

QCD uncertainties at NuTeV

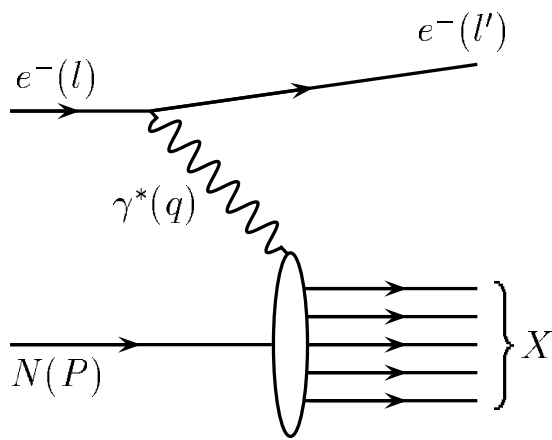
Sven-Olaf Moch

DESY Theorie
Zeuthen

1. **Deep-inelastic lepton-nucleon scattering**
 - **status of theory**
2. **The Paschos-Wolfenstein relation**
 - **parton distribution**
 - **charged current charm production**
 - **nuclear effects**
3. **Summary**

Introduction

Deep-inelastic lepton-hadron scattering



$$Q^2 = -q^2$$

negative 4-momentum transfer squared

$$x = \frac{Q^2}{2P \cdot q}$$

parton momentum in Breit frame

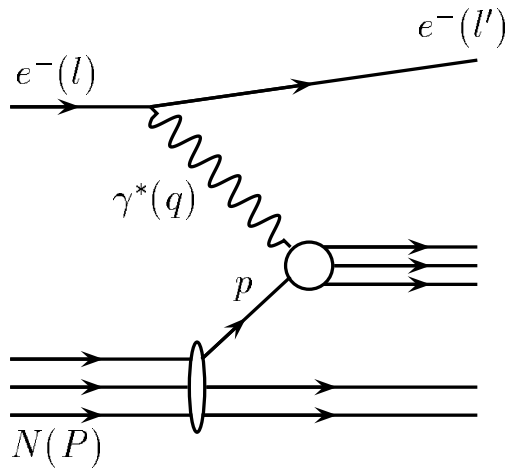
$$y = \frac{P \cdot q}{P \cdot l}$$

$\frac{E_\gamma}{E_l}$ in proton rest frame (inelasticity)

– Neutral current scattering

$$\frac{d^2\sigma^\pm}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[(1 + (1-y)^2) F_2 - y^2 F_L \mp (1 - (1-y)^2) xF_3 \right]$$

The quark parton model



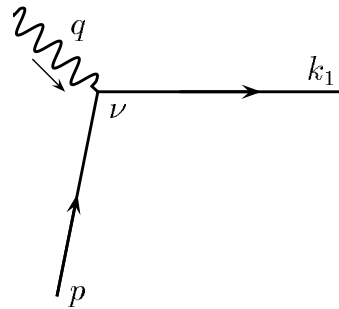
$$F_2 = \sum_q A_q x(q + \bar{q}) \quad \text{with} \quad A_q = e_q^2 + O\left(\frac{Q^2}{Q^2 + M_Z^2}\right)$$

$$F_3 = \sum_q B_q x(q - \bar{q}) \quad \text{with} \quad B_q = O\left(\frac{Q^2}{Q^2 + M_Z^2}\right)$$

