Hadronic vacuum polarization effects in $\alpha_{\rm em}(M_Z)$

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Mini Workshop

ELECTROWEAK PRECISION DATA AND THE HIGGS MASS

DESY Zeuthen, February 28-29, 2003

① Introduction

Fundamental "constant" α is a function of the energy scale

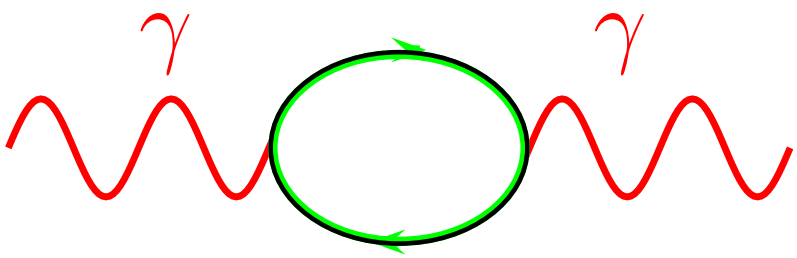
$\alpha \to \alpha(E)$ of a process: charge screening.

effect due to vacuum polarization in the photon propagator

$$e^2 D_{\mu\nu}(q) = \frac{g_{\mu\nu}\, e^2\, Z}{q^2\ (1+\Pi(q^2))} + \text{ gauge terms} \quad .$$

$$\Downarrow$$

$$e^2 \to e^2(q^2) = \frac{e^2 Z}{1+\Pi(q^2)}$$



virtual pairs

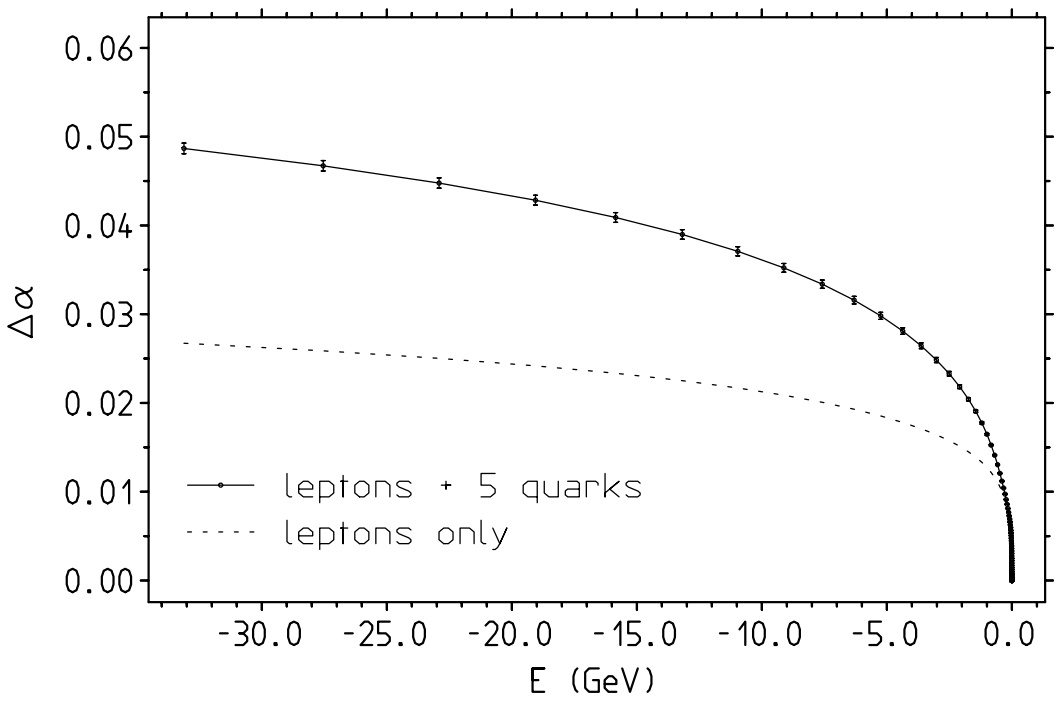
$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \to \gamma^*$$

Effective finestructure "constant" $\alpha(E)$

***appears in many places in physics!* large EWRC !**

Of particular interest:

$\alpha(M_Z)$ and $a_\mu \equiv (g-2)_\mu/2$



- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha}\ ;\quad \Delta\alpha = -e^2 \mathrm{Re}\ (\Pi'(s) - \Pi'(0))$$

$$\Delta\alpha = \Delta\alpha_{\rm lep} + \Delta^{(5)}\alpha_{\rm had} + \Delta\alpha_{\rm top} + \cdots$$

$$\Pi'_{\rm had}(q^2) \quad = \quad$$

1pi blob

② $\alpha_{\rm em}(s)$ in Precision Physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

α , G_μ, M_Z **most precise input parameters**

partially ↑

non-perturbative ⇕

relationship ↓

precision predictions

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters**

for VB physics (Z,W) etc.

$$\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$$

$$\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$$

$$\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4} \quad \text{(present)}$$

$$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad \text{(TESLA requirement)}$$

⇒ serious limiting factor for precision physics

$\Delta\alpha$ in electroweak precision physics

Calculate $\sin^2\Theta_i$ from α, G_μ and M_Z :

$$\sqrt{2}\,G_\mu\,M_Z^2\,\sin^2\Theta_i\,\cos^2\Theta_i = \pi\alpha\,\frac{1}{1-\Delta r_i}$$ **Sirlin 1980**

Various definitions of $\sin^2\Theta_i$, which coincide at tree level: From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:

$$\begin{aligned}
\sin^2\Theta_W &= 1-\frac{M_W^2}{M_Z^2}\\
\sin^2\Theta_g &= e^2/g^2\\
\sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\ ,\quad f\neq\nu
\end{aligned}$$

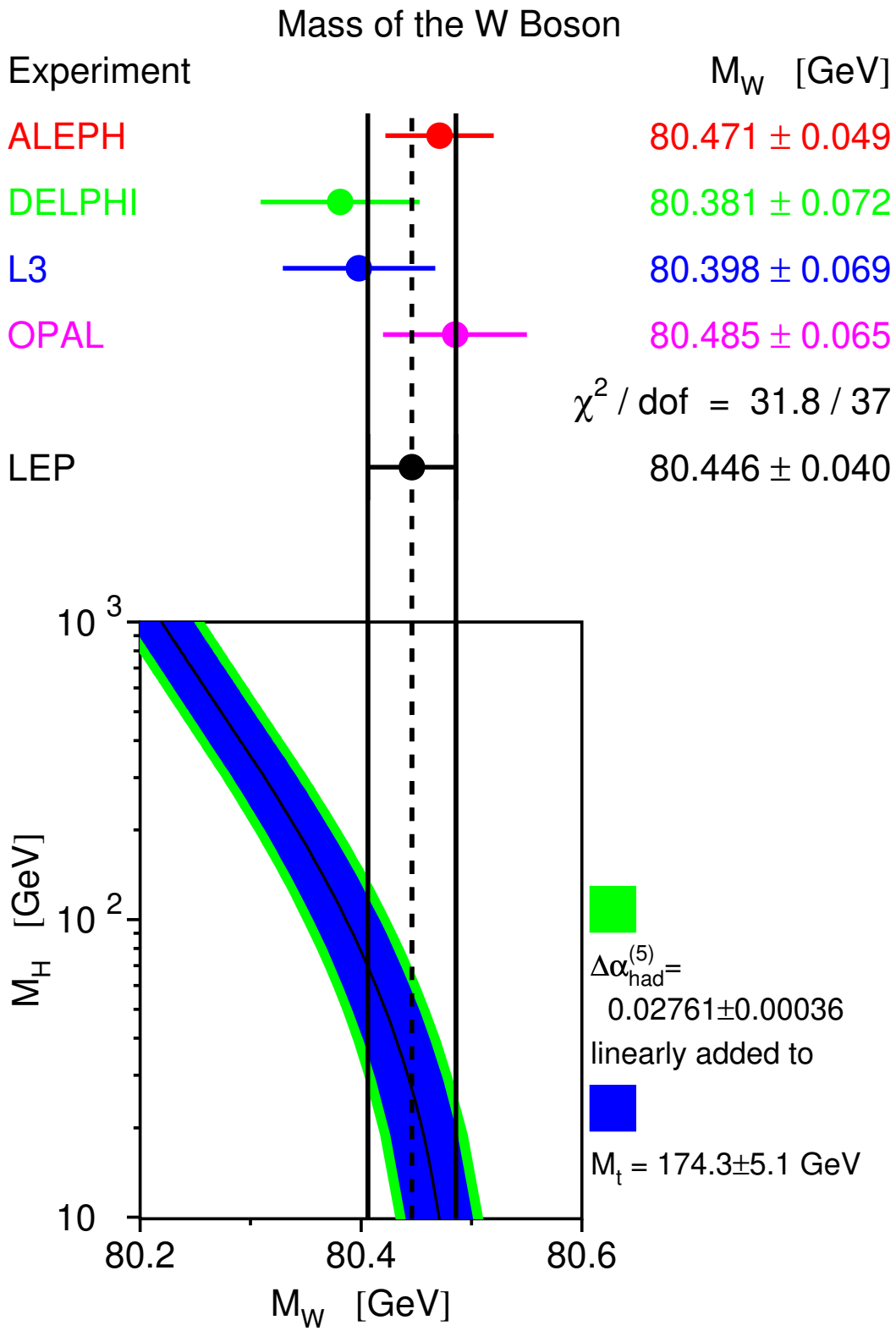
$$\begin{aligned}
\Delta r_i &= \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)\\
\Delta r_i &= \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\mathrm{rem}}
\end{aligned}$$

$\uparrow$

universal

$\rightarrow$ **predictions for M_W, A_{LR}, A_{FB}^f, Γ_f, $\cdots$**

$$\delta\Delta\alpha \rightarrow \delta M_W\ ,\quad \delta\sin^2\Theta_i\ ,\ \cdots$$



(LEP Electroweak Working Group: D. Abbaneo et al. 01)

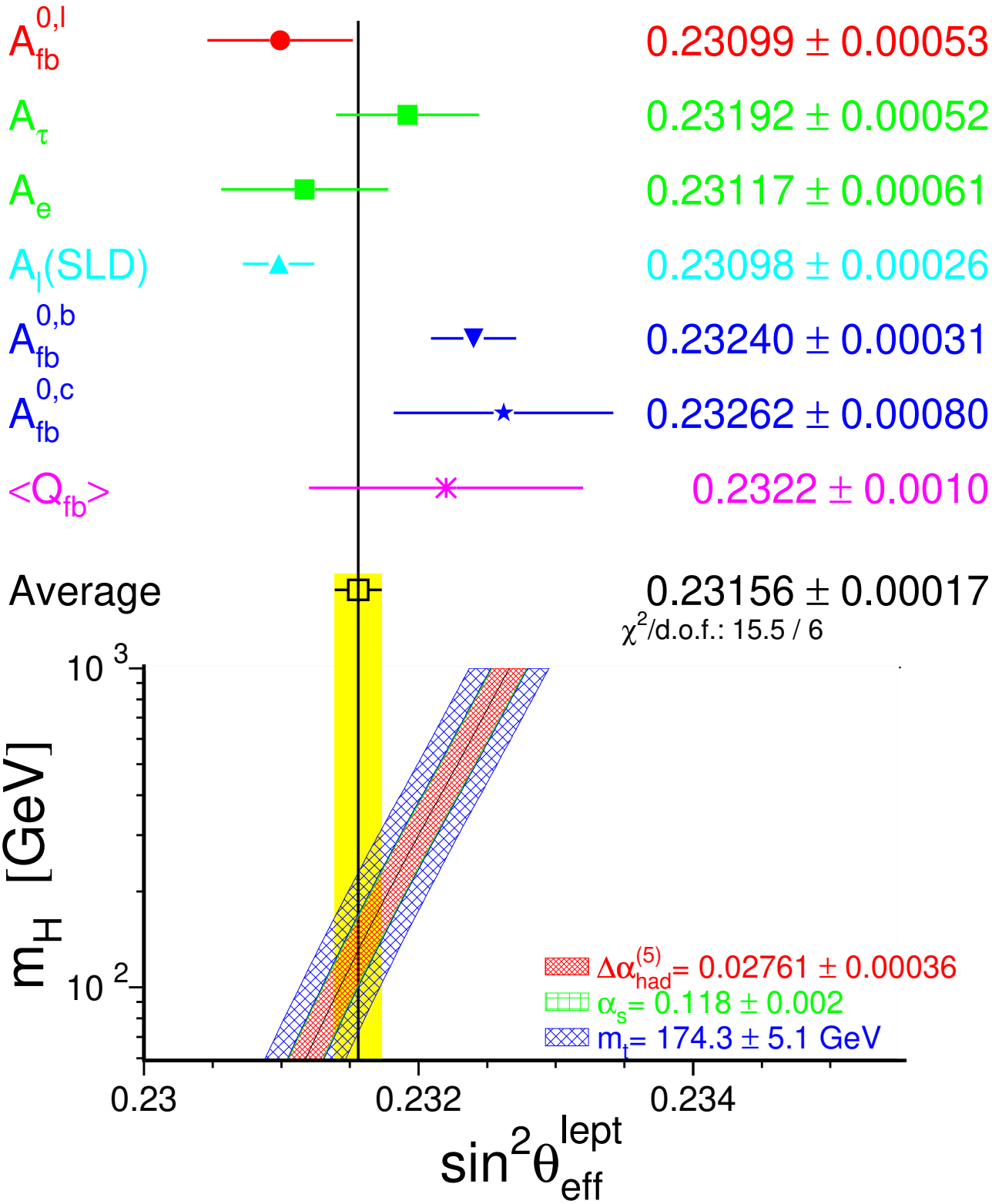
Indirect Higgs mass "measurement": $m_H = 88^{+53}_{-35}$ GeV

