MSSM Fit to Precision Data

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3. Precision tests: SM vs. MSSM
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1. Introduction

Electroweak precision measurements:

$$
\begin{array}{llll}
M_Z[\,\mathrm{GeV}] & = & 91.1875 \pm 0.0021 & 0.002\% \\
G_\mu[\,\mathrm{GeV}^{-2}] & = & 1.16637(1)\,10^{-5} & 0.0009\% \\
m_t[\,\mathrm{GeV}] & = & 174.3 \pm 5.1 & 2.9\% \\
 & & & \\
M_W[\,\mathrm{GeV}] & = & 80.449 \pm 0.034 & 0.04\% \\
\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} & = & 0.23148 \pm 0.00017 & 0.07\% \\
\Gamma_Z[\,\mathrm{GeV}] & = & 2.4952 \pm 0.0023 & 0.09\% \\
\ldots & & &
\end{array}
$$

Quantum effects of the theory:

Loop corrections: $\sim \mathcal{O}(1\%)$

SM:

M_H is free parameter

precise measurement of M_W, $\sin^2\theta_{\mathrm{eff}}$, ...

$\Rightarrow$ constraints on M_H

MSSM:

m_h is predicted

precise measurement of M_W, $\sin^2\theta_{\mathrm{eff}}$, m_h, ...

$\Rightarrow$ constraints on $m_{\tilde{t}}$, $\theta_{\tilde{t}}$, $m_{\tilde{b}}$, $\theta_{\tilde{b}}$, ...

Minimal Supersymmetric Standard Model (MSSM):

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \qquad \tilde{\chi}^{\pm}_{1,2} \qquad \tilde{\chi}^0_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector:

Two Higgs doublets, physical states: $h^0, H^0, A^0, H^{\pm}$

Unconstrained MSSM:

Parameterization of possible SUSY-breaking terms, no particular SUSY breaking mechanism assumed

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Specific models for soft-SUSY breaking:

"Hidden sector": SUSY breaking	$\longrightarrow$	Visible sector: MSSM

"Gravity-mediated": mSUGRA

"Gauge-mediated": GMSB

"Anomaly-mediated": AMSB

...

MSSM Higgs potential contains two Higgs doublets:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \underbrace{\frac{g'^2 + g^2}{8}} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1 \bar{H}_2|^2$$

gauge couplings, in contrast to SM

Five physical states: $h^0, H^0, A^0, H^\pm$

Input parameters: $\tan\beta = \frac{v_2}{v_1}$, M_A

$\Rightarrow$ m_h, m_H, mixing angle α, $m_{H^\pm}$: no free parameters

$\Rightarrow$ Prediction for m_h

MSSM tree-level bound (from gauge sector):

$m_h < M_Z$, excluded by LEP Higgs searches

Large radiative corrections (from Yukawa sector):

Dominant one-loop corrections: $\sim G_\mu m_t^4 \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

Bound including corrections up to two-loop order:
[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

$$m_h \lesssim 135 \text{ GeV}$$

$\Rightarrow$ m_h is a 'precision observable'

Prospects for experimental accuracies at the next generation of colliders:

$$\Delta M_W^{exp} / \text{ MeV}: \quad 34 \xrightarrow{\text{Tev/LHC}} 15 \xrightarrow{\text{LC}} 7$$

$$\Delta \sin^2\theta_{eff}^{exp} / 10^{-5}: \quad 17 \xrightarrow{\text{Tev/LHC}} 17 \xrightarrow{\text{LC}} 1$$

$$\Delta m_h^{exp} / \text{ GeV}: \quad ?? \xrightarrow{\text{Tev/LHC}} 0.2 \xrightarrow{\text{LC}} 0.05$$

$$\Delta m_t^{exp} / \text{ GeV}: \quad 5.1 \xrightarrow{\text{Tev/LHC}} 1.5 \xrightarrow{\text{LC}} 0.1$$

Status of precision observables in the SM:

Complete one-loop results available for all electroweak precision observables

QCD corrections:

$\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha\alpha_s^2)$ contributions known for many observables: M_W, $\sin^2\theta_{eff}$, Γ_l, $B \to X_s\gamma$, ...

EW 2-loop corrections:

Complete electroweak two-loop results available for

$g_\mu - 2$

[*A. Czarnecki, B. Krause, W. Marciano '95, '96*]

[*S. Peris, M. Perrottet, E. de Rafael '95*]

M_W (from muon decay):
fermionic / bosonic two-loop contributions

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

[*M. Awramik, M. Czakon '02*] [*A. Onishchenko, O. Veretin '02*]

Expansion in m_t up to $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ available for

$\sin^2\theta_{eff}$, Γ_l

[*G. Degrassi, P. Gambino, A. Sirlin '97*]

[*G. Degrassi, P. Gambino '00*]

Leading EW corrections beyond 2-loop:

Pure fermion-loop contributions up to 4-loop order

[*A. Stremplat '98*], [*G.W. '98*]

Leading 3-loop corrections to $\Delta\rho$:

$\mathcal{O}(G_\mu^3 m_\mathrm{t}^6)$, $\mathcal{O}(G_\mu^2 \alpha_\mathrm{s} m_\mathrm{t}^4)$ terms

limit: $M_\mathrm{H} = 0$

[*J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker '01*]

Remaining theoretical uncertainties:

[*P. Gambino '98*]

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

[*S. Heinemeyer, W. Hollik, G.W. '00*]

[*S. Heinemeyer, G.W. '01*]

[*U. Baur, R. Clare, J. Erler, S. Heinemeyer, D. Wackeroth, G. W., D. Wood '01*]

[*A. Ferroglia, G. Ossola, M. Passera, A. Sirlin '02*]

- from experimental errors of input parameters:

  $\Delta m_{\mathrm{t}} = \pm 5.1$ GeV

  $\Rightarrow \Delta M_{\mathrm{W}} \approx \pm 31$ MeV, $\Delta \sin^2 \theta_{\mathrm{eff}} \approx \pm 16 \times 10^{-5}$

  $\Delta(\Delta\alpha_{\mathrm{had}}) = \pm 3.6 \times 10^{-4}$

  $\Rightarrow \Delta M_{\mathrm{W}} \approx \pm 6.5$ MeV, $\Delta \sin^2 \theta_{\mathrm{eff}} \approx \pm 13 \times 10^{-5}$

- from unknown higher-order corrections:

  $\Delta M_{\mathrm{W}}^{\mathrm{SM}} \approx \pm 5$ MeV, $\Delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{SM}} \approx \pm 7 \times 10^{-5}$

Comparison: estimate of higher-order uncertainty for 'blue band plot' up to Moriond '01

[*D. Bardin, M. Grünewald, G. Passarino '99*]

$\Delta M_{\mathrm{W}}^{\mathrm{SM}} \approx \pm 3$ MeV, $\Delta \sin^2 \theta_{\mathrm{eff}}^{\mathrm{SM}} \approx \pm 3 \times 10^{-5}$

Towards complete electroweak two-loop results for Z-boson observables:

Pseudo-observables $\sin^2\theta_{\text{eff}}$, Γ_l, ... are based on effective Born structure

'SM remnants' (box contributions, γ–Z interference, imaginary parts) are used to obtain pseudo-observables from realistic observables

At $\mathcal{O}(\alpha^2) \approx \mathcal{O}(\alpha\,\Gamma_Z/M_Z)$:

Relation between theory predictions and pseudo-obs. $\sin^2\theta_{\text{eff}}$, Γ_l, ... becomes very involved

Gauge-invariant treatment requires expansion around complex pole $s = \mathcal{M}^2 = M_Z^2 - iM_Z\Gamma_Z$

In *ZFITTER* and *TOPAZ0*:

'various terms' of $\mathcal{O}(\alpha\,\Gamma_Z/M_Z)$ are neglected

necessary for obtaining Born-like structure of pseudo-observables

[*D. Bardin, M. Grünewald, G. Passarino '99*]

$\alpha\,\Gamma_Z/M_Z \approx 0.02\%$

Effects of new physics on precision observables

It is in general not sufficient to ristrict to S, T, U parameters or similar parameterizations

S, T, U parameters:
[*M. Peskin, T. Takeuchi '92*]

parameterize new physics contributions which only enter via vacuum-polarization effects to gauge-boson propagators

S, T, U are intrinsically one-loop quantities

– expressions are linearized in S, T, U

– beyond one-loop order there is no model of new physics that enters via vacuum-polarization effects only

Many kinds of new physics contributions cannot be described in terms of S, T, U parameters

E.g.: New physics contributions to $(g_\mu - 2)$

2. Status of precision observables in the MSSM

Additional sources for sizable radiative corrections in the MSSM:

- Mass and couplings of light $\mathcal{CP}$-even Higgs:

  Large Yukawa corr.: $\sim G_\mu m_\mathrm{t}^4 \ln\left(\frac{m_{\tilde{\mathrm{t}}_1} m_{\tilde{\mathrm{t}}_2}}{m_\mathrm{t}^2}\right)$, . . .

