FRAGMENTATION INTO SPIN-1 HADRONS

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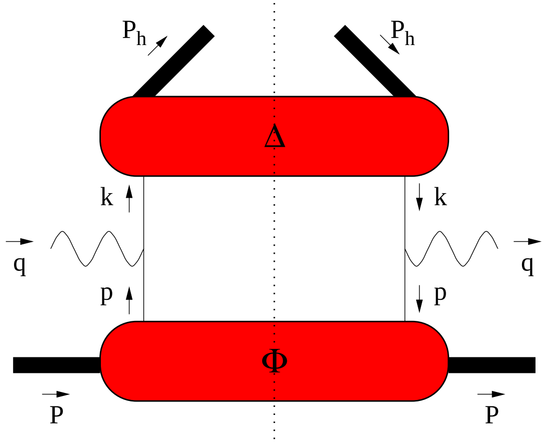
FOM

- Semi-inclusive DIS
- Fragmentation to spin-1 hadrons
- Transversity measurements
- Positivity bounds and modeling attempts

Semi-inclusive DIS

$$\mathrm{d}\sigma(l + H \to l' + h + X) \propto L_{\mu\nu}\, W^{\mu\nu}$$



$$2MW^{\mu\nu} \propto \mathrm{Tr}\left[\Phi(x_B)\,\gamma^\mu\,\Delta(z_h)\,\gamma^\nu\right]$$

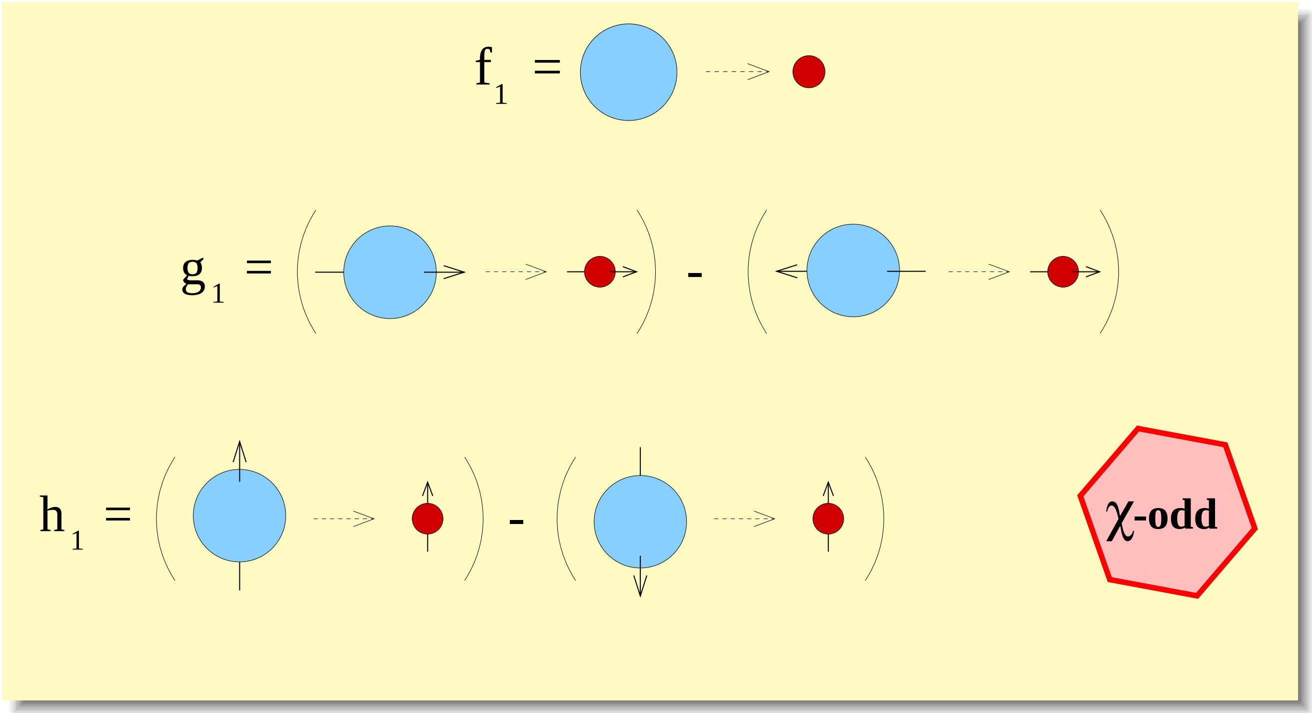
$$x_B = \frac{Q^2}{2\,P\cdot q} \qquad z_h = \frac{2\,P_h\cdot q}{Q^2}$$

$$\Phi(x) = \frac{1}{2}\int \mathrm{d}^2\boldsymbol{p}_T\,\mathrm{d}p^-\,\Phi(p,P)\Big|_{p^+=xP^+}$$

$$\Delta(z) = \frac{z}{4}\int \mathrm{d}^2\boldsymbol{k}_T\,\mathrm{d}k^+\,\Delta(k,P_h)\Big|_{k^-=\frac{P_h^-}{z}}$$

Distribution functions for spin-1/2

From decomposition of Φ correlation function.

