Two Loop Bosonic Corrections to the Muon Decay in the Standard Model

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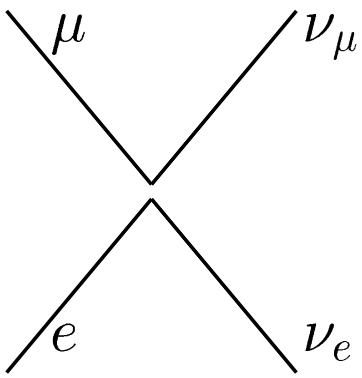
Plan:

1. Introduction
2. Matching and Projection
3. Renormalization and Gauge Invariance
4. Computation
5. Numerics versus Expansions
6. Conclusions

1. Introduction

- **Dynamics of the muon decay described in the Fermi Model:**

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{152\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1+\Delta q)$$



- **Point interactions corrected in the SM**

$$G_F = \frac{\alpha\pi}{2 s_W^2 M_W^2}(1+\Delta r)$$

- **High precision parameters** α, M_Z, G_F **give** W **boson mass in function of** M_H, M_t

$$M_W = M_Z \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_F M_Z^2}(1 + \Delta r)}}$$

- **Experiment**

$M_W = 80.423$	$\pm\ 0.039$	GeV	**present**
	$\pm\ 0.015$	GeV	**LHC**
	$\pm\ 0.006$	GeV	**Tesla**

versus theory

- **constrain Higgs boson mass**
- **test the Standard Model**

Previous results

- **One-loop corrections $\mathcal{O}(\alpha)$**

  A. Sirlin, W.J. Marciano and A. Sirlin

- **QCD corrections $\mathcal{O}(\alpha\alpha_S)$, $\mathcal{O}(\alpha\alpha_S^2)$**

  A. Djouadi, F. Halzen and B.A. Kniehl

  L. Avdeev et al.

  K. Chetyrkin, J. Kühn, M. Steinhauser

- **Two-loop electroweak Fermionic $\mathcal{O}(\alpha^2)$**

  - **expansions in M_t and M_H**

    J. v.d Bij, F. Hoogeveen

    R. Barbieri et al.

    J. Fleischer, O.V. Tarasov, F. Jegerlehner

    G. Degrassi, P. Gambino, A. Vicini

  - **exact**

    A. Freitas, W. Hollik, W. Walter, G. Weiglein

This work

- **Two-loop electroweak Bosonic $\mathcal{O}(\alpha^2)$**

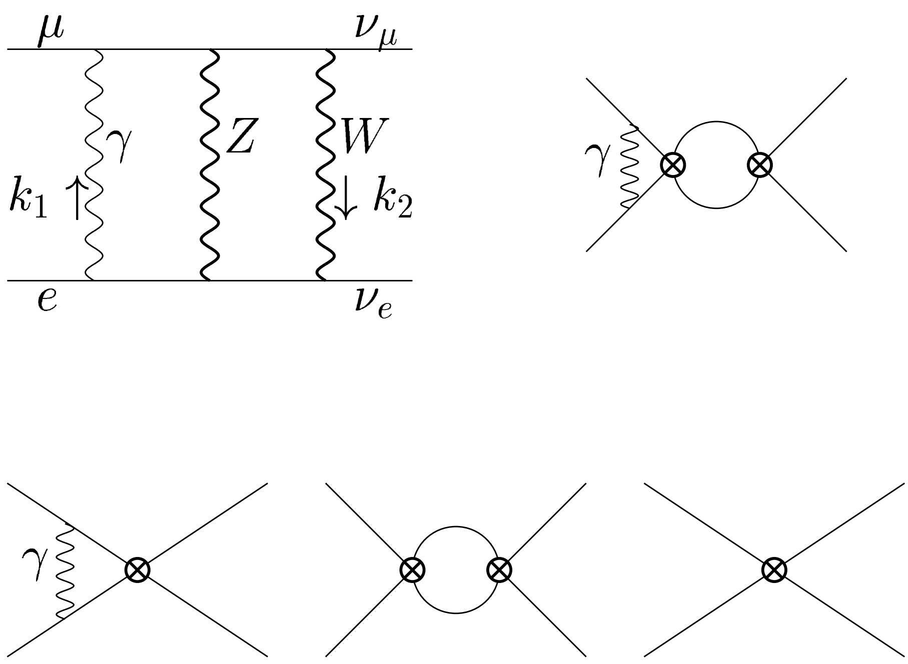
2. Matching and Projection

- **Use the factorization theorem to separate the scales.**

  Prescription:

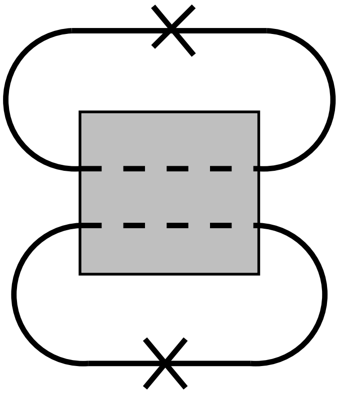
  put all external momenta and light fermion masses to zero.

- **Equivalent to Puli-Villars treatment**

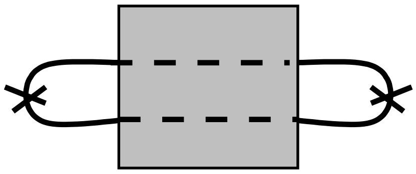


- Possible Projectors

double string



single string



- The double string projector was used in the calculation of the Fermionic contribution

  A. Freitas, W. Hollik, W. Walter and G. Weiglein

  The result is gauge invariant but problems :

  – divergencies remain

  – Pauli-Villars Λ scale does not cancel

  Solution: corrections based on algebraic arguments.

- The single string, used in this calculation,

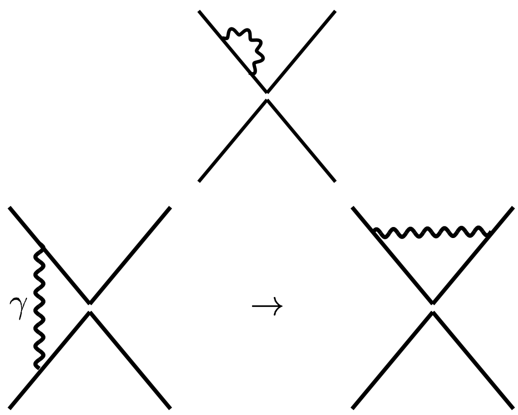
$$T_1 \otimes T_2 \to -\frac{1}{2d(d-2)} Tr[\Gamma_1 \gamma_\mu P_R \Gamma_2 \gamma^\mu P_R],$$

has the advantages:

– Respects Fierz symmetry in d dimensions

$$\gamma_\mu \gamma_\nu \gamma_\rho P_L \otimes \gamma^\rho \gamma^\nu \gamma^\mu P_L \sim \gamma_\mu \gamma_\nu \gamma_\rho \gamma^\nu \gamma^\mu P_L \otimes \gamma^\rho P_L$$

$\to$ Pauli-Villars contributions cancel trivially



$\to$ corresponds to the contribution of

T. van Ritbergen, R. G. Stuart

– The result is finite

– The algebraic requirements used to improve the double string projector are fulfilled.

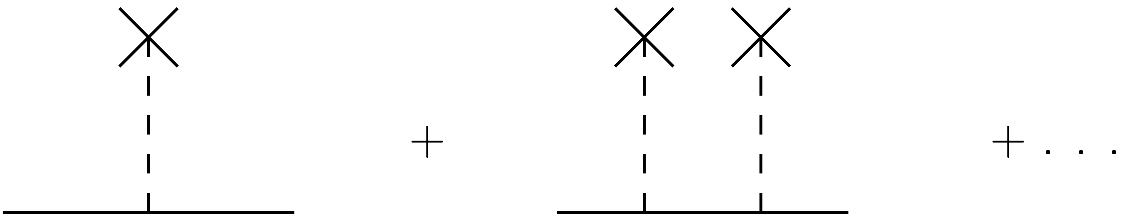
3. Renormalization and Gauge Invariance

- All parameters and fields are renormalized
  - finite Green functions
  - possibility to check partial results
- Gauge Fixing Lagrangian written in terms of renormalized parameters
  - no counter-terms generated
  - implies non-trivial renormalization of the ghost sector
- Gauge invariant masses and coupling counter-terms without explicit tadpole diagrams
  - less diagrams to evaluate
  - 1PI Green functions only