$e^+e^- \to \tau$: $\delta m_H \sim -19$ GeV

Direct lower bound: $m_H > 114$ GeV at 95% CL

Indirect upper bound: $m_H < 193$ GeV at 95% CL

Preliminary



(LEP Electroweak Working Group: D. Abbaneo et al. 01)

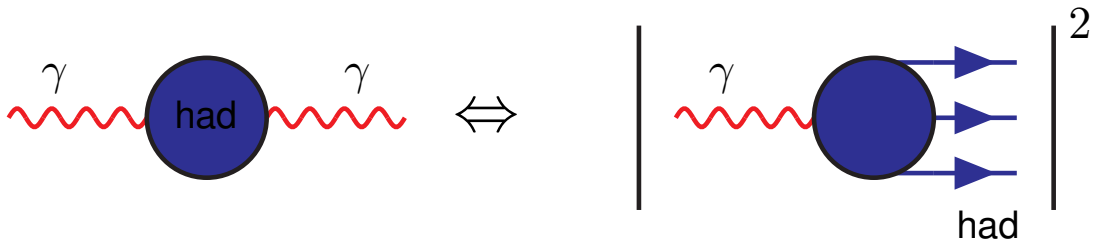
❑ **The way out:**

non–perturbative approach by dispersion relation

The total cross section

$$\sigma(e^+e^- \to hadrons)$$

for hadro-production in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$



$\Pi'_\gamma(q^2)$ $\sim \sigma^{\rm had}_{\rm tot}(q^2)$

causality (analyticity) + unitarity (optical theorem)

$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\; \mathrm{P}\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm QCD}_\gamma(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

Encoded in R ratio:

$$R(s) \equiv \frac{\sigma_{\rm had}(s)}{\sigma_{\mu\mu}^{(m_\mu=0)}(s)} = \frac{\sigma_{\rm had}^{(0)}(s)}{\sigma_{\mu\mu}^{(0,m_\mu=0)}(s)}$$

$$|| \qquad\qquad ||$$

$$\frac{4\pi\alpha^2(s)}{3s} \qquad \frac{4\pi\alpha^2}{3s}$$

$$\sigma_i^{(0)}(s) = \sigma_i(s)\,(\alpha/\alpha(s))^2$$

cross section undressed from QED corrections. Note:

$$\frac{\sigma_{\rm had}(s)}{\sigma_{\mu\mu}^{(m_\mu=0)}(s)} \neq \frac{\sigma_{\rm had}^{\rm obs}(s)}{\sigma_{\mu\mu}^{\rm obs}(s)}$$

convolution with γ radiation does not factor out !

Theory of $R(s) \Rightarrow$ (Chetyrkin, Kühn et.al.)

Recent progress:

- **2002: Final CMD-2 $\pi\pi$–data: syst error 1.4% $\longrightarrow$ 0.6 % !**
  some other CMD-2, SND data at $E < 1.4$ GeV
  (Akhmetshin et al.)
- **2001: BES-II R–data: syst error 20% $\longrightarrow$ 7% !**
  region 2.0 GeV to 5.0 GeV
  (Bai et al. 2000, 2002)
- **2000: τ–data**
  (ALEPH, OPAL, CLEO Collab., 1996-2002)
- **Iso-spin breaking effects τ–data vs. e^+e^-–data**
  (Cirigliano et al. 2002)
- **new situation: τ–data not compatible with e^+e^-–data at 10% level**

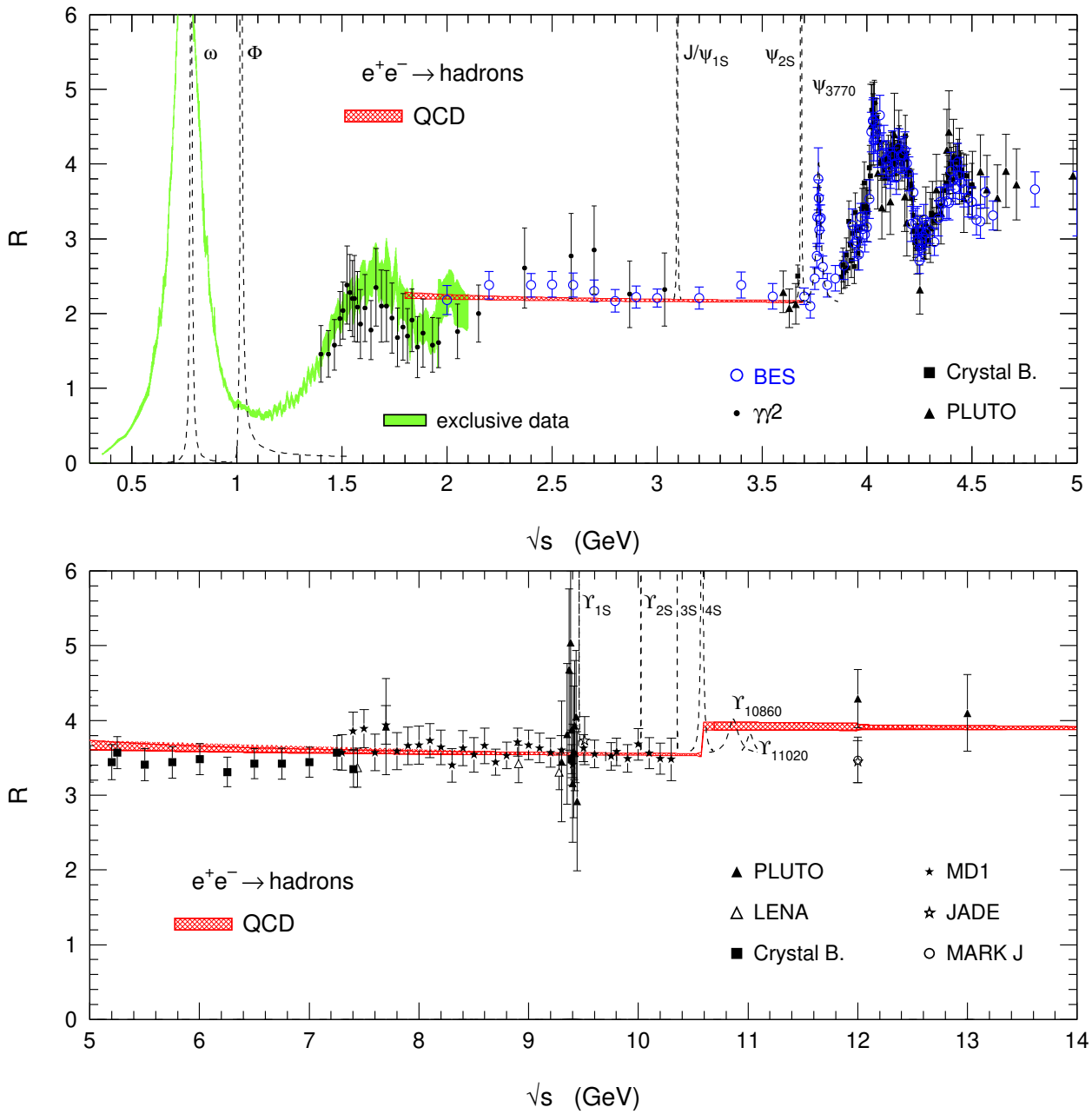
Evaluation FJ 2002 update: **at $M_Z =$ 91.19 GeV**

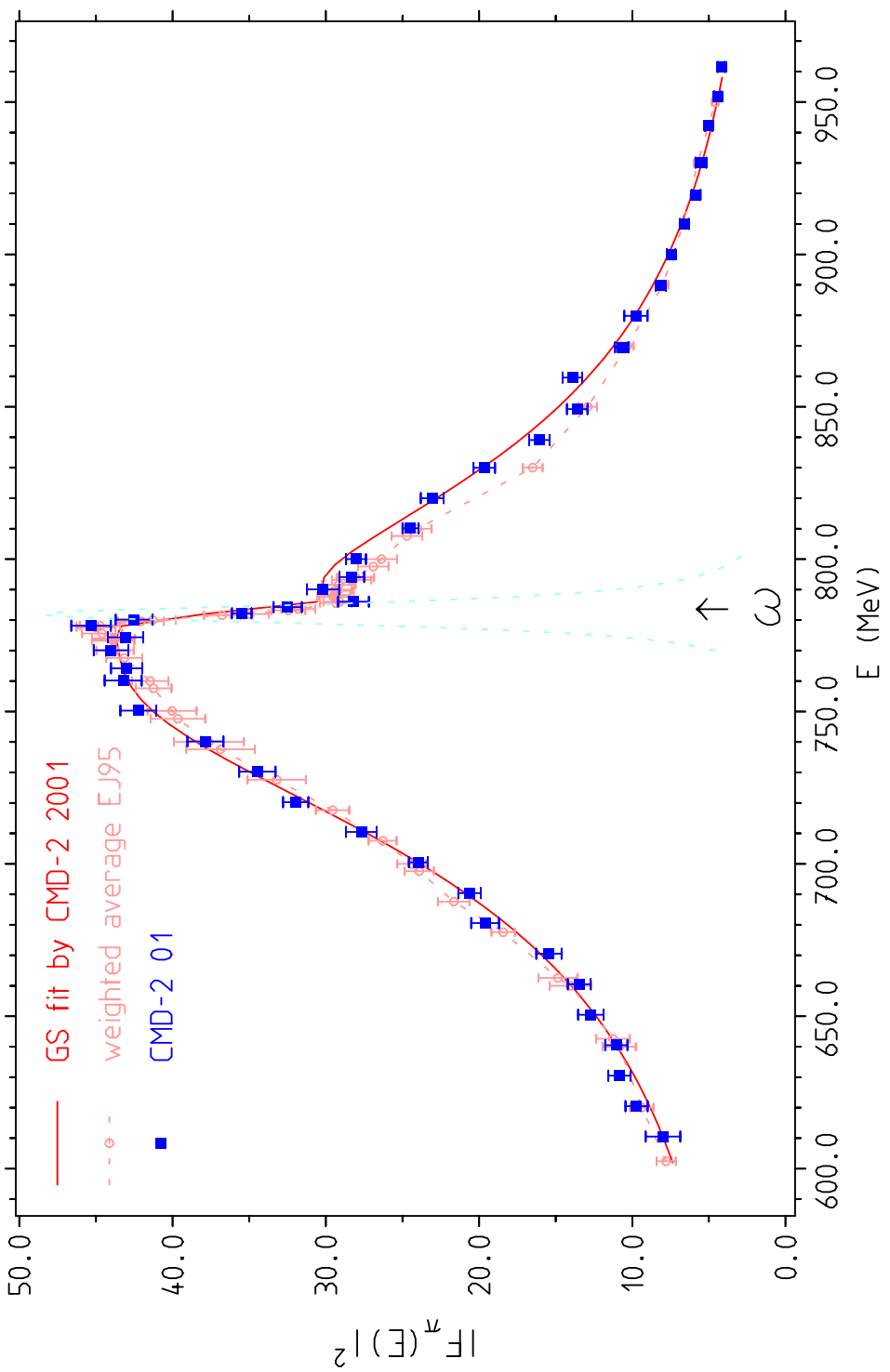
- **$R(s)$ data up to $\sqrt{s} = E_{cut} = 5$ GeV**
  and for Υ resonances region between 9.6 and 13 GeV
- **perturbative QCD from 5.0 to 9.6 GeV**
  and for the high energy tail above 13 GeV

$$\begin{aligned} \Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) &= 0.027572 \pm 0.000359 \\ &\quad 0.027610 \pm 0.000360 \quad \text{BP 01} \\ \alpha^{-1}(M_Z^2) &= 128.952 \pm 0.049 \\ &\quad 128.936 \pm 0.046 \quad \text{BP 01} \end{aligned}$$

