QCD corrections to the forward-backward asymmetry

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Abstract

We discuss the higher order QCD corrections to the forward backward asymmetry for bottom quark production which have been calculated in the past. There exist two definitions for this quantity. One is finite in the limit that the mass of the heavy quark becomes equal to zero whereas the other one is mass singular. Further we also investigate the validity of the massless quark approach of the second order correction.

1 Introduction

Experiments carried out at electron positron colliders like LEP and SLC have provided us with a wealth of information about the constants appearing in the standard model of the electroweak and strong interactions. One among them is the electroweak mixing angle defined by θ_W. The latter can be very accurately extracted from the forward-backward asymmetry A_{FB} which is measured in leptonic and heavy quark channels. However the measurements on the Z-peak show that the value of this quantity for the bottom quark given by $A_{FB}^b = 0.0994$ [1] deviates from the standard model fit $A_{FB}^b = 0.1038$ by 3.2 σ with 99.9 % confidence level [2]. Therefore it is important to evaluate the theoretical predictions which have been made in the past.

The forward backward asymmetry A_{FB}^b is measured in the following process

$$e^+ + e^- \to Z(\gamma^*)(q) \to b(\bar{b})(p) +' X' \,, \tag{1}$$

where X denotes any inclusive hadronic state. The unpolarized cross section is given by

$$\begin{aligned}\frac{d\sigma^{\rm H}(s)}{d\cos\theta} &= \frac{3}{8}(1+\cos^2\theta)\left[\sigma_{\rm VV}(s)f_T^v(\rho)+\sigma_{\rm AA}(s)f_T^a(\rho)\right] \\ &+\frac{3}{4}\sin^2\theta\left[\sigma_{\rm VV}(s)f_L^v(\rho)+\sigma_{\rm AA}(s)f_L^a(\rho)\right] \\ &+\frac{3}{4}\cos\theta\left[\sigma_{\rm VA}(s)f_A^a(\rho)\right]\,, \qquad \rho=\frac{4m^2}{s}\,. \end{aligned} \tag{2}$$

Here θ is the angle between the outgoing quark and the incoming electron and $\sqrt{s}$ and m denote the centre of mass energy and the quark mass respectively. Note that in the expression for the cross section above the electro-weak corrections are neglected and all QCD corrections are described by the structure functions f_k^l ($k=T,L,A$, $l=v,a$). The Born cross sections in Eq. (2) can be written as

$$\begin{aligned}\sigma_{VV}(s) &= \frac{4\pi\alpha^2}{3s}N\left[e_\ell^2e_q^2+\frac{2s(s-M_Z^2)}{|Z(s)|^2}\,e_\ell e_q C_{V,\ell}C_{V,q}\right. \\ &\left.+\frac{s^2}{|Z(s)|^2}\left(C_{V,\ell}^2+C_{A,\ell}^2\right)C_{V,q}^2\right]\,, \end{aligned} \tag{3}$$

$$\sigma_{AA}(s) = \frac{4\pi\alpha^2}{3s}N\left[\frac{s^2}{|Z(s)|^2}\left(C_{V,\ell}^2+C_{A,\ell}^2\right)C_{A,q}^2\right]\,, \tag{4}$$

$$\begin{aligned}\sigma_{VA}(s) &= \frac{4\pi\alpha^2}{3s}N\left[\frac{2s(s-M_Z^2)}{|Z(s)|^2}\,e_\ell e_q C_{A,\ell}C_{A,q}\right. \\ &\left.+4\frac{s^2}{|Z(s)|^2}\,C_{A,\ell}C_{A,q}C_{V,\ell}C_{V,q}\right]\,, \end{aligned} \tag{5}$$

where N denotes the number of colours in the case of the gauge group $SU(N)$ (in QCD one has $N=3$). Furthermore in the expressions above we adopt for the Z-propagator the energy independent width approximation

$$Z(s)=s-M_Z^2+iM_Z\Gamma_Z\,. \tag{6}$$

The charges of the lepton and the quark are given by e_ℓ and e_q respectively and the electro-weak angle θ_W appears in the electroweak constants as follows.