$$F_L = 0$$

– Pure electromagnetic interactions for $Q^2 \ll M_Z^2$

– Leading order hard scattering process



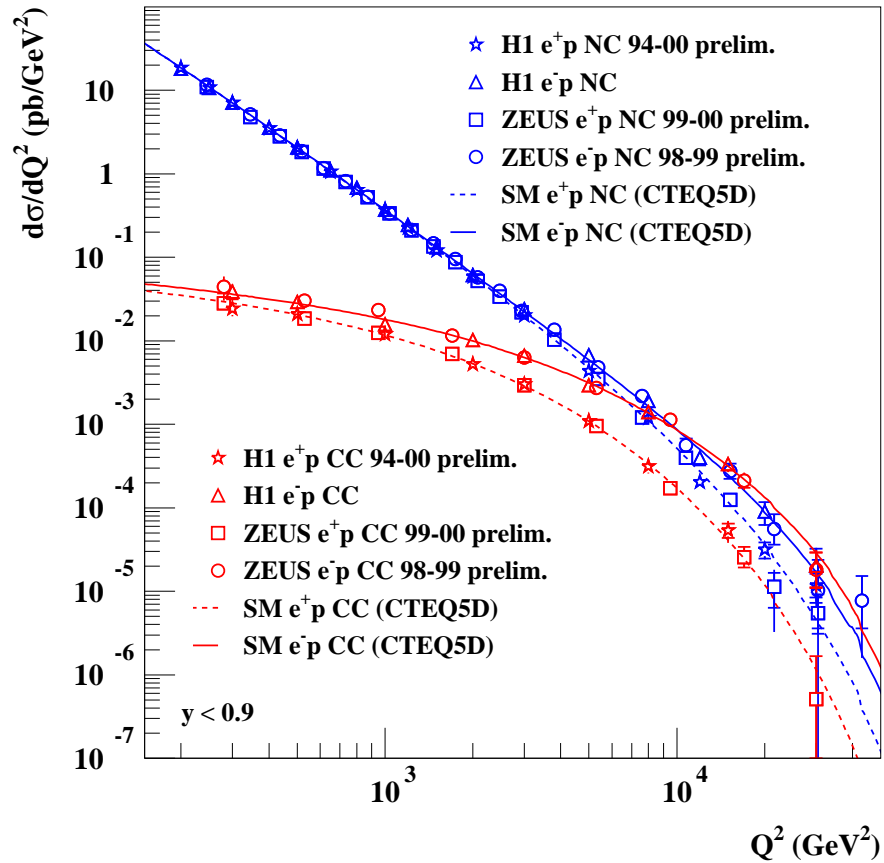
- Charged current scattering

$$\frac{d^2\sigma^\pm}{dx dQ^2} = \frac{G_F^2}{4\pi x} \frac{1}{(1 + Q^2/M_W^2)^2} \left[(1 + (1-y)^2) F_2 - y^2 F_L \mp (1 - (1-y)^2) xF_3 \right]$$

- The parton model in charged current scattering

$$\begin{aligned} F_1^{W^+} &= \bar{u} + d + s + \bar{c} & F_1^{W^-} &= u + \bar{d} + \bar{s} + c \\ F_2^{W^+} &= 2x(\bar{u} + d + s + \bar{c}) & F_2^{W^-} &= 2x(u + \bar{d} + \bar{s} + c) \\ xF_3^{W^+} &= 2x(-\bar{u} + d + s - \bar{c}) & xF_3^{W^-} &= 2x(u - \bar{d} - \bar{s} + c) \end{aligned}$$

Electroweak measurements at HERA



- neutral/charged current cross sections become equal at high Q^2
- space-like electroweak precision test
 - M_W measurement
 - parton substructure
 - ...

DIS in the QCD-improved parton model

- Structure function $F_2(x, Q)$ given as convolution

$$F_2(x, Q) = \sum_{p=\text{partons}} \int_x^1 \frac{dz}{z} f_{p/N}\left(\frac{x}{z}, \mu\right) C_{2p}\left(z, \frac{Q}{\mu}; \alpha_s\right)$$

- parton distributions $f_{p/N}(z, \mu)$
- coefficient functions $C_{2p}(z, Q/\mu; \alpha_s)$ calculable in QCD

What are the parameters ?

- QCD input parameters
 - strong coupling $\alpha_s(M_Z) = 0.1183 \pm 0.0027$ [Bethke hep-ex/0211012](#)
 - charm quark mass :
 - $m_c(m_c) = 1.0 - 1.4 \text{ GeV}$ [Hagiwara et al. PDG 2002](#)
 - $m_c(m_c) = 1.304(27) \text{ GeV}$ [Kühn, Steinhauser hep-ph/0109084](#)
- parton distributions in the nucleon $u(x), \bar{u}(x), d(x), \bar{d}(x), s(x), \dots, G(x)$

What has been done ?

- QCD corrections for DIS structure functions with massless quarks
- LO :
 - anomalous dimensions/splitting functions Gross, Wilczek '73 ; Altarelli, Parisi '77
- NLO :
 - complete one loop F_2 and F_L Bardeen, Buras, Duke, Muta '78
 - two loop anomalous dimensions/splitting functions Floratos, Ross, Sachrajda '79 ; Gonzalez-Arroyo, Lopez, Ynduráin '79 ; Curci, Furmanski, Petronzio '80 ; Furmanski, Petronzio '80
 - two loop F_L Duke, Kimel, Sowell '82 ; Devoto, Duke, Kimel, Sowell '85 ; Kazakov, Kotikov '88 ; Kazakov, Kotikov, Parente, Sampayo, Sanchez Guillen '90
- NNLO :
 - complete two loop F_2, F_3 and F_L Zijlstra, van Neerven '92 ; S.M., Vermaseren '99
 - fixed Mellin moments of F_2, F_3 and F_L at three loops Larin, Nogueira, van Ritbergen, Vermaseren '97 ; Retey, Vermaseren '00
 - approximate three loop splitting functions van Neerven, Vogt '00
 - fermionic contribution to three loop non-singlet anomalous dimensions/splitting functions S.M, Vermaseren, Vogt '02

What has been done ? (cont'd)

- QCD corrections for DIS structure functions with massive quarks
- Neutral current $O(\alpha_s)$:
 - complete structure functions F_2 and F_L Witten '76 ; Glück, Reya '79
- Neutral current $O(\alpha_s^2)$:
 - complete $O(\alpha_s^2)$ F_2 and F_L Laenen, Riemersma, van Neerven, Smith '92
 - Monte-Carlo HVQDIS Harris, Smith '92
- Charged current $O(\alpha_s)$:
 - complete corrections to νN DIS Gottschalk '81 ; van der Bij, Oldenborgh '91 ; Kramer, Lampe '92
 - Monte Carlo Kretzer, Mason, Olness '01
- Charged current $O(\alpha_s^2)$:
 - $O(\alpha_s^2 \ln^n(Q^2/m^2))$ contributions Buza, van Neerven '97
- Treatment of heavy quarks (c -threshold) Aivazis, Collins, Olness, Tung '94 ; Thorne, Roberts '98 ; Chuvakin, Smith, van Neerven '00