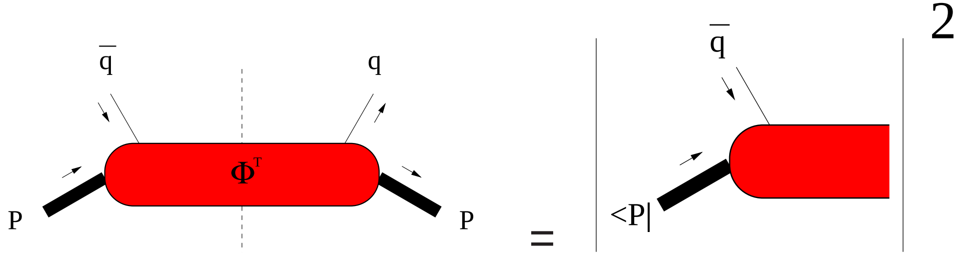


Quark+target spin structure

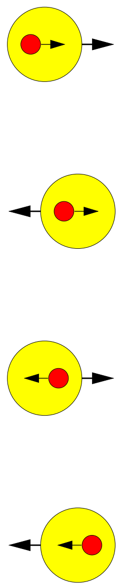
Full scattering matrix $M = (\Phi\gamma^+)^T$

for $\overline{q} + P \to X$

in parton $\otimes$ hadron helicity spaces



$$\begin{pmatrix} f_1 + g_1 & 0 & 0 & 2\,h_1 \\ 0 & f_1 - g_1 & 0 & 0 \\ 0 & 0 & f_1 - g_1 & 0 \\ 2\,h_1 & 0 & 0 & f_1 + g_1 \end{pmatrix}$$



Soffer bound

Positive definiteness of this matrix requires:

Positivity of diagonal elements $\Rightarrow$ trivial bounds

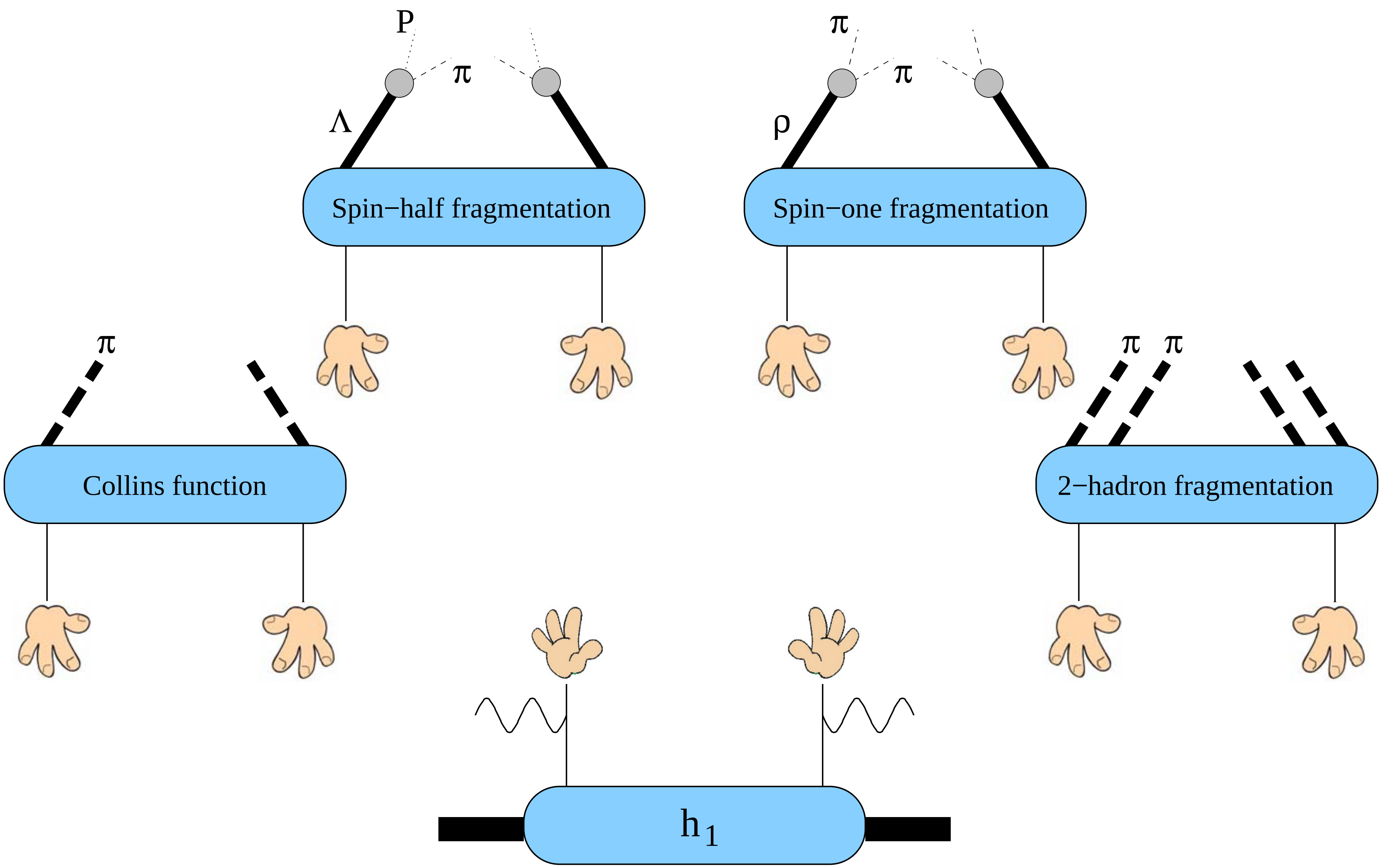
$$f_1(x) \geq 0$$

$$|g_1(x)| \leq f_1(x)$$

Positivity of eigenvalues $\Rightarrow$ less trivial (Soffer) bound

$$|h_1(x)| \leq \frac{1}{2}\left(f_1(x) + g_1(x)\right)$$

Chiral-odd partners for the transversity distribution



Fragmentation to spin-1 hadrons

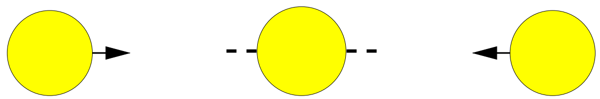
We need to measure the polarization of the outgoing hadron by means of VECTOR AND TENSOR ANALYZING POWERS of the decay

$$\Delta_{ij}(k, P_h, A_h^i, A_h^{ij}) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}k\cdot\xi} \langle 0|\psi_i(0)|P_h, A_h^i, A_h^{ij}\rangle\langle P_h, A_h^i, A_h^{ij}|\overline{\psi}_j(\xi)|0\rangle$$

or as a matrix in the hadron's helicity space

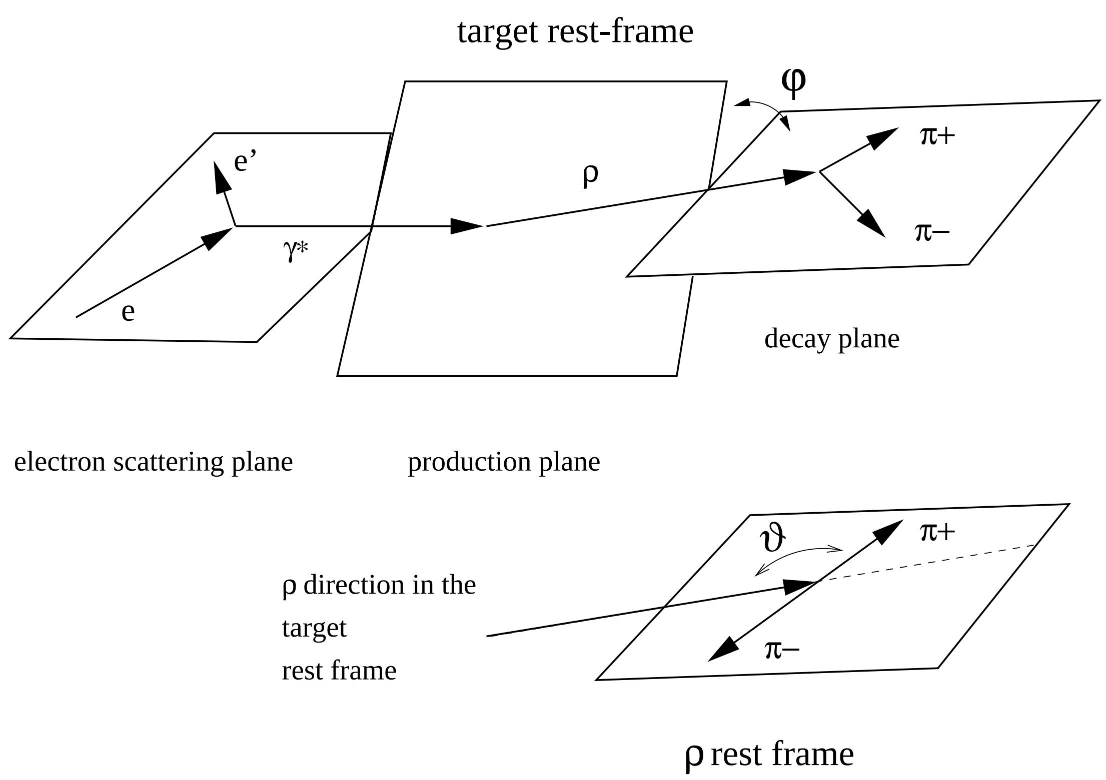
$$\Delta_{ij;\Lambda\Lambda'}(k, P_h) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}k\cdot\xi} \langle 0|\psi_i(0)|P_h, \Lambda\rangle\langle P_h, \Lambda'|\overline{\psi}_j(\xi)|0\rangle$$