- t / $\tilde{\mathrm{t}}$ loops

- b / $\tilde{\mathrm{b}}$ loops for large $\tan\beta$

- Corrections to relation between bottom mass and bottom Yukawa coupling:

$$y_b = \frac{\sqrt{2}}{v\cos\beta}\frac{m_\mathrm{b}}{1+\Delta_b},$$

  $\Delta_b = \mu\tan\beta\,(\alpha_\mathrm{s} I(\ldots) + \alpha_\mathrm{t} I(\ldots))$

  $\Rightarrow$ Coupling non-perturbative for $\Delta_b \to -1$

- Loop contributions from light SUSY particles

- . . .

Theoretical uncertainties quoted below do not hold in non-perturbative region of y_b

Complete one-loop results available for m_{h}, M_{W}, Z observables, $(g_\mu - 2)$, $B \to X_s\gamma$, ...

Available higher-order corrections:

<u>m_{h}:</u>

Dominant $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ and $\mathcal{O}(\alpha^2)$ contributions

[*G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. W. '02*]

Implemented in *FeynHiggs*

[*S. Heinemeyer, W. Hollik, G. W. '98, '00, '02*]

`http://www.feynhiggs.de`

Remaining theoretical uncertainties:

– From uncertainties in input parameters

$m_{\mathrm{t}}, \ldots, M_{\mathrm{A}}, \tan\beta, m_{\tilde{\mathrm{t}}_1}, m_{\tilde{\mathrm{t}}_2}, \theta_{\tilde{\mathrm{t}}}, m_{\tilde{\mathrm{g}}}, \ldots$

$\Delta m_{\mathrm{t}} \approx 5$ GeV $\Rightarrow$ $\Delta m_{\mathrm{h}} \approx 5$ GeV

– From unknown higher-order corrections:

$\Rightarrow$ $\Delta m_{\mathrm{h}} \approx 3$ GeV

$\Rightarrow$ LC accuracy on m_{t} crucial for keeping theo. unc. in MSSM Higgs sector under control

More than factor 10 improvement in unc. from unknown higher orders needed to match prospective LHC accuracy

M_W and Z observables

Dominant $\mathcal{O}(\alpha\alpha_s)$ contributions
[A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger, G. W. '97]
[S. Heinemeyer, W. Hollik, G. W. '98]

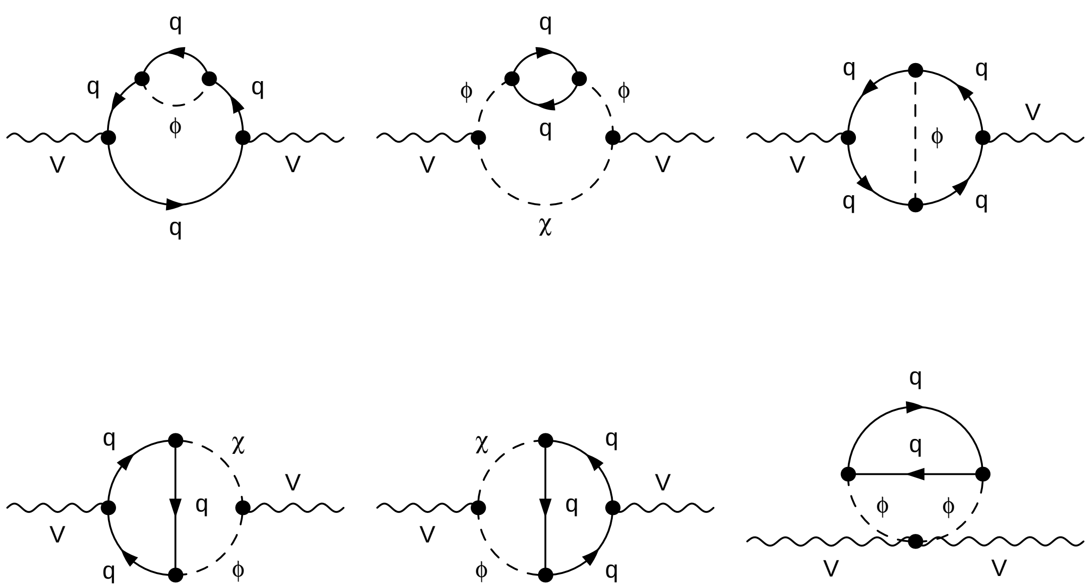
Leading contributions to M_W, $\sin^2\theta_{\rm eff}, \ldots$ from mass splitting between isospin doublet fields enter via

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2},$$

$$\Delta M_W \approx \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho, \qquad \Delta\sin^2\theta_{\rm eff} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho$$

Leading $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ corrections to $\Delta\rho$ in limit $M_{\rm SUSY} \to \infty$
[S. Heinemeyer, G. W. '01, '02]

Generic types of two-loop diagrams:

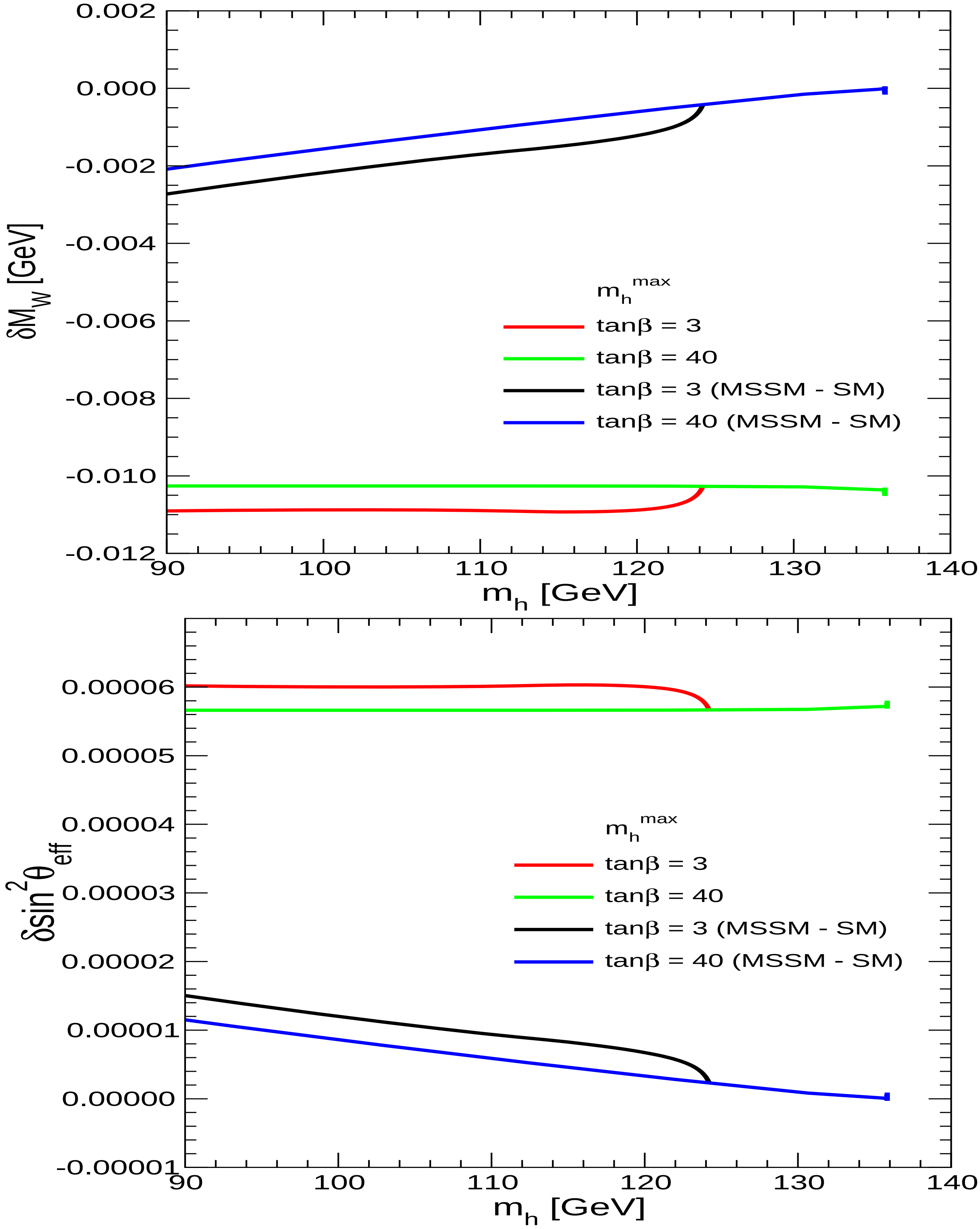


$V = Z, W^{\pm}$, $q = t, b$, $\phi, \chi = h^0, H^0, A^0, H^{\pm}, G^0, G^{\pm}$

Corrections to M_W and $\sin^2\theta_{eff}$:

Contribution from $\Delta\rho^{MSSM}_{\alpha_t^2}$ and comparison with SM contrib. as fct. of m_h (M_A varied), $\tan\beta = 3, 40$:

[S. Heinemeyer, G. W. '02]



Corrections to M_W, $\sin^2\theta_{eff}$ up to -12 MeV, 6×10^{-5}

$|\delta M_W^{MSSM-SM}| \lesssim 3$ MeV, $\delta\sin^2\theta_{eff}^{MSSM-SM} \lesssim 3\times10^{-5}$

In gauge-less limit (gauge couplings neglected):

$m_{\mathrm{h}}^2 = 0$, $\sin\alpha = -\cos\beta$, $\cos\alpha = \sin\beta$

$m_{\mathrm{H}}^2 = M_{\mathrm{A}}^2$, $m_{\mathrm{H}^\pm}^2 = M_{\mathrm{A}}^2$, $m_{\mathrm{G}}^2 = m_{\mathrm{G}^\pm}^2 = 0$

$\Rightarrow$ no SUSY effects in $hf\bar{f}$ couplings

$\Delta\rho(m_{\mathrm{h}} = 0)$ is bad approximation

$\Rightarrow$ Formally non-leading two-loop contributions are numerically important

Theoretical uncertainties of M_{W}, $\sin^2\theta_{\mathrm{eff}}$, ... in the MSSM from unknown higher-order corrections:

about twice as large as in the SM

<u>$B \to X_s\gamma$:</u>

Leading higher-order contributions (enhanced by $\tan\beta$ factors, $\ln(M_{\mathrm{SUSY}}/M_{\mathrm{W}})$

[*G. Degrassi, P. Gambino, G. Giudice '00*]

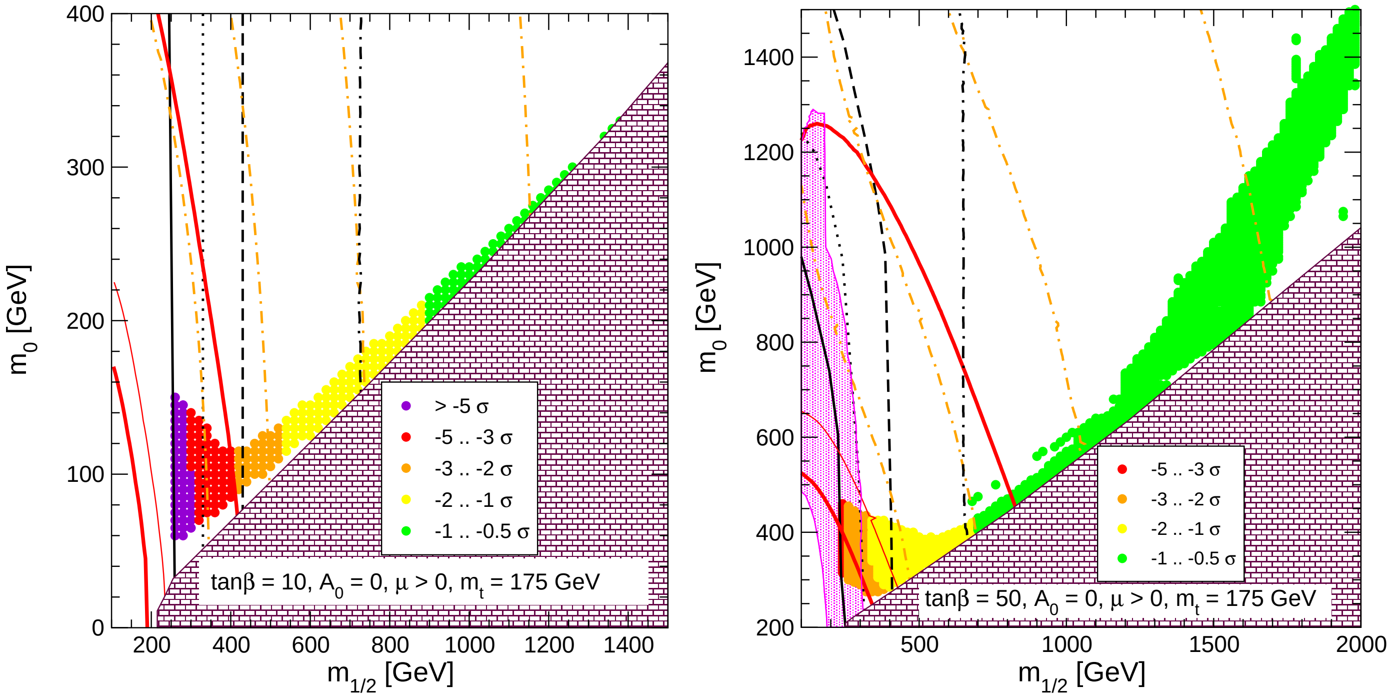
[*M. Carena, D. Garcia, U. Nierste, C. Wagner '00*]

3. Precision tests: SM vs. MSSM

Is mSUGRA scenario (4 parameters and a sign instead of 105 parameters of unconstrained MSSM) consistent with constraints from Higgs search, CDM, $(g_\mu - 2)$, $B \to X_s\gamma$?

Prediction for $\sigma(e^+e^- \to Zh) \times \mathrm{BR}(h \to WW^*)$ in parameter region allowed by cosmology:
comparison mSUGRA – SM:
[*J. Ellis, S. Heinemeyer, K. Olive, G. W. '02*]

$\mu > 0$, $\tan\beta = 10, 50$:



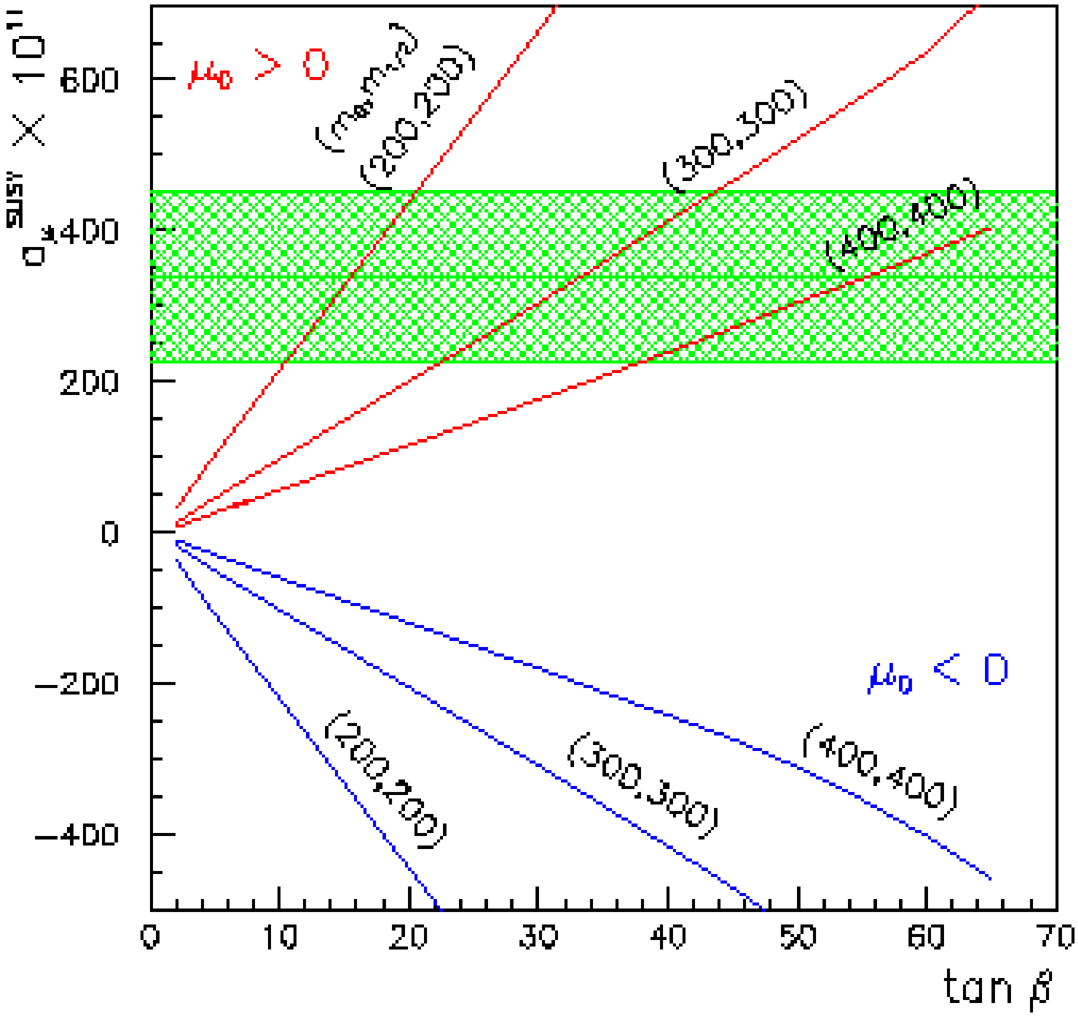
$\Rightarrow$ mSUGRA scen. consistent with exp. constraints

In allowed parameter space: sizable deviations from SM predictions for precision observables in the Higgs sector possible

mSUGRA predictions for $(g_\mu - 2)$:

[*W. de Boer, C. Sander '02*]

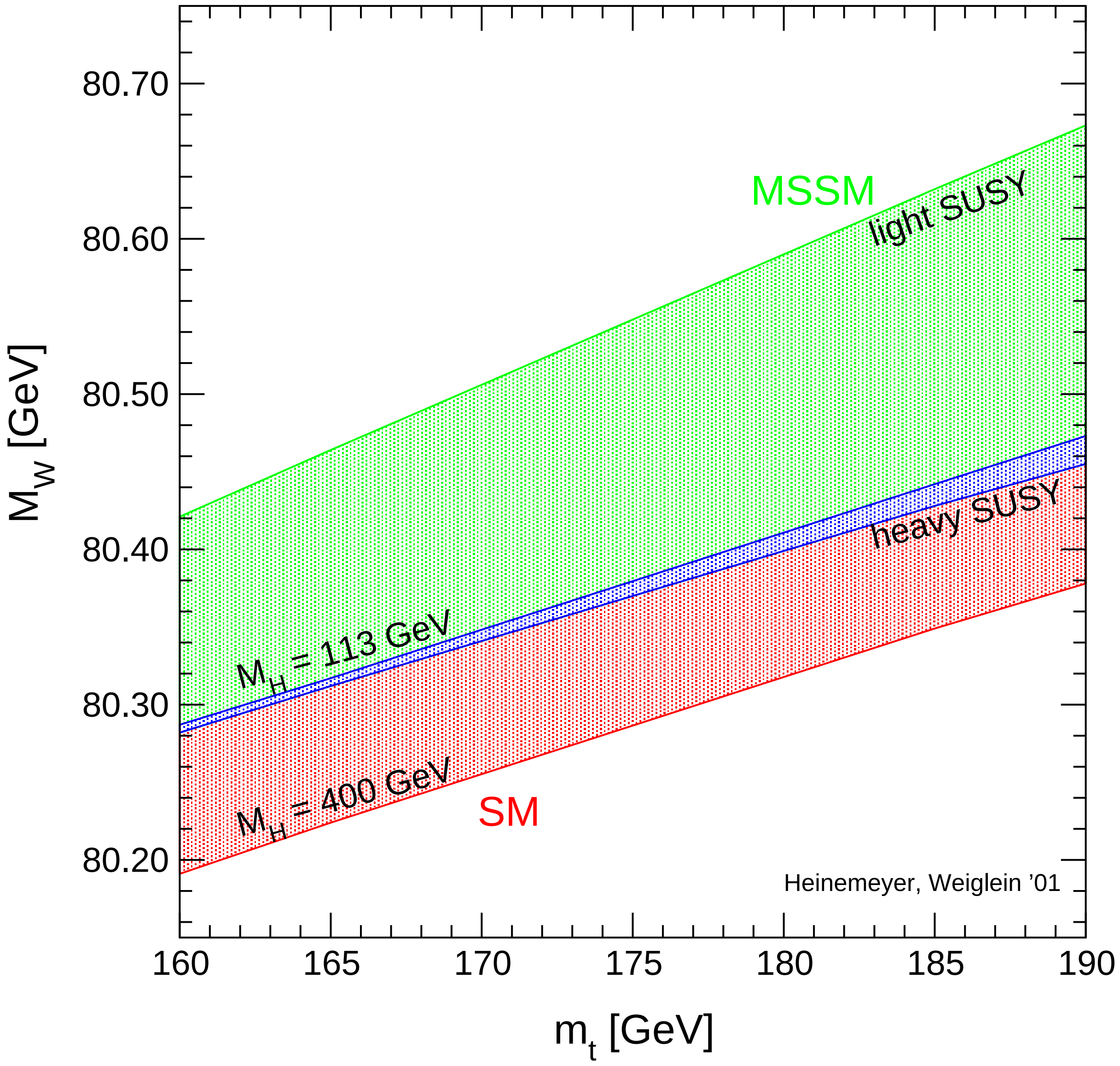
$$a_\mu = \frac{1}{2}(g_\mu - 2);\ \ a_\mu - a_\mu^{\rm SM} = (338 \pm 112) \times 10^{-11}$$



$\Rightarrow$ Preference for $\mu > 0$

Prediction for M_{W} in the SM and the MSSM:

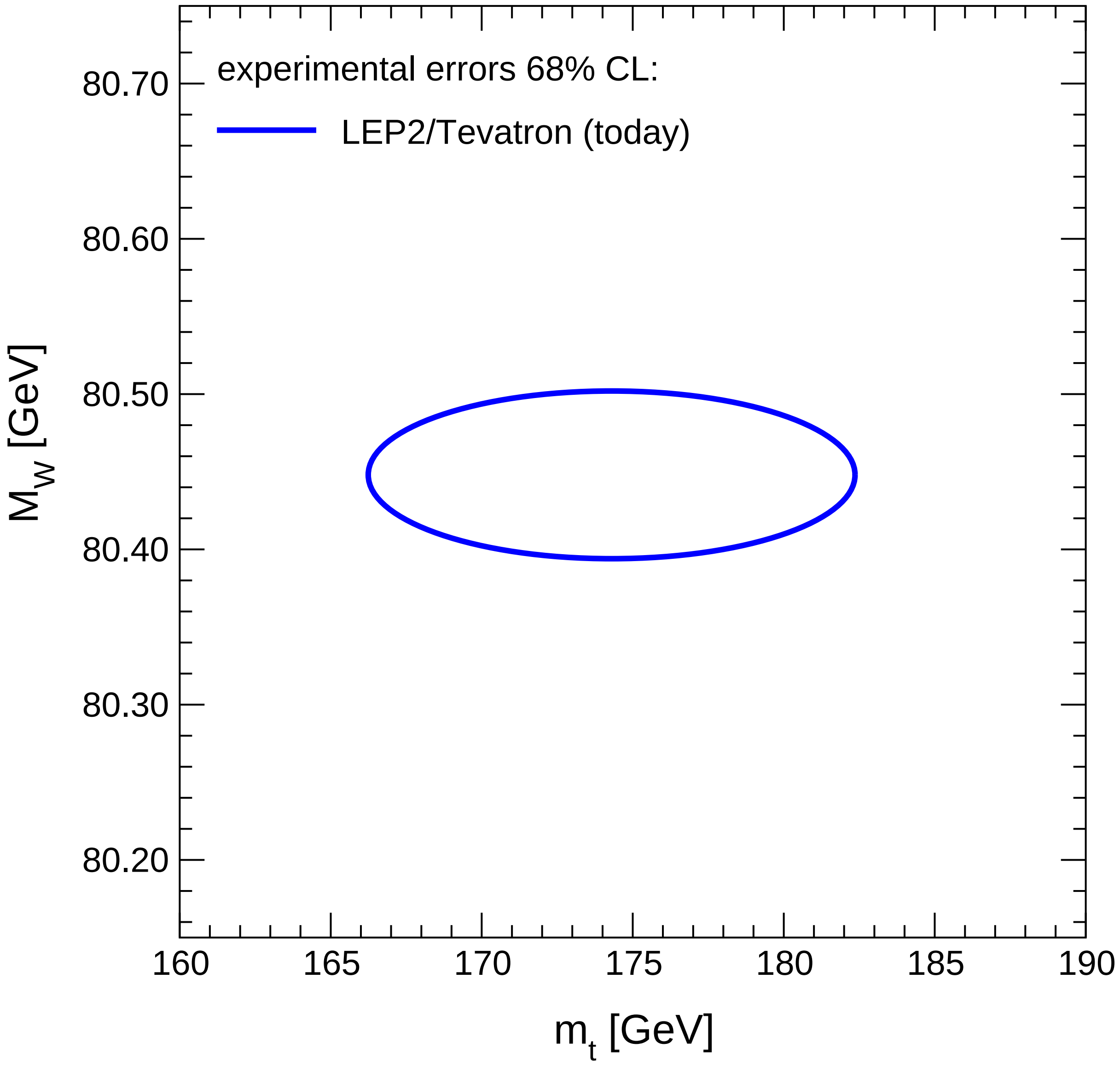
[*S. Heinemeyer, G. W. '01*]

