Two Loop QCD Corrections to Gluon Fusion into two Photons

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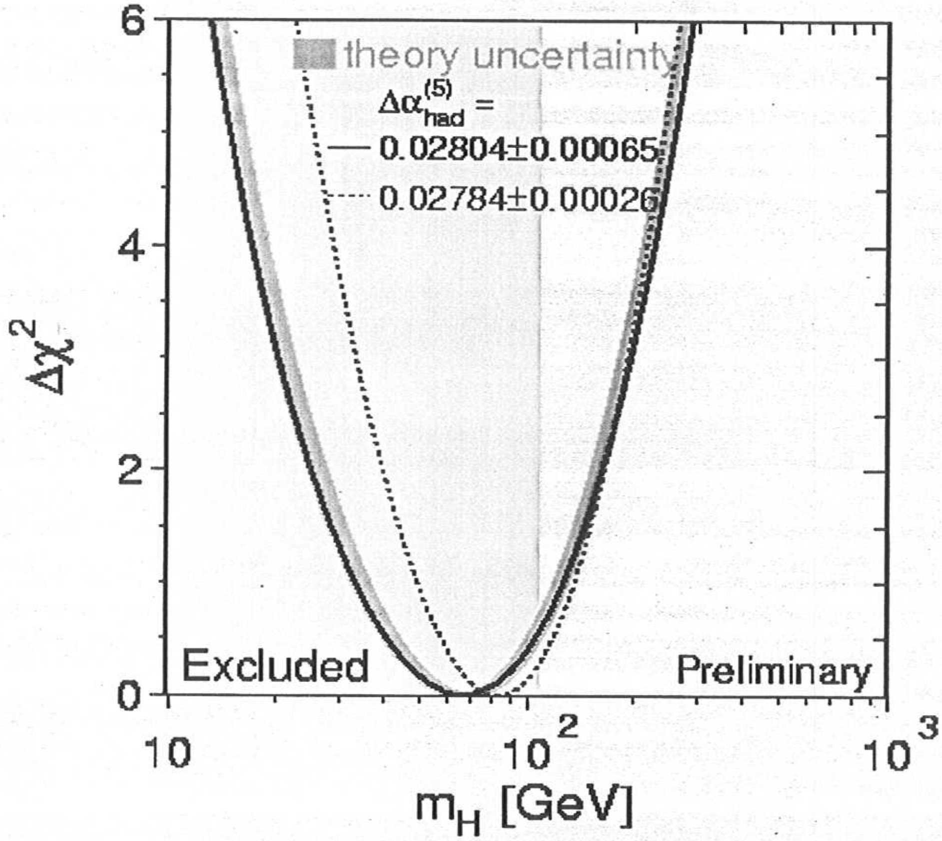
Where is the Higgs Boson?

Jnder the assumption of a Standard Model Higgs, virtual loop :orrections pin down the Higgs mass.



$m_H < 196$ GeV at 95% confidence level.

.EP found a hint of the Higgs at the end point of its energy each: $M_H = 115$ GeV. If true, this is good for upersymmetry.

Two-Loop Amplitudes

- $e^+e^- \longrightarrow e^+e^-$ — Bern, Dixon, Ghinculov

- $q\bar{q} \longrightarrow q\bar{q}$
- $q\bar{q} \longrightarrow gg$
- $gg \longrightarrow gg$

Anastasiou, Glover, Oleari, Tejeda-Yeomans.