where Λ and Λ' are hadron's helicities (1,0,-1)



$$\Delta(k, P_h, A_h^i, A_h^{ij}) = \mathrm{Tr}\left[R_h^{\text{(decay)}}(A_h^i, A_h^{ij})\, \Delta(k, P_h)\right]$$

Analyzing powers



$$A_{LL} = \frac{1}{3}\left(\cos^2\theta + \cos 2\theta\right)$$

$$A_{LT}^{x} = -\sin 2\theta \ \cos\varphi \qquad A_{LT}^{y} = -\sin 2\theta \ \sin\varphi$$

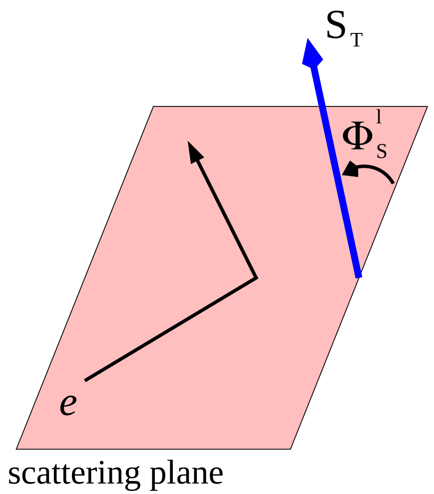
$$A_{TT}^{xx} = -\sin^2\theta \ \cos 2\varphi \qquad A_{TT}^{xy} = -\sin^2\theta \ \sin 2\varphi$$

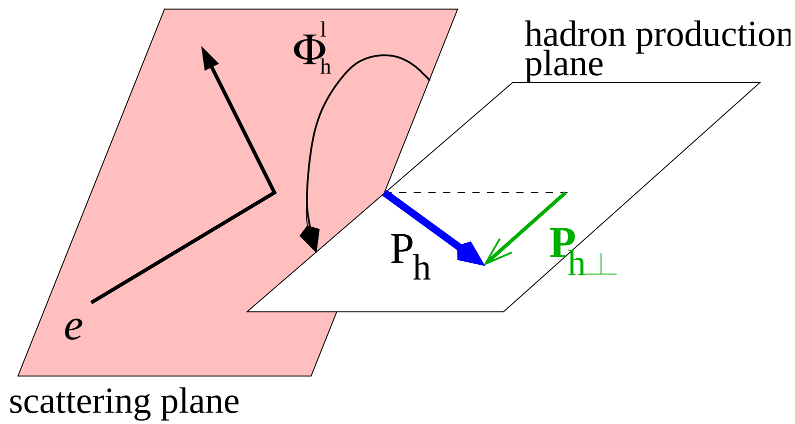
Transversity and asymmetries

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = f\left(\phi_S^{\ell}, |P_{h\perp}|, \phi_h^{\ell}, x_B, z_h\right)$$

$$\left\langle \begin{matrix}\text{Weight} \\ \text{Function}\end{matrix} \right\rangle_T (x_B, z_h) = \int \mathrm{d}\phi_S^{\ell}\, \mathrm{d}|P_{h\perp}|\, \mathrm{d}\phi_h^{\ell} \begin{matrix}\text{Weight} \\ \text{Function}\end{matrix} \left(\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}\right)$$