The Paschos-Wolfenstein relation

- exact relation for massless quarks and isospin zero target

Paschos, Wolfenstein '73; Llewellyn Smith '83

$$R^- = \frac{\sigma_{\text{NC}}^{\text{v}} - \sigma_{\text{NC}}^{\bar{\text{v}}}}{\sigma_{\text{CC}}^{\text{v}} - \sigma_{\text{CC}}^{\bar{\text{v}}}} = \frac{1}{2} - \sin^2 \theta_W$$

QCD corrections to the Paschos-Wolfenstein relation

- Second moments of PDFs $q^- = \int dx x(q - \bar{q})$, expand in isoscalar combination $u^- + d^-$

Davidson, Forte, Gambino, Rius, Strumia hep-ph/0112302

$$R^- = \frac{1}{2} - \sin^2 \theta_W + \left[1 - \frac{7}{3} \sin^2 \theta_W + \frac{4\alpha_s}{9\pi} \left(\frac{1}{2} - \sin^2 \theta_W \right) \right] \left(\frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-} \right)$$

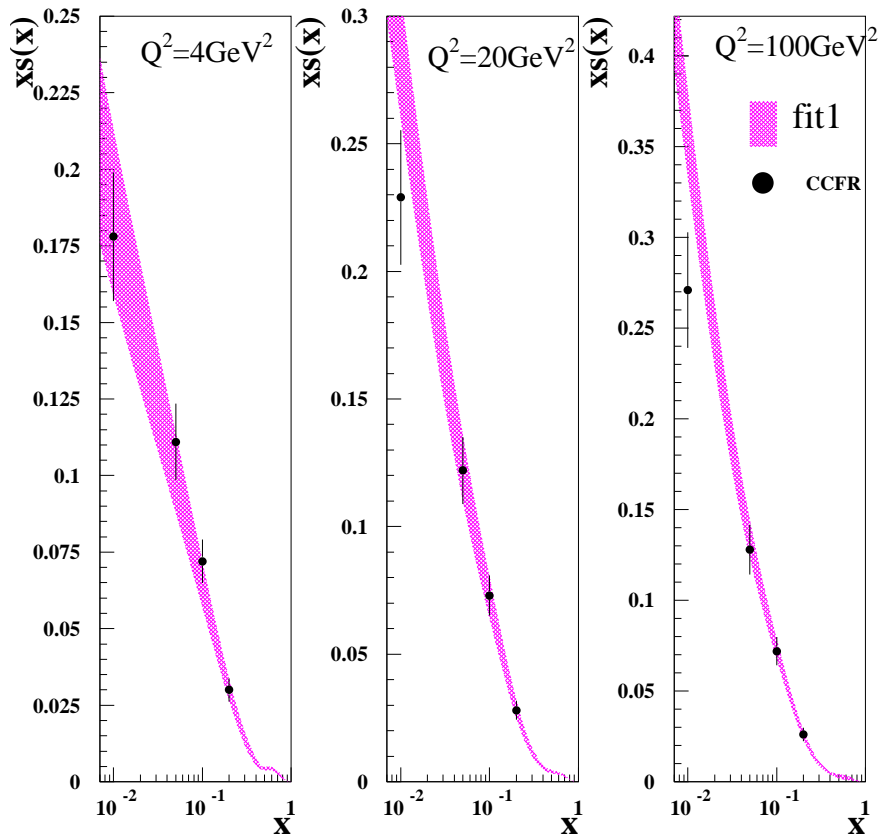
- R^- is ideal observable
 - NLO QCD corrections multiply isovector combination $u^- - d^-$ and asymmetries s^-, c^-
- Situation is different for separate ratios $\frac{\sigma_{\text{NC}}^{\text{v}}}{\sigma_{\text{CC}}^{\text{v}}}$ and $\frac{\sigma_{\text{NC}}^{\bar{\text{v}}}}{\sigma_{\text{CC}}^{\bar{\text{v}}}}$
 - QCD corrections proportional to dominant isoscalar combination $u^- + d^-$

Parton distributions

- Unpolarized PDF's well known from small to relatively large x
Glück, Reya, Vogt '98 ; Martin, Roberts, Stirling, Thorne '02 ; Lai et al. (CTEQ) '02
- Less information on polarized PDF's
Blümlein, Böttcher '02
- Recent dedicated analyses of lepton-nucleon DIS (including neutrino data)
Barone, Pascaud, Zomer hep-ph/9907512 ; Botje hep-ph/9912439 ; Alekhin hep-ph/0011002
- Scheme dependence
 - $\overline{\text{MS}}$ -scheme vs. DIS-scheme \longrightarrow affects radiative corrections in coefficient functions
 - e.g. $c_{2,q}^{(1)} = c_{2,g}^{(1)} = 0$ for coefficient functions, but different splitting functions, large logarithms for $x \rightarrow 1$ in PDFs
- Scheme independent evolution \longrightarrow fit F_2 and dF_2/dQ^2
Blümlein, Vogt '98
- PDF uncertainties
 - account for correlated errors in PDFs
Botje hep-ph/9912439 ; Giele, Keller, Kosower hep-ph/0104052

