I \ell_J}$	**GigaZ potential, independently of origin of SUSY breaking**

with constraints dominantly from $\ell_J \to \ell_I \gamma$ since:

$$\text{BR}(\ell_J \to 3\ell_I) \approx \alpha_{em} \text{BR}(\ell_J \to \ell_I \gamma), \quad \text{R}(\mu\text{Ti} \to e\text{Ti}) \approx 5 \times 10^{-3} \times \text{BR}(\mu \to e\gamma)$$

General Lorentz Structure $V\bar{\ell}_I\ell_J$

- **Amplitudes** for **on–shell** external legs: one–loop parametrization $[\alpha_W = g^2/(4\pi)]$

$$\mathcal{M}_Z = -ig\,\frac{\alpha_W}{4\pi}\,\varepsilon^\mu_Z\bar{u}_{\ell_I}(p_2)\left[\gamma_\mu(f_V^Z - f_A^Z\gamma_5) + \frac{\sigma_{\mu\nu}q^\nu}{M_W}(if_M^Z + f_E^Z\gamma_5)\right]u_{\ell_J}(p_1)$$

$$\mathcal{M}_\gamma = -ie\,\frac{\alpha_W}{4\pi}\,\varepsilon^\mu_\gamma\bar{u}_{\ell_I}(p_2)\left[\gamma_\mu \underbrace{f_V^\gamma}_{=0\ \text{for}\ I\neq J} + \frac{\sigma_{\mu\nu}q^\nu}{m_{\ell_J}}(if_M^\gamma + f_E^\gamma\gamma_5)\right]u_{\ell_J}(p_1)$$

Dipole form factors (f_M, f_E) are chirality flipping (proportional to a fermion mass)

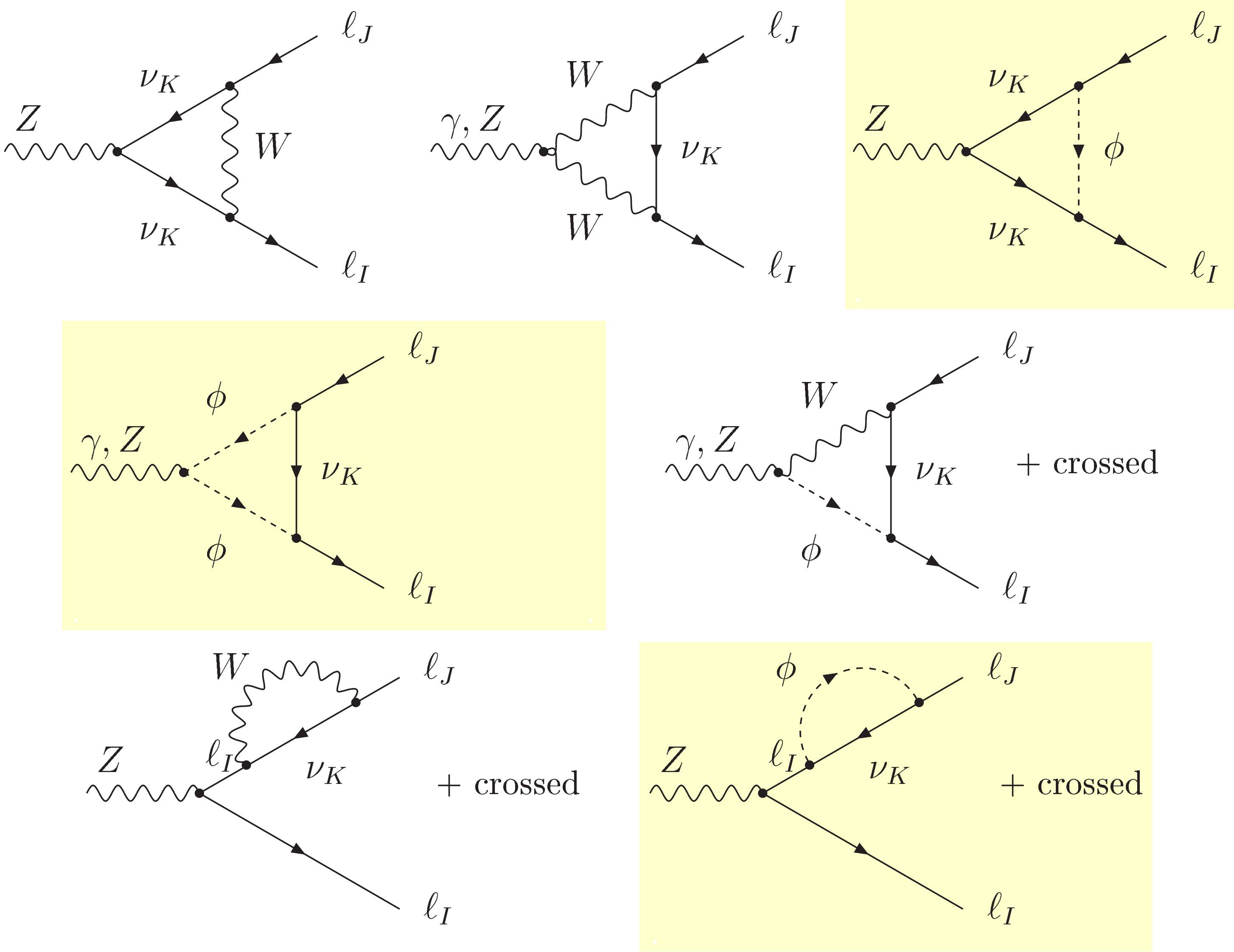
- **Branching ratios** $[f_{L,R} = f_V \pm f_A]$