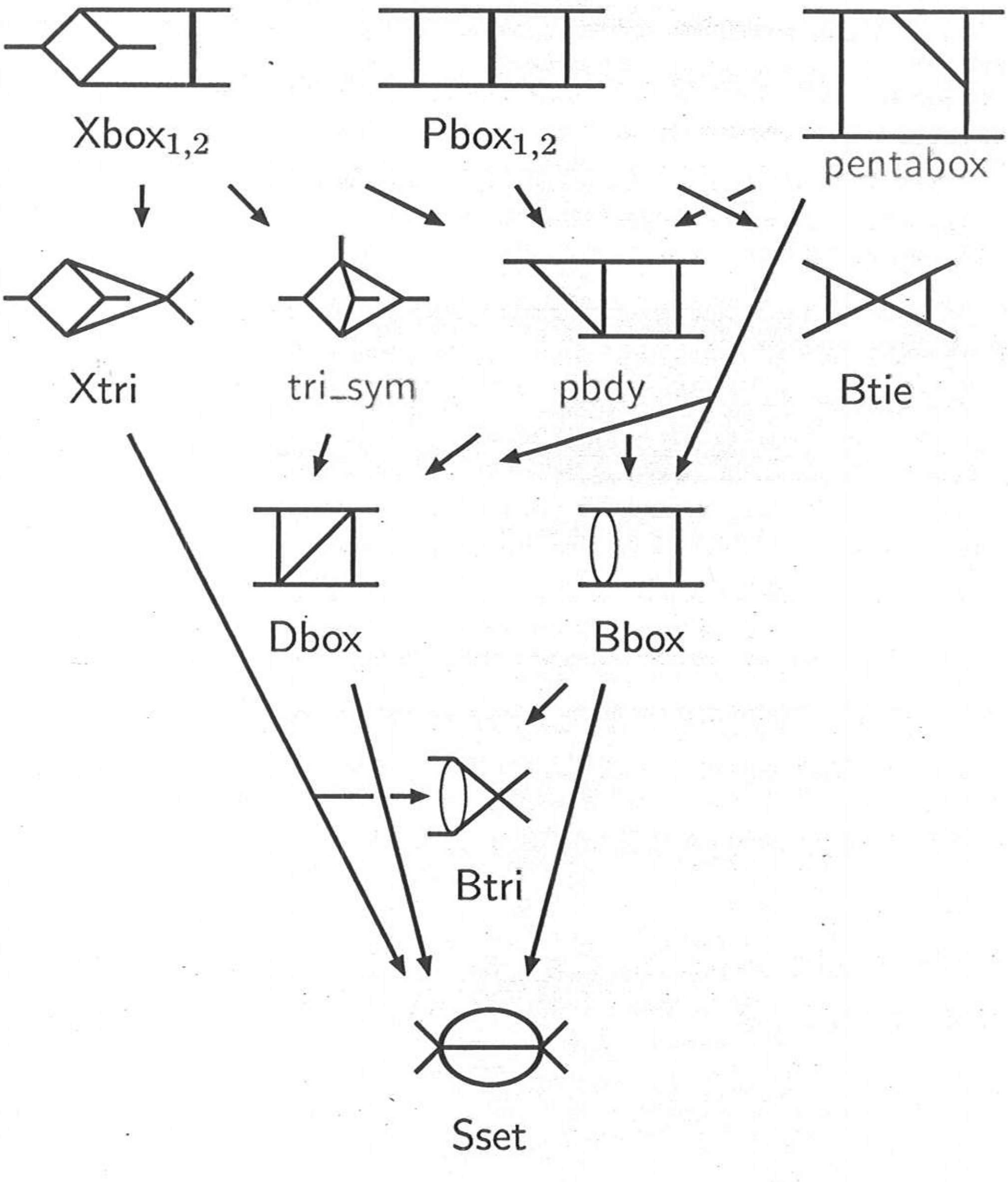
- $\gamma\gamma \longrightarrow \gamma\gamma$ — Bern, me, Dixon, Ghinculov, Wong.
- $gg \longrightarrow \gamma\gamma$ — Bern, me, Dixon

- $q\bar{q} \longrightarrow \gamma\gamma$
- $q\bar{q} \longrightarrow g\gamma$
- $e^+e^- \longrightarrow \gamma\gamma$

Anastasiou, Glover, Tejeda-Yeomans.

- $e^+e^- \longrightarrow 3$ partons — Garland, Gehrmann, Glover, Koukoutsakis, Remiddi; Moch, Uwer, Weinzierl.

Two-loop integral inheritance chart



Some All-Massless Integrals References

Xbox_1, tri_sym: Tausk, hep-ph/9909506

$\text{Xbox}_{1,2}$, Xtri: Anastasiou et al., hep-ph/0003261

$\text{Pbox}_{1,2}$, Bbox, Dbox: Smirnov, hep-ph/9905323; Smirnov & Veretin, hep-ph/9907385

pentabox, Dbox: Anastasiou, Glover, Oleari, hep-ph/9912251

Bbox: Anastasiou, Glover, Oleari, hep-ph/9907523

Basic identities:

- Integration by parts

  Tkachov, PL100B(1981)65; Chetyrkin & Tkachov, NPB192(1981)159

- Lorentz invariance Gehrmann & Remiddi, hep-ph/9912329

Act in space of integrals with multiple propagators.
Used judiciously, reduce tensor integrals to master integrals plus simpler boundary integrals.