Total strange distribution

- The BPZ fit compared to CCFR data [Barone, Pascaud, Zomer hep-ph/9907512](#)

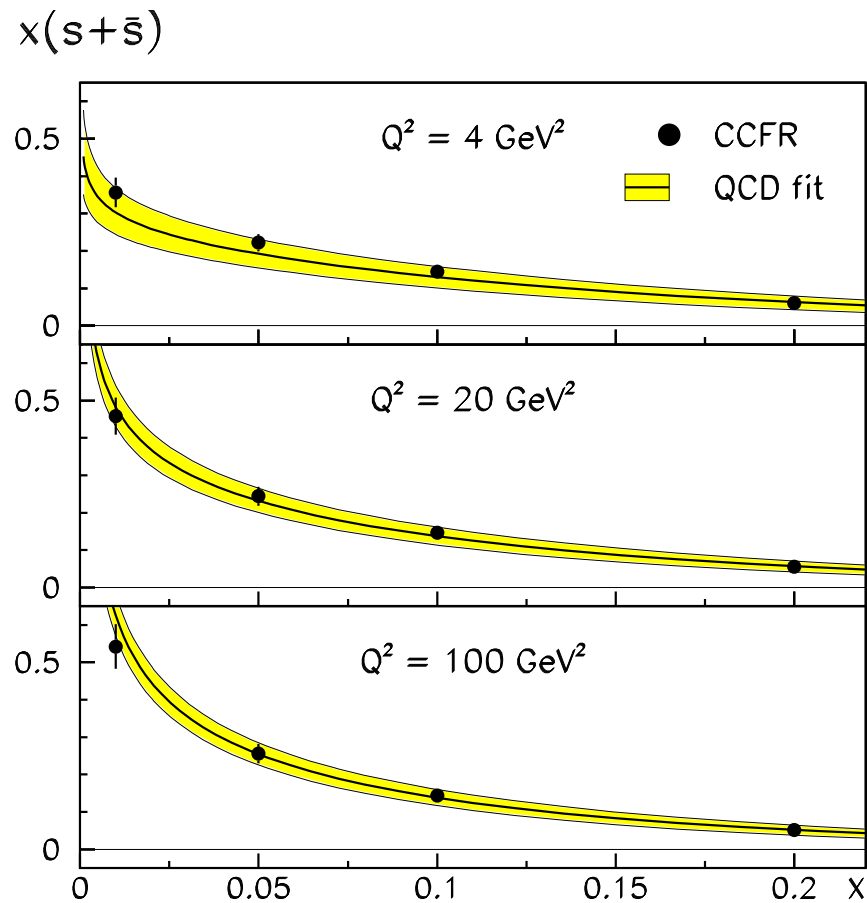


- standard assumptions in global fits
 - no strangeness in the proton
 - $\longrightarrow s(x) = \bar{s}(x)$
 - isospin symmetry $\longrightarrow u_P(x) = d_N(x)$

Total strange distribution (cont'd)

- Error on total strange distribution in the QCD fit

Botje hep-ph/9912439

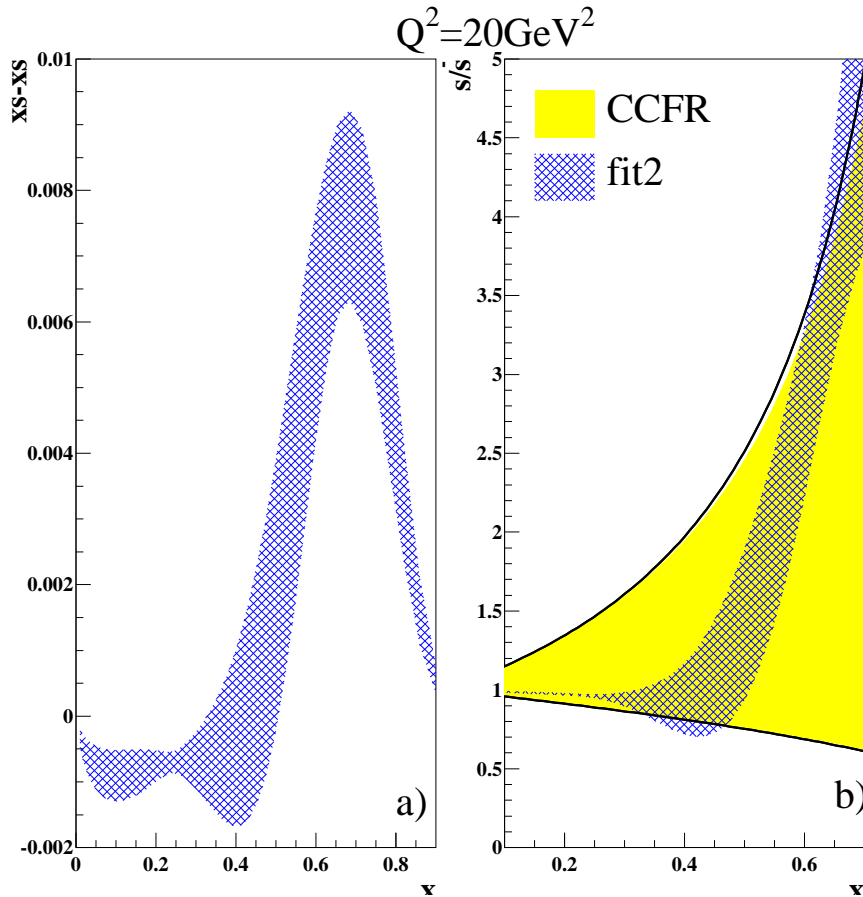


Typical uncertainties in PDF analysis :

- Input errors in PDFs
 - error on α_s
 - deuterium/iron nuclear corrections
 - strange quark content of the proton
 - charm threshold
- Theoretical error
 - variation of scales :
renormalization/factorization scales
 $\mu_{r,f}^2, \quad Q^2/2 < \mu_{r,f}^2 < 2Q^2$

Indications for a strangeness asymmetry ?