$$\begin{aligned}
&C_{A,\ell} = -\frac{1}{2\sin 2\theta_W}\,, && C_{V,\ell} = C_{A,\ell}\,(1-4\sin^2\theta_W)\,,\\
&C_{A,u} = -C_{A,d} = -C_{A,\ell}\,, && \\
&C_{V,u} = -C_{A,\ell}\,(1-\frac{8}{3}\sin^2\theta_W)\,, && C_{V,d} = C_{A,\ell}\,(1-\frac{4}{3}\sin^2\theta_W)\,.
\end{aligned} \tag{7}$$

The functions f_k^l $(k = T, L, A;\ l = v, a)$ in Eq. (1) can be computed order by order in perturbative QCD

$$\begin{aligned}
f_k^l(\rho) &= \int_{\sqrt{\rho}}^1 dx\, \mathcal{C}_{k,q}^{l,\mathrm{NS}}\left(x,\rho,\frac{s}{\mu^2}\right)\int_0^1 dz\,\Gamma_{qq}^{\mathrm{NS}}\left(z,\frac{\mu^2}{m^2}\right)\,,\\
x &= \frac{s}{2p\cdot q} \qquad s = q^2\,,
\end{aligned} \tag{8}$$

where μ stands for the factorization as well as the renormalization scale. and p denotes the momentum of the outgoing quark (see Eq. (1)). The non-singlet quark coefficient functions, represented by $\mathcal{C}_{k,q}^{l,\mathrm{NS}}$ appear in the fragmentation functions $F_k(x,s)$, where now p represents the momentum of the outgoing hadron, which have been calculated up to order α_s^2 for massless quarks in [3]. The kernel $\Gamma_{qq}^{\mathrm{NS}}$ which arises from mass factorization contain all mass singularities of the type $\ln\mu^2/m^2$. The forward backward asymmetry is then given by

$$A_{\mathrm{FB}}^{\mathrm{H}}(s) = \frac{3}{4}\frac{\sigma_{\mathrm{VA}}(s)\,f_A^a(\rho)}{\sigma^b(s)}\,, \tag{9}$$

with

$$\sigma^b(s) = \sigma_{\mathrm{VV}}(s)\Big[f_T^v(\rho)+f_L^v(\rho)\Big] + \sigma_{\mathrm{AA}}(s)\Big[f_T^a(\rho)+f_L^a(\rho)\Big]\,. \tag{10}$$

In the literature there exist two different type of definitions for the functions f_k^v and f_k^a $(k = T, L)$. The one given in [4], hereafter called RN, leads to the expression

$$f_k^l(\rho) = \int_{\sqrt{\rho}}^1 dx\, \mathcal{C}_{k,q}^{l,\mathrm{NS},-}\left(x,\rho,\frac{s}{\mu^2}\right)\int_0^1 dz\,\Gamma_{qq}^{\mathrm{NS},-}\left(z,\frac{\mu^2}{m^2}\right)\,, \tag{11}$$

with

$$\int_0^1 dz\,\Gamma_{qq}^{\rm NS,-}\left(z,\frac{\mu^2}{m^2}\right)=1\,. \tag{12}$$

The relation above holds in all orders of perturbation theory which follows from the conservation of the vector current (flavour number conservation). This definition has as consequence that σ^b is finite in the limit $m\to 0$ and it becomes equal to

$$\sigma^b(s)=\sigma_{tot}(s)\underset{m\to 0}{=}\Big(\sigma_{\rm VV}(s)+\sigma_{\rm AA}(s)\Big)\,R_{e^+e^-}\,, \tag{13}$$

where $R_{e^+e^-}=\sigma(e^+e^-\to hadrons)/\sigma(e^+e^-\to\mu^+\mu^-)$. The second definition, hereafter called CS, is given in [5]. In this case f_k^l becomes

$$f_k^l(\rho) \;=\; \int_{\sqrt{\rho}}^1 dx\,\mathcal{C}_{k,q}^{l,\rm NS,+}\left(x,\rho,\frac{s}{\mu^2}\right)\int_0^1 dz\,\Gamma_{qq}^{\rm NS,+}\left(z,\frac{\mu^2}{m^2}\right) \tag{14}$$