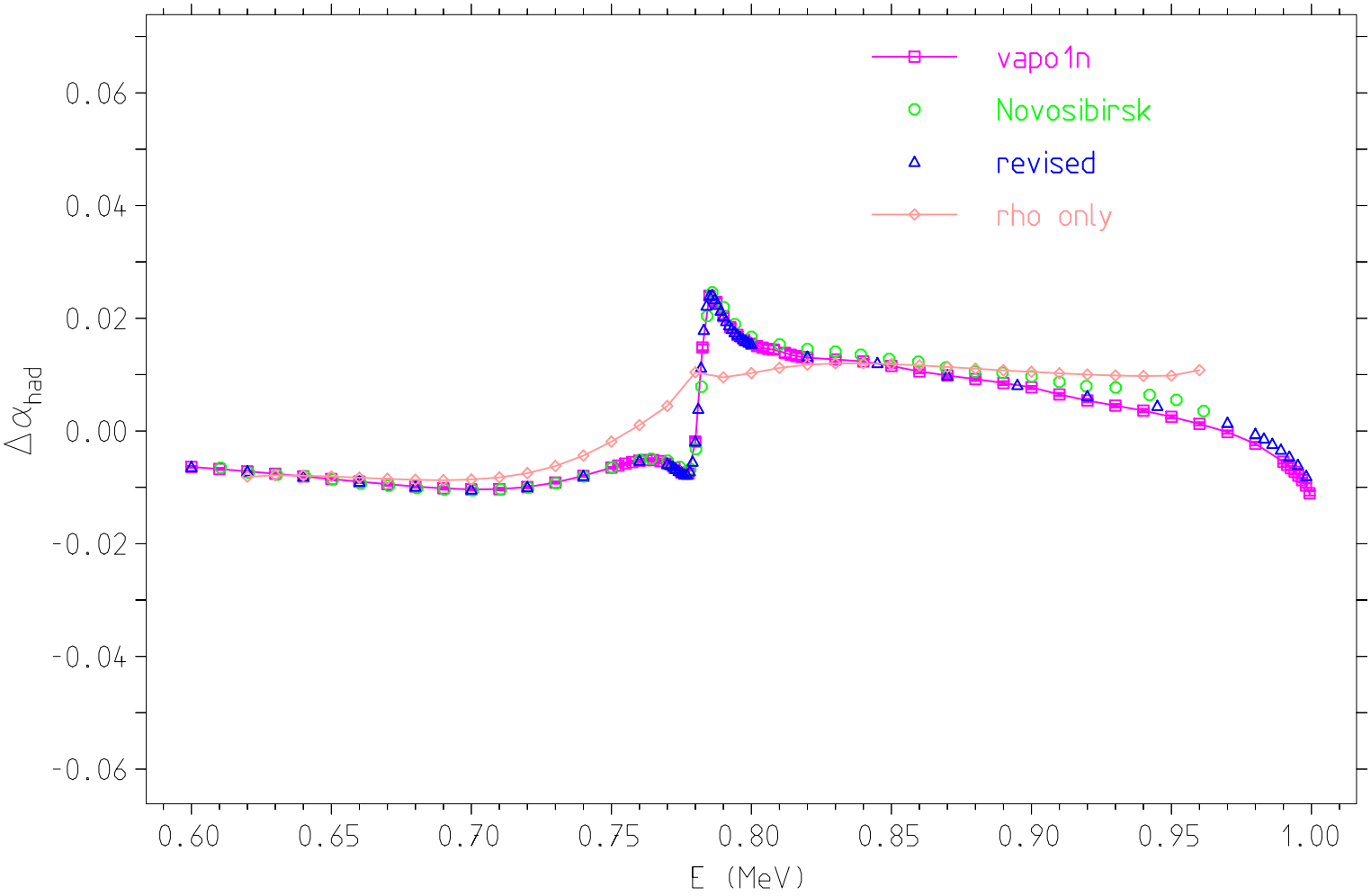
Compilation: Davier, Eidelman et al. 02

Theory = pQCD: Groshny et al. 91, Chetyrkin et al. 97




GS fit by CMD-2 2001
weighted average EJ95
CMD-2 01
$|F_\pi(E)|^2$
E (MeV)
ω
0.0
10.0
20.0
30.0
40.0
50.0
600.0
650.0
700.0
750.0
800.0
850.0
900.0
950.0

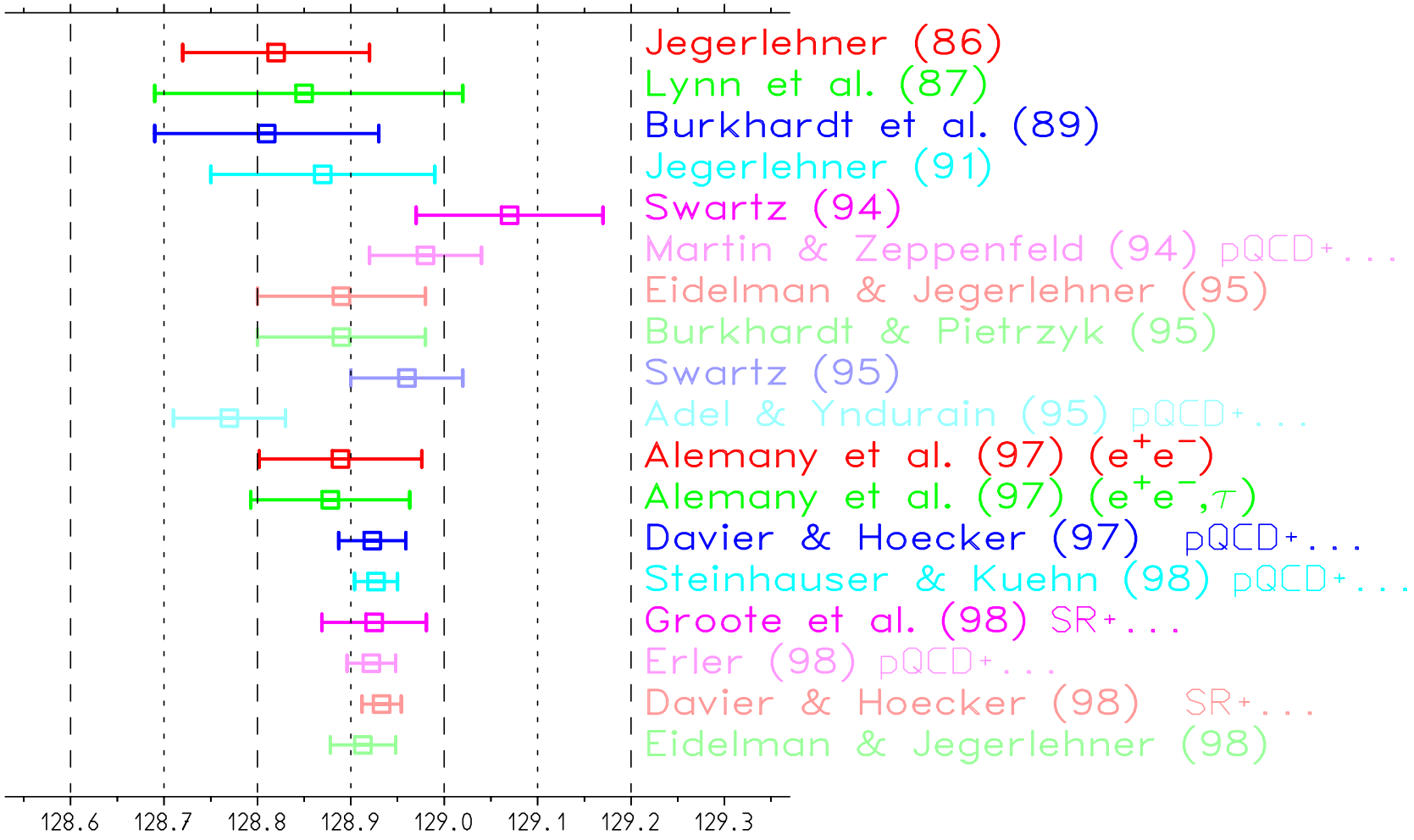
- **substantially lower than old e^+e^-–data !**

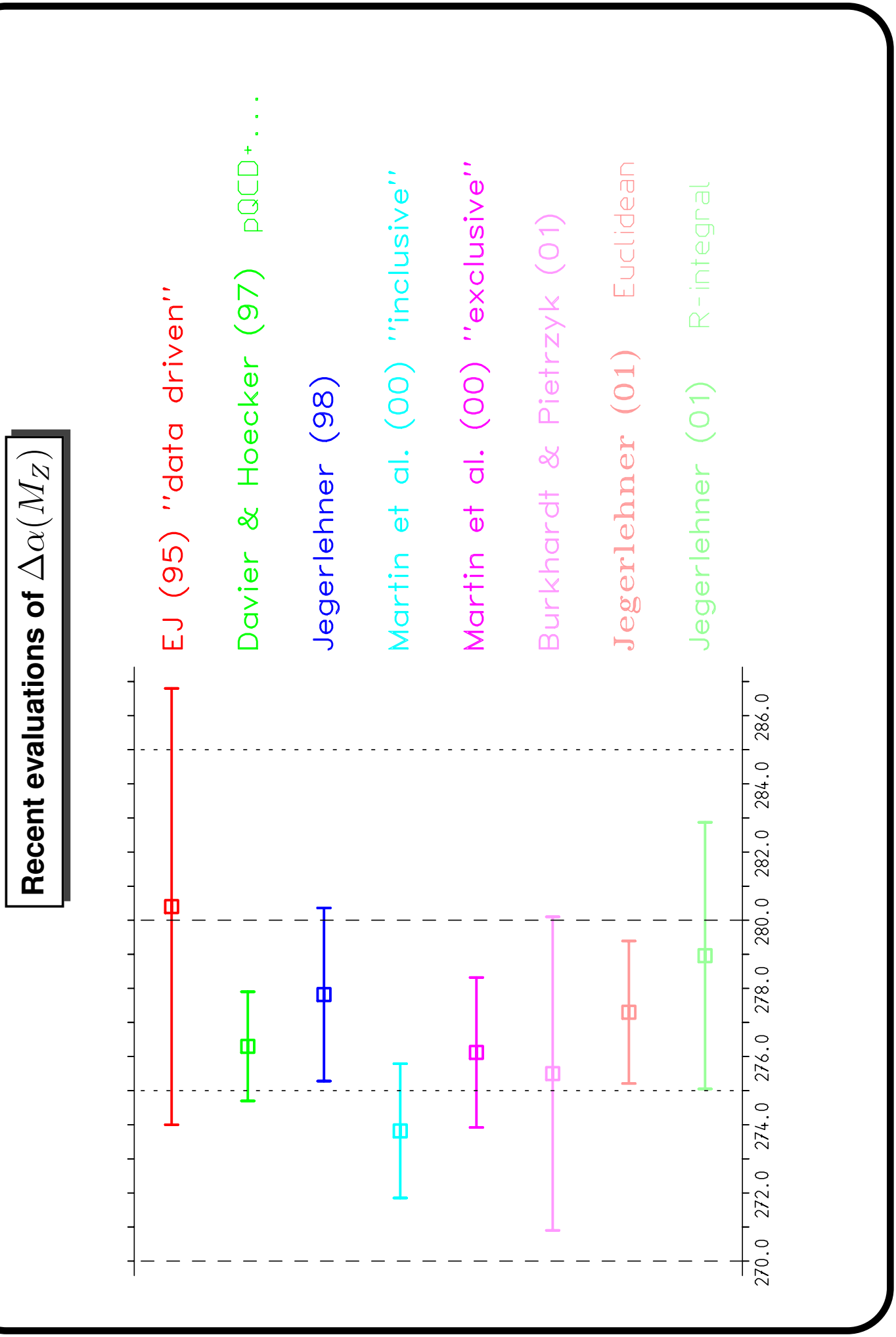


checking VP subtraction by CMD-2 group !

