❻ Supersymmetry

- In the SM enormous fine-tuning is required to keep m_{H} in the 100 GeV range
- Way out: couple bosons and fermions to protect $m_{\mathrm{H}} \rightarrow$ Supersymmetry
- the quadratic divergences of fermion- and sfermion-loops cancel
  $\Rightarrow$ Higgs remains light

Particle content:

- all known particles
- SUSY needs two Higgs doublets to give masses to up- and down-type particle
  $\Rightarrow$ 5 Higgs particles $\rightarrow$ Higgs section
- each fermion has a scalar partner (where left- and right-handed fermions have to be counted separately)
- each boson has a fermionic partner:
  - Two charginos $\chi^{\pm}_{1,2}$ ($m_{\chi^{\pm}_1} < m_{\chi^{\pm}_2}$), partner of $W^{\pm}, H^{\pm}$, mixed
  - Four neutralinos $\chi^{0}_{1,2,3,4}$ ($m_{\chi^{0}_1} < \ldots < m_{\chi^{0}_4}$), partner of γ, Z, h, H, mixed
  - gluinos ($\tilde{g}$), gravitino ($\tilde{G}$)

However $m_{\text{Particle}} \neq m_{\text{Partner}}$ $\Rightarrow$ SUSY is broken

Need $m_{\text{SUSY}} < 1\text{TeV}$ to solve hierarchy-problem

In general > 100 new free parameters $\Rightarrow$ have to make some assumptions how they are correlated

SUSY-breaking parameters in the minimal model (MSSM):

- $U(1), SU(2), SU(3)$ Gaugino-masses $M_{1,2,3}$
- Higgsino mass-parameter μ
- Scalar-masses m_i (or universal m_0)
- Sfermion-Higgs couplings A_i, B_i

R-parity $R = (-1)^{2S+L+3B}$

- SUSY-particles only in pairs
- lightest SUSY particle (LSP) is stable

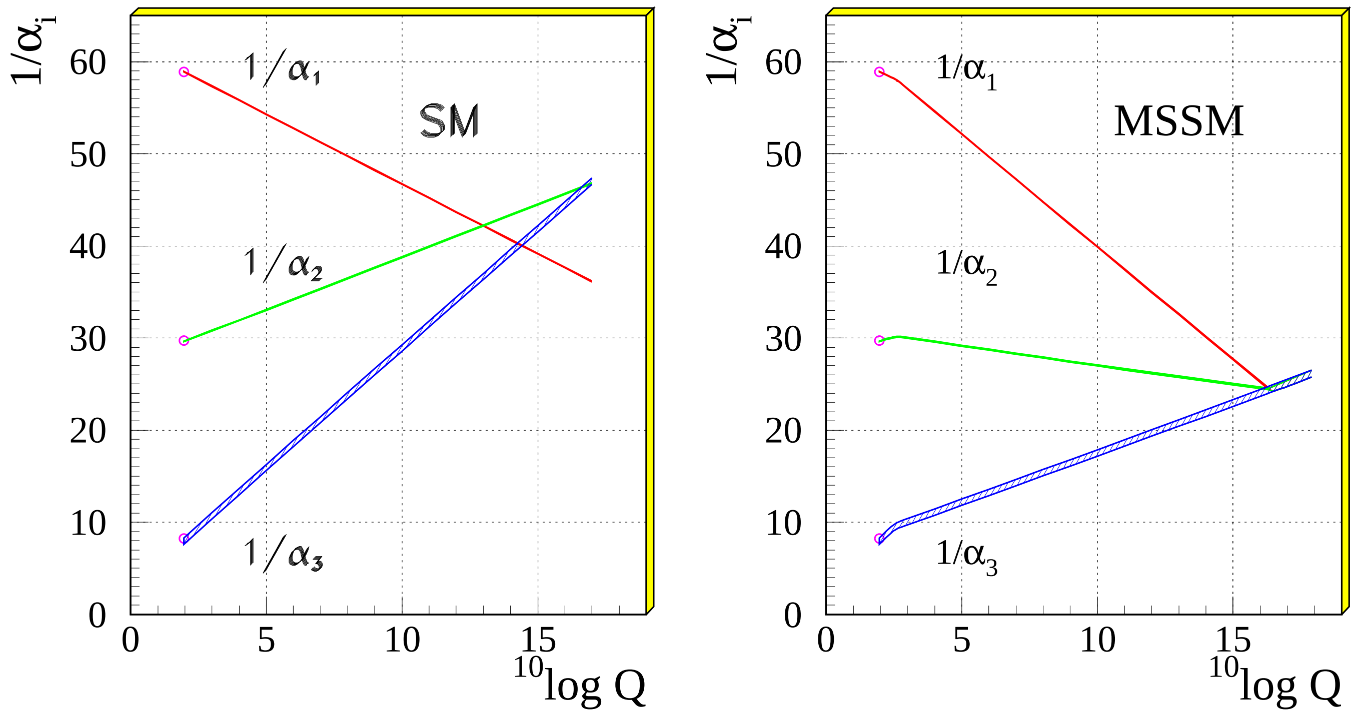
➠ Excellent dark matter candidate

R-parity can also be broken

- very rich phenomenology
- however care has to be taken to avoid proton decay

Other virtues of SUSY:

- SUSY can be a new source of CP-violation
  ➠ may explain the matter/anti-matter asymmetry in the universe
- String theories are the only known way to connect gravity with quantum mechanics
  ➠ all string theories are supersymmetric
- SUSY enables unification of forces at a high scale



SUSY breaking schemes

Gravity mediated SUSY breaking

- SUSY is broken at a high scale by gravitational interaction to a hidden sector
- Gauge coupling unification at the GUT scale ($m_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$) possible

➨ Common gaugino mass $m_{1/2}$ at m_{GUT}
$\Rightarrow \frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}$ at the weak scale

- often also universal scalar mass m_0 assumed
- slepton masses:

$$
\begin{aligned}
M^2_{\tilde{\nu}} &= m_0^2 + 0.77 M_2^2 + 0.5 m_Z^2 \cos 2\beta \\
M^2_{\tilde{\ell}_L} &= m_0^2 + 0.77 M_2^2 - 0.27 m_Z^2 \cos 2\beta \\
M^2_{\tilde{\ell}_R} &= m_0^2 + 0.22 M_2^2 - 0.27 m_Z^2 \cos 2\beta
\end{aligned}
$$

- squark masses similar with M_3^2 term
- L-R sfermion mixing $\propto m_f(A_f - \mu \tan\beta)$ only relevant for 3rd generation
- chargino mass matrix