Barone, Pascaud, Zomer [hep-ph/9907512](https://arxiv.org/abs/hep-ph/9907512)



- Relax constraint $s(x) = \bar{s}(x)$
 → improved χ^2 in BPZ fit to CCFR data

- Ansatz for NLO QCD parametrization

$$xs(x, Q_0^2) = A_s x^{B_s} (1-x)^{C_s} (1 + D_s x^{E_s})$$

$$x\bar{s}(x, Q_0^2) = A_{\bar{s}} x^{B_{\bar{s}}} (1-x)^{C_{\bar{s}}} (1 + D_{\bar{s}} x^{E_{\bar{s}}})$$

- put $A_s = A_{\bar{s}}$ and $B_s = B_{\bar{s}}$
- impose no net strangeness

$$\int_0^1 dx (s(x) - \bar{s}(x)) = 0$$

The NuTeV analysis of dimuon production cross section

Goncharov et al. hep-ex/0102049

– Measurement of $\nu N, \bar{\nu} N \longrightarrow \mu^+ \mu^- + X$

– uncertainties in fragmentation

Peterson, Schlatter, Schmitt, Zerwas '83 ; Collins, Spiller '85

– NLO fragmentation of heavy quarks

Cacciari, Greco '97

– Parametrization of the strange distribution at LO in QCD

$$s(x, Q^2) = \kappa_\nu \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1-x)^{\alpha_\nu}$$

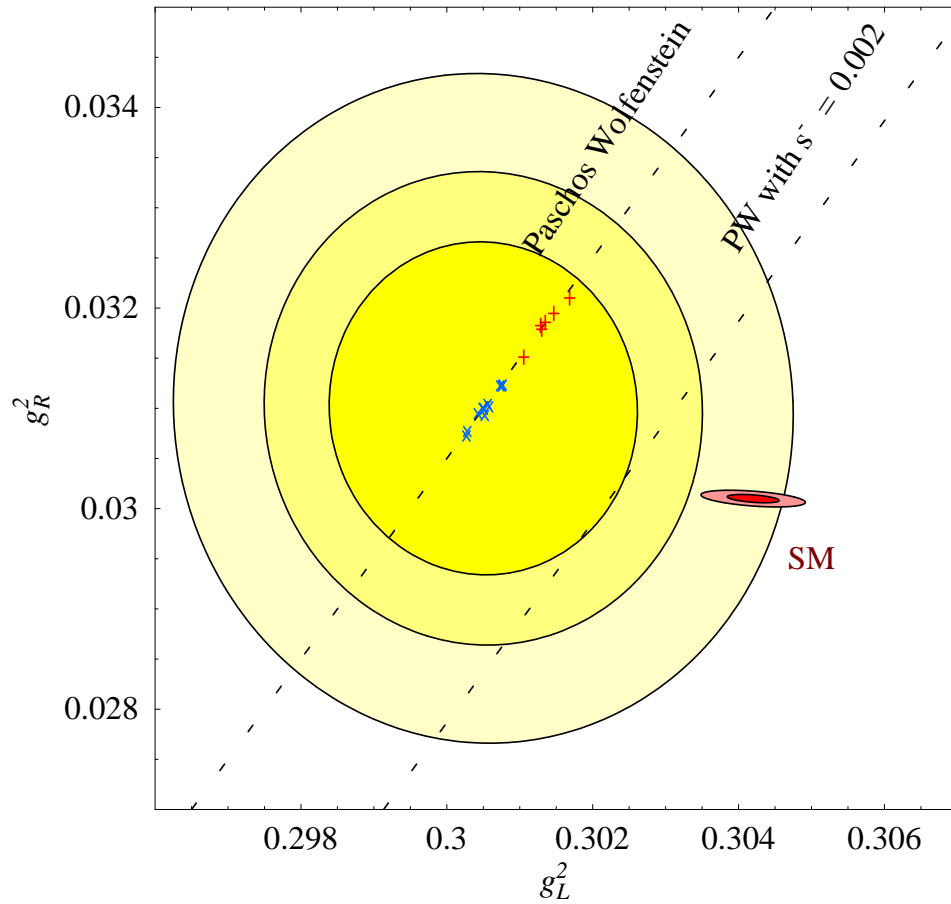
$$\bar{s}(x, Q^2) = \kappa_{\bar{\nu}} \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1-x)^{\alpha_{\bar{\nu}}}$$

The current status

The issue of $s(x) \neq \bar{s}(x)$ is still not settled.