This expression becomes mass singular in second order perturbation theory because

$$\int_0^1 dz\,\Gamma_{qq}^{\rm NS,+}\left(z,\frac{\mu^2}{m^2}\right) \;=\; 1+a_s^2(\mu^2)\int_0^1 dz\left[P_{q\bar q}^{\rm NS,(2)}(z)\ln\frac{\mu^2}{m^2}+a_{q\bar q}(z)\right],$$

$$a_s(\mu^2) \;=\; \frac{\alpha_s(\mu^2)}{4\pi}\,. \tag{15}$$

In contrast to the denominator the numerator in Eq. (9) is the same for both definitions and f_A^a given by

$$f_A^a(\rho) \;=\; \int_{\sqrt{\rho}}^1 dx\,\mathcal{C}_{A,q}^{l,\rm NS,-}\left(x,\rho,\frac{s}{\mu^2}\right)\int_0^1 dz\,\Gamma_{qq}^{\rm NS,-}\left(z,\frac{\mu^2}{m^2}\right)\,, \tag{16}$$

is finite for $m\to 0$ by virtue of Eq. (12). Notice that the experimentalists measure A_{FB}^b according to the second definition given in [5] (CS). For the bottom quark this has no numerical consequences as we will see later on.

2 Calculation

In the Born approximation we have to compute

$$V\to b+\bar b\quad\text{with}\quad V=\gamma^*,Z\,. \tag{17}$$

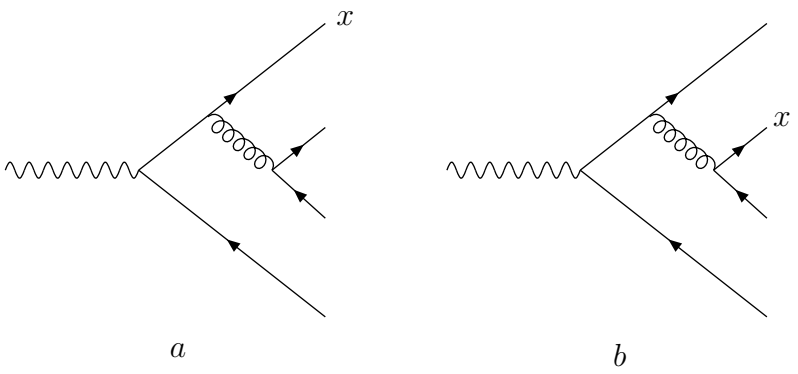

Figure 1: process $V \to b + \bar{b} + b + \bar{b}$. x denotes the detected quark.

The order α_s corrections are obtained from the one-loop corrections to the reaction in Eq. (17) and the gluon bremsstrahlung process

$$V \to b + \bar{b} + g\,. \tag{18}$$

Both contributions have been calculated for general masses in [6]. For the order α_s^2 corrections a semi analytical formula only exists for the total cross section σ_{tot} in Eq. (13) see [7]. Here one has used the optical theorem which relates the total cross section to the imaginary part of the current-current correlator. This theorem cannot be applied to the quantities σ_{VA} in Eq. (9) and σ^b if one uses the definition in Eqs. (14), (15). Because of the complexity due to the presence of the mass the order α_s^2 corrections to the latter quantities have been calculated in the limit $m \to 0$ in [4] ,[5], [8]. except when a mass singularity occurs which reveals itself as $\ln s/m^2$. We can distinguish between dominant and small corrections. The dominant ones are given by the two-loop virtual corrections to the Born process in Eq. (17), the one-loop corrections to the bremsstrahlung reaction in (18) and the two-gluon bremsstrahlung process given by

$$V \to b + \bar{b} + g + g\,. \tag{19}$$

In addition we have a reaction which will play a very important role in the definition of σ^b in Eq. (9). In this case two bottom and two anti-bottom

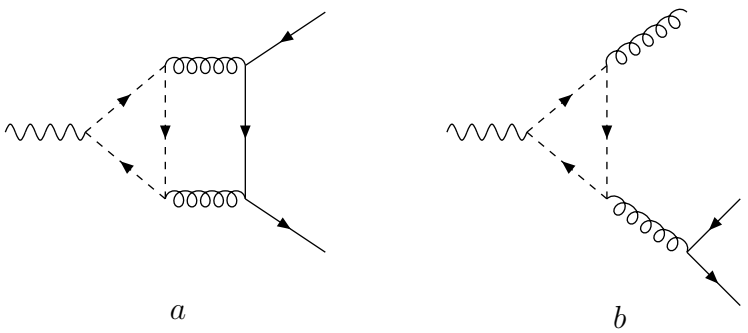

Figure 2: triangular quark loop corrections to $V \to b+\bar{b}$ (a) and $V \to b+\bar{b}+g$ (b).