We used the <u>helicity formalism</u>:

$$\varepsilon_\mu^{\pm}(k;q) = \pm \frac{\langle k\pm|\gamma^\mu|q\pm\rangle}{\sqrt{2}\,\langle k\mp|q\pm\rangle}$$

where $|k\pm\rangle$ are Weyl spinors of momentum k and positive and negative helicity, respectively. q is a reference momentum.

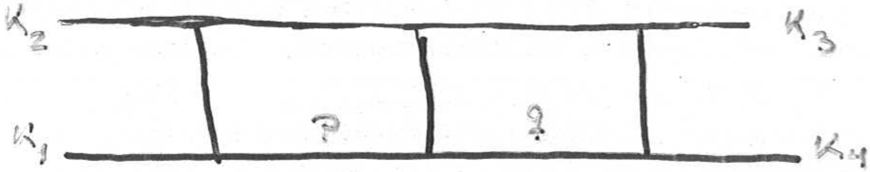
The <u>interference formalism</u> becomes cumbersome in the $gg \to \gamma\gamma$ case, since the tree level amplitude vanishes.

We worked in the 't-Hooft – Veltman scheme.

Integrals with helicity:

Helicity formalism introduces extra irreducible integrands.

Example: Two-Loop planar double box



- In an interference, summed over all helicities, integrand is a polynomial in $p\cdot k_i$ and $q\cdot k_i$
- Easily reduce to polynomial in $p\cdot k_4$ and $q\cdot k_1$.
- In helicity amplitude, also encounter $p\cdot \varepsilon_i$.
- Expand: $p\cdot\varepsilon = c_1 k_1\cdot\varepsilon + c_2 k_2\cdot\varepsilon + c_3 k_3\cdot\varepsilon + c_4 v\cdot\varepsilon$

where $c_i \propto p\cdot k_i,\ q\cdot k_i$ $\quad c_4 \propto p\cdot v$ and

$$v_\mu = \varepsilon_{\mu\nu\sigma\rho}\, k_1^\nu\, k_2^\sigma\, k_3^\rho$$

$p\cdot v$ not in the class of integrals that are handled by previous paper, but not difficult to evaluate.

Used unitarity cuts.

previously applied to:

- four-gluon identical helicity.
  (hep-ph/0001001)
- then to the other helicity configurations.
- Light by Light scattering.

Basic Idea:

$$S^\dagger S = 1$$

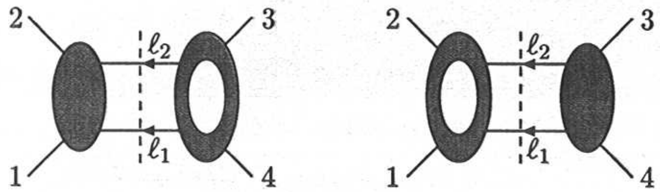
$$S = 1 + iT \implies i(T - T^\dagger) = T^\dagger T$$

$$\implies \langle a | i(T - T^\dagger) | b \rangle = \sum_c \langle a | T^\dagger | c \rangle \langle c | T | b \rangle.$$

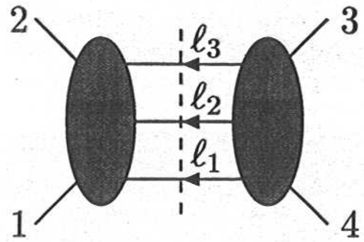
This means that in a perturbative expansion in the coupling g, we can get the g^n term by knowing all the terms up to g^{n-1}.

Generalized Cuts

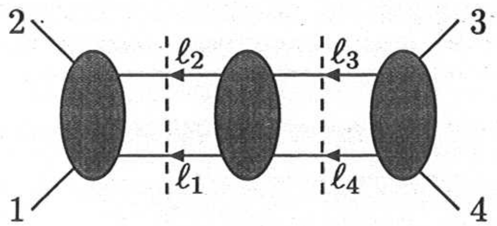
Two-particle cuts:



Three-particle cuts:



Generalized double two-particle cut:



This does *not* mean "imaginary part of imaginary part". It should be interpreted as demanding that cut propagators do not cancel.