Stirling @ NuFact'02 Workshop

The NuTeV $\sin^2 \theta_W$ result and global PDFs



- Error estimates from parton distributions for $\sin^2 \theta_W$ at NuTeV

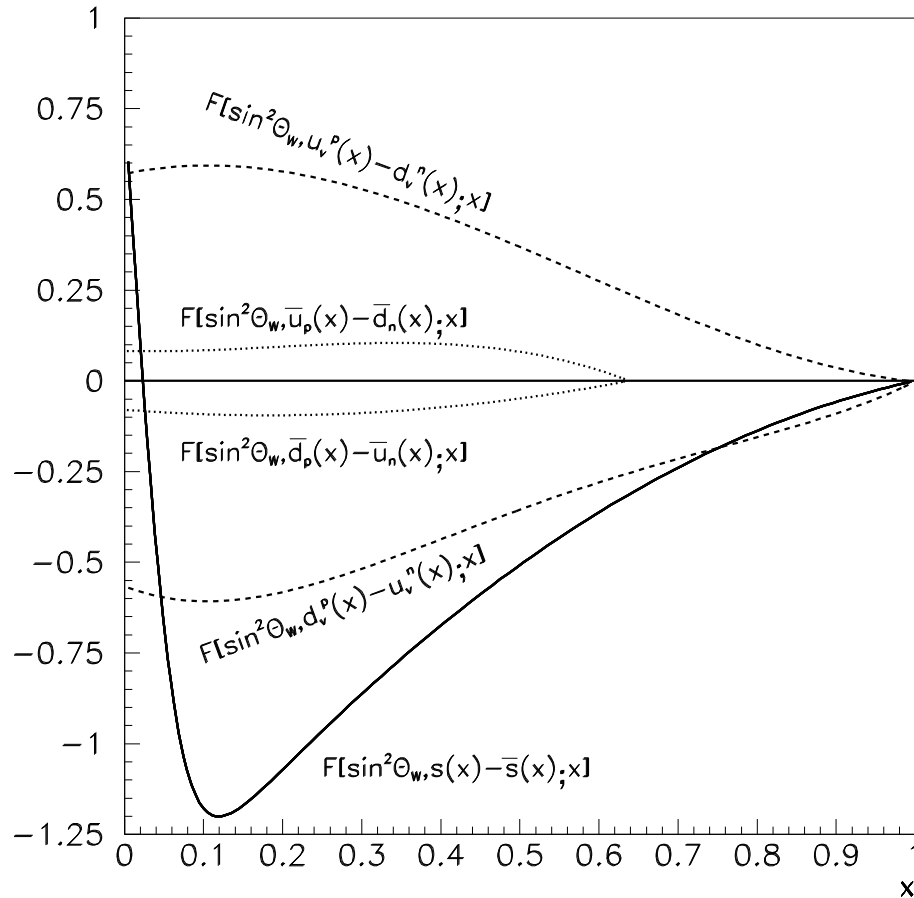
Davidson, Forte, Gambino, Rius, Strumia
[hep-ph/0112302](https://arxiv.org/abs/hep-ph/0112302)

The explanation of the NuTeV result through an asymmetric strange sea [...] is only supported by an after-the-fact analysis of statistically poor data applied [...] in the wrong kinematic region.

Bernstein (NuTeV) [hep-ex/0210061](#)

The PDF error functional from NuTeV

Zeller, McFarland et al. (NuTeV) hep-ex/0203004



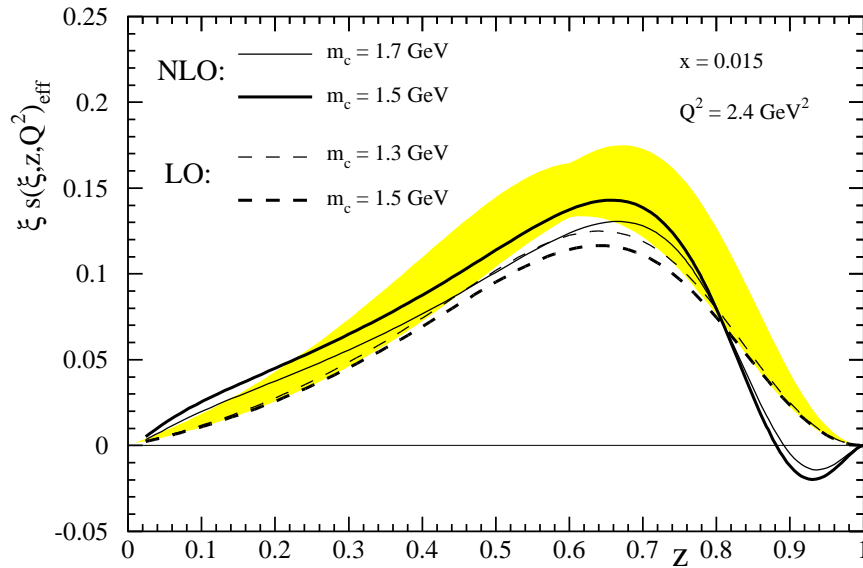
$$\Delta \sin^2 \theta_W = \int_0^1 dx F_s(x) (s(x) - \bar{s}(x)) + \int_0^1 dx F_I(x) (u_P(x) - d_N(x)) + \dots$$

– in particular :