$$\propto h_1(x_B) \times ?(z_h)$$



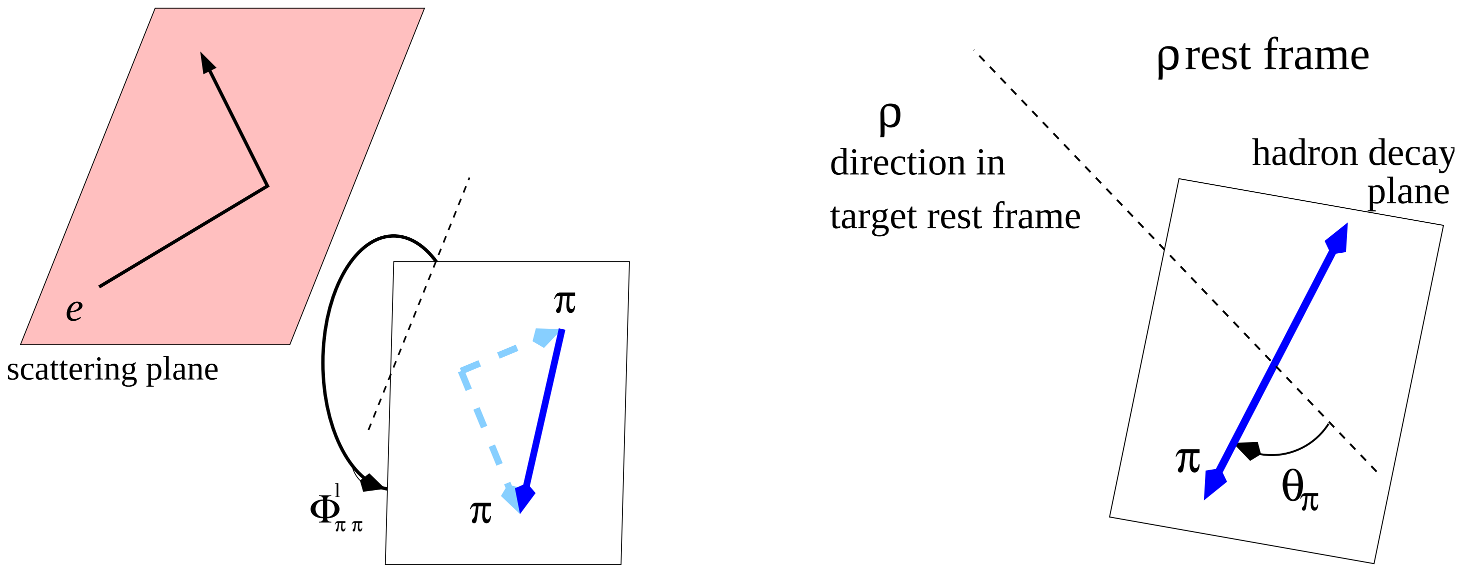


Asymmetries in spin-1 production

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} = f\left(\theta_\pi, \phi^\ell_{\pi\pi}, \phi^\ell_S, |P_{\rho\perp}|, \phi^\ell_\rho, x_B, z_\rho\right)$$

$$\left\langle \frac{\text{Weight}}{\text{Function}} \right\rangle_T (x_B, z_\rho) = \int \mathrm{d}\theta_\pi\, \mathrm{d}\phi^\ell_{\pi\pi}\, \mathrm{d}\phi^\ell_S\, \mathrm{d}|P_{\rho\perp}|\, \mathrm{d}\phi^\ell_\rho \frac{\text{Weight}}{\text{Function}} \left(\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}\right)$$

$$\propto h_1(x_B) \times\ ?\ (z_\rho)$$



Asymmetries containing the transversity distribution

$$\left\langle \sin\left(\phi^{\ell}_{\pi\pi} + \phi^{\ell}_{S}\right)\right\rangle_T \;\propto\; h_1(x_B)\, H_{1LT}(z_\rho)$$

χ-odd
T-odd

- Requires measurement of $\phi^{\ell}_{\pi\pi}, \phi^{\ell}_{S}, x_B, z_\rho$
- No complications due to presence of hadron transverse momentum