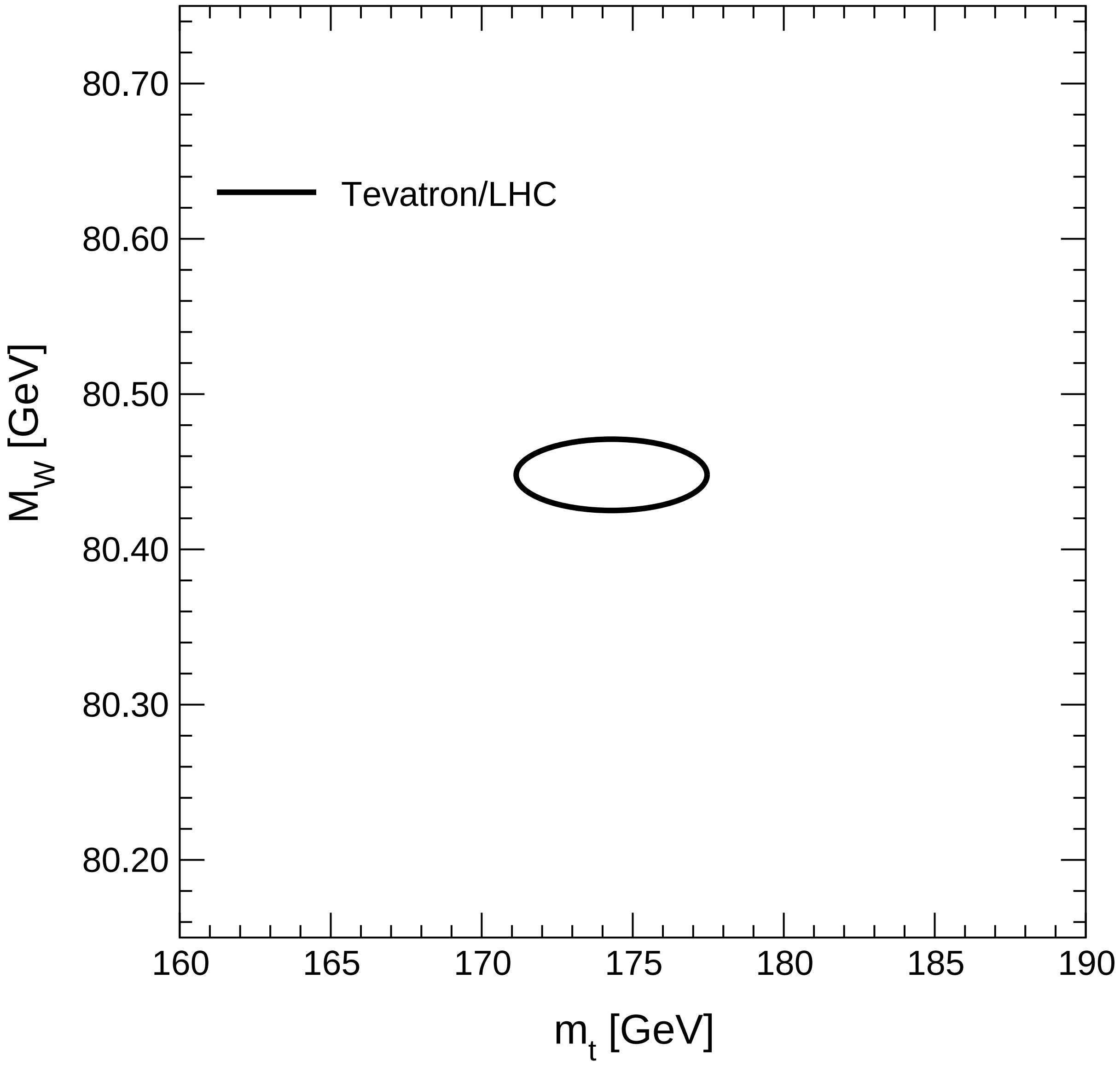


SM: M_{H} varied

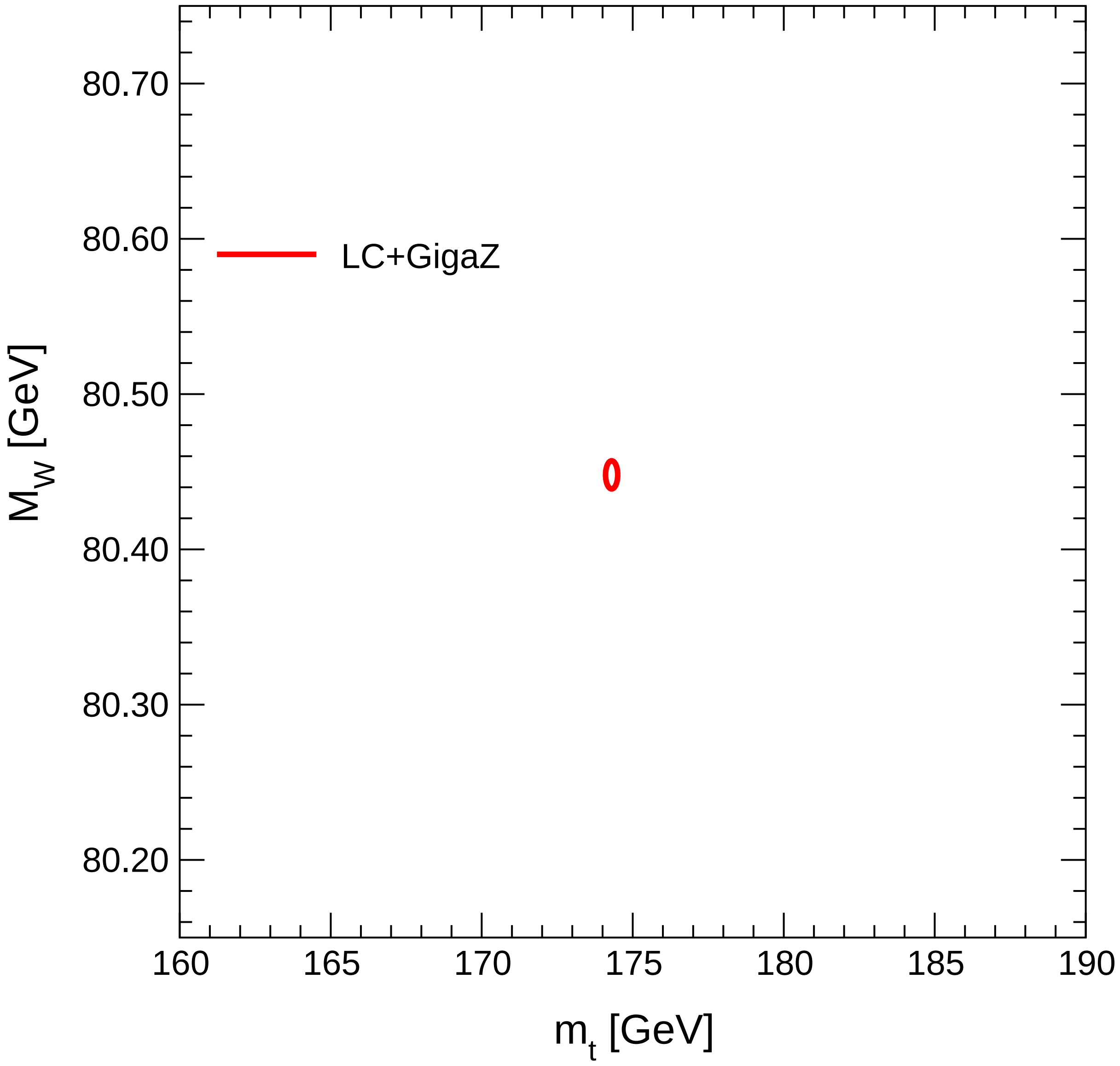
MSSM: SUSY parameters varied

$\Rightarrow$ Slight preference for the MSSM


experimental errors 68% CL:
LEP2/Tevatron (today)
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]

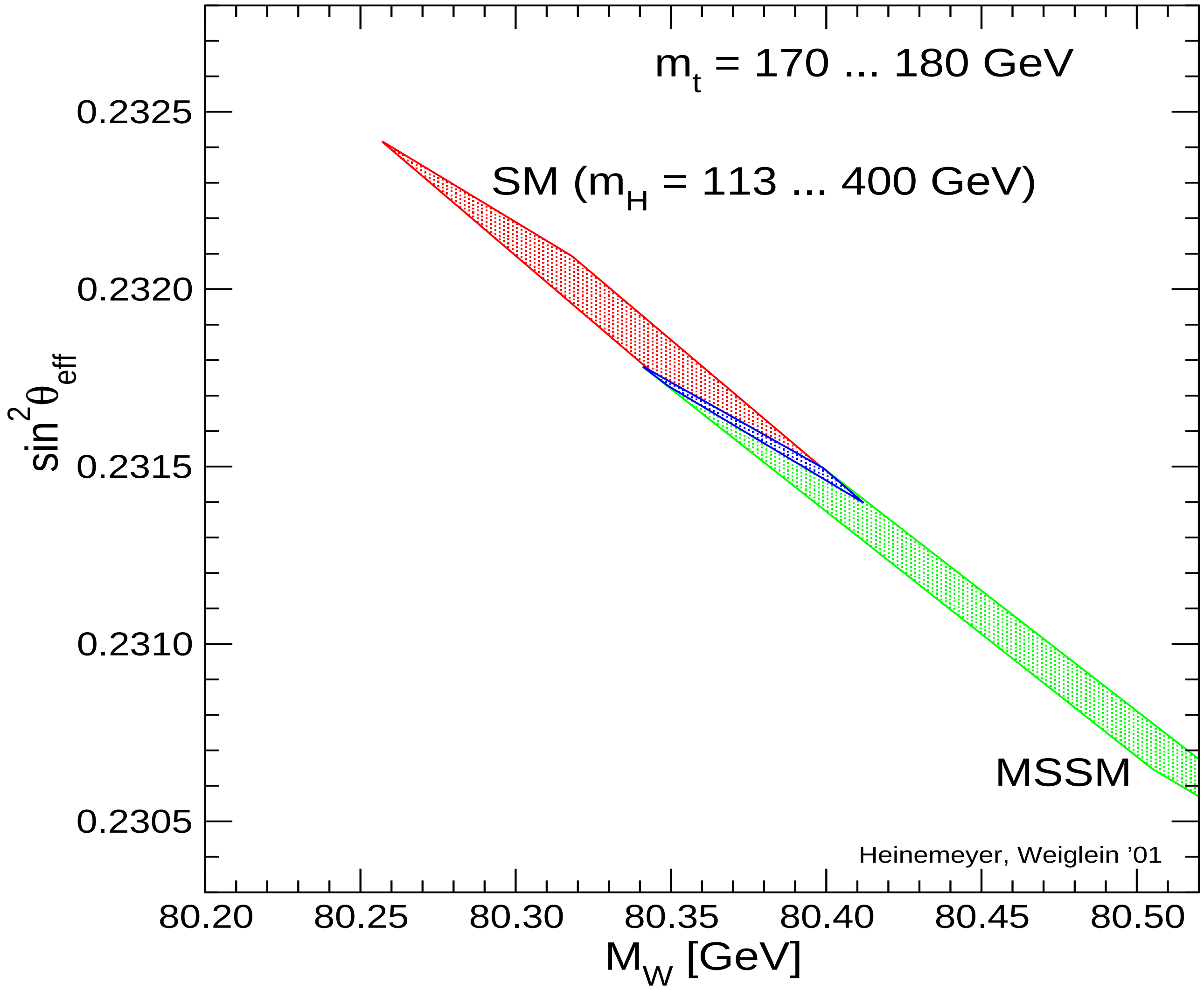


80.70
80.60
80.50
80.40
80.30
80.20
Tevatron/LHC
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]