$$\text{BR}(Z\to\bar{\ell}_I\ell_J + \ell_I\bar{\ell}_J) = \underbrace{\frac{\alpha_W^3 M_Z}{48\pi^2\Gamma_Z}}_{\mathcal{O}(10^{-6})}\left[|f_L^Z|^2 + |f_R^Z|^2 + \frac{1}{c_W^2}(|f_M^Z|^2 + |f_E^Z|^2)\right]$$

$$\text{BR}(\ell_J\to\ell_I\gamma) = \frac{12\alpha}{\pi}\frac{M_W^4}{m_{\ell_J}^4}\left(|f_M^\gamma|^2 + |f_E^\gamma|^2\right)\times\underbrace{\text{BR}(\ell_J\to\ell_I\nu_J\bar{\nu}_I)}_{=1/0.17/0.17\ \text{for}\ \mu e/\tau e/\tau\mu}$$

- For $\ell\equiv\ell_I=\ell_J$, **anomalous magnetic dipole moment** $\quad a_\ell = (g_\ell - 2)/2 = \dfrac{\alpha_W}{4\pi}f_M^\gamma$

Standard Model + **massive neutrinos**



$$\mathcal{M} \propto \sin^2 2\theta \, \frac{\Delta m^2_{IJ}}{M^2_W}$$

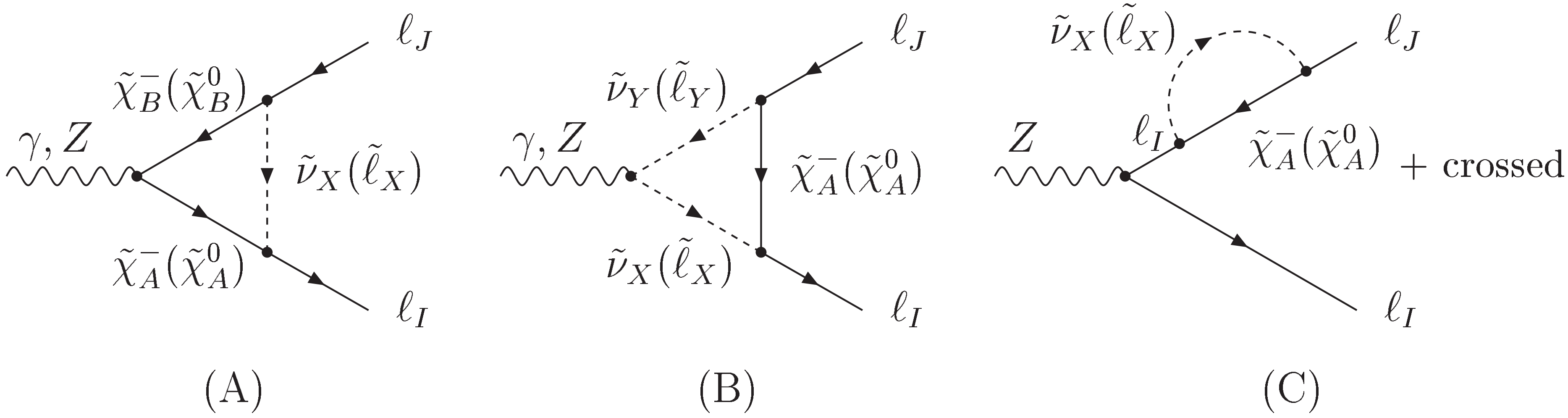
Oscillation data $\Rightarrow$

$\mathrm{BR}(\ell_J \to \ell_I \gamma) \lesssim 10^{-48}$

$\mathrm{BR}(Z \to \ell_I \ell_J) \lesssim 10^{-54}$

hopeless!

MSSM



- Charginos–sneutrinos $(\tilde{\chi}^\pm_{A=1,2}$; $\tilde{\nu}_{X=1,2,3}$ 'L')
  Neutralinos–charged sleptons $(\tilde{\chi}^0_{A=1,2,3,4}$; $\tilde{\ell}_{X=1,\dots,6}$ 'L' and 'R')
- (A)+(B)+(C): ultraviolet finite and decoupling of heavy SUSY particles
- (C) no contribution to $\ell_J \to \ell_I \gamma$ (no dipole structure)
- FCNC/LFV $(\ell_I \neq \ell_J) \Leftrightarrow \begin{cases} \text{slepton family mixing} \\ \text{slepton mass splitting (non–degeneracy)} \end{cases}$ (LL, RR, LR)