Evaluations of $\alpha(M_Z)$ by different authors (before 2000)

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.




Recent evaluations of $\Delta\alpha(M_Z)$
EJ (95) "data driven"
Davier & Hoecker (97) pQCD+...
Jegerlehner (98)
Martin et al. (00) "inclusive"
Martin et al. (00) "exclusive"
Burkhardt & Pietrzyk (01)
Jegerlehner (01) Euclidean
Jegerlehner (01) R-integral
270.0
272.0
274.0
276.0
278.0
280.0
282.0
284.0
286.0

④ The τ vs. e^+e^- problem

① Additional data: τ–data + CVC

ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 96

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^-\to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \le M_\tau$$

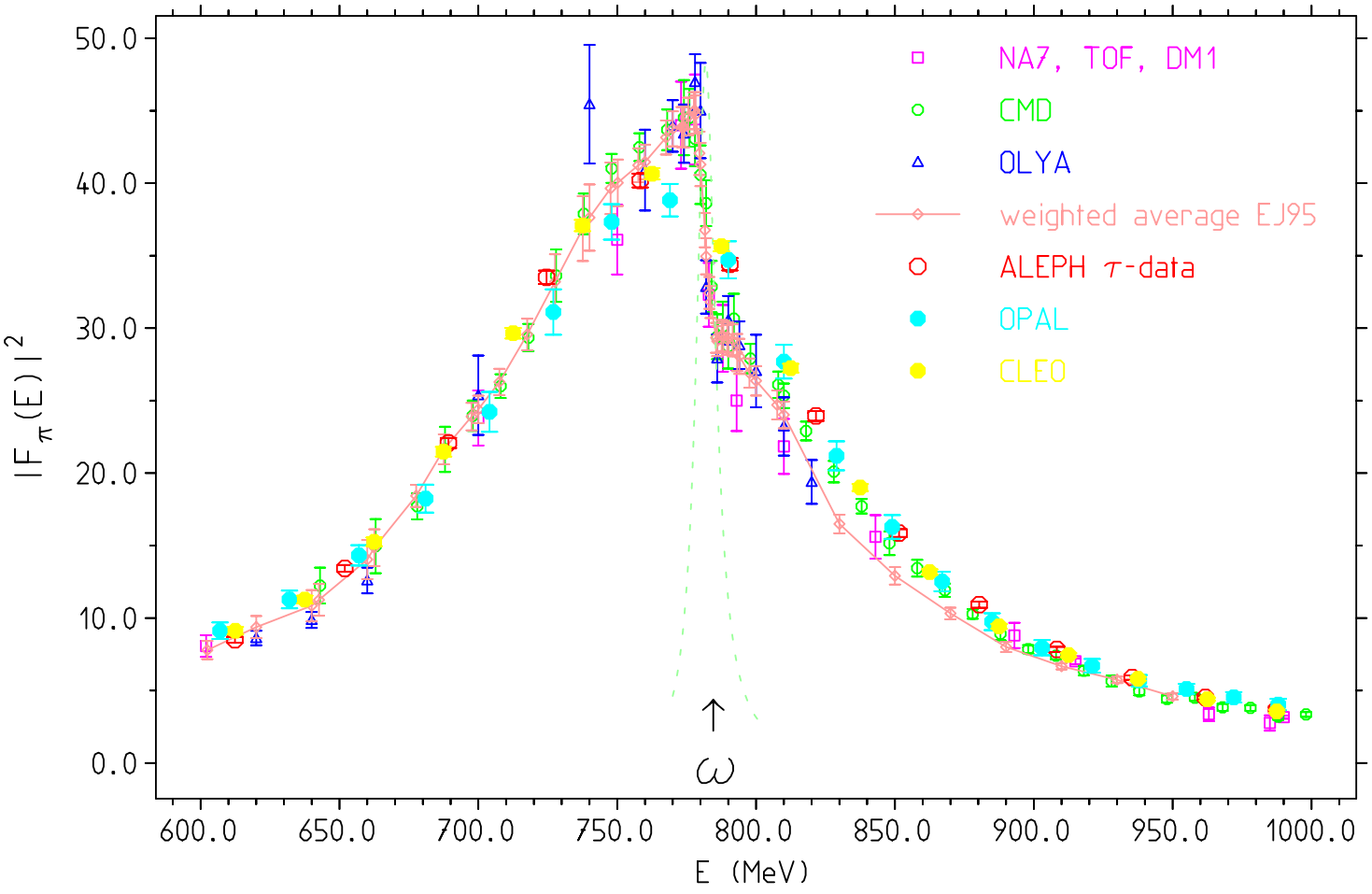
in terms of the τ spectral function v_1.

- **mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)**
- **which is dominating in $a_\mu^{\rm had}$ (72%)**

$$I = 1 \sim 75\% ; \; I = 0 \sim 25\%$$

τ–data cannot replace e^+e^-–data

$$\begin{aligned} \delta a_\mu &: \; 15.6 \times 10^{-10} \; \to \; 10.2 \times 10^{-10} \\ \delta\Delta\alpha &: \; 0.00067 \; \to \; 0.00065 \end{aligned}$$



systematic deviations: can they be understood?

e^+e^-–data*= data corrected for isospin violations:

$$|F(s)|^2 \text{ = (}|F(s)|^2\text{–data) / } \left|\left(1+\frac{\epsilon s}{(s_\omega - s)}\right)\right|^2$$

with

$$s_\omega = (M_\omega - \tfrac{i}{2}\Gamma_\omega)^2$$

ϵ **determined by fit to the data:** $\epsilon = 0.00172$

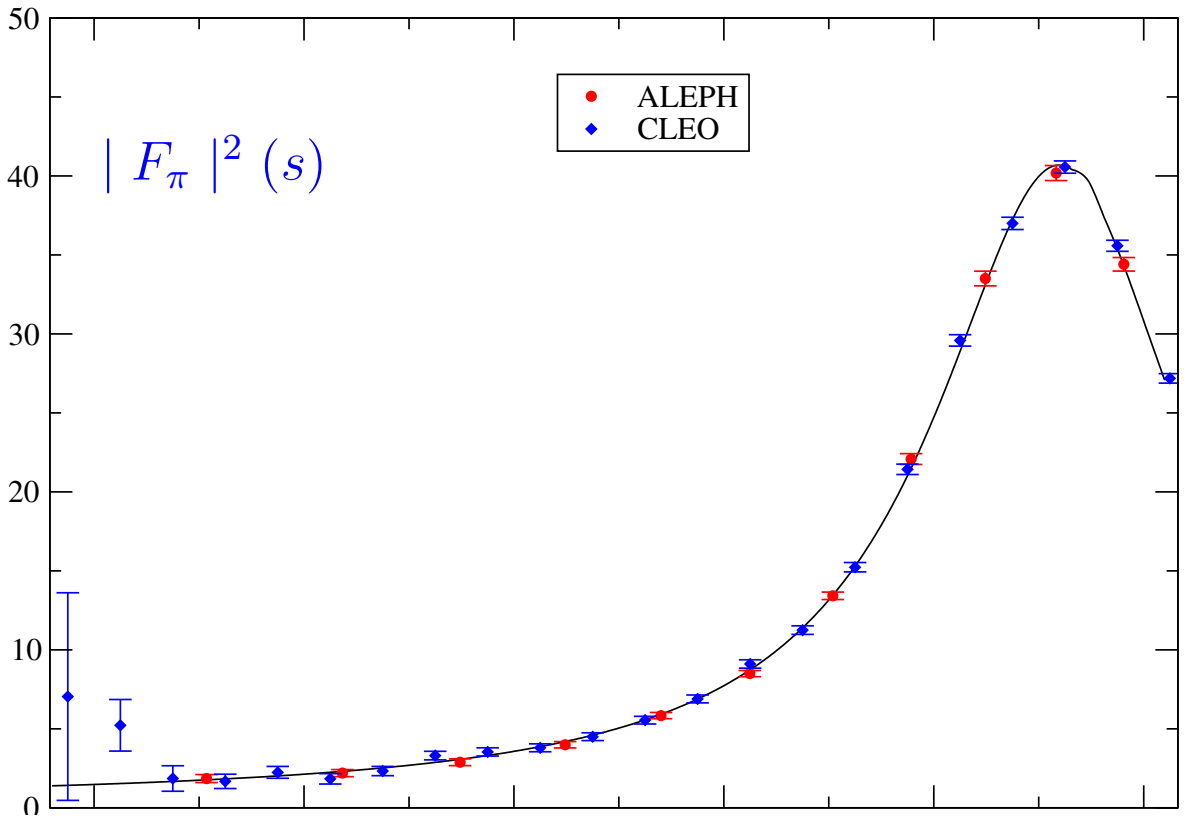
(Leutwyler 00)

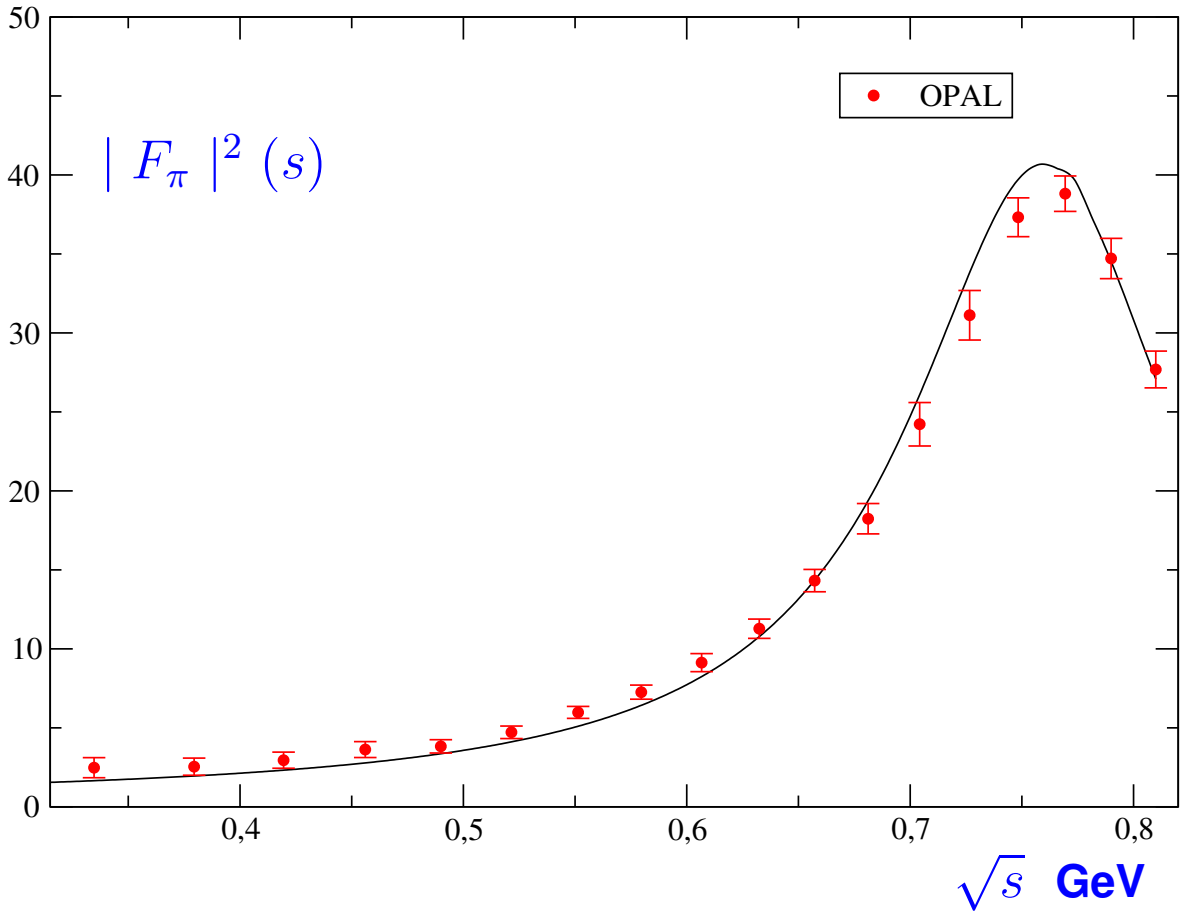
After correction and shifts of normalizations no systematic deviations between CLEO/ALEPH and DM1/OLYA/CDM1/CDM2 data beyond fluctuations of e^+e^- data, while OPAL data show substantial deviations below 800 MeV:

work in progress

I. Caprini, G. Colangelo, J. Gasser, F. J., H. Leutwyler

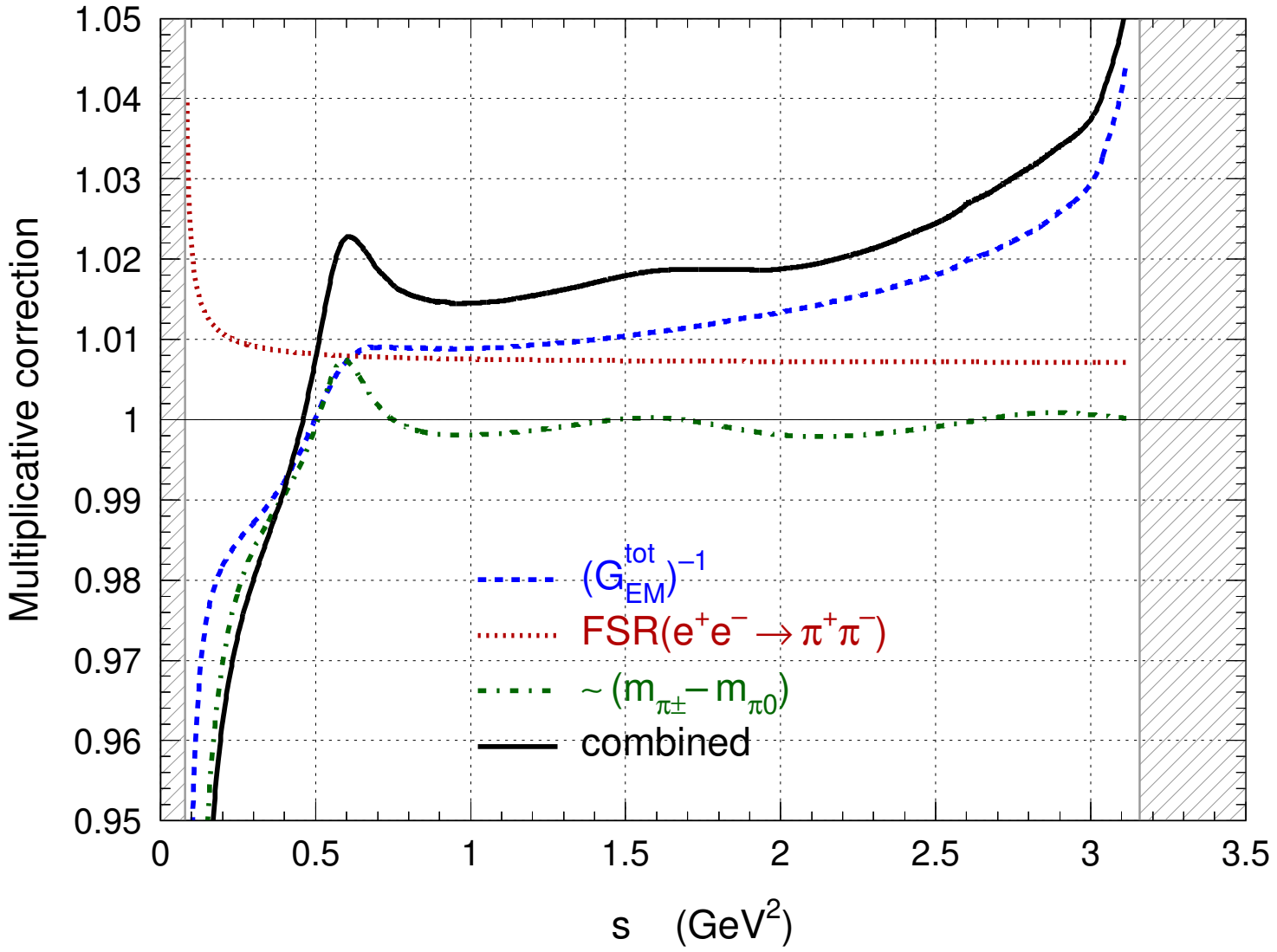
NA7+CMD-2 e^+e^-–data analytized+unitarized+chiral limit: solid line vs. τ –data





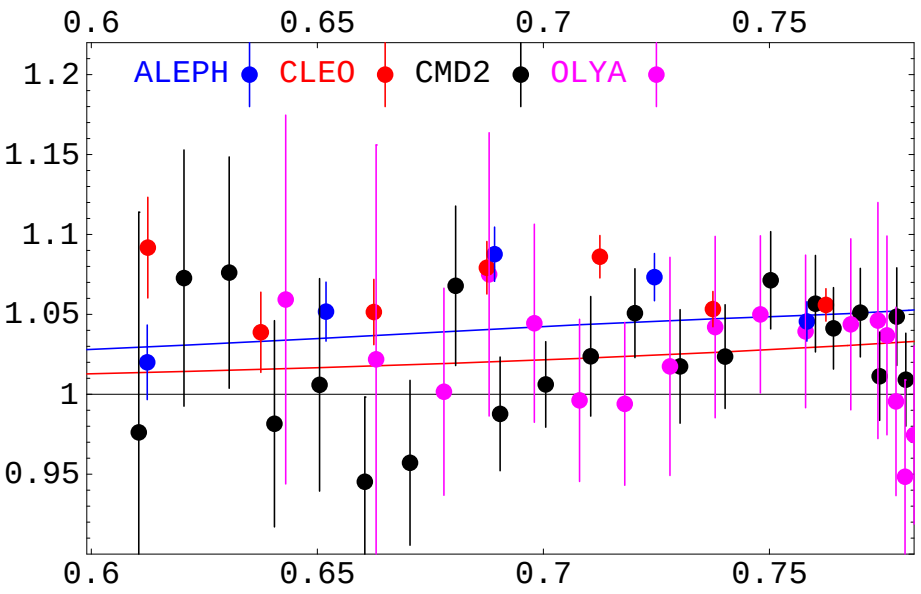
Isospin breaking effects: (Cirigliano et al.)

Surprisingly, after corrections at larger energies 10% deviations !

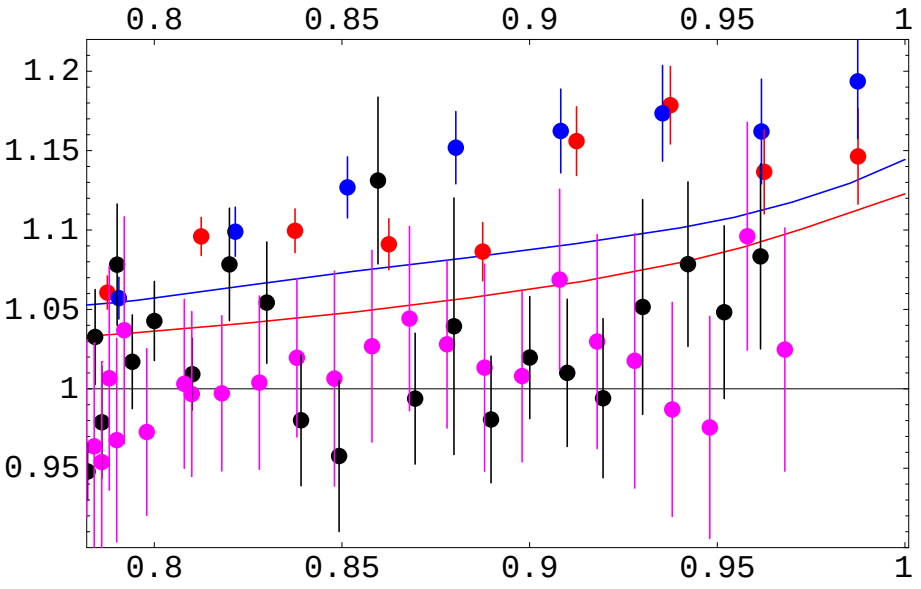


- **τ vs. e^+e^-**

Revised, 24. 8. 2002



Region below the ω $(0.6 < E < 782$ MeV)



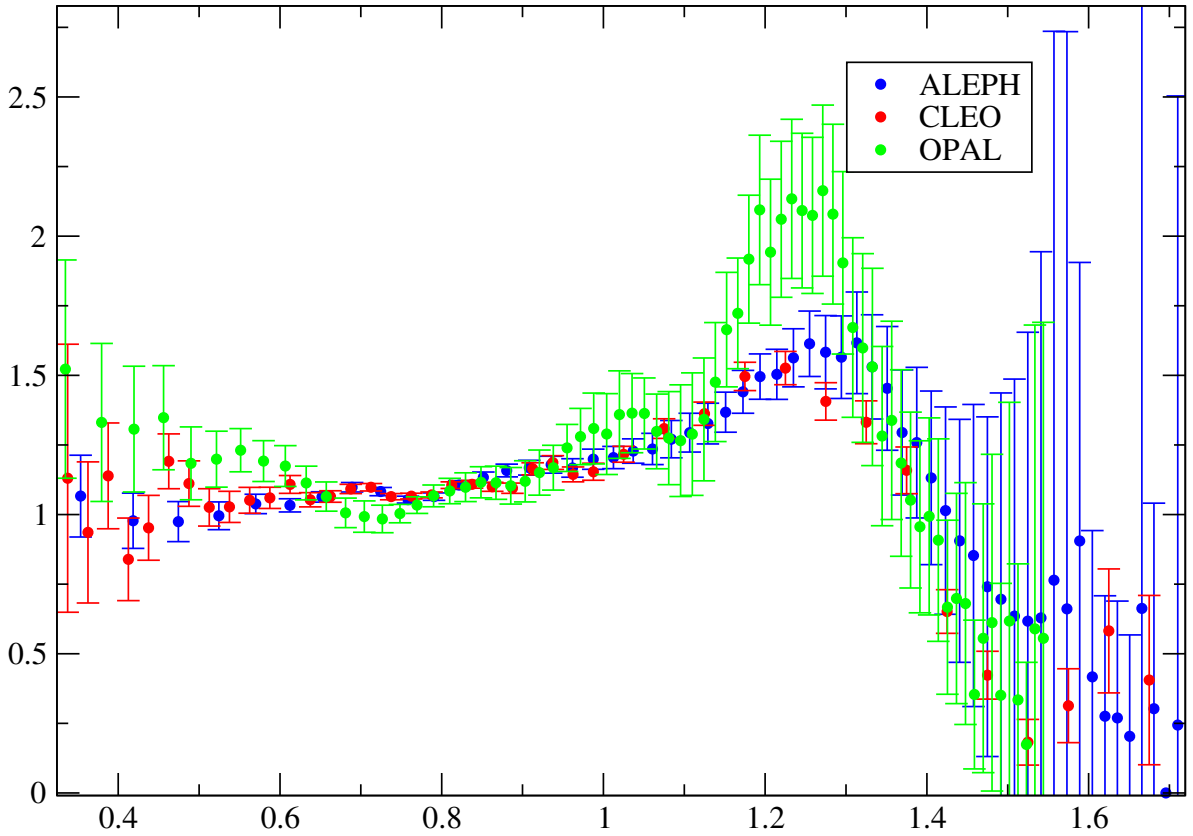
Region above the ω $(782\,\text{MeV} < E < 1$ GeV)