quarks appear in the final state i.e.

$$V \to b + \bar{b} + b + \bar{b}\,. \tag{20}$$

This reaction is represented by two types of graphs. In graph Fig. 1a the (anti-)bottom quark is a primary quark because the detected quark is attached to the vector boson V whereas in graph Fig. 1b the (anti-)bottom quark becomes a secondary quark because it is mediated by a gluon. The difference between the definitions for A_{FB} in Eqs. (11) and (14) can be traced back to the interference terms between Figs. 1a and 1b. For the total cross section one has two pairs of identical quarks which provides us with a statistical factor $(1/2!)^2$. In this case the logarithmic mass singularities $\ln s/m^2$ cancel when these interference terms are added to the remaining contributions arising from the other reactions discussed above. However if one of the (anti-)bottom quarks is detected in the final state then only one pair of identical quarks or anti-quarks is left and the statistical factor becomes $1/2!$. In this case the cancellation of the mass singularities will not occur and we are left with the term in Eq. (15).

Besides the dominant corrections there are also small contributions to the forward backward asymmetry. One is given by the process in in Fig. 2a which contains the triangular quark loop computed in [9]. The latter also

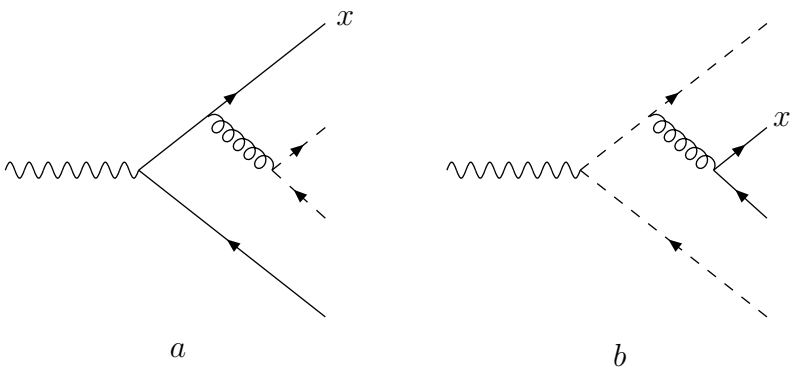

Figure 3: process $V \to b + \bar{b} + q + \bar{q}$. x denotes the detected quark.

appears in Fig. 2b which is calculated in [10]. Finally we have the process

$$V \to b + \bar{b} + q + \bar{q}\,, \tag{21}$$

where q represents all quarks lighter than the bottom quark. The square of all graphs corresponding to the process in Fig. 2a are added to the analogous type of graphs in Fig. 1a which give rise to the term proportional to n_f (number of light flavours) in the expression for A_{FB}. The interference term between Fig. 2a and Fig. 2b is very small (see [8]). The same holds for the squared diagrams corresponding to Fig. 2b which are calculated in [11]. If one adopts the definition in Eq. (11) the result is finite in the limit $m \to 0$ so that one can put the mass equal to zero right from the start and use n-dimensional regularization (see [4]). The expression for the forward-backward asymmetry becomes very simple and it reads

$$\begin{aligned} A_{FB}^{\mathrm{RN,b}}(s) &= A_{FB}^{b,(0)}(s)\left[1 - 3\,a_s(\mu^2)\,C_F + a_s^2(\mu^2)\left\{\frac{21}{2}\,C_F^2 + C_A\,C_F\,\left(11\,\ln\frac{s}{\mu^2}\right.\right.\right. \\ &\left.\left.\left. -\frac{123}{2}\right) + n_f\,C_F\,T_f\,\left(-4\,\ln\frac{s}{\mu^2} + 22\right)\right\}\right], \end{aligned} \tag{22}$$

where $C_A = 3$, $C_F = 4/3$ and $T_f = 1/2$ denote the colour factors and n_f represents the number of light flavours.