Slepton mass matrices

$\mathbf{M}^2_{\tilde\nu} = (\mathbf{m}^2_{\tilde\nu_L})_{3\times 3}$	$\mathbf{M}^2_{\tilde\ell} = \begin{pmatrix} \mathbf{m}^2_{LL} & \mathbf{m}^{2T}_{LR} \\ \mathbf{m}^2_{LR} & \mathbf{m}^2_{RR} \end{pmatrix}_{6\times 6}$ (symmetric)
$(\mathbf{m}^2_{\tilde\nu_L})_{IJ} = (\mathbf{m}^2_{L})_{IJ} + \frac{1}{2} M_Z^2 c2\beta\ \delta_{IJ}$	$(\mathbf{m}^2_{LL})_{IJ} = (\mathbf{m}^2_{L})_{IJ} + \left[m^2_{\ell_I} + \left(-\frac{1}{2} + s_W^2\right) M_Z^2 c2\beta\right] \delta_{IJ}$
	$(\mathbf{m}^2_{RR})_{IJ} = (\mathbf{m}^2_{R})_{IJ} + \left[m^2_{\ell_I} - M_Z^2 c2\beta s_W^2\right] \delta_{IJ}$
$(I, J = 1, 2, 3)$	$(\mathbf{m}^2_{LR})_{IJ} = (\mathbf{A}_\ell)_{IJ}\, v\cos\beta/\sqrt{2} - m_{\ell_I}\mu\tan\beta\ \delta_{IJ}$

Assume two–generation mixing (IJ) and no alignment between fermion and scalar fields $\Rightarrow$

$$\mathbf{M}^2_{\tilde\nu} = \tilde m^2 \begin{pmatrix} 1 & \cdot \\ \delta^{\tilde\nu\, IJ}_{LL} & 1 \end{pmatrix} ; \qquad \mathbf{M}^2_{\tilde\ell} = \tilde m^2 \begin{pmatrix} 1 & \cdot & \cdot & \cdot \\ \delta^{\tilde\ell\, IJ}_{LL} & 1 & \cdot & \cdot \\ \delta^{\tilde\ell\, II}_{LR} & \delta^{\tilde\ell\, IJ}_{LR} & 1 & \cdot \\ \delta^{\tilde\ell\, JI}_{LR} & \delta^{\tilde\ell\, JJ}_{LR} & \delta^{\tilde\ell\, IJ}_{RR} & 1 \end{pmatrix} \qquad \sin 2\theta = 1$$

and only one $\delta \neq 0$: $\quad \tilde m^2_{1,2} = \tilde m^2(\sqrt{1+\delta^2} \mp \delta) \quad \Longleftrightarrow \quad \delta = \frac{\tilde m_2^2 - \tilde m_1^2}{2\tilde m^2}\ ; \quad \tilde m^2 = \tilde m_1 \tilde m_2$

Calculation

- **Analytical expressions** in terms of generic couplings and one–loop tensor integrals, implemented with complete MSSM Feynman rules (full chargino and neutralino mixings)

- **Constraints on SUSY masses** [PDB]:

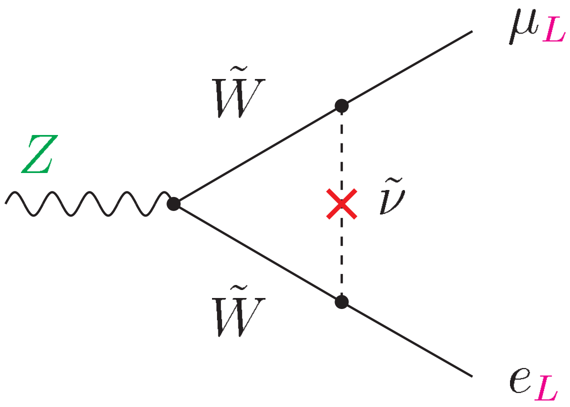
lightest slepton $(\tilde{m}_1)$	$m_{\tilde{\nu}} > 45$ GeV $m_{\tilde{\ell}_{L,R}} > 90$ GeV *
lightest chargino	$m_{\tilde{\chi}_1^+} > 75$ GeV, if $m_{\tilde{\nu}} > m_{\chi_1^+}$ > 45 GeV, otherwise
lightest neutralino	$m_{\tilde{\chi}_1^0} > 35$ GeV

* $m_{\tilde{\nu}}^2 = m_{\tilde{\ell}_L}^2 + M_Z^2 c_W^2 \cos 2\beta \Rightarrow m_{\tilde{\nu}} > 65\ (40)$ GeV for $\tan\beta = 2\ (50)$

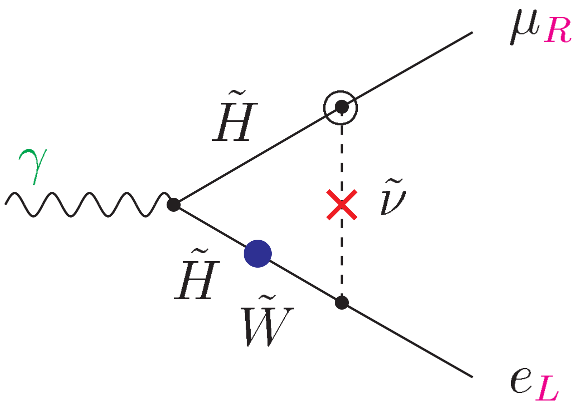
- **Inputs**: M_2, μ, $\tan\beta$ (spectra and couplings of charginos/neutralinos),
  $\tilde{m}_1$ (relevant lightest scalar mass), δ (LFV parameter)

- **Dominating diagrams** (mass insertion method): proportional to δ (if small)
  diag