$$
\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}
$$

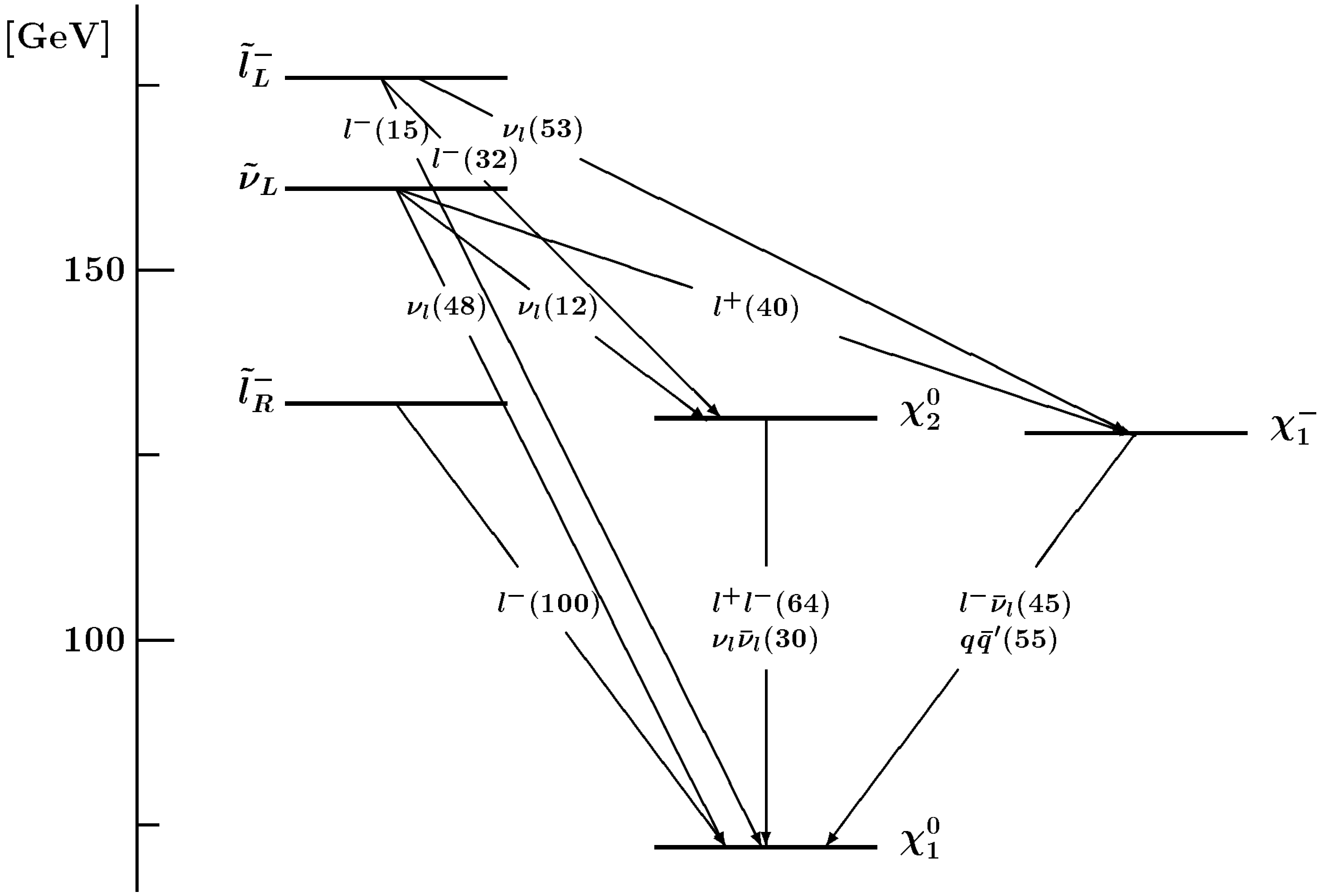
detailed properties of $\chi^\pm_{1,2}$ (gaugino-,Higgsino-like) depend on values of parameters

- neutralinos similar

“Typical” mass spectrum

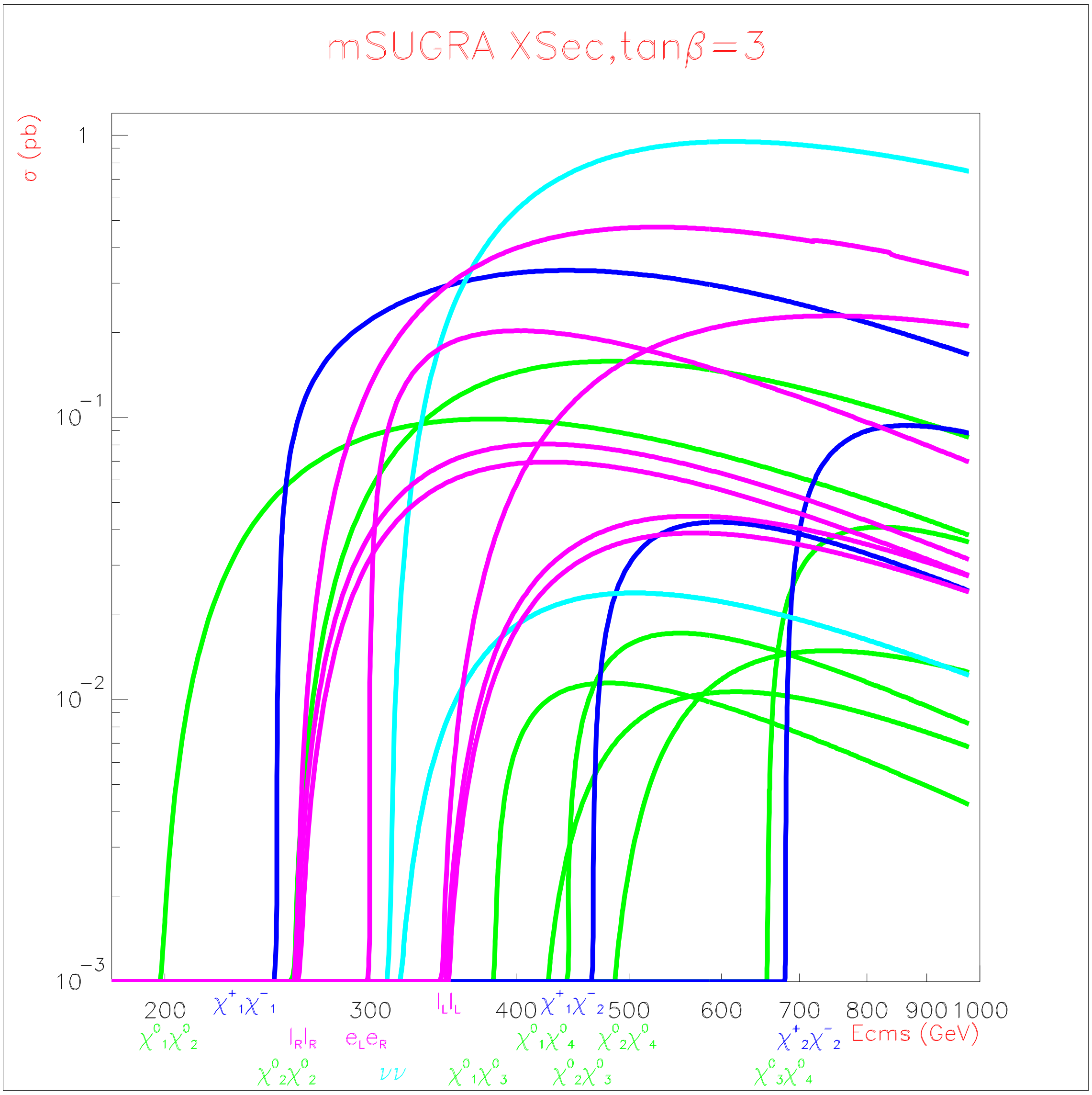
$(m_0 = 100\,\text{GeV}, m_{1/2} = 200\,\text{GeV})$

$$m_{\chi_1^0} \sim 70\,\text{GeV}$$
$$m_{\chi_1^\pm,\chi_2^0} \sim 130\,\text{GeV}$$
$$m_{\chi_2^\pm,\chi_{3,4}^0} \sim 350\,\text{GeV}$$
$$m_{\tilde{\ell}} \sim 150\,\text{GeV}$$
$$m_{\tilde{q}} \sim 430\,\text{GeV}$$



- of course all moves with m_0, $m_{1/2}$
- $m_{\tilde{t}_1}$ can be moved arbitrarily by changing A