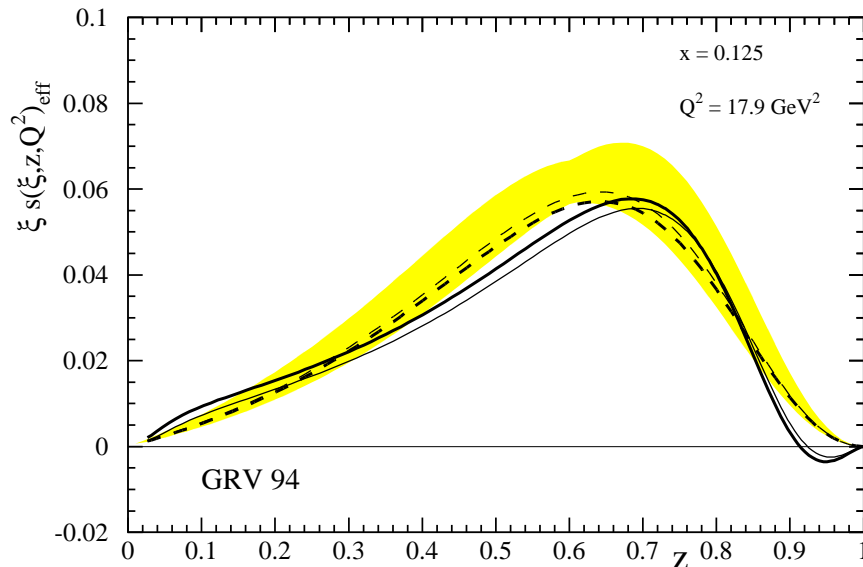
$$s > \bar{s} \longrightarrow \sin^2 \theta_W \text{ decreases}$$

– dimuon cross section puts further constraints on s, \bar{s}

Charged current charm production



- plot momentum distribution of D mesons
- momentum fraction z
- Peterson fragmentation function
 Peterson, Schlatter, Schmitt, Zerwas '83



$$\xi_s(z)_{\text{eff}} \simeq \text{const.} \frac{d^3\sigma^{(c\bar{s})}}{dx dy dz}$$

- yellow band
 → “data” from CCFR
- dependence on charm mass

Nuclear effects

- nuclear shadowing at low Q^2 ; vector meson dominance and higher twist
- modified Paschos-Wolfenstein relation in nuclei; isoscalar corrections for iron
- nuclear charge symmetry breaking; MIT bag model, $m_u - m_d$ mass difference
- ...

Some references

Miller, Thomas [hep-ex/0204007](#)

Kovalenko, Schmidt, Yang [hep-ph/0207158](#)

Melnitchouk, Thomas [hep-ex/0208016](#)

Kumano [hep-ph/0209200](#)

Londergan, Thomas [hep-ex/0301147](#)

Kulagin [hep-ph/0301045](#)

Zeller, McFarland et al. (NuTeV) [hep-ex/0207052](#)

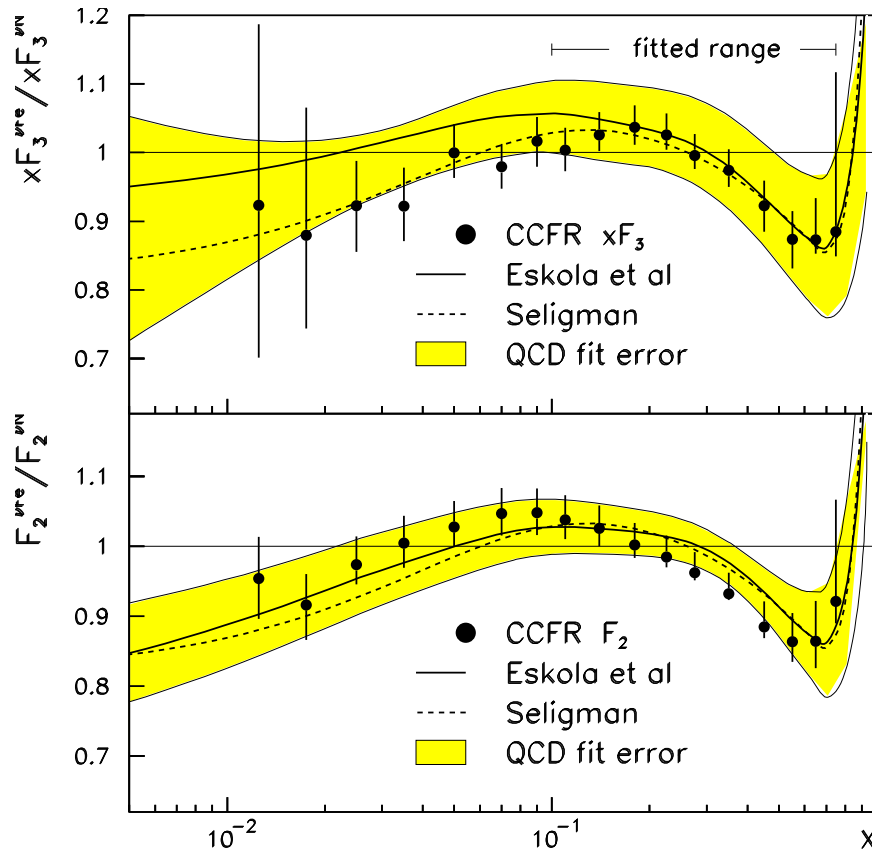
Bernstein (NuTeV) [hep-ex/0210061](#)



... to be continued ...