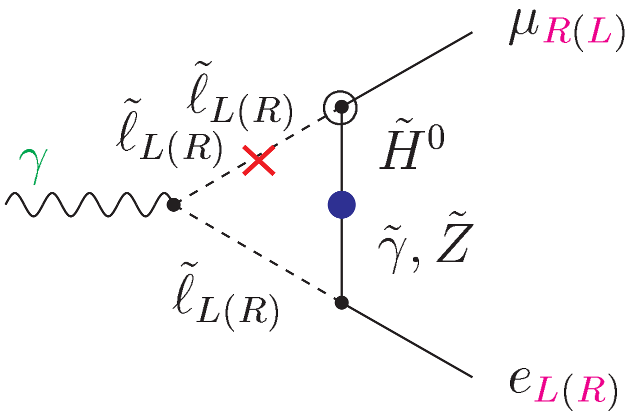
Dominating diagrams



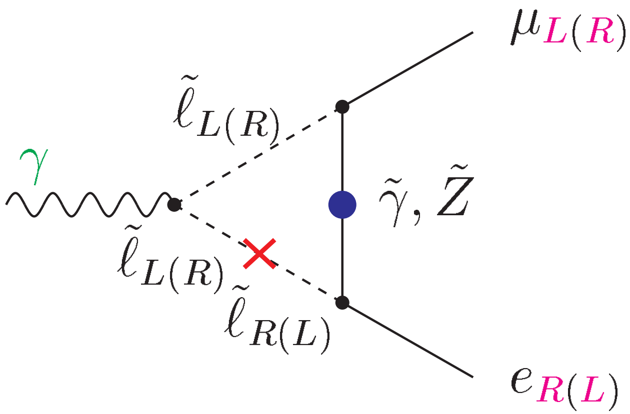
(a) $\delta_{LL}^{\tilde{\nu}\,12}$



(b1) $\delta_{LL}^{\tilde{\nu}\,12}$ $m_\mu \tan\beta$



(b2) $\delta_{LL(RR)}^{\tilde{\ell}\,12}$ $m_\mu \tan\beta$



(b3) $\delta_{LR}^{\tilde{\ell}\,12}$ $m_{\tilde{\gamma},\tilde{Z}}$

SUSY predictions for $Z \to \ell_I \ell_J$

- **Ignoring other LFV processes**:
  - Maximal effects for a decoupled second family $(\delta^{IJ} \to \infty)$
  - Chargino–sneutrino diagrams dominate by $\mathcal{O}(10)$