- **Construction of the Lagrangian with Gauge invariant bare parameters:**
  - **couplings from on-shell processes**
  - **masses from the poles of the S-matrix (in our case the pole is on the real axis)**
  - **VEV of the Higgs field satisfies**

$$\frac{1}{2}v_0(\frac{1}{2}v_0^2\lambda_0 - \mu_0^2) = 0$$

$\rightarrow$ **no contributions of the type:**



- **Additional renormalization of the VEV**

$$v_0 \rightarrow v_0 Z_v^{\frac{1}{2}}$$
$$M_W^0 \rightarrow M_W^0 Z_v^{\frac{1}{2}}$$
$$M_Z^0 \rightarrow M_Z^0 Z_v^{\frac{1}{2}}$$

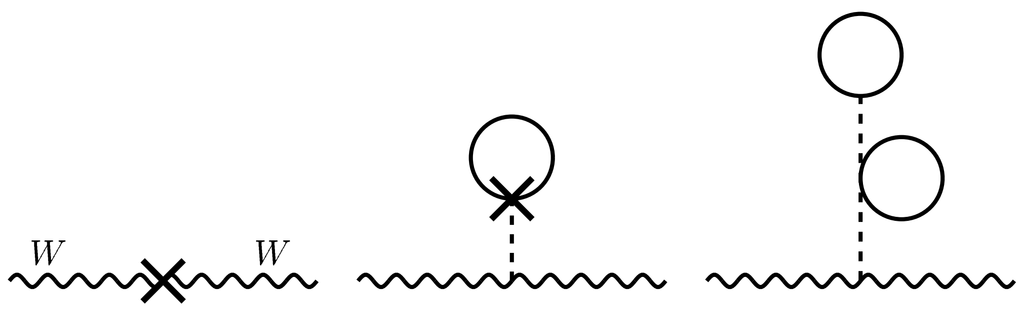
  - **fix Z_v to cancel tadpoles**

- **Complicated counter-term at two loop order**

$$\delta Z_v^{(1)} = \frac{e}{\sin\theta_W M_W M_H^2}\Pi_H^{(1)}$$

$$\delta Z_v^{(2)} = \frac{e}{\sin\theta_W M_W M_H^2}\Pi_H^{(2)} - \frac{1}{2}\delta Z_v^{(1)}\left(\delta Z_v^{(1)} + \delta Z_H^{(1)} + 2\frac{\delta M_H^{2(1)}}{M_H^2} + \frac{\delta M_W^{2(1)}}{M_W^2} + 2\frac{\delta \sin\theta_W^{(1)}}{\sin\theta_W} - \delta Z_e^{(1)}\right)$$

  - **reproduces the calculation with explicit tadpoles**



4. Computation

- **Calculation in on-shell scheme**

- **Calculation in $\overline{MS}$ scheme translated to on-shell**

- **Exact results reexpanded for comparison with**
  O.Veretin A.Onishchenko

- R_ξ **gauge**

- calculation without expansions
  - only 3 parameters (M_H, M_W, M_Z)
    $\rightarrow$ no problem with FORM
  - the result is exact
- Tensor reduction of the propagator integrals with the algorithm of

  G. Weiglein, R. Scharf and M. Böhm
- Vacuum integrals with Partial Integration identities

  K.G. Chetyrkin, F.V. Tkachov

  A.I. Davydychev, J.B. Tausk
- Numeric evaluation in quadrupole precision in C++ with help of single integral representation