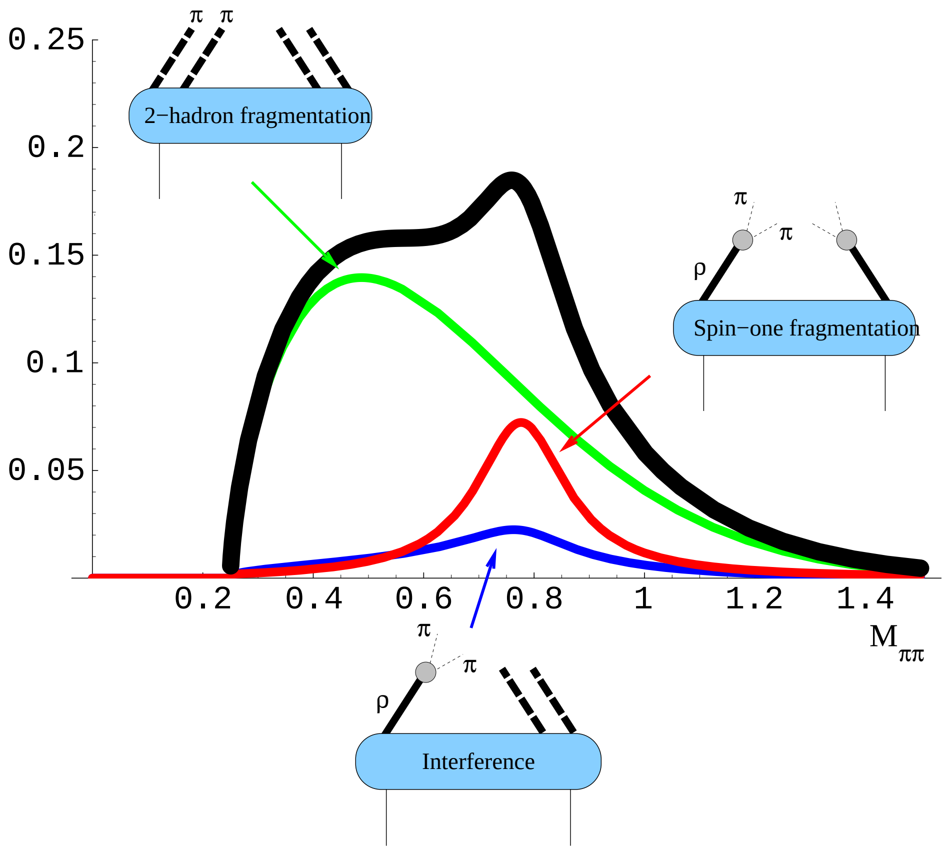
$$\left\langle \frac{|P_{\rho\perp}|}{2\,z\,M_h}\,\sin\left(2\phi^{\ell}_{\pi\pi} + \phi^{\ell}_{S} - \phi^{\ell}_{\rho}\right)\right\rangle_T \;\propto\; h_1(x_B)\, H^{(1)}_{1TT}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|}{2\,z\,M_h}\,\sin\left(\phi^{\ell}_{\rho} + \phi^{\ell}_{S}\right)\right\rangle_T \;\propto\; h_1(x_B)\, H^{\perp(1)}_{1LL}(z_\rho)$$

$$\left\langle \frac{|P_{\rho\perp}|^2}{2\,z^2\,M_h^2}\,\sin\left(\phi^{\ell}_{\pi\pi} - \phi^{\ell}_{S} - 2\phi^{\ell}_{\rho}\right)\right\rangle_T \;\propto\; h_1(x_B)\, H^{\perp(2)}_{1LT}(z_\rho)$$

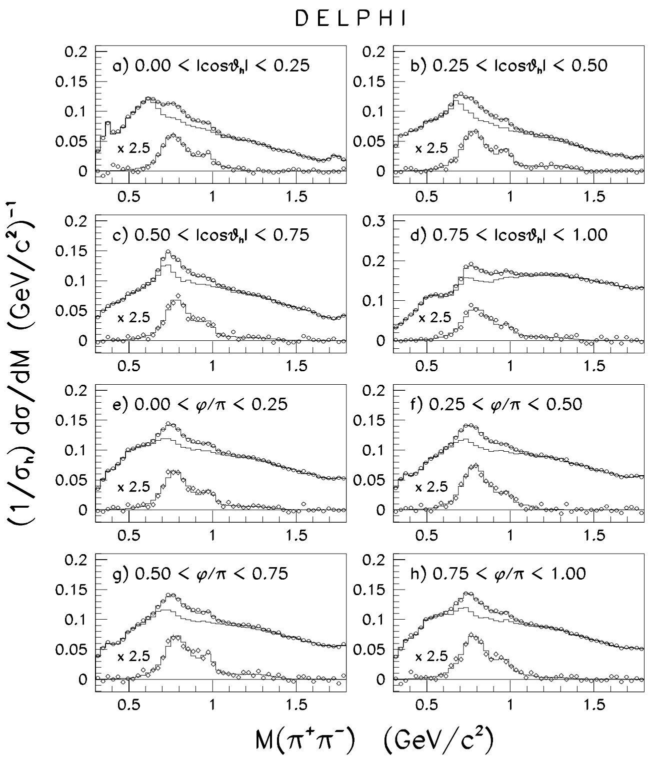
Where to observe these functions?