    Results almost independent of lepton masses

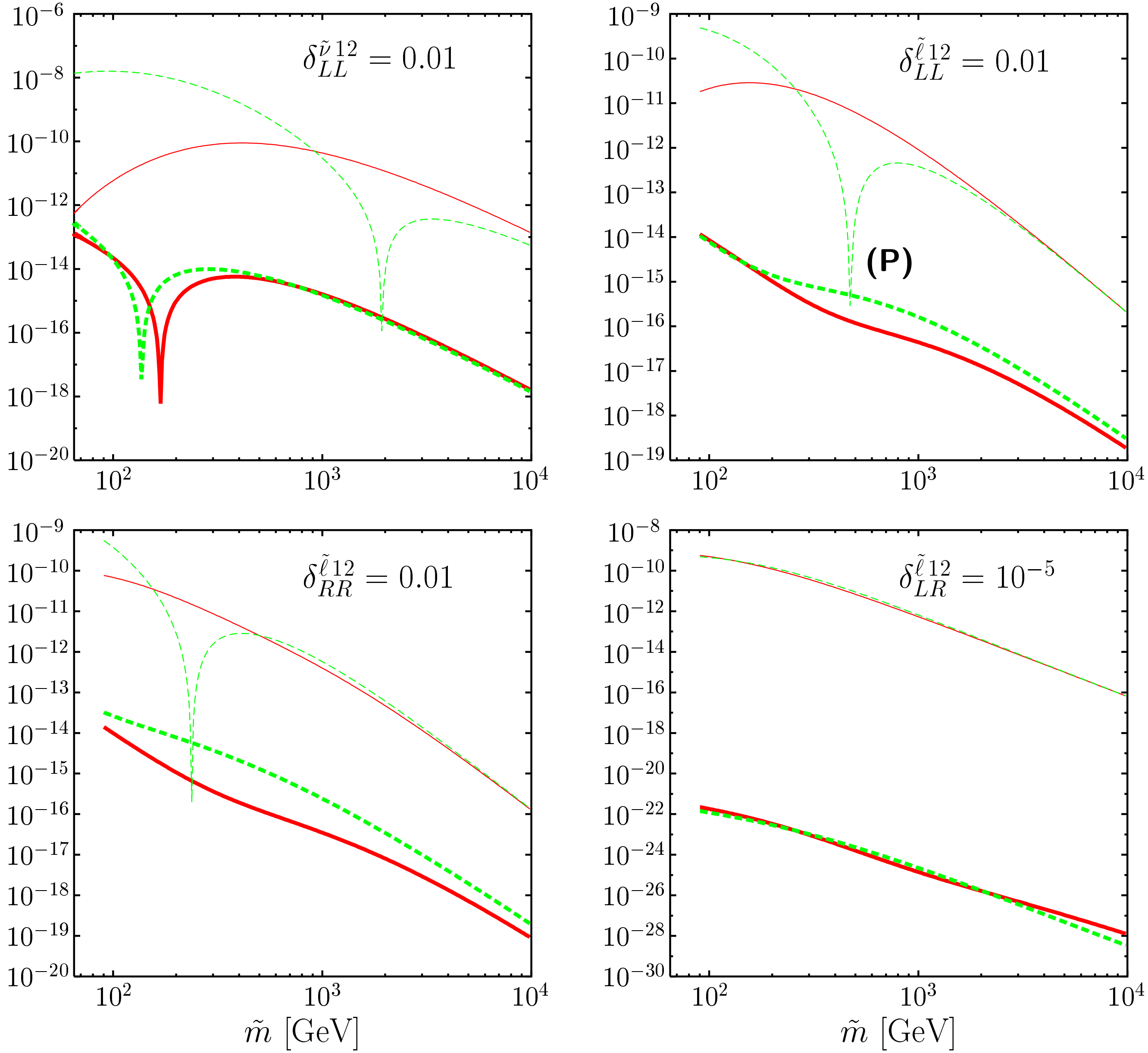
    Mild dependence on $\tan\beta$

  - $\surd$ BR$(Z \to \ell_I \ell_J)$ up to 2.5×10^{-8} (7.5×10^{-8}) for $\tan\beta = 2$ (50)
  - $\surd$ BR$(Z \to \ell_I \ell_J) > 2 \times 10^{-9}$ (2×10^{-8}) for $m_{\tilde{\nu}} < 305$ (85) GeV, $m_{\tilde{\chi}_1^+} < 270$ (105) GeV
  - **Cancellations**: contributions from different particles have opposite signs `plot`

- **Correlation with $\ell_J \to \ell_I \gamma$**:
  - BR$(\mu \to e\gamma) < 1.2 \times 10^{-11} \Rightarrow$ BR$(Z \to \mu e) < 1.5 \times 10^{-10}$
  - BR$(Z \to \tau e;\ \tau\mu)$ **at reach** of best projection of GigaZ $(\kappa < 1)$

    with BR$(\tau \to e\gamma;\ \mu\gamma) \ll$ present limits `plot`

BR($Z \to \mu e$) & BR($\mu \to e\gamma$)

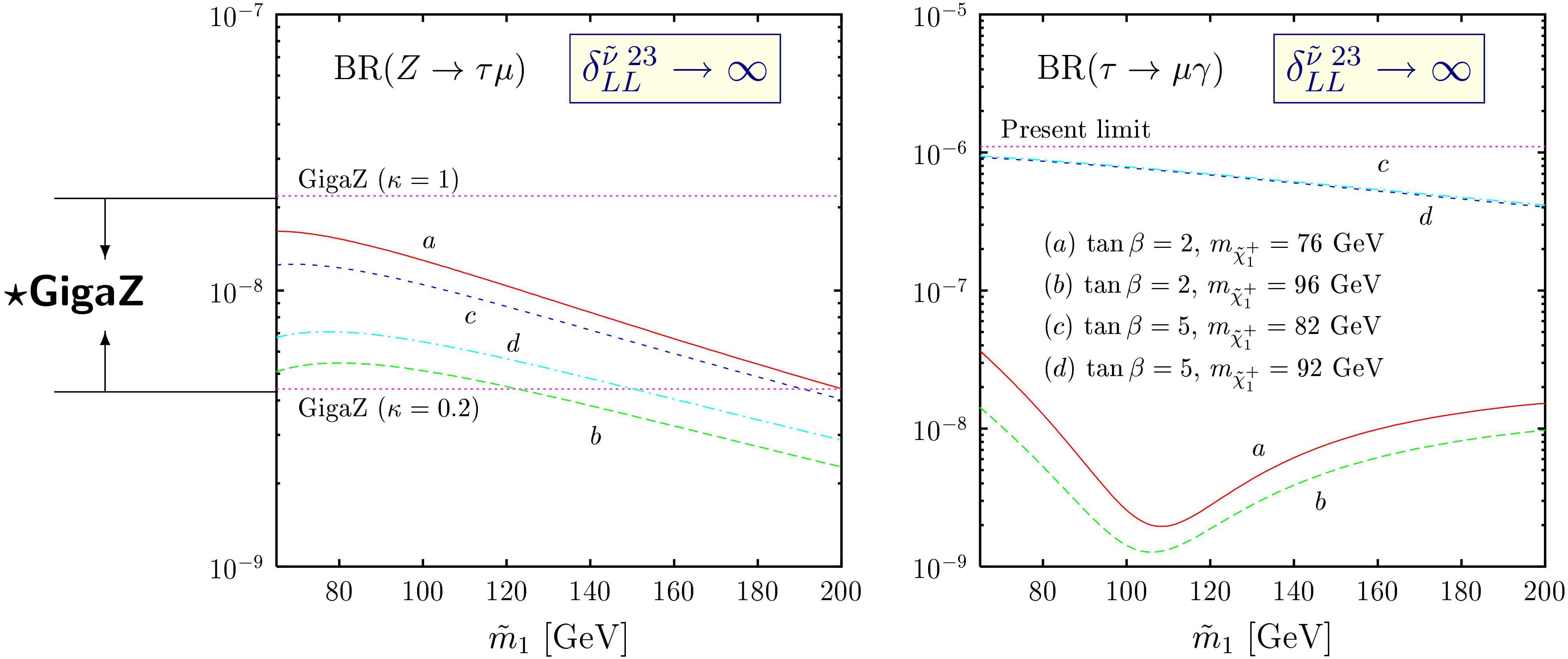


	$\tan\beta = 2$
solid	$M_2 = 150$ GeV
	$\mu = -500$ GeV
dashed	$M_2 = \mu = 150$ GeV

asymptotically $\propto \dfrac{\delta^2}{\tilde{m}^4}$

(small δ)

BR($Z \to \tau\mu$) & BR($\tau \to \mu\gamma$)