The plots show the square of the form factor divided by the Omnes factor. In the case of the e^+e^- data, the contributions from ω exchange and vacuum polarization are removed and only the statistical errors are shown. In the case of the τ data, the error bars represent the diagonal elements of the error matrix.

cannot be understood within SM !

likely an experimental problem

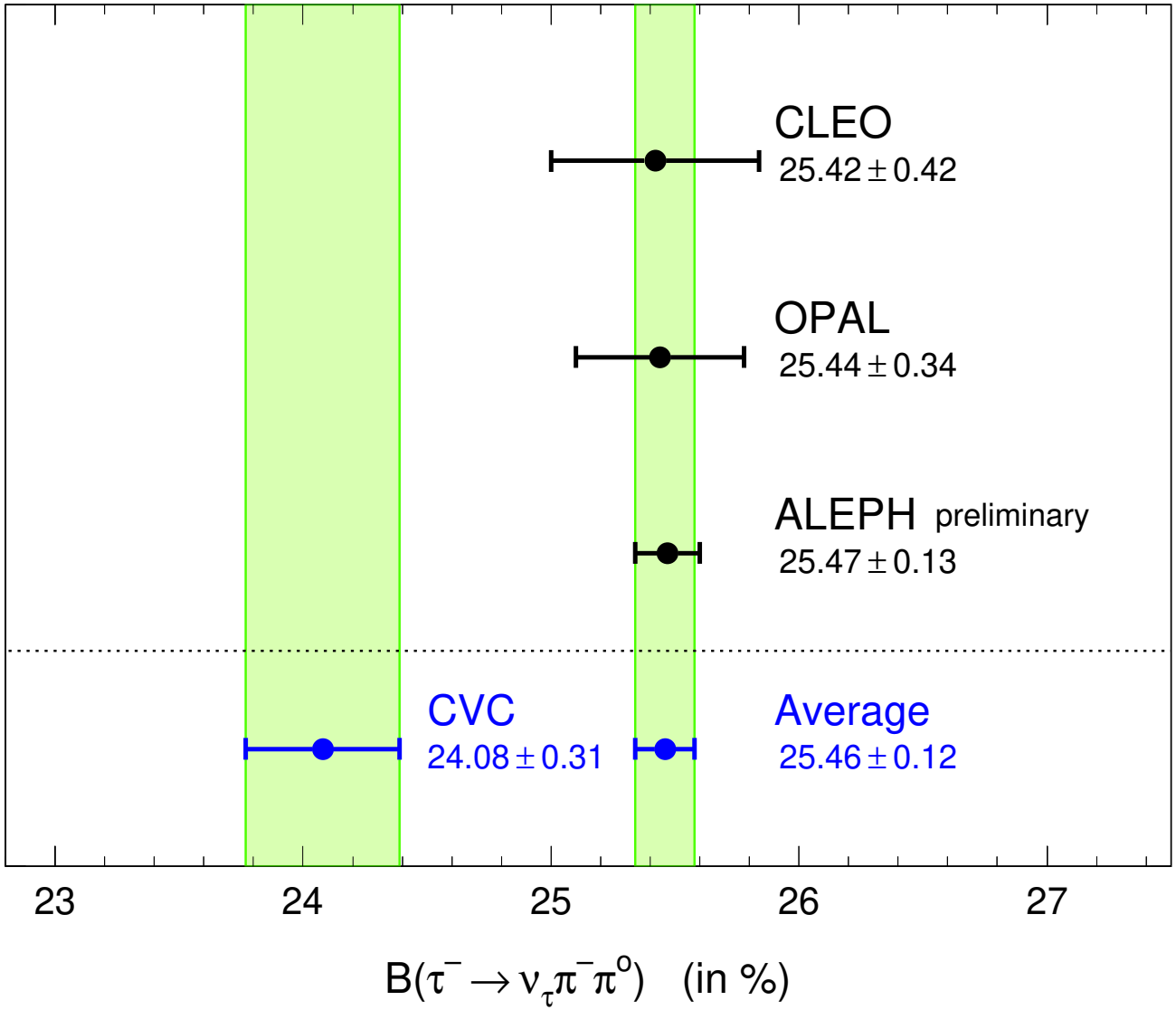
- **Comparison of τ–data:**



τ–data may be not so easy; DELPHI, L3 could not measure τ spectral–functions;

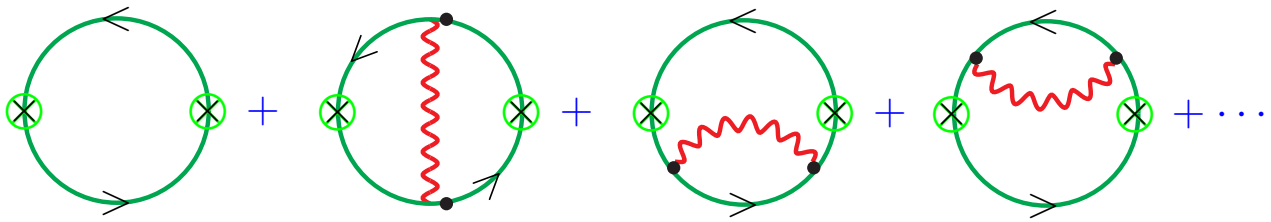
ALEPH vs. OPAL no good agreement.

The measured branching ratios for $\tau \longrightarrow \nu_\tau \pi^- \pi^0$ compared to the prediction from the $e^+e^- \longrightarrow \pi^+\pi^-$ after applying the isospin breaking correction



⑤ Controlling pQCD via the Adler function

❶ pQCD calculations of vacuum polarization amplitudes



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

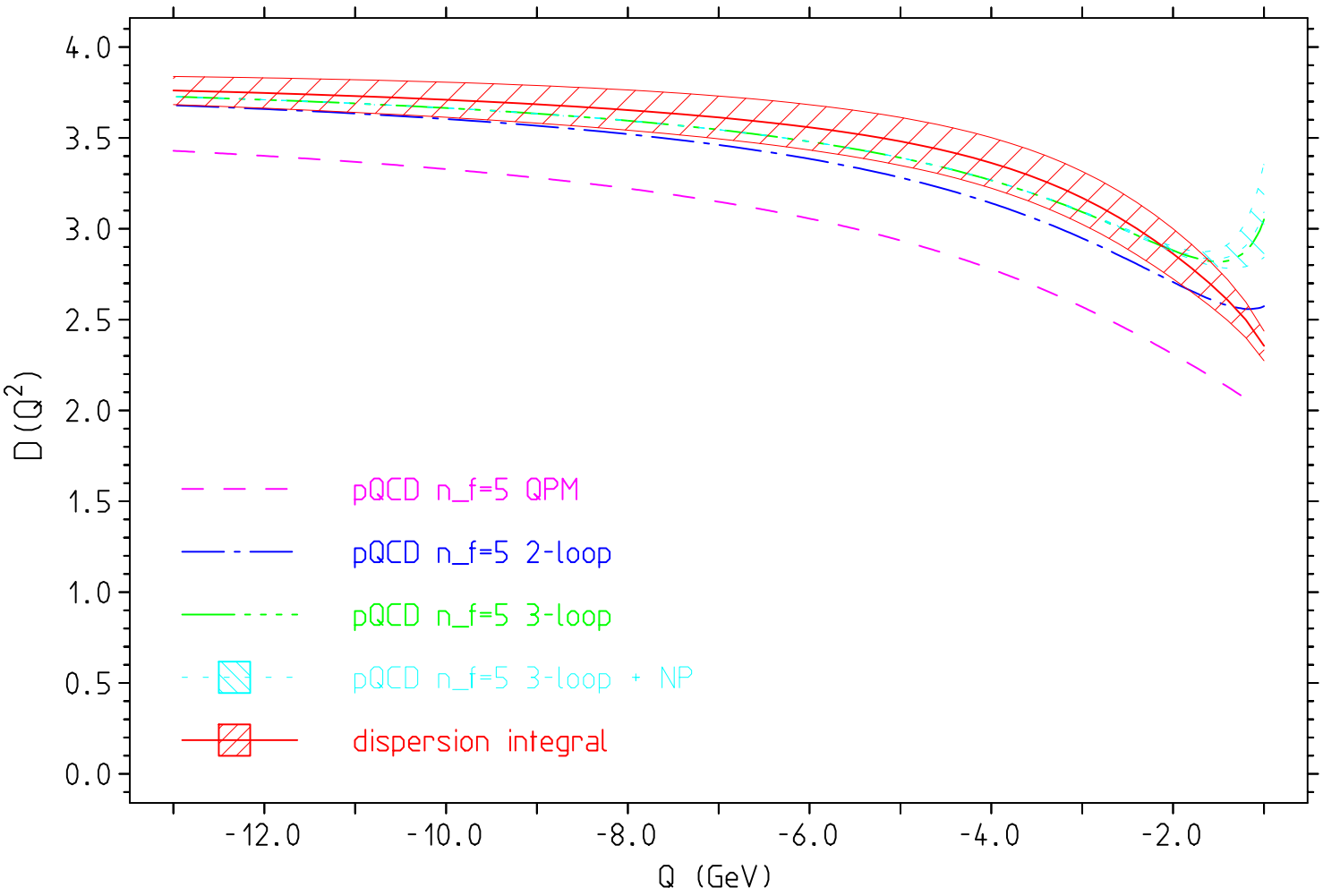
❷ use old idea: testing non–perturbative effects with help of the Adler function

Eidelman, F. J., Kataev, Veretin 98

$$
\begin{aligned}
D(-s) &\doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) \\
&= -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds} \\
\Rightarrow \qquad D(Q^2) &= Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}
\end{aligned}
$$