  S. Bauberger

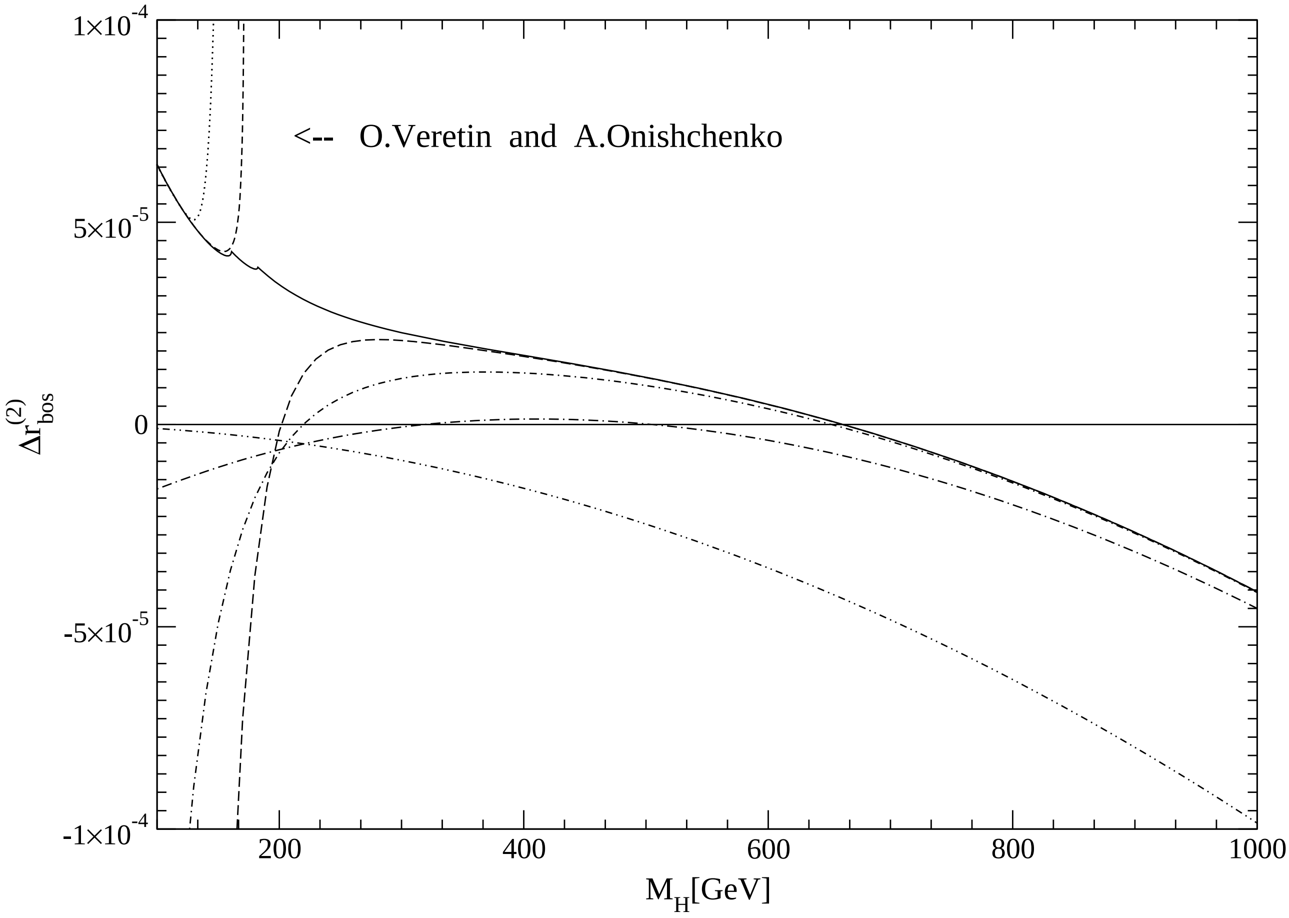
DiaGen

Diagram generation preformed with DiaGen

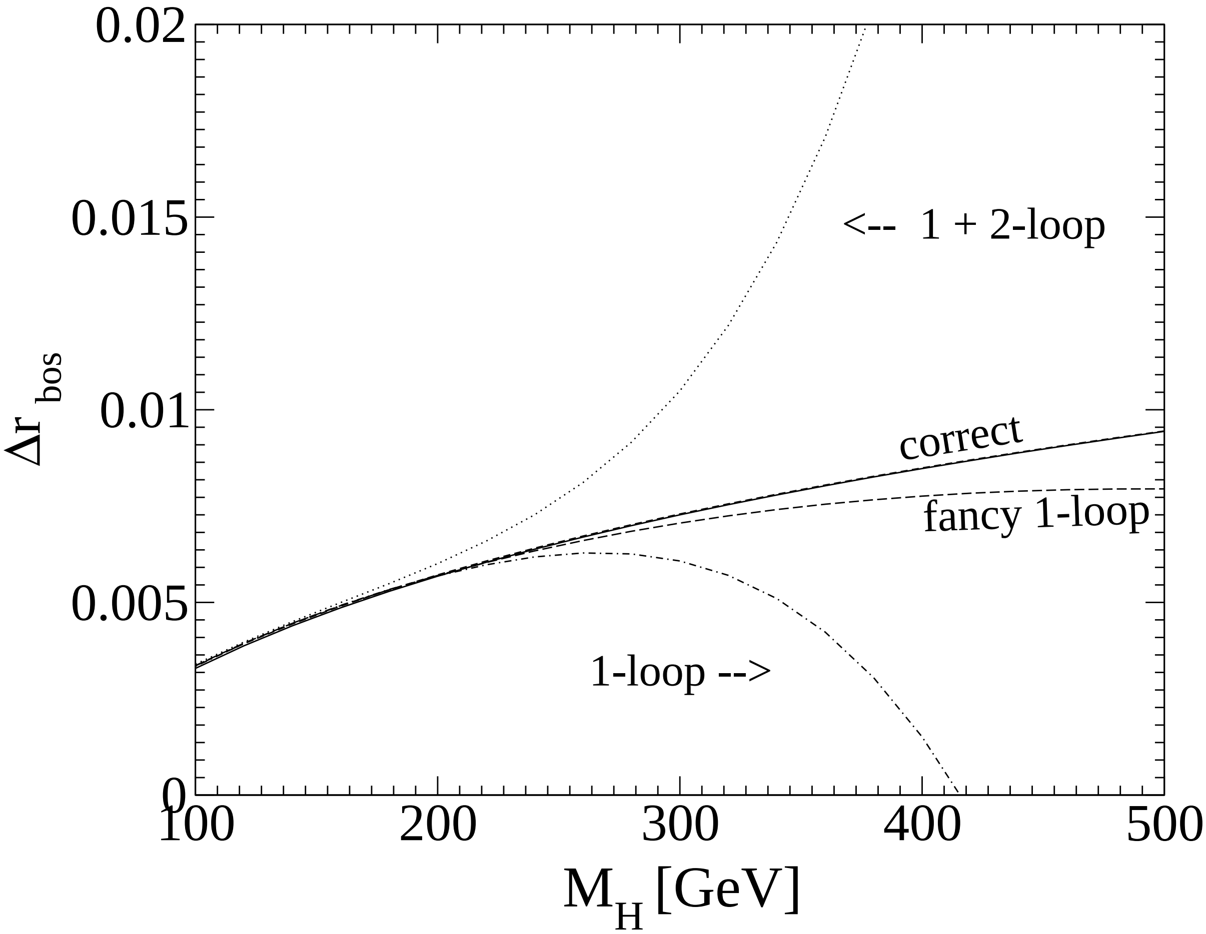
M. Czakon

- **C++ library for graphs**
- **Allows for**
  - **topology generation (draws topologies)**
  - **topological identification**
  - **momentum remapping**
  - **generation of the output suitable for direct evaluation in e.g. FORM**
  - **generation of diagrams with counter-term insertions**
  - **handling of theories with Majorana neutrinos**
- **very fast (4500 diagrams in $\sim$ 3s)**
- **to be publicly available at the end of the year**

5. Numerics versus Expansions



- **when preformed numerically the transition $\overline{MS} \rightarrow$ on-shell leads to problems :**