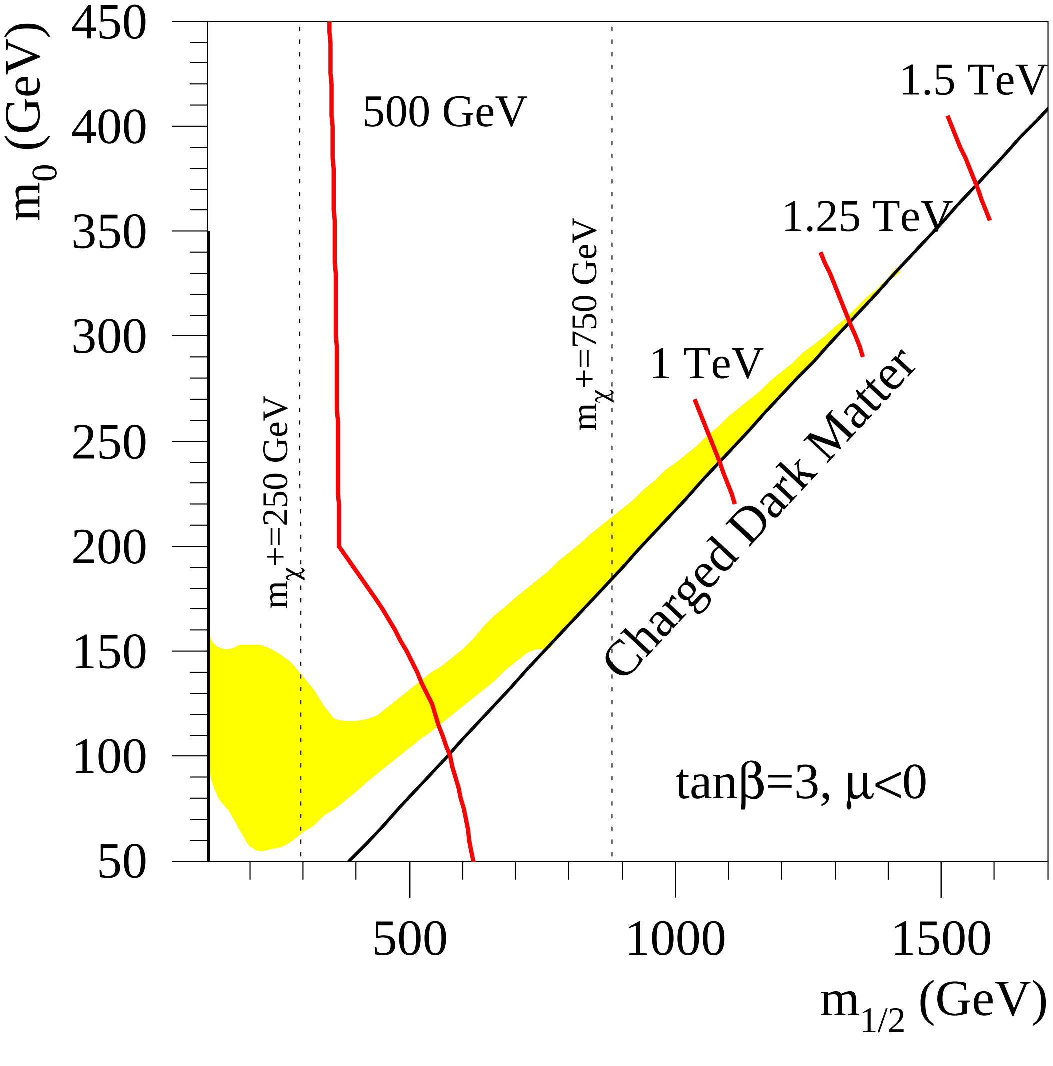
typical cross sections:



- all channels have cross sections $\sim 10 - 1000\,\mathrm{fb}$
- all channels have visible decays of at least 50%

Where do we expect SUGRA?

- naturalness suggests $\tilde{m} < 1\,\text{TeV}$, however only logarithmic dependence
- recent analysis looks into correct neutralino density as dark matter



- “natural” region predicts SUSY well below 500 GeV
- however some tails due to coannihilation
- mostly covered at 1 TeV

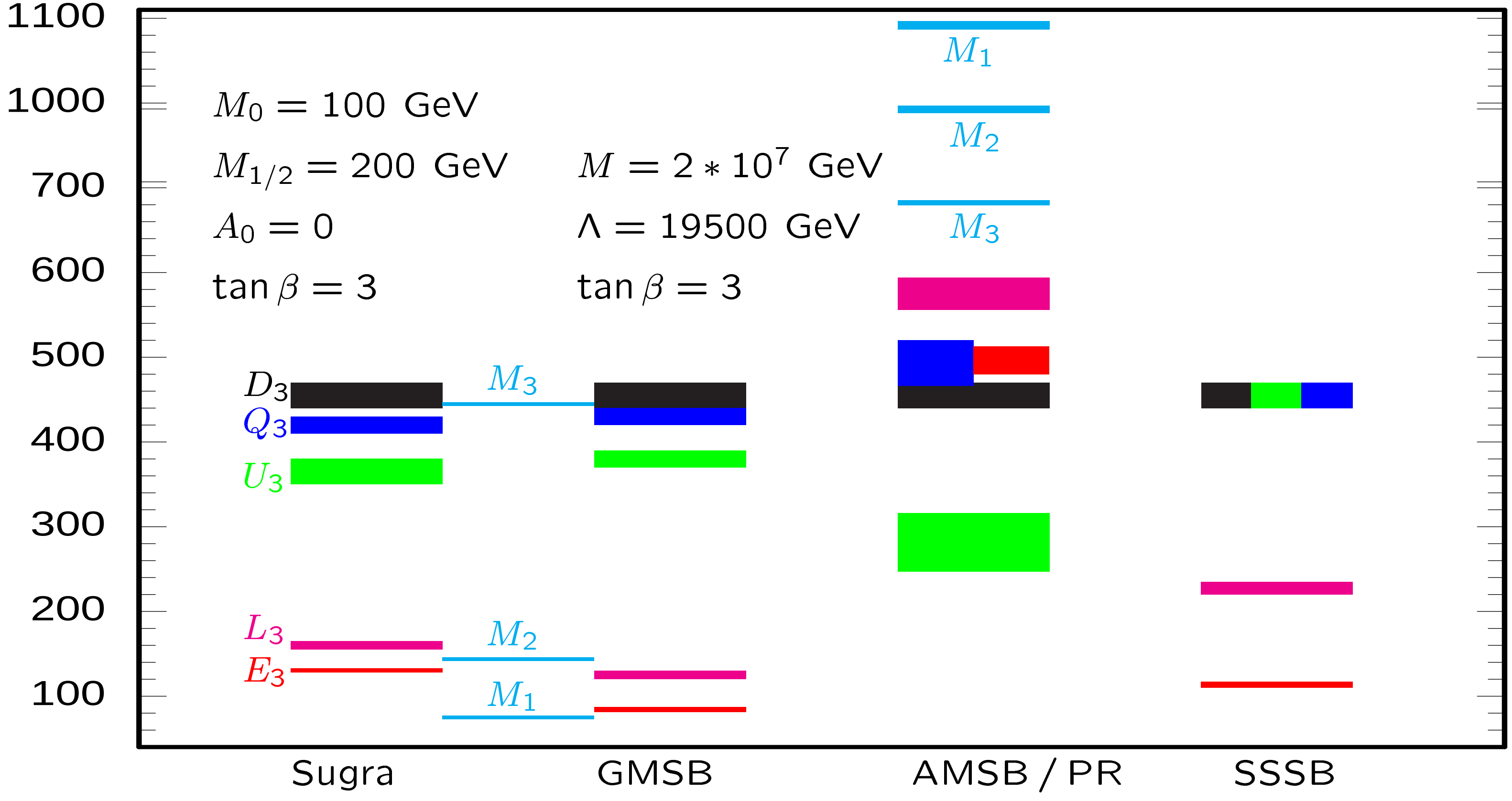
Gauge mediated SUSY breaking

- SUSY is broken at intermediate scales ($10^3 - 10^8$ GeV) by gauge interactions involving messengers between the visible and the hidden sector
- main free parameters:

$M_{\rm mess}$	messenger mass scale
$N_{\rm mess}$	number of messenger generations
Λ	universal soft braking scale
$\tan\beta$	
sign(μ)	

- main differences to SUGRA
  - very light gravitino $\sim$ eV
  - NLSP either χ_1^0 with $\chi_1^0 \to \tilde{G}\gamma$ or $\tilde{\ell}$ with $\tilde{\ell} \to \tilde{G}\ell$ (if mixing is large in 2nd case, $\tilde{\tau}_1$ is NLSP) in both cases NLSP lifetime can be significant
  - sfermion masses $\propto \alpha_i$, $i =$QED,QCD $\Rightarrow$ larger mass splitting between sleptons and squarks

Gaugino and Sfermion Mass Parameters



The complementarity LC/LHC

LHC:

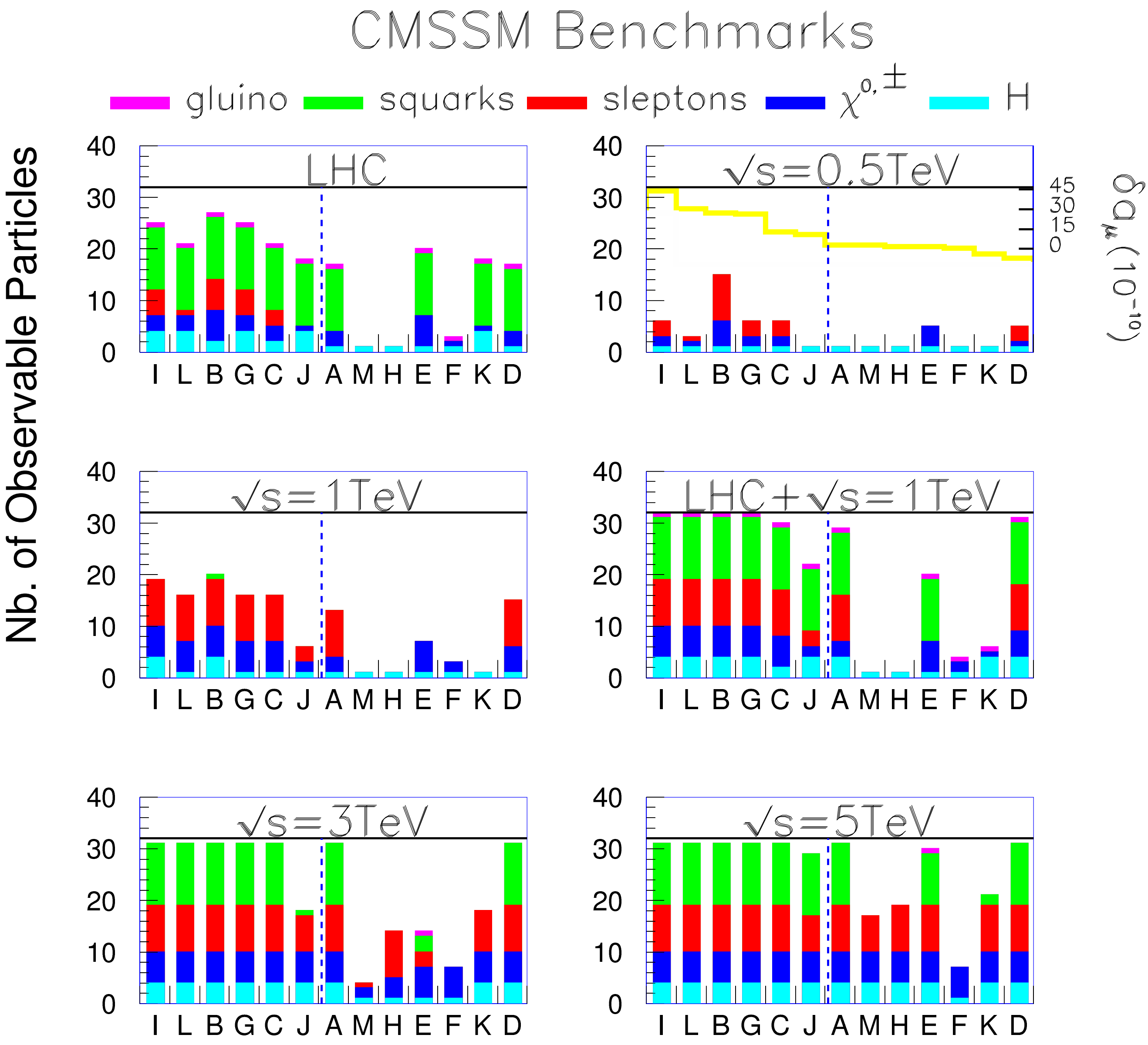
- Mass reach $\mathcal{O}(1\,\text{TeV})$
- Squarks are produced strongly ⇒ huge cross section
- Sleptons and gauginos are produced weakly or in cascades ⇒ maybe difficult to see
- LSP cannot be reconstructed completely due to missing information ⇒ mainly sensitive to mass differences

LC:

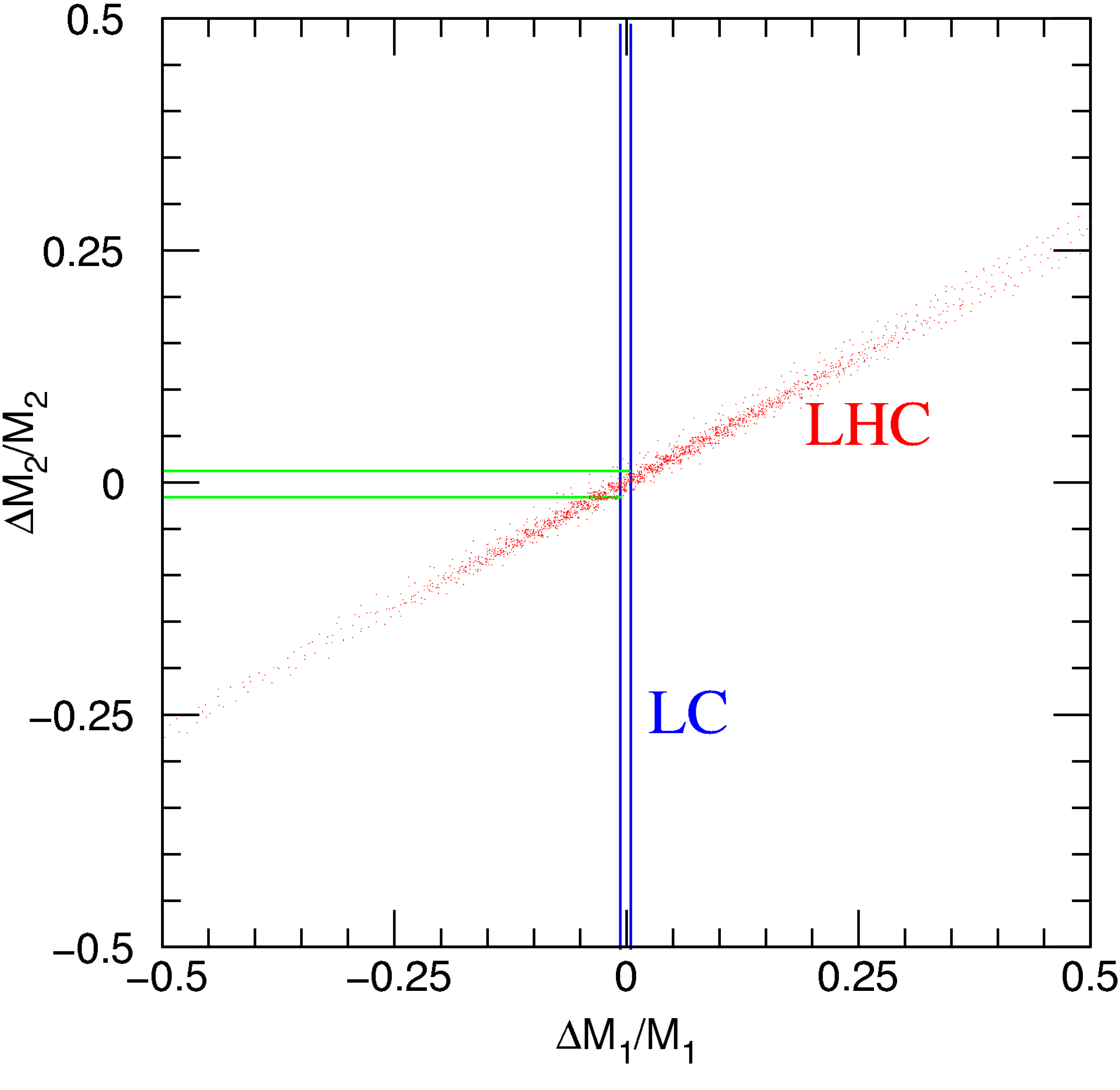
- all SUSY processes within mass reach have similar cross section
  ⇒ all particles can be cleanly reconstructed
- LSP can be reconstructed from kinematic quantities ⇒ all masses can be measured absolute
- all particles are produced in electroweak processes that can be calculated accurately ⇒ particle couplings can be measured
- squarks and gluinos are probably too heavy to be produced at LC

LC+LHC:

If SUSY light enough all masses and the lepton and gaugino couplings can be measured with good precision



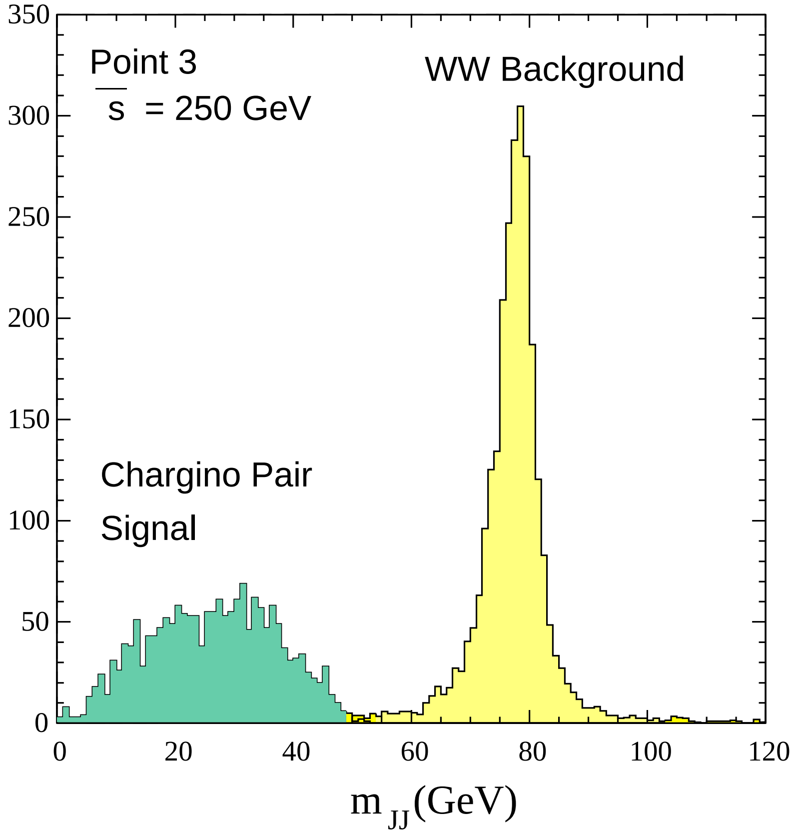
Example: $m(\chi_1^0) - m(\chi_2^0)$ from LHC combined with $m(\chi_1^0)$ from LC



Typical signals in SUSY

SUGRA signals:

- due to stable LSP always missing mass and missing p_t
- most simple decay $\tilde{f} \to f\chi_1^0$:
  two identical lept. or jets and missing mass and p_t
- decay $\chi_i \to ff'\chi_j$:
  four leptons/jets and missing mass and p_t
- in general cascade decays can have many leptons+jets
- good detector resolution separates SUSY-signals from known physics



GMSB signals:

- if the NLSP lifetime is large: like SUGRA
- due to NLSP decay the missing quantities are smaller
- this is compensated by the additional visible γ/lepton

Signals with R-parity violation:

- The LSP decays into ordinary particles
  - ➠ also LSP pair productions is visible
  - ➠ the LSP needs not to be neutral
- SUSY breaking can be in any scheme
- experimentally the missing mass/energy is replaced by LSP-reconstruction
  $\Rightarrow$ similar efficiencies