$A_{\rm FB}^c$			
	$m_c = 0$ GeV/c^2	$m_c = 1.50$ GeV/c^2	$m_c = 0.0$ GeV/c^2
	QCD	QCD	QED
$A_{\rm FB}^{(0)}$	0.0712	0.0712	0.0712
$A_{\rm FB}^{(1)}$	0.0685 (- 3.78 %)	0.0687 (- 3.50 %)	0.0547
$A_{\rm FB}^{(2)}$	0.0675 (- 5.16 %)	0.0677 (- 4.87 %)	0.0610

Table 1: The forward-backward asymmetry of the charm quark at $\sqrt{s} = M_Z$.

$A_{\rm FB}^b$			
	$m_b = 0.0$ GeV/c^2	$m_b = 4.50$ GeV/c^2	$m_b = 0.0$ GeV/c^2
	QCD	QCD	QED
$A_{\rm FB}^{(0)}$	0.1002	0.1002	0.1002
$A_{\rm FB}^{(1)}$	0.0964 (- 3.78 %)	0.0972 (- 2.99 %)	0.0938
$A_{\rm FB}^{(2)}$	0.0950 (- 5.16 %)	0.0958 (- 4.36 %)	0.0962

Table 2: The forward-backward asymmetry of the bottom quark at $\sqrt{s} = M_Z$.

For the definition in Eq. (14) which becomes mass singular in second order we obtain [5]

$$A_{FB}^{\rm CS,b}(s) = A_{FB}^{\rm RN,b}(s) + 4\,A_{FB}^{b,(0)}(s)\,C_F\left(CF - \frac{1}{2}\,C_A\right)a_s^2(\mu^2)\left[\left(\frac{13}{2} - 6\zeta(2) + 4\,\zeta(3)\right)\ln\frac{s}{m^2} - 8.179\right]. \quad (23)$$

3 Results

In tables 1 and 2 we have shown the forward backward asymmetries for charm and bottom quark respectively. The percentage of the higher order QCD correction are put between parentheses. For comparison we have also presented the initial state QED corrections in leading log approximation. They are of the same order as the QCD corrections. Further we infer that the approximation by putting the mass of the heavy flavour equal to zero is excellent for charm production but less good for bottom production. Since no exact calculation of the second order corrections to A_{FB} exists we can estimate the error of the approximation by comparing the order α_s corrected result $A_{FB}^{(1)}$ for $m = 0$ and $m \neq 0$. For charm production the difference between the massive and massless quark approach is 0.28% which is much smaller than the difference between $A_{FB}^{(1)}$ and $A_{FB}^{(2)}$ which is about 1.37%. The latter is the same for the bottom quark but now the approximation becomes worse and the difference between the result for $m_b = 0$ and $m_b = 4.50\ \mathrm{GeV/c^2}$ becomes 0.79%. Therefore in the case of top-quark production all higher order radiative corrections have to be calculated for massive quarks. The second issue is the difference between the definitions given in [4] and [5] leading to expressions $A_{FB}^{\mathrm{RN,b}}$ and $A_{FB}^{\mathrm{CS,b}}$ presented in Eqs. (22) and (23) respectively. In the case of bottom production a miraculous cancellation occurs in the second term of Eq. (23) so that $A_{FB}^{(2)}$ is the same for both approaches. This is due to the input parameters used to produce the results in table 2 in particular to the chosen value of the bottom mass. If the mass becomes smaller like in the case of charm production (see table 1) one observes a small difference $A_{FB}^{\mathrm{RN,c}} - A_{FB}^{\mathrm{CS,c}} = -0.0002$. It is clear that the formula for A_{FB}^{CS} in Eq. (23) cannot be applied to light quark production like u, d, s for which $m \sim 0$. In this case one has to observe hadrons rather than quarks in the final state. However this entails the introduction of a fragmentation function $D_q^H(z, \mu^2)$ describing the fragmentation of a quark into a hadron. Hence the definition of the functions $f_k^l(\rho)$ in Eq. (8) has to be changed and it reads

$$f_k^l(\rho) \;=\; \int_{\sqrt{\rho}}^1 dx\, \mathcal{C}_{k,q}^{l,\mathrm{NS}}\left(x, \rho, \frac{s}{\mu^2}\right) \int_0^1 dz\, D_q^H(z, \mu^2)\,. \tag{24}$$

Finally we want to make a remark on the numerical results given above. All formulae and results are presented when A_{FB} refers to the quark axis. If the

thrust axis is chosen, the result in table 2 becomes $A_{FB}^{(2)} = 0.0962$ instead of $A_{FB}^{(2)} = 0.0958$

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