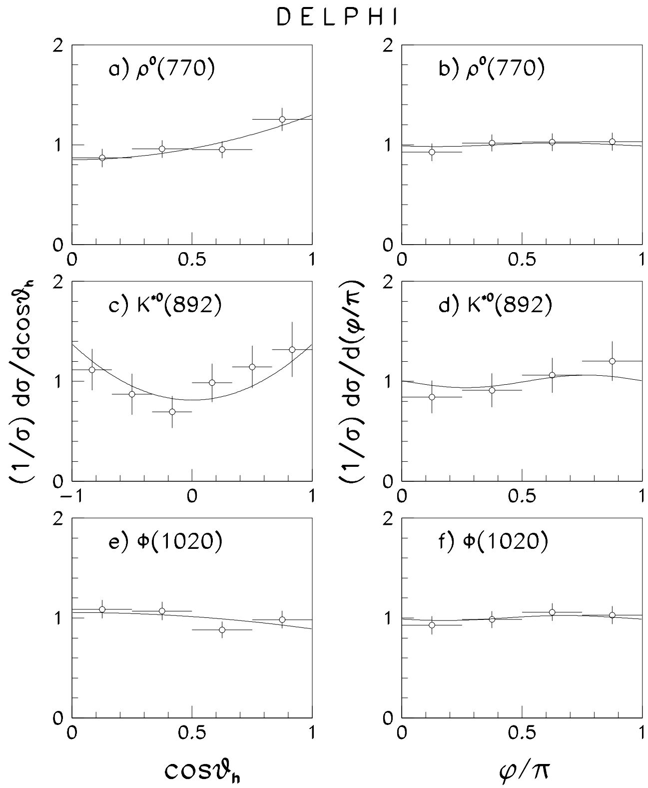
Mass spectrum of pion couples



Measurements in $e^+ e^-$ annihilation

Delphi collaboration Phys. Lett. B 406 (1997) 271

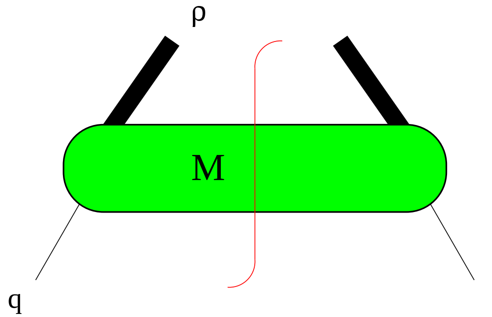




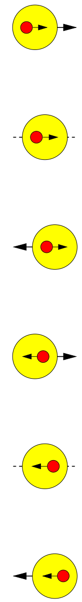
Measurement of decay distribution seems to be feasable.

Quark+hadron spin structure

Full scattering matrix M for $q \to \rho + X$



$$\begin{pmatrix}
D_1 + G_1 - \frac{D_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}\,(H_1 + \mathrm{i}H_{1LT}) & 0 \\
0 & D_1 + \frac{2\,D_{1LL}}{3} & 0 & 0 & 0 & \sqrt{2}\,(H_1 - \mathrm{i}H_{1LT}) \\
0 & 0 & D_1 - G_1 - \frac{D_{1LL}}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & D_1 - G_1 - \frac{D_{1LL}}{3} & 0 & 0 \\
\sqrt{2}\,(H_1 - \mathrm{i}H_{1LT}) & 0 & 0 & 0 & D_1 + \frac{2\,D_{1LL}}{3} & 0 \\
0 & \sqrt{2}\,(H_1 + \mathrm{i}H_{1LT}) & 0 & 0 & 0 & D_1 + G_1 - \frac{D_{1LL}}{3}
\end{pmatrix}$$



Positivity bounds

Positive definiteness of this matrix requires:

A.B., P.J. Mulders, hep-ph/0104176

Positivity of diagonal elements ⇒ trivial bounds

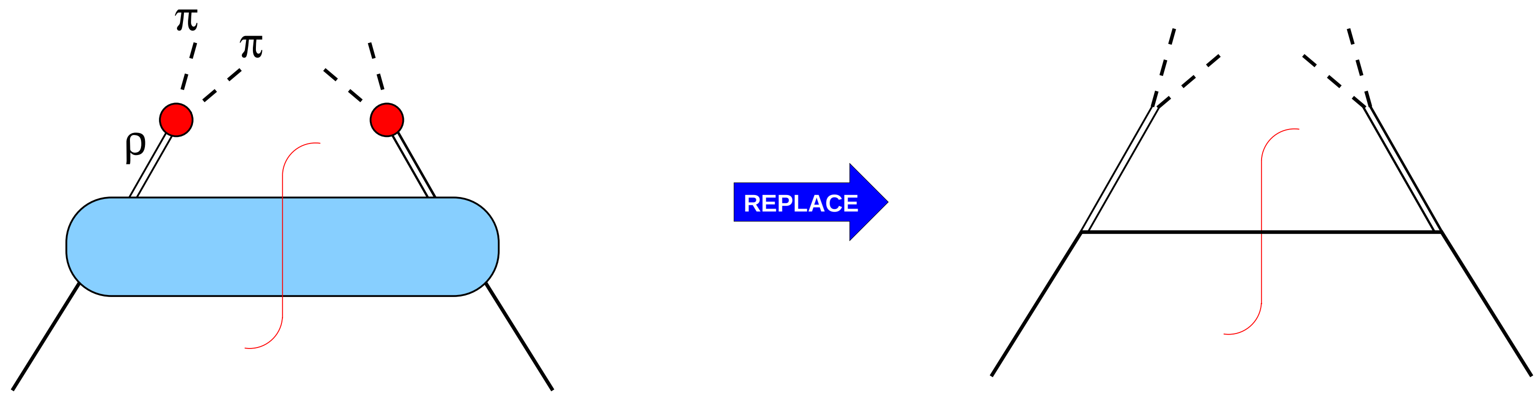
$$
\begin{aligned}
D_1(z) &\geq 0 \\
-\frac{3}{2} D_1(z) &\leq D_{1LL}(z) \leq 3\, D_1(z) \\
|G_1(z)| &\leq D_1(z) - \frac{1}{3} D_{1LL}(z) \leq \frac{3}{2} D_1(z)
\end{aligned}
$$

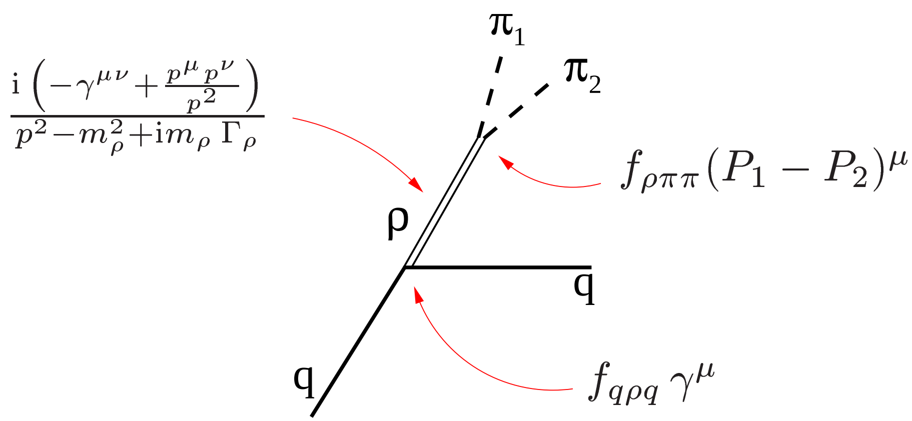
Positivity of 2-dimensional minors ⇒ (Soffer-like) bound

$$
\left[H_{1LT}(z)\right]^2 \leq \left[D_1(z) + \frac{2}{3} D_{1LL}(z)\right] \left[D_1(z) - \frac{1}{3} D_{1LL}(z)\right] \leq \frac{9}{2} \left[D_1(z)\right]^2 .
$$

Generating non-zero T-odd functions

Take a simple model for the fragmentation process



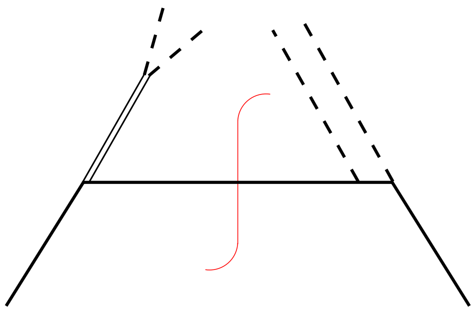


Generating non-zero T-odd functions

Add one-loop corrections (see also talk by Rajen Kundu)

T-odd functions $\neq 0$

"Competing" or "complementary" mechanism: interference fragmentation functions (see talk by Marco Radici)



Conclusions

- Fragmentation to spin-1 hadron is a viable option for transversity measurements, at the same level as Collins function and interference fragmentation functions.

- Since polarimetry on the final hadron requires the observation of two decay products, it represents a specific contribution to the more general case of two hadron production.

- Information on the unknown fragmentation functions can be extracted from positivity bounds and simple models.