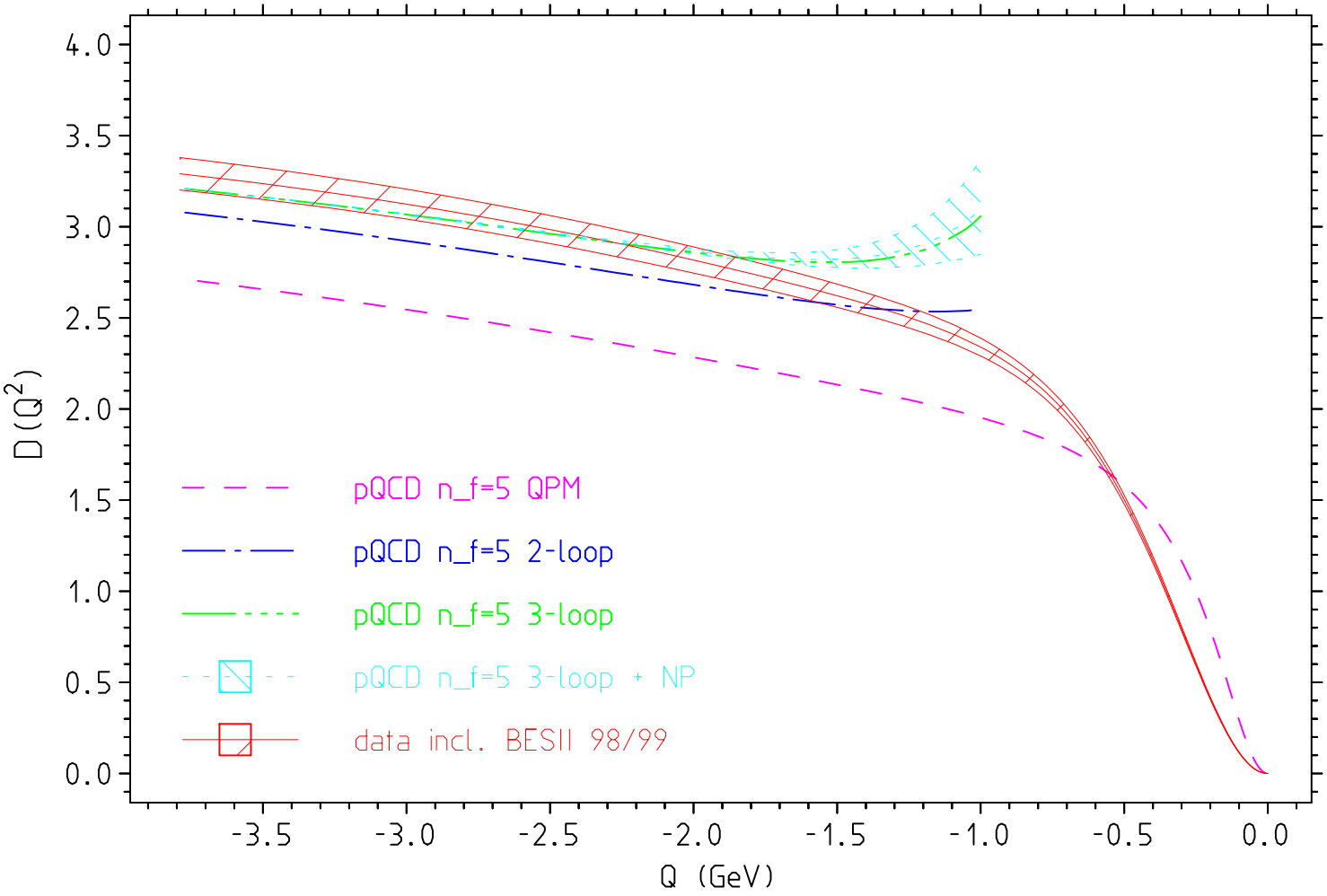
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

"Experimental" Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98)

"Experimental" Adler–function versus theory (pQCD + NP)



(Update 2000: EJKV98 + BESII 98/99 prel.)

$\Rightarrow$ **pQCD works well to predict** $D(Q^2)$ **down to** $s_0 = (2.5\,\textbf{GeV})^2$ **(not down to** m_τ **! however); use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\textbf{GeV})^2$**:** **(FJ 98/02)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007340 \pm 0.000093$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.02745 \pm 0.00018$$

$$\alpha^{-1}(-M_Z^2) =$$

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean,** $Q^2 \gtrsim$ **2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder"** $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ **is mainly sensitive to low energy data !!!**

The second error comes from the variation of the pQCD parameters. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

$$\Rightarrow \quad \delta\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.00015$$

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$: Including this term yields

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02737 \pm 0.00020$$

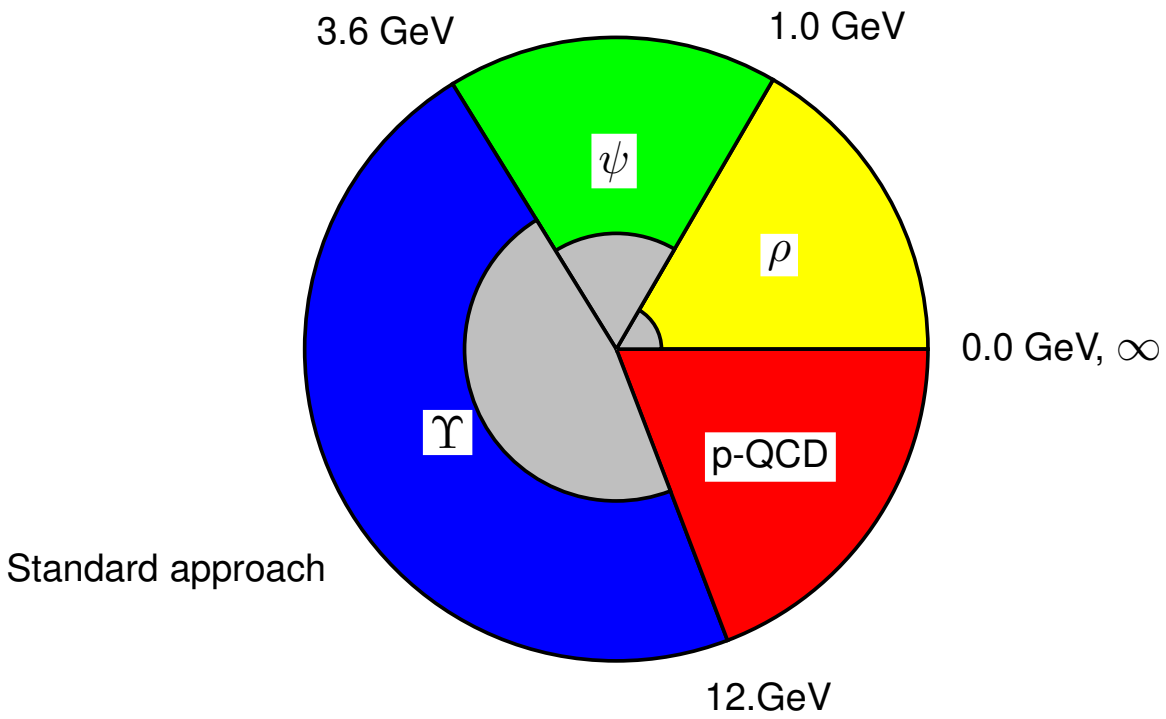
$$\alpha^{-1}(M_Z^2) =$$

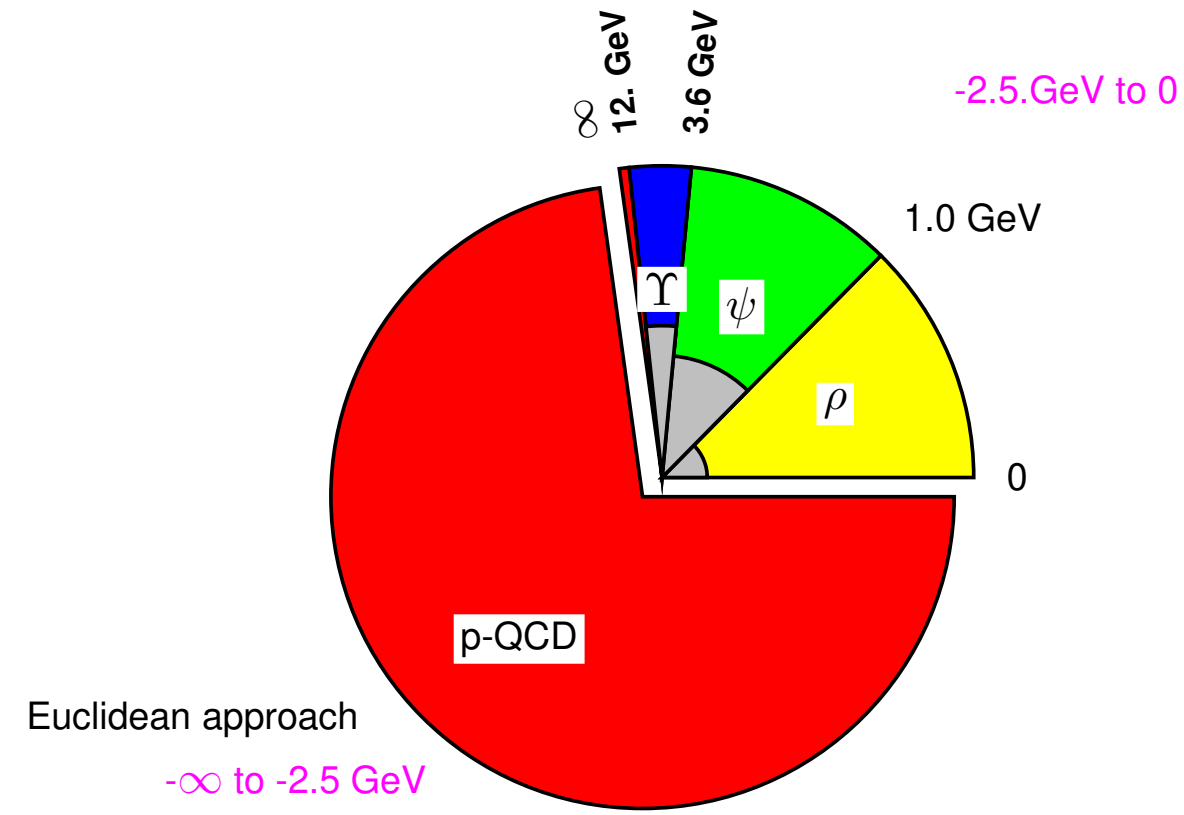
One may ask the question whether terms like Δ should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. Note that the Dyson resummation in general is not a systematic resummation of leading, sub-leading etc. terms as the RG resummation is.

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

e^+e^- data based approach:

Based on available data up to 12 GeV (FJ 00)





Shaded area: 10 $\times$ error (scaled up)

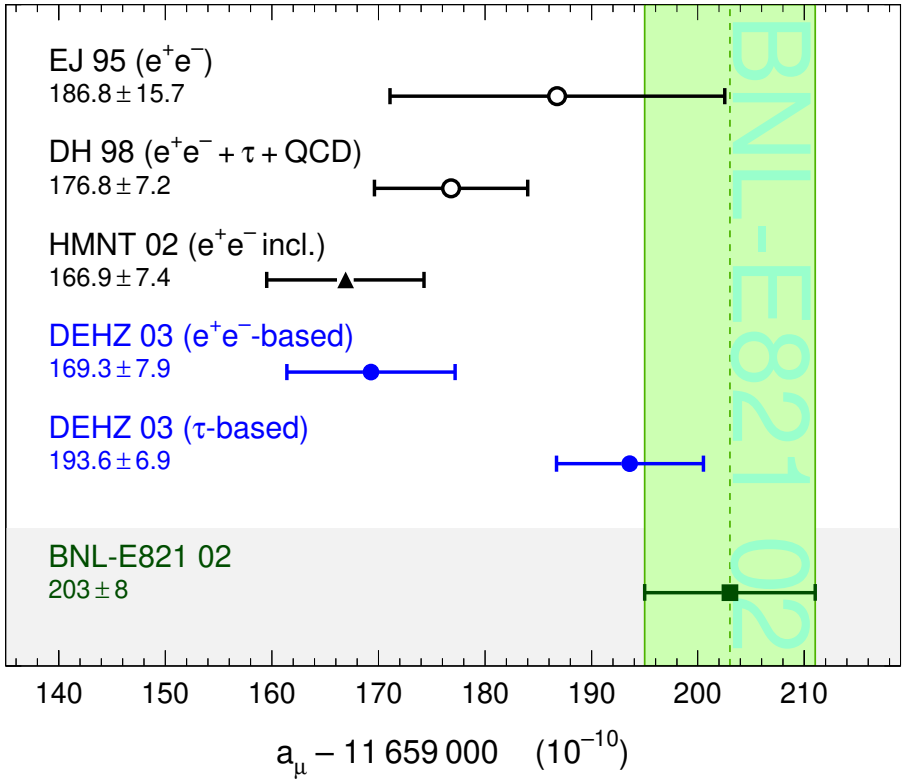
⑥ Status and Outlook

$a_\mu^{\rm had}$

Maximum confusion ! Likely only new experiments can settle the problems

$\Rightarrow$ **Photon tagging experinents at KLOE, BARAR**

a_μ status


EJ 95 (e+e−)
186.8 ± 15.7
DH 98 (e+e− + τ + QCD)
176.8 ± 7.2
HMNT 02 (e+e− incl.)
166.9 ± 7.4
DEHZ 03 (e+e−-based)
169.3 ± 7.9
DEHZ 03 (τ-based)
193.6 ± 6.9
BNL-E821 02
203 ± 8
BNL-E821 02
140
150
160
170
180
190
200
210
a_μ – 11 659 000 (10^−10)

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	$\delta\Delta\alpha$	$\delta \sin^2\Theta_f$	δM_W	Method	Ref.
0.0280	0.00065	0.000232	12.0	data $< 12.$ GeV	[EJ95]
0.02777	0.00017	0.000061	3.2	data < 1.8 GeV	[KS98]
0.02763	0.00016	0.000057	3.0	data < 1.8 GeV	[DH98b]
0.027572	0.000359	0.000128	6.6	[EJ95] + new data CMD & BES	[FJ01/03]
0.02737	0.00020	0.000071	3.7	Euclidean > 2.5 GeV	[FJ98/03]
0.027426	0.000190	0.000070	3.6	scaled data, pQCD 2.8-3.7, 5-∞	[MOR00]
0.027649	0.000214	0.000078	4.0	same but "exclusive"	[MOR00]
0.02761	0.00036	0.000128	6.6	data-driven incl. BES	[BP01]
-	0.00007	0.000025	1.3	$\delta\sigma \lesssim 1\%$ up to J/ψ	
-	0.00005	0.000018	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
world average		0.00017	39.0	PDG 2002	

Table 1: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ and its uncertainties in different evaluations. Two entries show what can be reached by increasing the precision of cross section measurements to 1%. δM_W in MeV.