The amplitudes are constructed by combining all the cuts into a single function with all cuts correct.

Universal Two-Loop Infrared Singularities

Catani, hep-ph/9802439

- Given in terms of QCD color-space operators:

$$\mathcal{M}_4(\alpha_s(\mu)) = 4\pi\alpha_s(\mu^2)\left[\mathcal{M}_4^{(0)} + \frac{\alpha_s(\mu)}{2\pi}\mathcal{M}_4^{(1)} + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \mathcal{M}_4^{(2)} + \dots\right]$$

$$|\mathcal{M}_n^{(2)}\rangle_{R.S.} = I^{(1)}(\varepsilon)\,|\mathcal{M}_n^{(1)}\rangle_{R.S.} + I^{(2)}_{R.S.}(\varepsilon)\,|\mathcal{M}_n^{(0)}\rangle_{R.S.} + |\mathcal{M}_n^{(2)\mathrm{fin}}\rangle_{R.S.}$$

where

$$I^{(1)}(\varepsilon) = \frac{1}{2}\frac{e^{-\varepsilon\psi(1)}}{\Gamma(1-\varepsilon)}\sum_{i=1}^{n}\sum_{j\neq i}^{n} T_i\cdot T_j\left[\frac{1}{\varepsilon^2} + \frac{\gamma_i}{T_i^2}\frac{1}{\varepsilon}\right]\left(\frac{\mu^2 e^{-i\lambda_{ij}\pi}}{2p_i\cdot p_j}\right)^{\varepsilon}$$

and

$$I^{(2)}_{R.S.}(\varepsilon) = -\frac{1}{2}I^{(1)}(\varepsilon)\left(I^{(1)}(\varepsilon) + \frac{4\pi\beta_0}{\varepsilon}\right) +$$

$$+ \frac{e^{\varepsilon\psi(1)}\Gamma(1-2\varepsilon)}{\Gamma(1-\varepsilon)}\left(\frac{2\pi\beta_0}{\varepsilon} + K_{R.S.}\right)I^{(1)}(2\varepsilon) + H^{(2)}_{R.S.}(\varepsilon)$$

One-Loop amplitudes:

$$\mathcal{M}^{1\text{-Loop}}_{gg\to\gamma\gamma} = 4\alpha\alpha_s(\mu)\,\delta^{a_1 a_2}\left(\sum_i Q_i^2\right) M^{(1)}$$

$M^{(1)}$ can be gotten in terms of the $gg\to gg$ subamplitudes:

$$\mathcal{M}^{(1)}_{gg\to gg}(1,2,3,4) = N_f \sum_{\sigma\in S_3} \mathrm{Tr}\left[T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}T^{a_{\sigma(3)}}T^{a_{\sigma(4)}}\right]$$
$$\times A^{(1)}(\sigma(1),\sigma(2),\sigma(3),\sigma(4))$$

$$M^{(1)} = \sum_{\sigma\in S_3} A^{(1)}(\sigma(1),\sigma(2),\sigma(3),\sigma(4))$$

$$M^{(1)}_{++++} = 1 + \mathcal{O}(\varepsilon)$$

$$M^{(1)}_{-+++} = M^{(1)}_{+-++} = M^{(1)}_{++-+} = M^{(1)}_{+++-} = 1 + \mathcal{O}(\varepsilon)$$

$$M^{(1)}_{--++} = -\frac{1}{2}\frac{t^2+u^2}{s^2}\left[\ln^2\left(\frac{t}{u}\right)+\pi^2\right] - \frac{t-u}{s}\ln\left(\frac{t}{u}\right) - 1 + \mathcal{O}(\varepsilon)$$

$$M^{(1)}_{-+-+} = -\frac{1}{2}\frac{t^2+s^2}{u^2}\ln^2\left(-\frac{t}{s}\right) - \frac{t-s}{u}\ln\left(-\frac{t}{s}\right) - 1$$
$$-i\pi\left[\frac{t^2+u^2}{u^2}\ln\left(-\frac{t}{s}\right) + \frac{t-s}{u}\right] + \mathcal{O}(\varepsilon)$$

Two minor points:

- Catani's formula is valid in the CDR scheme where

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{10}{9} T_A N_f$$

However, in general, we have found K to be scheme dependent, e.g.:

$$K_{FDH} = K_{CDR} - C_A \left(\frac{1}{6} + \frac{4}{9}\varepsilon\right)$$

- Extra term in $H^{(2)}_{R.S.}(\varepsilon)$:

$$H^{(2)\,\text{extra}}_{R.S.}(\varepsilon) = -\frac{1}{\varepsilon} \ln\left(\frac{s}{t}\right) \ln\left(\frac{t}{u}\right) \ln\left(\frac{u}{s}\right) \left[T_1 \cdot T_2, T_2 \cdot T_3\right]$$

This is OK because this term vanishes in the color-summed interference,

$$\langle \mathcal{M}_4^{(0)} | \left[T_1 \cdot T_2, T_2 \cdot T_3\right] | \mathcal{M}_4^{(0)} \rangle = 0$$

corresponds to terms found in soft gluon radiation at 1-loop

Catani, Grazzini, hep-ph/0007142

Catani's formula also allows us to make regularization scheme conversions:

$$|\mathcal{M}^{(2)}\rangle\Big|_{div} = \mathbf{I}^{(1)}(\varepsilon)|\mathcal{M}^{(1)}\rangle + \mathbf{I}^{(2)}_{R.S.}(\varepsilon)|\mathcal{M}^{(0)}\rangle$$

shifts in the finite terms are given by the $\mathcal{O}(\varepsilon)$ terms in $\mathcal{M}^{(1)}$ and $\mathcal{M}^{(0)}$, so:

$$\mathcal{M}^{(2)}_{CDR} - \mathcal{M}^{(2)}_{HV} = \mathcal{M}^{(2)}_{CDR}\Big|_{div} - \mathcal{M}^{(2)}_{HV}\Big|_{div}$$

Result in terms of Catani's formula plus finite terms:

$$\mathcal{M}^{2\text{-Loop}}_{gg\to\gamma\gamma} = \frac{2\alpha\alpha_s^2(\mu)}{\pi}\delta^{a_1a_2}\left(\sum_i Q_i^2\right)\Bigg\{$$

$$\left[I^{(1)}(\varepsilon) + \frac{11N-2N_f}{6}\left(\ln\left(\frac{\mu^2}{s}\right)+i\pi\right)\right]M^{(1)}$$

$$+ N F^{L}(s,t) - \frac{1}{N}F^{SL}(s,t)\Bigg\}$$

where

$$I^{(1)}(\varepsilon) = -N\frac{e^{-\varepsilon\psi(1)}}{\Gamma(1-\varepsilon)}\left[\frac{1}{\varepsilon^2}+\left(\frac{11}{6}-\frac{1}{3}\frac{N_f}{N}\right)\frac{1}{\varepsilon}\right]\left(\frac{\mu^2}{-s}\right)^{\varepsilon}$$

and

$$\mathcal{M}^{1\text{-Loop}}_{gg\to\gamma\gamma} = 4\alpha\alpha_s(\mu)\delta^{a_1a_2}\left(\sum_i Q_i^2\right)M^{(1)}$$

Subleading finite remainders

Define $$x=\frac{t}{s},\quad y=\frac{u}{s},\quad X=\ln\Big(-\frac{t}{s}\Big),\quad Y=\ln\Big(-\frac{u}{s}\Big)$$

$$F^{\rm SL}_{++++}=-\frac{3}{2}$$

$$F^{\rm SL}_{-+++}=\frac{1}{8}\Big[\frac{x^2+1}{y^2}((X+i\pi)^2+\pi^2)+\frac{1}{2}(x^2+y^2)((X-Y)^2+\pi^2)$$
$$-4\Big(\frac{1}{y}-x\Big)(X+i\pi)\Big]+\Big\{t\leftrightarrow u\Big\}$$