Error in nuclear effects for ν -Fe scattering

- ratios of $\frac{F_2^{\nu\text{Fe}}}{F_2^{\nu N}}$ and $\frac{x F_3^{\nu\text{Fe}}}{x F_3^{\nu N}}$ with errors Botje hep-ph/9912439



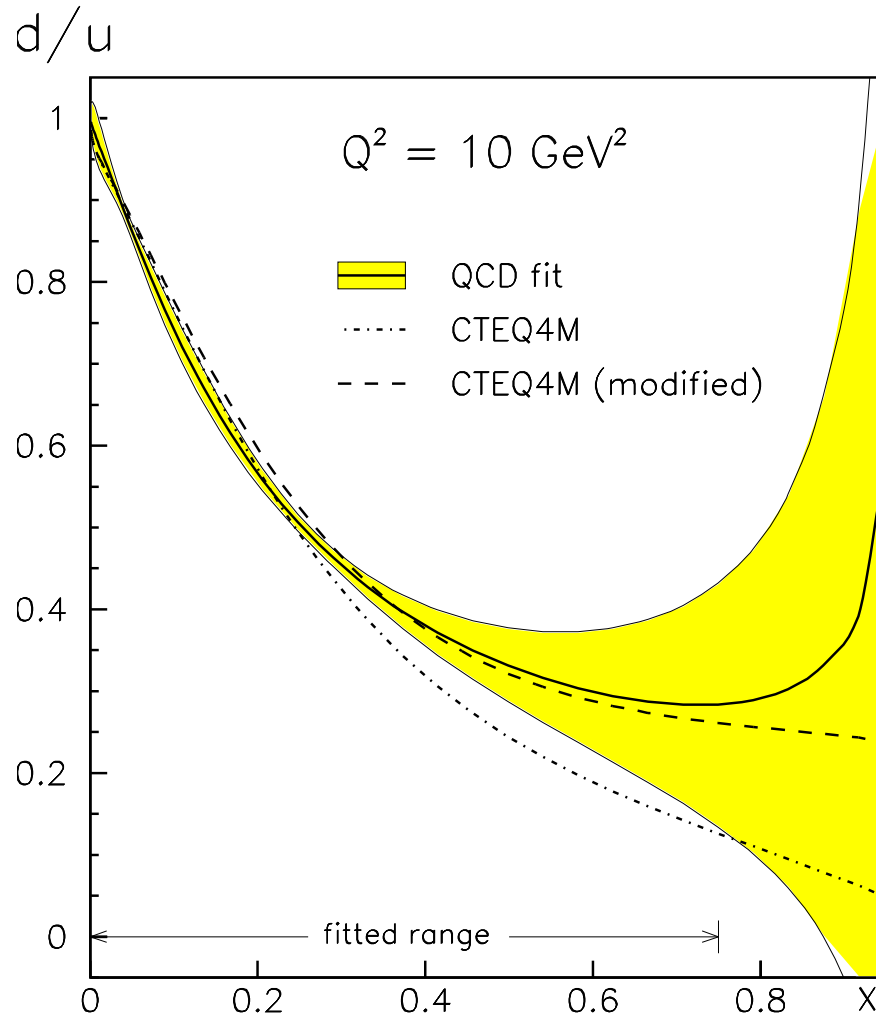
- $x F_3^{\nu\text{Fe}} = x F_3^{\nu N} [1 + K_{\text{Fe}}(R_{\text{Fe}} - 1)]$

- $R_{\text{Fe}} = \text{parametrization of } \frac{x F_3^{\nu\text{Fe}}}{x F_3^{\nu N}}$

- Assume 50% error ($K_{\text{Fe}} = 1 \pm 0.5$)

The d/u ratio

- Error band for ratio of d/u Botje hep-ph/9912439



- ratio $\frac{F_2^P}{F_2^N}$ is sensitive to d/u at large x

$$\frac{F_2^P}{F_2^N} \simeq \frac{1 + 4d/u}{4 + d/u} \text{ as } x \rightarrow 1$$

- d/u badly constrained at large x
- $d/u \rightarrow \infty$ from QCD fit Botje '99
- $d/u \rightarrow 0.2$ Bodek, Yang '98
- $d/u \rightarrow 0$ from CTEQ Lai et al. '96

Summary

Summary

- NuTeV has thoroughly investigated QCD effects, both in the nucleon and in nuclear matter
- PDF uncertainties should be addressed in global analyses
- NLO QCD predictions should be incorporated in future analyses

In this situation, it is worthwhile relying on [...] NuTeV data for cross-sections and SFs [...] to clarify the situation with the help of independent analyses [...], which should include all types of QCD effects, including those related to NLO perturbative QCD corrections.

[Kataev, Kumano hep-ph/0211052](#); [WG 3 summary NuFact'02 Workshop](#)

Chances and opportunities

- νN DIS at NuTeV can
 - help in understanding the quark flavor content of the nucleon
 - provide important input for the issue of $s(x) \neq \bar{s}(x)$