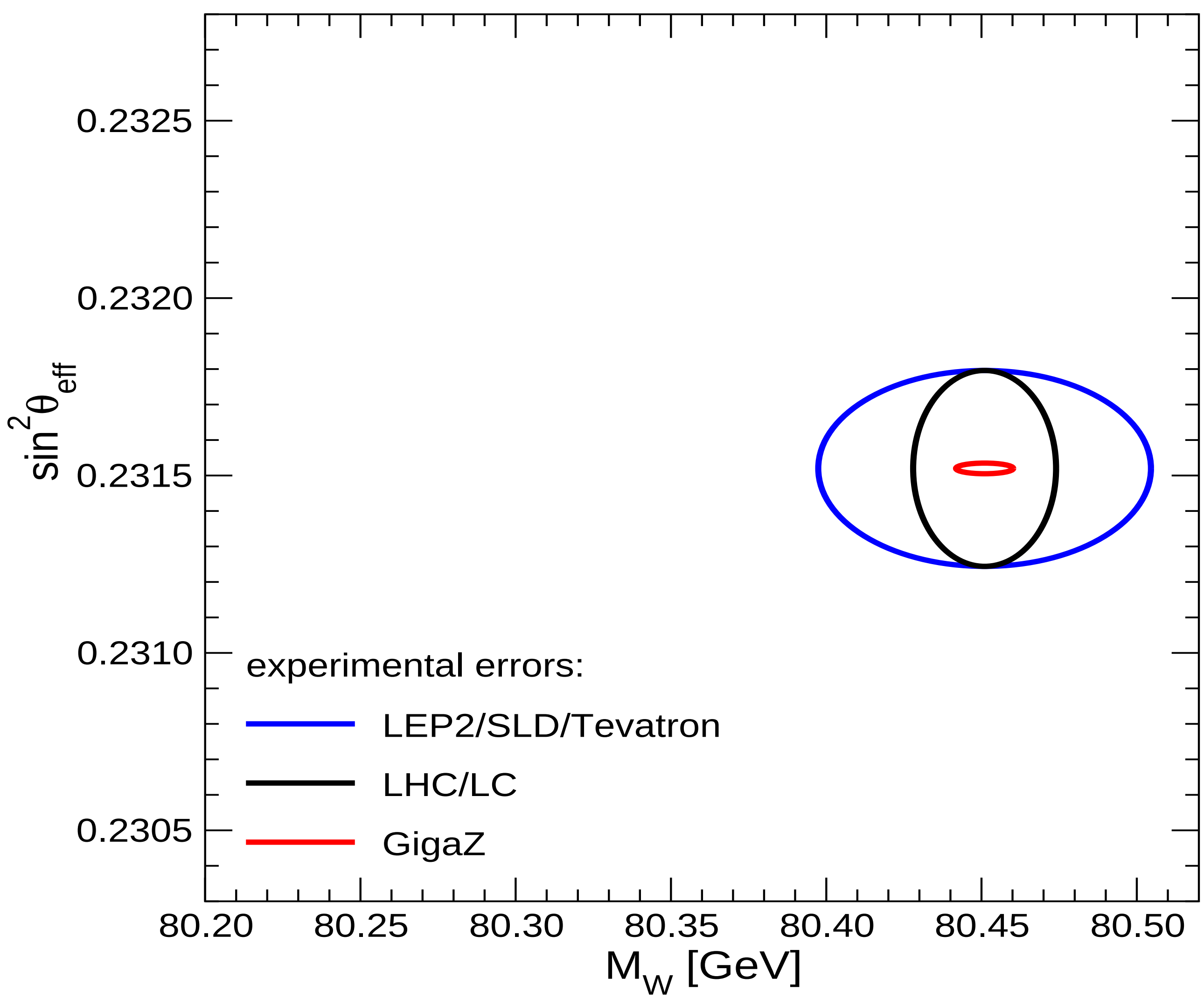

80.70
80.60
80.50
80.40
80.30
80.20
LC+GigaZ
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]

Prediction for M_{W}, $\sin^2\theta_{\mathrm{eff}}$ in SM and MSSM:

[S. Heinemeyer, G. W. '01]



$\Rightarrow$ High sensitivity to deviations both from the SM and the MSSM

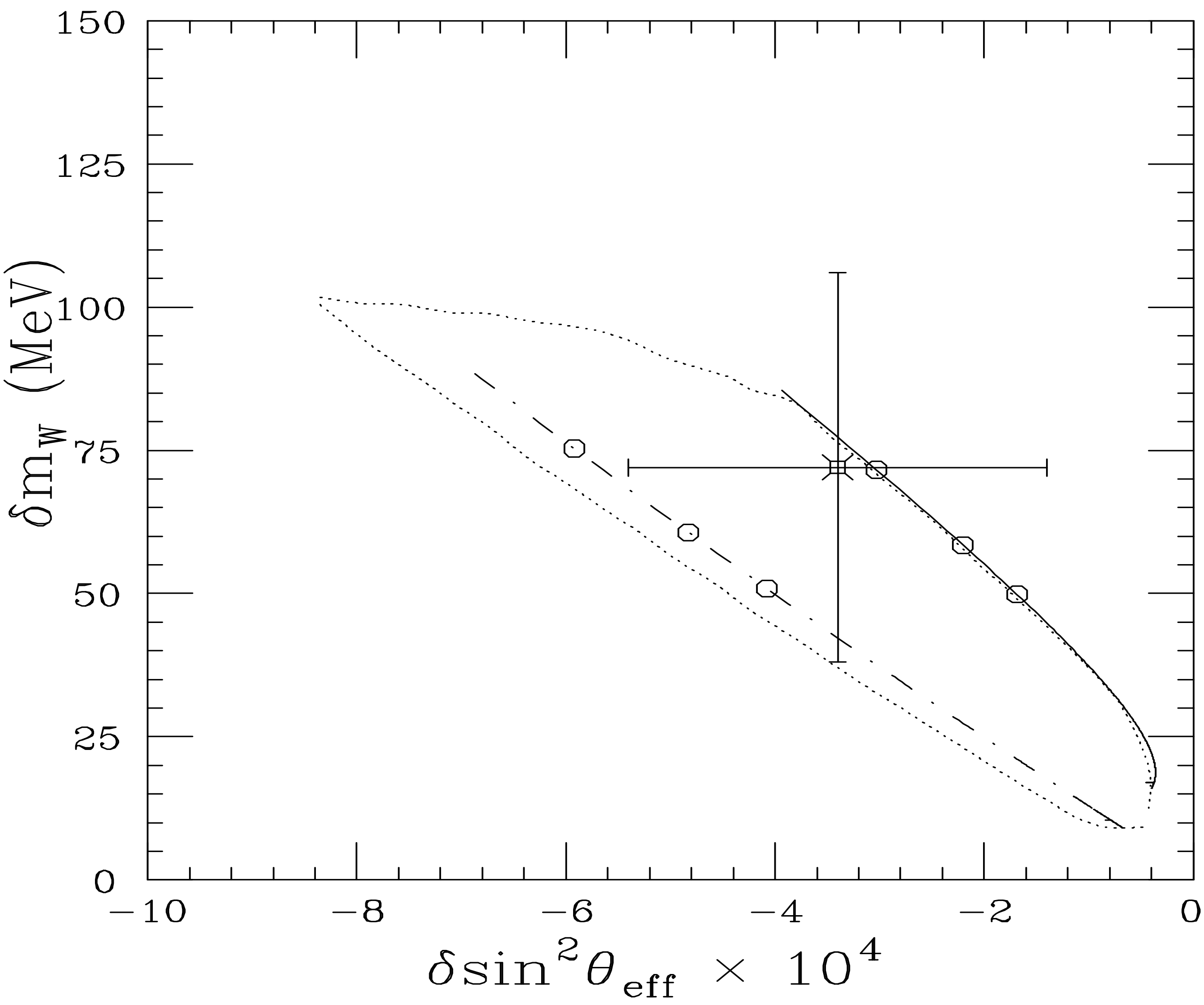


0.2325
0.2320
0.2315
0.2310
0.2305
$\sin^2\theta_{eff}$
80.20
80.25
80.30
80.35
80.40
80.45
80.50
M_W [GeV]
experimental errors:
LEP2/SLD/Tevatron
LHC/LC
GigaZ

Scenario where measurement of $A_{\mathrm{FB}}^{\mathrm{b}}$ is discarded:

[*G. Altarelli, F. Caravaglios, G. Giudice, P. Gambino, G. Ridolfi '01*]