$$F^{\rm SL}_{--++}=-2x^2\Big[\mathrm{Li}_4(-x)+\mathrm{Li}_4(-y)-(X+i\pi)\Big(\mathrm{Li}_3(-x)+\mathrm{Li}_3(-y)\Big)$$
$$+\frac{1}{12}X^4-\frac{1}{3}X^3Y+\frac{\pi^2}{12}XY-\frac{4}{90}\pi^4+i\frac{\pi}{6}X\Big(X^2-3XY+\pi^2\Big)\Big]$$
$$-(x-y)\Big(\mathrm{Li}_4(-x/y)-\frac{\pi^2}{6}\mathrm{Li}_2(-x)\Big)$$
$$-x\Big[2\mathrm{Li}_3(-x)-\mathrm{Li}_3(-x/y)-3\zeta_3-2(X+i\pi)\mathrm{Li}_2(-x)$$
$$+(X-Y)(\mathrm{Li}_2(-x/y)+X^2)+\frac{1}{12}(5(X-Y)+18i\pi)((X-Y)^2+\pi^2)$$
$$-\frac{2}{3}X(X^2+\pi^2)-i\pi(Y^2+\pi^2)\Big]$$
$$+\frac{1-2x^2}{4y^2}((X+i\pi)^2+\pi^2)-\frac{1}{8}(2xy+3)((X-Y)^2+\pi^2)+\frac{\pi^2}{12}$$
$$+\Big(\frac{1}{2y}+x\Big)(X+i\pi)-\frac{1}{4}+\Big\{t\leftrightarrow u\Big\}$$

$$
\begin{aligned}
F^{\mathrm{SL}}_{-+-+} = & -2\frac{x^2+1}{y^2}\Big[\mathrm{Li}_4(-x/y) - \mathrm{Li}_4(-y) + \frac{1}{2}(X-2Y-i\pi)(\mathrm{Li}_3(-x)-\zeta_3) \\
& \qquad + \frac{1}{24}(X^4 + 2i\pi X^3 - 4XY^3 + Y^4 + 2\pi^2 Y^2) + \frac{7}{360}\pi^4\Big] \\
& - 2\frac{x-1}{y}\Big[\mathrm{Li}_4(-x) - \zeta_4 - \frac{1}{2}(X+i\pi)(\mathrm{Li}_3(-x)-\zeta_3) \\
& \qquad + \frac{\pi^2}{6}\Big(\mathrm{Li}_2(-x) - \frac{\pi^2}{6} - \frac{1}{2}X^2\Big) - \frac{1}{48}X^4\Big] \\
& + \Big(2\frac{x}{y}-1\Big)\Big[\mathrm{Li}_3(-x) - (X+i\pi)\mathrm{Li}_2(-x) + \zeta_3 - \frac{1}{6}X^3 - \frac{\pi^2}{3}(X+Y)\Big] \\
& + 2\Big(2\frac{x}{y}+1\Big)\Big[\mathrm{Li}_3(-y) + (Y+i\pi)\mathrm{Li}_2(-x) - \zeta_3 + \frac{1}{4}X(2Y^2+\pi^2) \\
& \qquad - \frac{1}{8}X^2(X+3i\pi)\Big] - \frac{1}{4}(2x^2-y^2)((X-Y)^2+\pi^2) \\
& - \frac{1}{4}\Big(3+2\frac{x}{y^2}\Big)((X+i\pi)^2+\pi^2) - \frac{2-y^2}{4x^2}((Y+i\pi)^2+\pi^2) + \frac{\pi^2}{6} \\
& + \frac{1}{2}(2x+y^2)\Big[\frac{1}{y}(X+i\pi) + \frac{1}{x}(Y+i\pi)\Big] - \frac{1}{2}
\end{aligned}
$$

$$
F^{\mathrm{SL}}_{+--+}(s,t,u) = F^{\mathrm{SL}}_{-+-+}(s,u,t)
$$

$$
F^{\mathrm{SL}}_{++-+}(s,t,u) = F^{\mathrm{SL}}_{+-++}(s,t,u) = F^{\mathrm{SL}}_{+++-}(s,t,u) = F^{\mathrm{SL}}_{-+++}(s,t,u)
$$

Checks on Results

- Checked gauge invariance: replaced polarization vectors by longitudinal ones.

- Agreement with predicted infrared divergences provides stringent check also of finite parts.

- Supersymmetry Ward Identities. (Related to 4-gluon identical helicity case.)

Summary

- The $gg \to \gamma\gamma$ process is an important one: background to the Higgs.

- Used Catani's formula to write (verify) amplitudes
  - * found minor surprises in the way.
  - * as an extra bonus we can convert from HV to CDR.

- Calculated the two-loop helicity amplitudes for gluon fusion into two photons using:
  - * Unitarity cuts methods.
  - * Integration techniques developed in the last few years.