(Best) **GigaZ reach**: $55 < m_{\tilde{\nu}} < 215$ GeV

$75 < m_{\tilde{\chi}_1^+} < 100$ GeV

$\tan\beta < 7$

Bounds from $\ell_J \to \ell_I \gamma$

- Provide most stringent constraints on slepton LFV mass terms (today)
- Establish how severe is the **flavour problem** in the lepton sector of the MSSM
- **Aim**: update works by [Masiero *et alter* '86–'96] (only photino–mediated diagrams) with complete MSSM $\Rightarrow$ frequent cancellations `e.g. plot`
- Bounds involving the **first two families** are very restrictive `table`

$\delta_{LL}^{\tilde{\nu}\,12}$, $\delta_{LL}^{\tilde{\ell}\,12}$, $\delta_{RR}^{\tilde{\ell}\,12}$	:	$\mathcal{O}(10^{-3}\ldots10^{-5})$	stronger for high $\tan\beta$
$\delta_{LR}^{\tilde{\ell}\,12}$	:	$\mathcal{O}(10^{-6})$	independent of $\tan\beta$

- Bounds involving the **third family** are much weaker (or unexistent!)

		low $\tan\beta$	high $\tan\beta$
$\delta_{LL}^{\tilde{\nu}\,I3}$	:	∞	0.03 to 1.3
$\delta_{LL}^{\tilde{\ell}\,I3}$	:	∞	0.14 to ∞
$\delta_{RR}^{\tilde{\ell}\,I3}$	:	∞	0.11 to ∞
$\delta_{LR}^{\tilde{\ell}\,I3}$	:	0.05 to ∞	

Bounds from $\mu \to e\gamma < 1.2 \times 10^{-11}$

$\tilde{m}_1$	M_2	$\delta_{LL}^{\tilde{\nu}\,12}$ $\mu = -500$	$\mu = -150$	$\mu = 150$	$\mu = 500$	$\delta_{LL}^{\tilde{\ell}\,12}$ $\mu = -500$	$\mu = -150$	$\mu = 150$	$\mu = 500$
		$\tan\beta = 2$				$\tan\beta = 2$			
100	150	14×10^{-3}	1.0×10^{-3}	0.3×10^{-3}	1.0×10^{-3}	7.5×10^{-3}	2.6×10^{-3}	1.7×10^{-3}	5.0×10^{-3}
	500	33×10^{-3}	3.0×10^{-3}	1.7×10^{-3}	13×10^{-3}	84×10^{-3}	11×10^{-3}	7.0×10^{-3}	41×10^{-3}
500	150	3.7×10^{-3}	1.1×10^{-3}	1.2×10^{-3}	1.7×10^{-3}	14×10^{-3}	8.5×10^{-3}	**0.12 (P)**	32×10^{-3}
	500	7.3×10^{-3}	2.6×10^{-3}	2.3×10^{-3}	4.1×10^{-3}	24×10^{-3}	20×10^{-3}	32×10^{-3}	26×10^{-3}
		$\tan\beta = 50$				$\tan\beta = 50$			
100	150	9.3×10^{-5}	2.1×10^{-5}	2.0×10^{-5}	8.6×10^{-5}	2.4×10^{-4}	0.8×10^{-4}	0.8×10^{-4}	2.4×10^{-4}
	500	80×10^{-5}	9.0×10^{-5}	8.8×10^{-5}	77×10^{-5}	24×10^{-4}	3.5×10^{-4}	3.4×10^{-4}	23×10^{-4}
500	150	9.7×10^{-5}	4.5×10^{-5}	4.5×10^{-5}	9.4×10^{-5}	7.4×10^{-4}	6.4×10^{-4}	6.9×10^{-4}	7.6×10^{-4}
	500	22×10^{-5}	9.6×10^{-5}	9.5×10^{-5}	21×10^{-5}	9.7×10^{-4}	9.6×10^{-4}	9.8×10^{-4}	9.7×10^{-4}

$\tilde{m}_1$	M_2	$\delta_{RR}^{\tilde{\ell}\,12}$ $\mu = -500$	$\mu = -150$	$\mu = 150$	$\mu = 500$	$\delta_{LR}^{\tilde{\ell}\,12}$ $\mu = -500$	$\mu = -150$	$\mu = 150$	$\mu = 500$
		$\tan\beta = 2$				$\tan\beta = 2$			
100	150	4.2×10^{-3}	1.5×10^{-3}	1.8×10^{-3}	3.7×10^{-3}	1.6×10^{-6}	1.5×10^{-6}	1.6×10^{-6}	1.7×10^{-6}
	500	11×10^{-3}	3.2×10^{-3}	2.5×10^{-3}	8.3×10^{-3}	4.5×10^{-6}	4.4×10^{-6}	4.7×10^{-6}	4.6×10^{-6}
500	150	22×10^{-3}	10×10^{-3}	22×10^{-3}	**0.33**	1.3×10^{-6}	1.2×10^{-6}	1.2×10^{-6}	1.2×10^{-6}
	500	19×10^{-3}	12×10^{-3}	**0.33**	35×10^{-3}	7.6×10^{-6}	7.5×10^{-6}	7.6×10^{-6}	7.7×10^{-6}
		$\tan\beta = 50$				$\tan\beta = 50$			
100	150	1.6×10^{-4}	0.6×10^{-4}	0.6×10^{-4}	1.5×10^{-4}	1.6×10^{-6}	1.5×10^{-6}	1.6×10^{-6}	1.6×10^{-6}
	500	3.8×10^{-4}	1.1×10^{-4}	1.1×10^{-4}	3.8×10^{-4}	4.5×10^{-6}	4.5×10^{-6}	4.6×10^{-6}	4.5×10^{-6}
100	150	16×10^{-4}	13×10^{-4}	15×10^{-4}	17×10^{-4}	1.3×10^{-6}	1.2×10^{-6}	1.2×10^{-6}	1.3×10^{-6}
	500	9.4×10^{-4}	8.1×10^{-4}	8.7×10^{-4}	9.6×10^{-4}	7.7×10^{-6}	7.6×10^{-6}	7.6×10^{-6}	7.7×10^{-6}