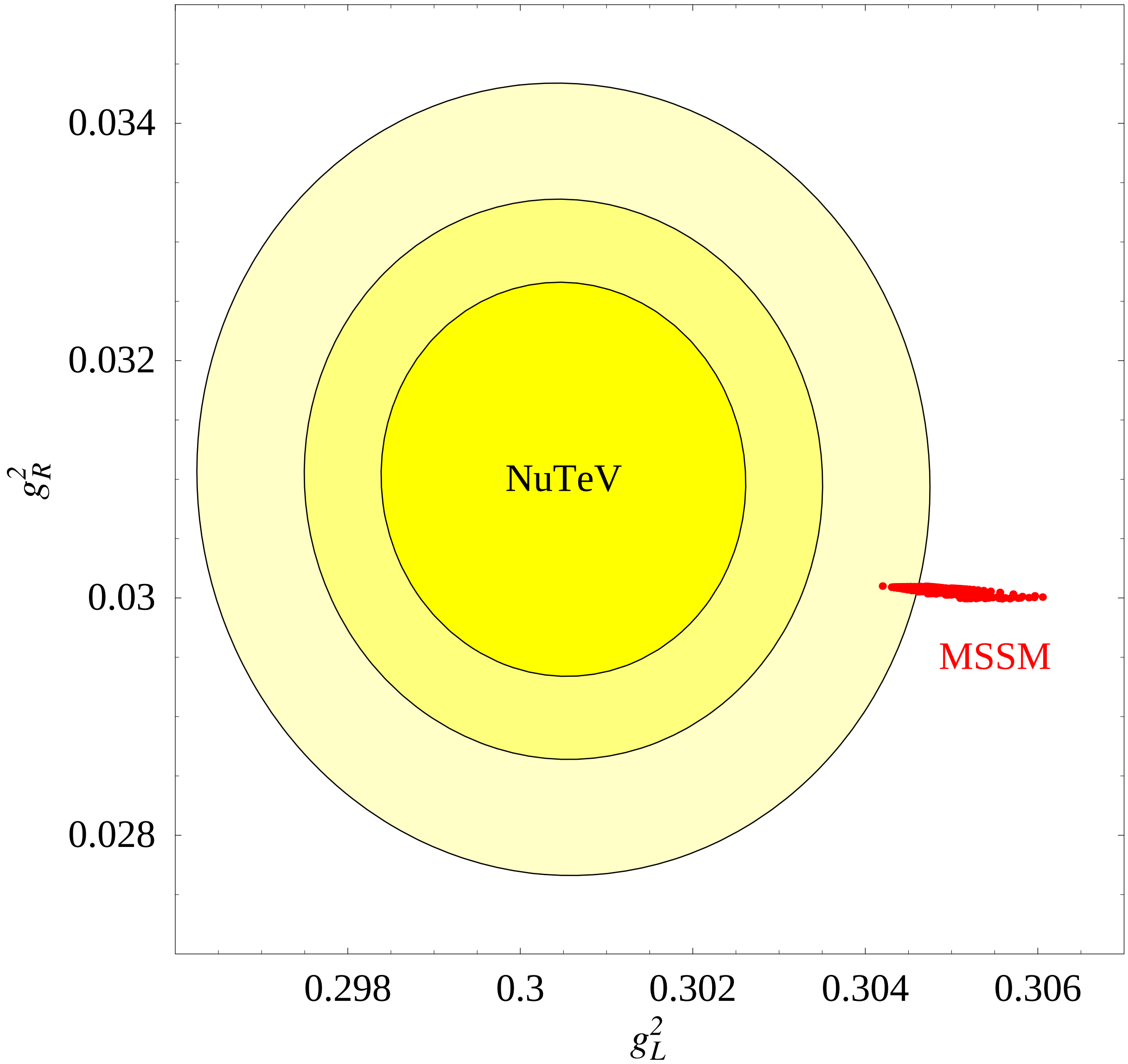
MSSM with $m_{\mathrm{h}} = 113$ GeV, $m_{\mathrm{t}} = 174.3$ GeV, possible shifts in M_{W}, $\sin^2\theta_{\mathrm{eff}}$ from light sleptons:



⇒ MSSM predictions with $m_{\mathrm{h}} = 113$ GeV can be brought in agreement with the data on M_{W} and $\sin^2\theta_{\mathrm{eff}}$ even if hadronic contributions to $\sin^2\theta_{\mathrm{eff}}$ are discarded

MSSM prediction for neutrino–nucleon scattering (measured at NuTeV):

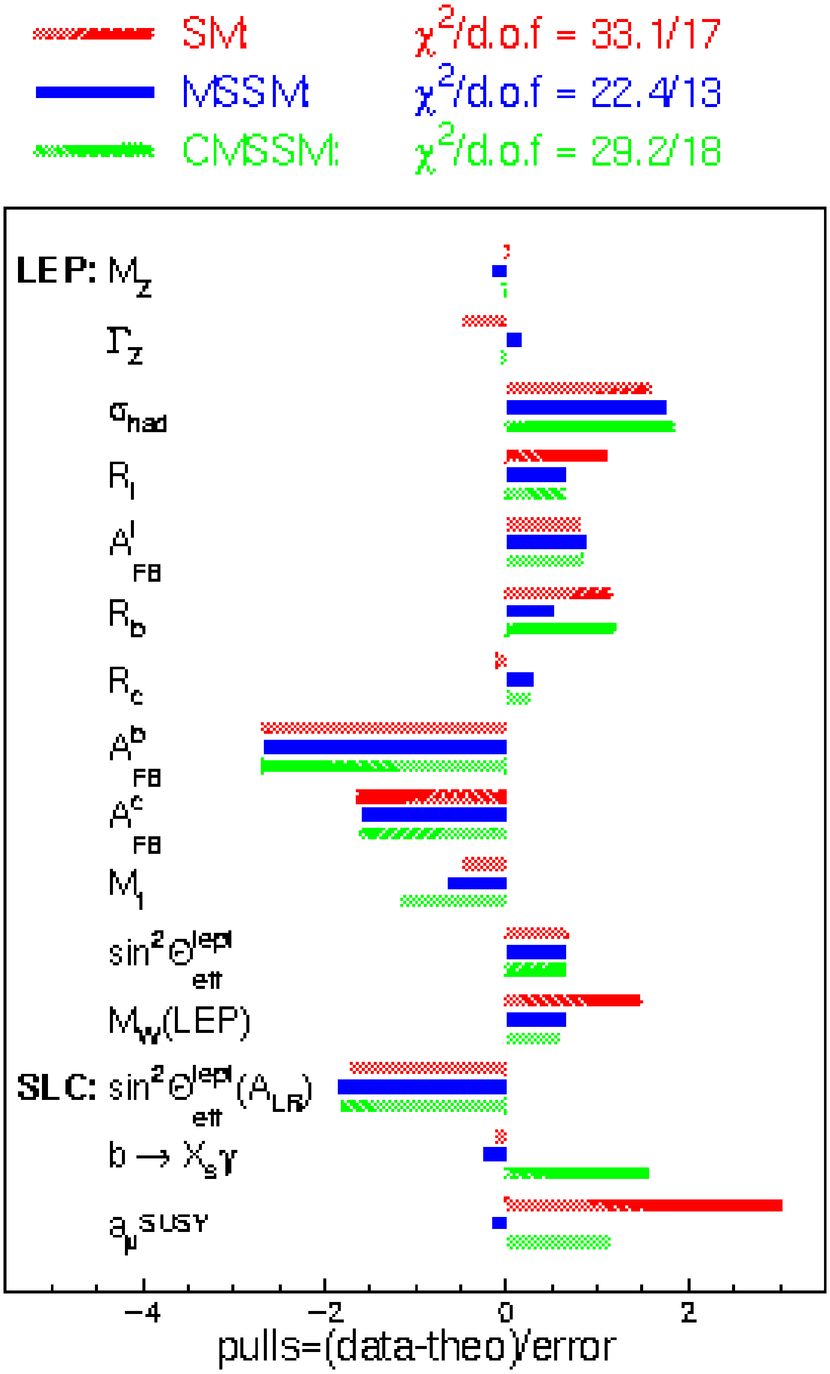
[*S. Davidson, S. Forte, P. Gambino, N. Rius, A. Strumia '02*]



$\Rightarrow$ Discrepancy cannot be explained in MSSM (light slepton corrections)

Global fit: SM / unconstrained MSSM / mSUGRA

[*W. de Boer. C. Sander '02*]



$\Rightarrow$ MSSM contributions can improve agreement with data for $g_\mu - 2$ (for large $\tan\beta$) and M_W

no significant improvement for A^b_{FB} and neutrino–nucleon cross section

Similar quality of global fit (slightly better in MSSM than in SM)

4. Conclusions

- In SUSY: m_{h} is precision observable

- SUSY sector of the MSSM contains several new sources for sizable radiative corrections

- mSUGRA in agreement with exp. constraints from Higgs search, CDM, $(g_\mu - 2)$, $B \to X_s\gamma$

- Global fit:

  better agreement of MSSM with data for $g_\mu - 2$, M_{W} than SM

  no improvement for $A_{\mathrm{FB}}^{\mathrm{b}}$, NuTeV

  similar fit quality

- Estimate of theoretical uncertainties from unknown higher orders:

  M_{W}, Z observables: $\approx$ twice as large as in SM

  m_{h}: need improvement by more than factor 10 to match LHC accuracy, crucial to get m_{t} from LC

  $\Rightarrow$ Large improvements needed for precision tests of the MSSM

- Does our current treatment of pseudo-observables need refinement when we head for complete electroweak two-loop results?