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$:

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	δM_W (MeV)	Ref.	Method
0.00065	12.0	[EJ95]	data $< 12.$ GeV
0.00016	3.0	[DH98]	data < 1.7 GeV
0.00025	4.6	[FJ98]	Euclidean > 2.5 GeV
0.000197	3.6	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	4.0	[MOR00]	same but "exclusive"
0.000460	8.5	[BP00]	data
0.00007	1.3	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	22.0	Osaka 2000	
MegaW	6.0	TESLA	

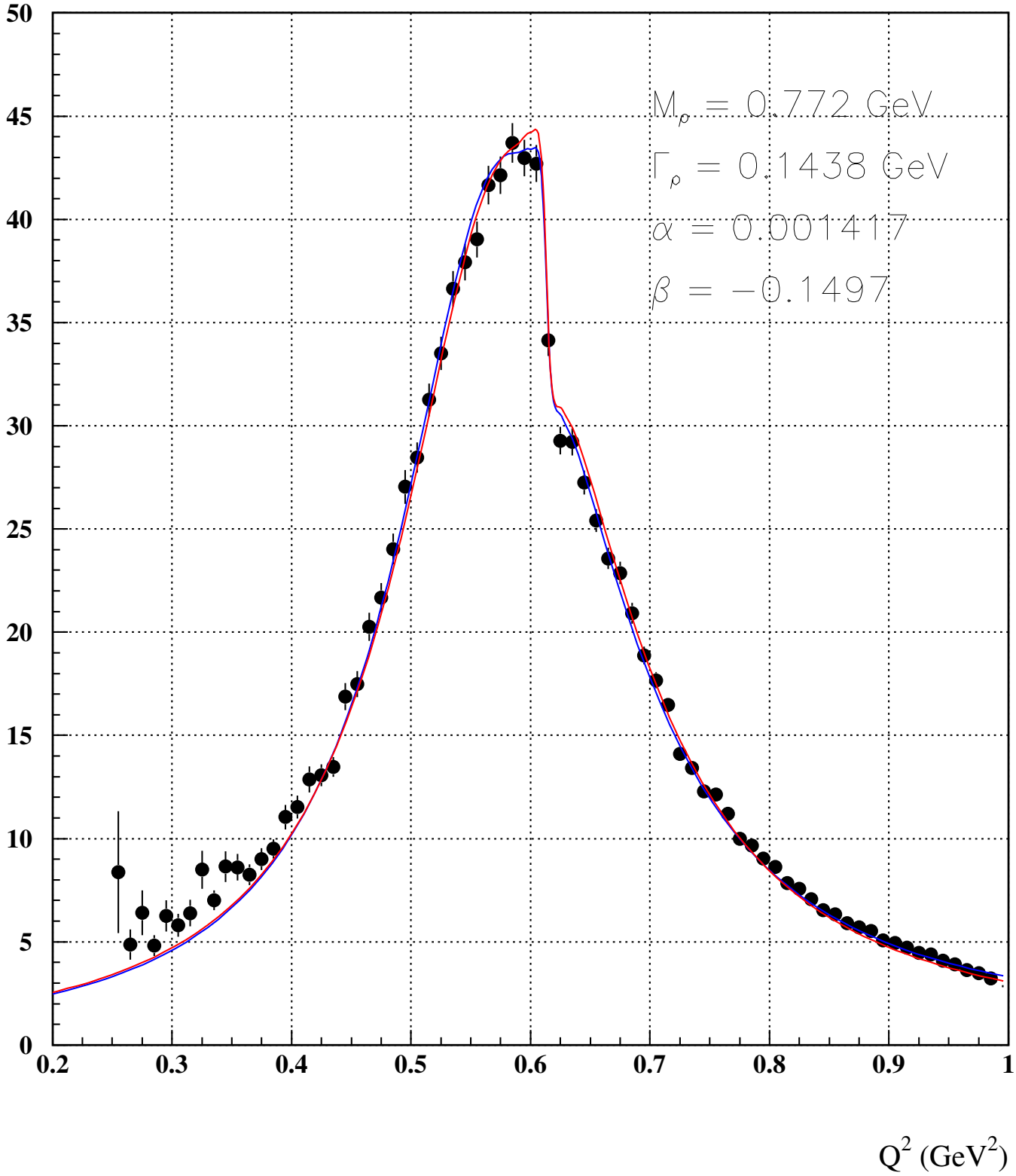
$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta \sin^2\theta$	Ref.	Method
0.00065	**0.000232**	**[EJ95]**	**data** $< 12.$ **GeV**
0.00016	**0.000057**	**[DH98]**	**data** < 1.7 **GeV**
0.00025	**0.000089**	**[FJ98]**	**Euclidean** > 2.5 **GeV**
0.000197	**0.000070**	**[MOR00]**	**rescaled data, pQCD 2.8-3.7, 5-**∞
0.000220	**0.000078**	**[MOR00]**	**same but "exclusive"**
0.000460	**0.000164**	**[BP00]**	**data**
0.00007	**0.000025**	$\delta\sigma \lesssim 1\%$ **up to** J/ψ	
0.00005	**0.000018**	$\delta\sigma \lesssim 1\%$ **up to** Υ	
WA	**0.000160**	**LEP+SLD**	
GigaZ	**0.000013**	**TESLA**	

Future:

- **KLOE (DAΦNE Frascati): expect a 1% measurement for $\sigma(e^+e^- \to hadrons)$ at $\sqrt{s} \lesssim$ 1 GeV**
  - **photon tagging method** **theory: $\sim$ 2%**
  - **$\pi^+\pi^-$ inv. mass spectrum** **theory: $\sim$ 0.5%**
  - ✗ **very important new masurement with different systematics (cross check of Novosibirsk measurements)**
  - ✗ **still a lot of theory efforts necessary !**
    - **large angle Bhabha ?**
    - **RC to photon tagging ?**
    - **photon radiation from hadrons ?**
  - ✗ **eagerly awaiting for fist release of KLOE data big attention for new input for $g-2$ now !**
- **CMD-2, SND (VEPP-2M Novosibirsk): CMD-2 final data analysis 0.6% systematics ? upgrade up to 2 GeV**
- **BES (BEPC Beijing): upgrade, higher luminosity, improved detector 2 to 5 GeV region**
- **SLAC/BABAR: $R(s)$ measurement in range $M_\Phi < E_{\rm cm} < M_{J/\psi}$**
  Also: radiative return $\pi\pi$ inv. mass spectrum at B factories (BABAR/SLAC, BELLE/KEK)

Pion Form Factor from ISR -73pb^{-1}



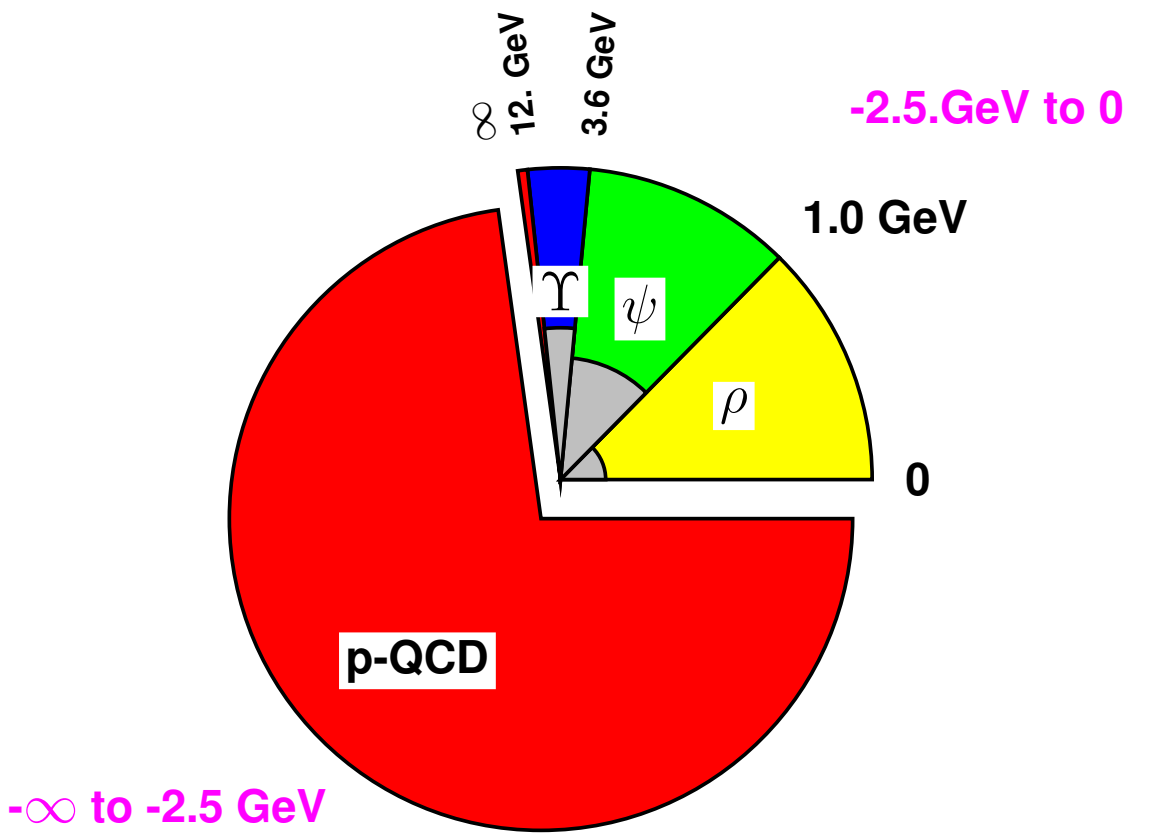
Data: KLOE Collaboration (A. Aloisio et al.)

Fits: KLOE/Frascati; CMD-2/Novosibirsk

Long term: need $\sigma(e^+e^- \to hadrons) \lesssim$ 1% up to 3.6 GeV both for $a_\mu^{\rm had}$ and for $\Delta\alpha^{\rm had}$:

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

"Adler function based" approach (EJKV99,FJ 00):



Shaded area: 10 × error (scaled up)

- **τ–charm factory**

  able to perform an energy scan between 2 and 3.6 GeV

  would satisfy requirements of future precision experiments

  g-2, GigaZ,...

- **Last but not least: need further progress in theory**
  - ✗ **more careful study of isospin breaking in τ–data vs. e^+e^-–data**
  - ✗ **constraints to $F_\pi(s)$ from χPT, unitarity and analyticity below the ρ**
  - ✗ **still a theoretical challenge: the hadronic light–by–light scattering contribution !**

Concerning τ–data vs. e^+e^-–data discrepancy: need more careful check of radiative corrections which have been applied !