LFV and $(g_\mu - 2)$

World average: $a_\mu^{\rm exp} = \underbrace{11\,659\,202.3\,(15.1)\times 10^{-10}}_{11\,659\,203\,(8)\times10^{-10}}$

including new BNL:

SM prediction: $a_\mu^{\rm SM} = 11\,659\,179.2\;\;(9.4)\times 10^{-10}$

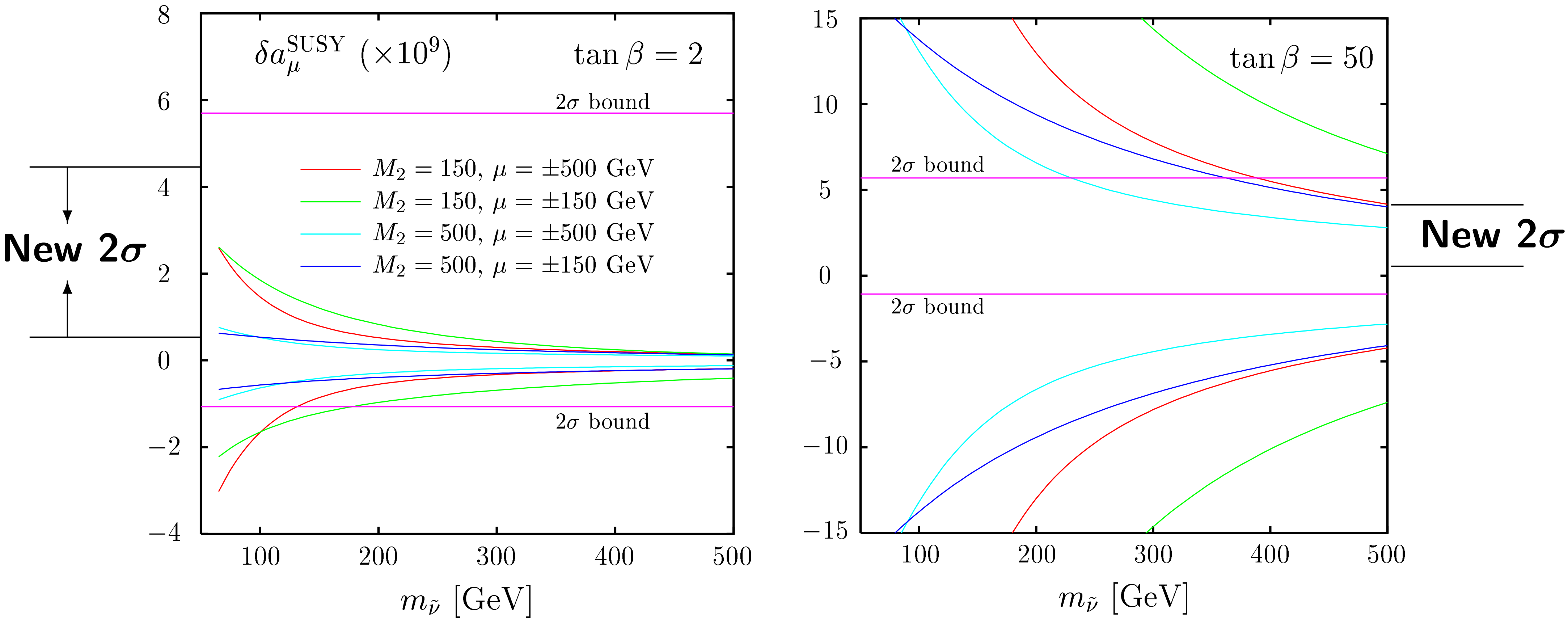
$\Rightarrow \delta a_\mu = (\underbrace{23.1}_{24} \pm \underbrace{16.9}_{10}) \times 10^{-10}$ new:

- $(g_\mu - 2) \approx$ 'normalization' of LFV lepton decays (small deltas):
  (similar diagrams $\Rightarrow$ replace $\delta_{LR}^{\tilde{\ell}\,IJ}$ by $\delta_{LR}^{\tilde{\ell}\,22}$ and remove all other delta insertions)

- $\boxed{\delta a_\mu^{\rm SUSY} = \delta a_\mu}$ $\Rightarrow$ **large uncertainties**
  - SM prediction
  - Extra assumptions:
    - take same mass scales $\tilde{m}$ involved in $(g_\mu - 2)$ as in $\mu \to e\gamma$ or $\tau \to \mu\gamma$
    - soft–breaking terms m_L and m_R involved at the same time in $(g_\mu - 2)$