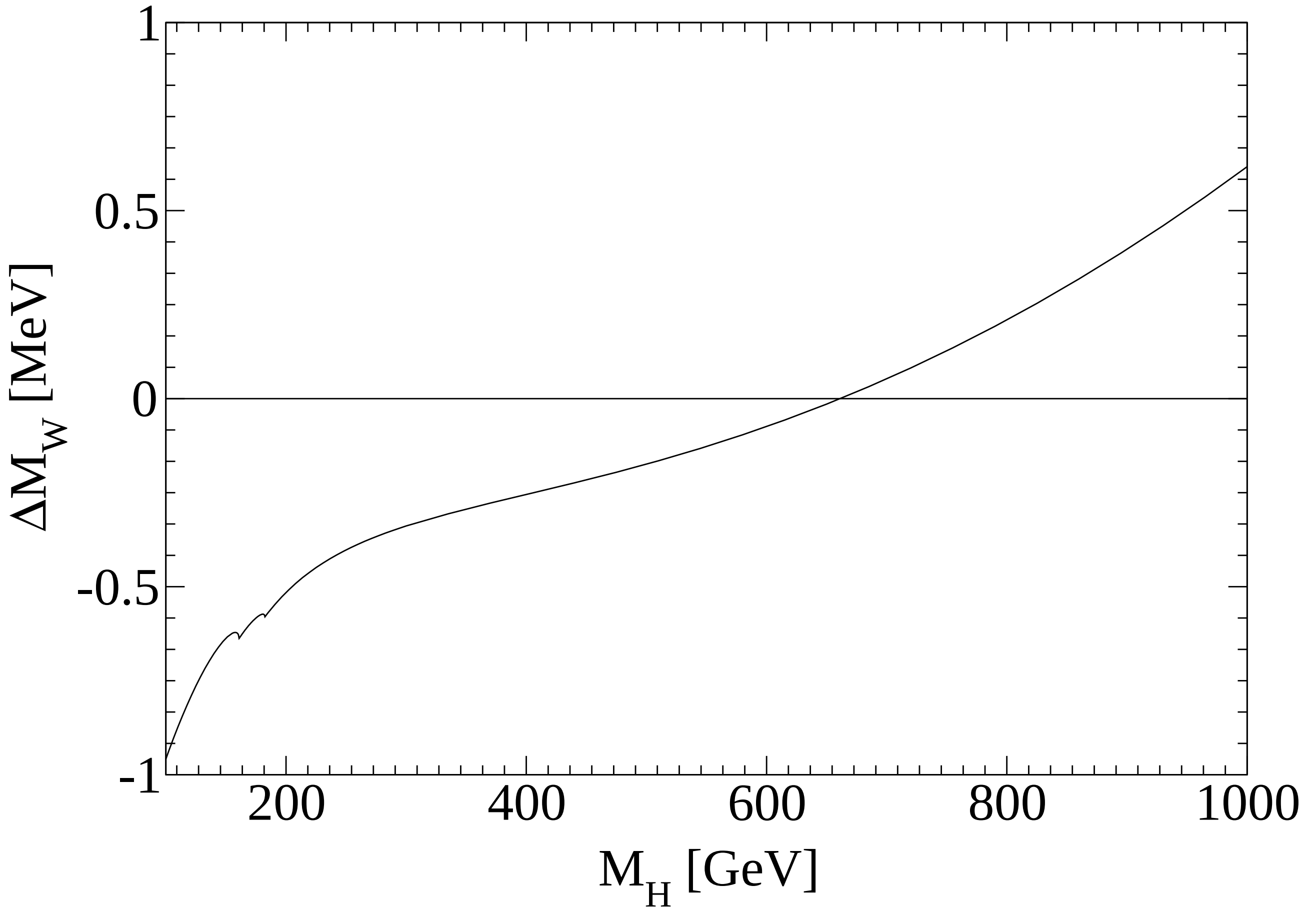


$$\frac{\alpha\pi}{2M_W^2 \sin^2\theta_W}\left(1+\Delta r\right)$$

- **The mass shift can be obtained from**

$$M_W = M_Z \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_F M_Z^2}(1+\Delta r)}}$$

$$\Delta M_W = -(1.491 + 1.779\ \overline{\Delta r}) \times 10^4 \Delta r^{(2)}_{bos} \text{[MeV]}$$



6. Conclusions

- Simple framework to calculate Δr to all orders
- Specific treatment of tadpoles to allow $1PI$ Green functions
- Calculation in on-shell and $\overline{MS}$
- Expansions compared with exact numeric results
- W boson mass shift negligible in view of the current experimental precision