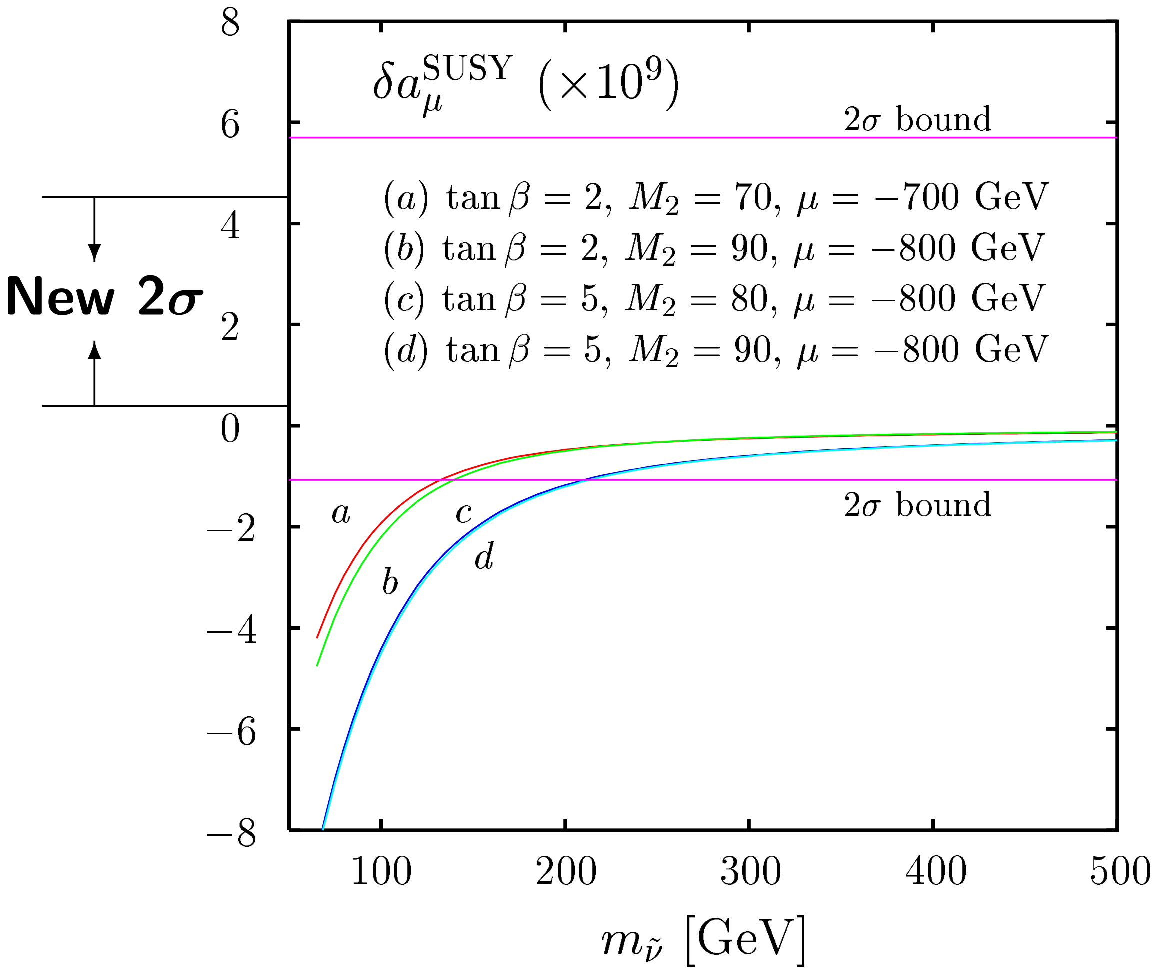
- **But taking present results seriously ...**

$(g_\mu - 2)$ and SUSY



- Positive Higgsino mass parameter μ preferred (now even more with new BNL data)

BR($Z \to \tau\mu$) & ($g_\mu - 2$): very speculative!



- Assuming $m_L = m_R$: the favourite scenario for $Z \to \tau\mu$ in GigaZ restricted to higher slepton masses (or practically ruled out by new BNL data)

Summary and conclusions

- **Slepton sector of SUSY** (soft–breaking terms) provides a **natural source of LFV**:

  effects ≈ proportional to slepton mass squared differences (degeneracy δ)

- **Our calculation**:
  - $Z \to \ell_I \ell_J$ realistically correlated with $\ell_J \to \ell_I \gamma$ in SUSY models (no SUSY–breaking mechanism assumed and exact beyond mass insertion approx.)
  - update to include complete MSSM (several δ types) and more recent experiments

- **GigaZ reach**:
  - $Z \to \mu e$ excluded by $\mu \to e\gamma$ not by limits on SUSY masses
  - $Z \to \tau e; \tau\mu$ possible ($\kappa < 1$) thanks to chargino–sneutrino contributions

- **Best bounds from $\ell_J \to \ell_I \gamma$ ⇒ SUSY lepton flavour problem?**
  - Non–observation of $\mu \to e\gamma \Rightarrow$ $\delta^{\tilde{\nu}|\tilde{\ell}\ 12}_{LL,RR,LR} \approx 10^{-3} \ldots 10^{-6}$ (large degeneracy!)

    . . . maybe justifiable by weakness of Yukawas of light lepton sector
  - Non–observation of $\tau \to e\gamma; \mu\gamma \Rightarrow$ very weak (even unexistent) constraints on δ^{I3}s