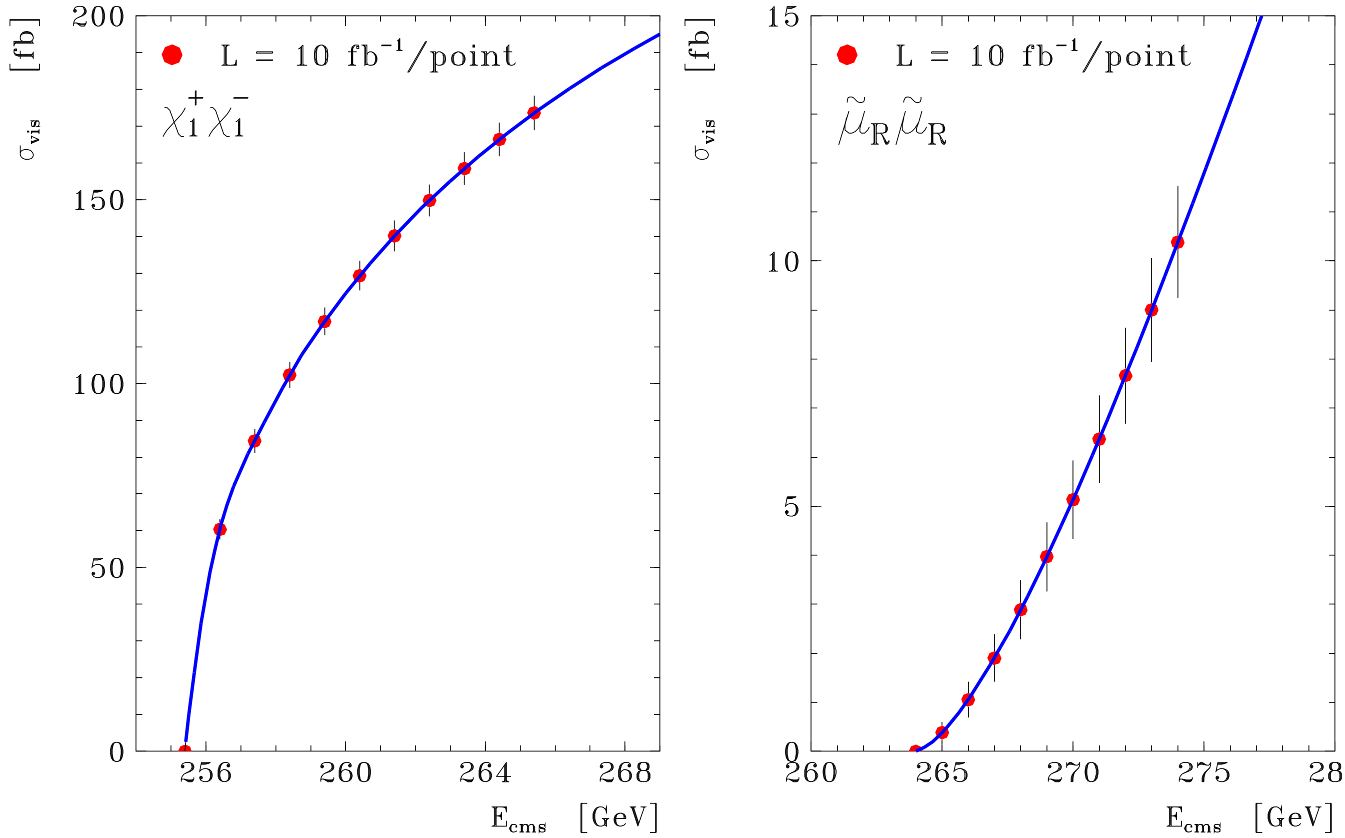
Mass measurements

Two principle methods:

- threshold scan
- reconstruction

Threshold scan:

- gauginos: threshold suppression $\propto \beta$
  $\Rightarrow$ good precision
- sfermions: threshold suppression $\propto \beta^3$
  $\Rightarrow$ precision relatively worse
- $\tilde{e}, \tilde{\nu}_1$: mixture of β^3 from s-channel Z,γ- and β from t-channel χ-exchange $\rightarrow$ model dependent



Reconstruction:

Decay of scalar particle $\tilde{\ell} \to \ell\chi$:

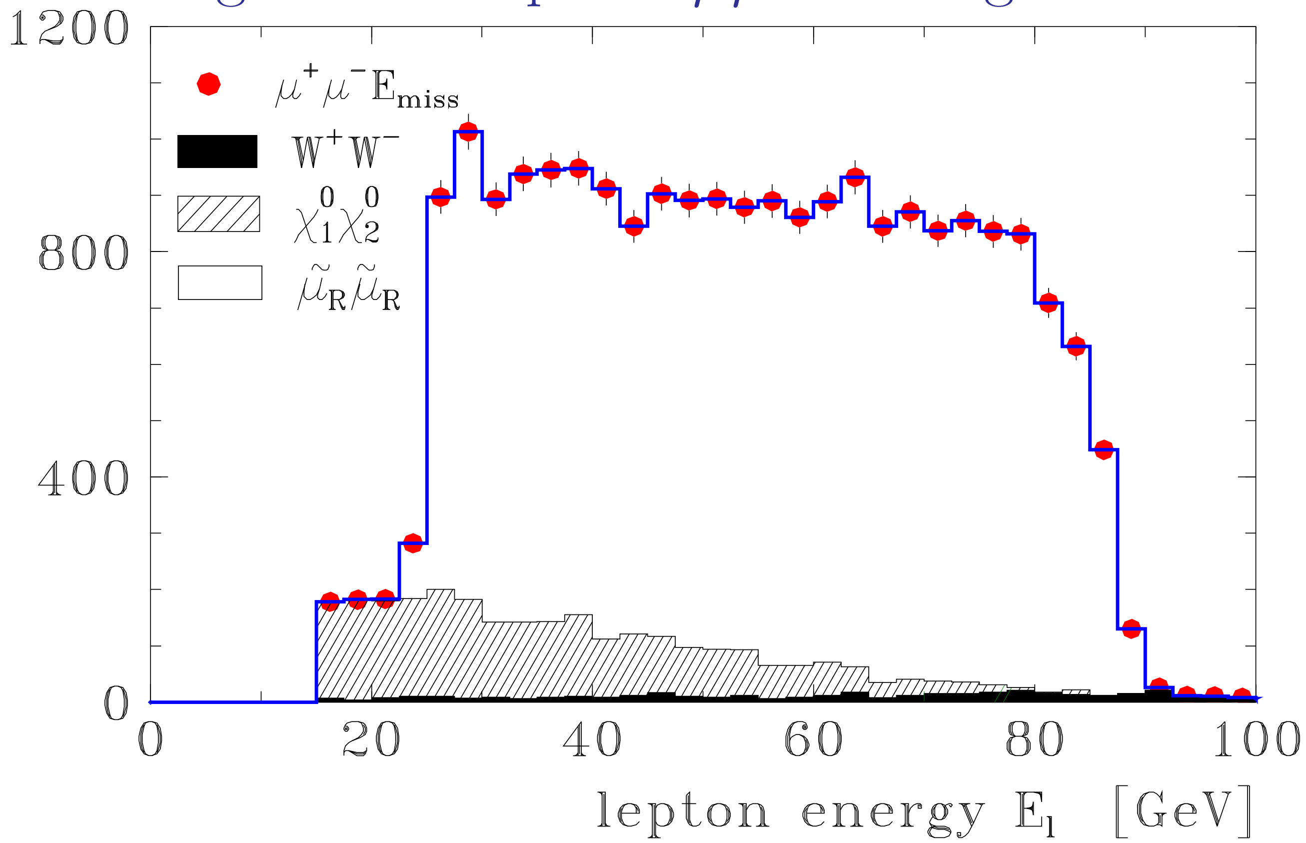
Flat energy distribution of ℓ between

$$\frac{E_\ell}{E_{\rm beam}} = \frac{1}{2}(1 \pm \beta)\left(1 - \frac{m_\chi^2}{m_{\tilde{\ell}}^2}\right)$$

$\Rightarrow m_\chi$ and $m_{\tilde{\ell}}$ can be obtained in a model independent way

e.g. $\mathrm{e^+e^-} \to \tilde{\mu}_R^+\tilde{\mu}_R^- \to \mu^+\chi_1^0\mu^-\chi_1^0$:

low background sample in $\mu\mu$+missing mass



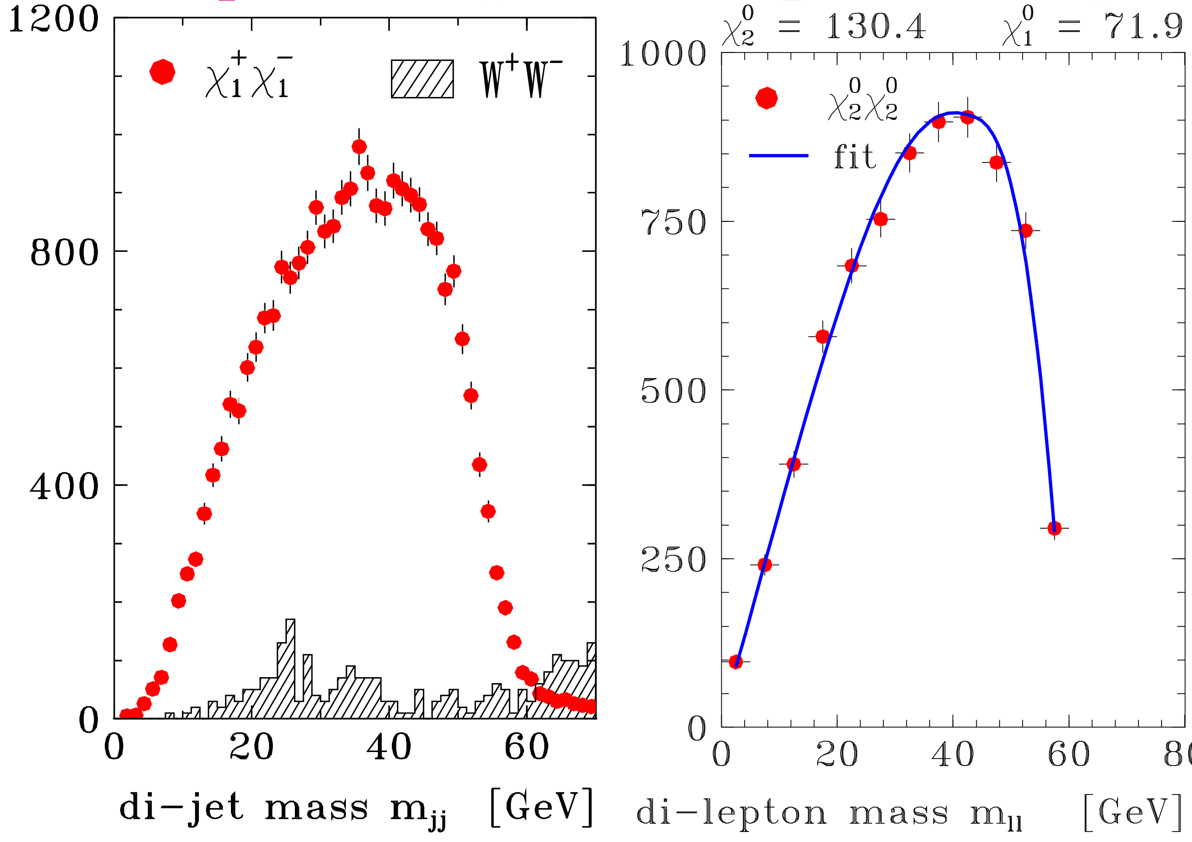
$\Rightarrow$ Can measure $m_{\chi_1^0}$ and $m_{\tilde{\mu}_R}$ to 0.3% with $\sqrt{s} =$ 320 GeV, $\mathcal{L} = 160\,\mathrm{fb}^{-1}$

Gauginos decay in 3-prongs and have spin $\Rightarrow$ mass determination from gaugino production not so easy

However for decay chain $\chi' \to f f' \chi$ $m(f f')$ gives accurate measurement of mass difference $m_{\chi'} - m_{\chi}$

Measurements can be done with gauginos from direct production and decays

$\chi_1^{\pm} \to q\bar{q}'\chi_1^0$ $\chi_2^0 \to \ell^+\ell^-\chi_1^0$



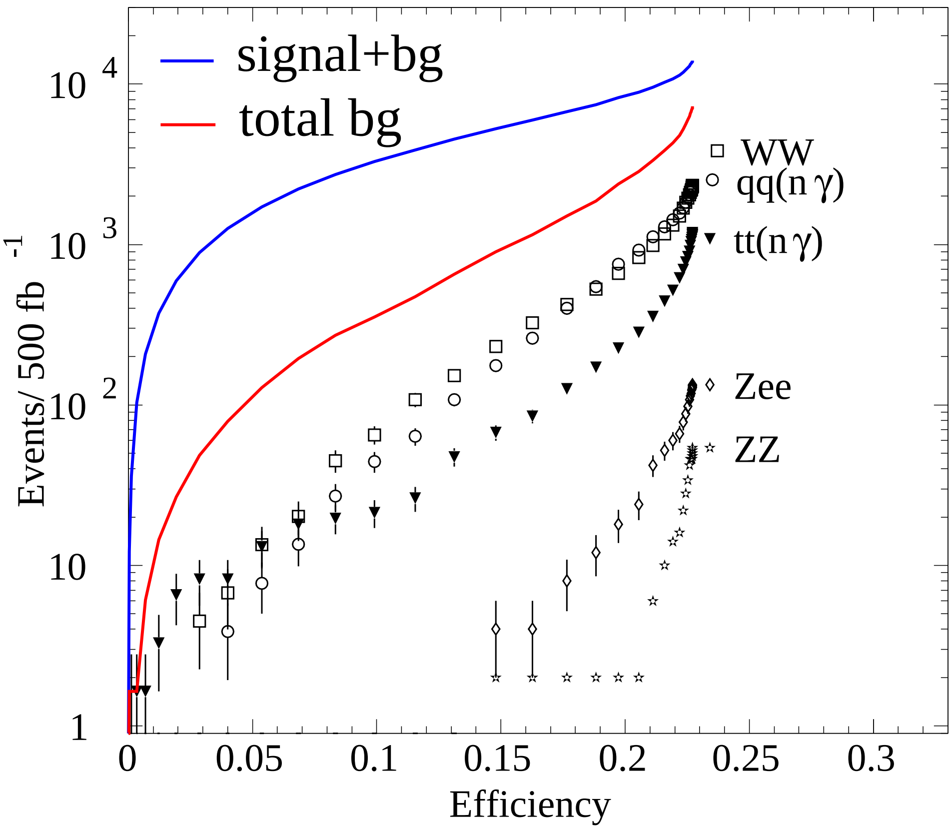
Both mass differences can be measured to 50 MeV

Study of stop production

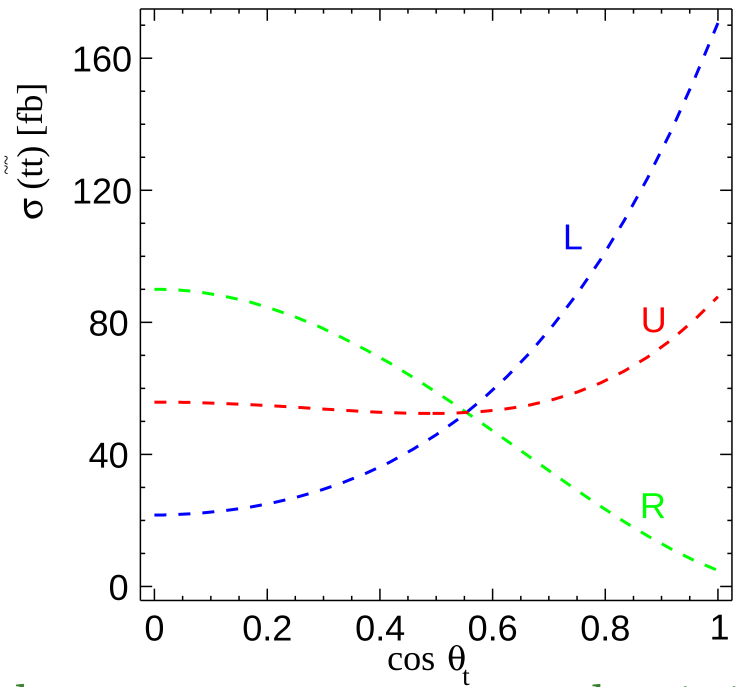
- due to mixing effects $\tilde{t}_1$ can be very light ($\tilde{t}_2$ then very heavy)
- $\tilde{t}_1$ decays into $\chi_1^+ b$ if kinematically allowed, otherwise into $\chi_1^0 c$

Analysis with 180 GeV $\tilde{t}_1 \to \chi_1^0 c$:

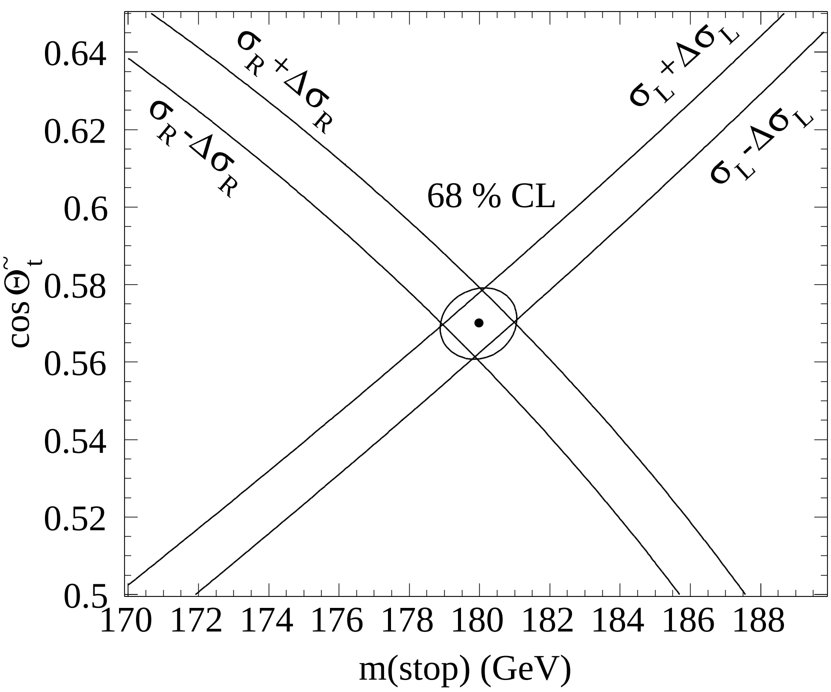
- iterative discriminant analysis using event shapes and jet energies
- $10 - 20\%$ efficiency with $\sim 90\%$ purity can be achieved



- cross section depends on $\tilde{t}_1$ mass and $\tilde{t}_L - \tilde{t}_R$ mixing angle
- dependence different for different beam polarization



Can be used to measure mass and mixing angle:



$$\delta m_{\tilde{t}_1} \approx 1\,\mathrm{GeV},\ \delta \cos\theta_{\tilde{t}} \approx 0.01$$

Analyzes of charginos and neutralinos

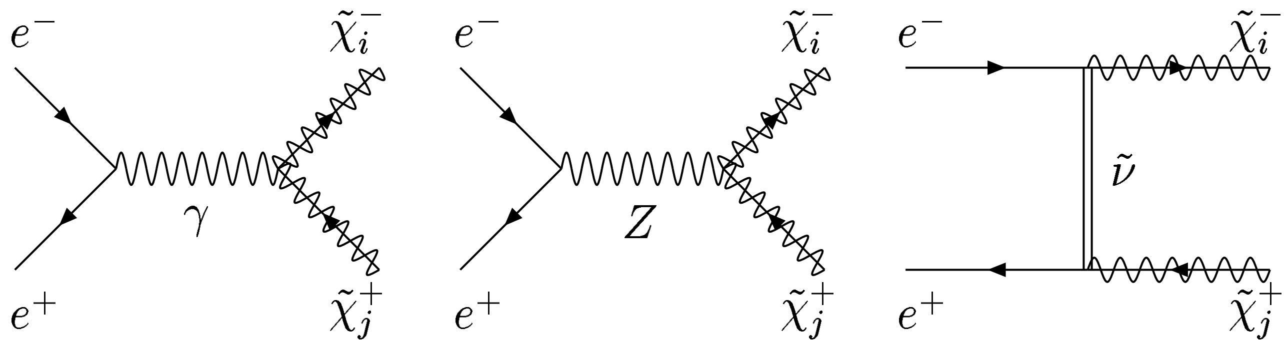
Chargino mass matrix is given by

$$\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

Matrix not symmetric $\Rightarrow$ need two mixing angles $\Phi_{L,R}$ for left- and right-handed states

Mixing angles and and chargino masses are (complicated) functions of M_2, μ and $\tan\beta$

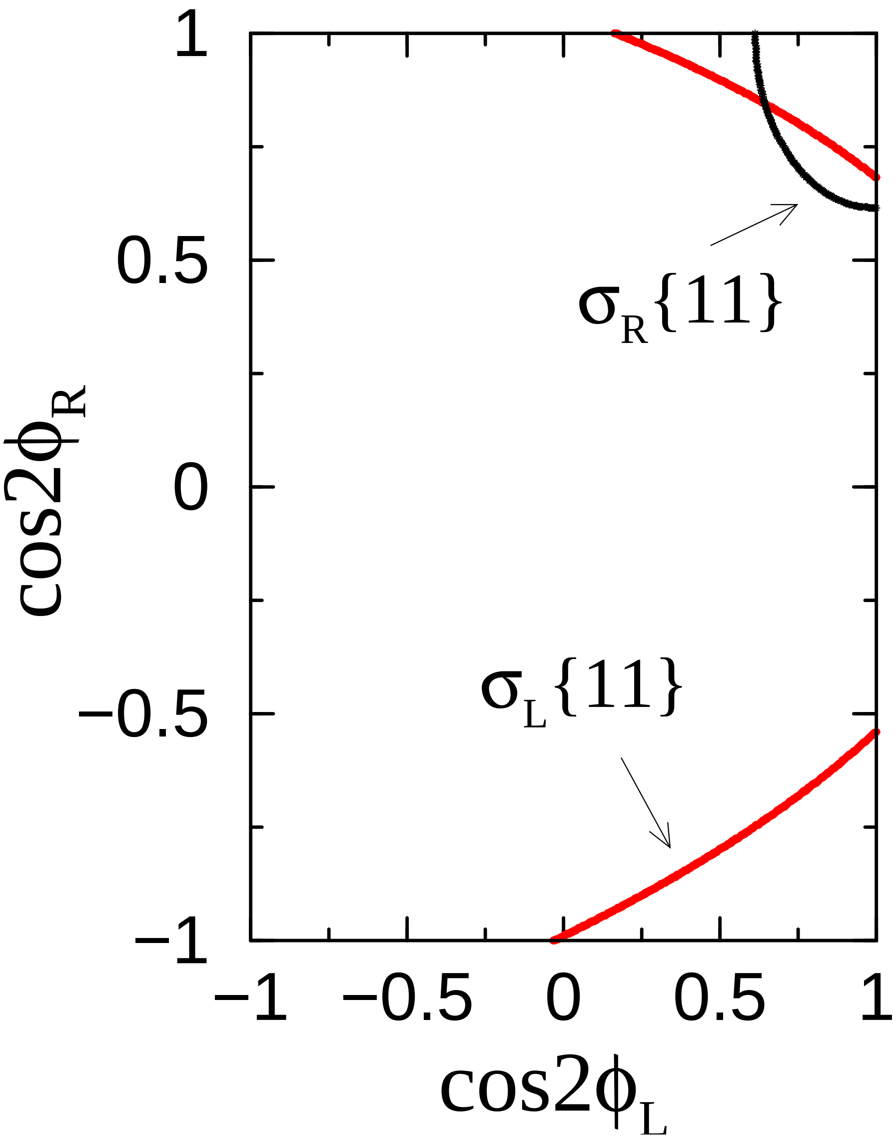
Chargino production via Z,γ s-channel and $\tilde{\nu}_e$ t-channel exchange



Need to know sneutrino mass to calculate cross section

Cross sections of $\mathcal{O}(100\,\mathrm{fb})$ $\Rightarrow$ expect several$\times 10^4$ events per channel

Even if only $\chi_1^+\chi_1^-$ channel is accessible (and $m_{\tilde{\nu}}$ known) can reconstruct mixing angles from polarized cross section

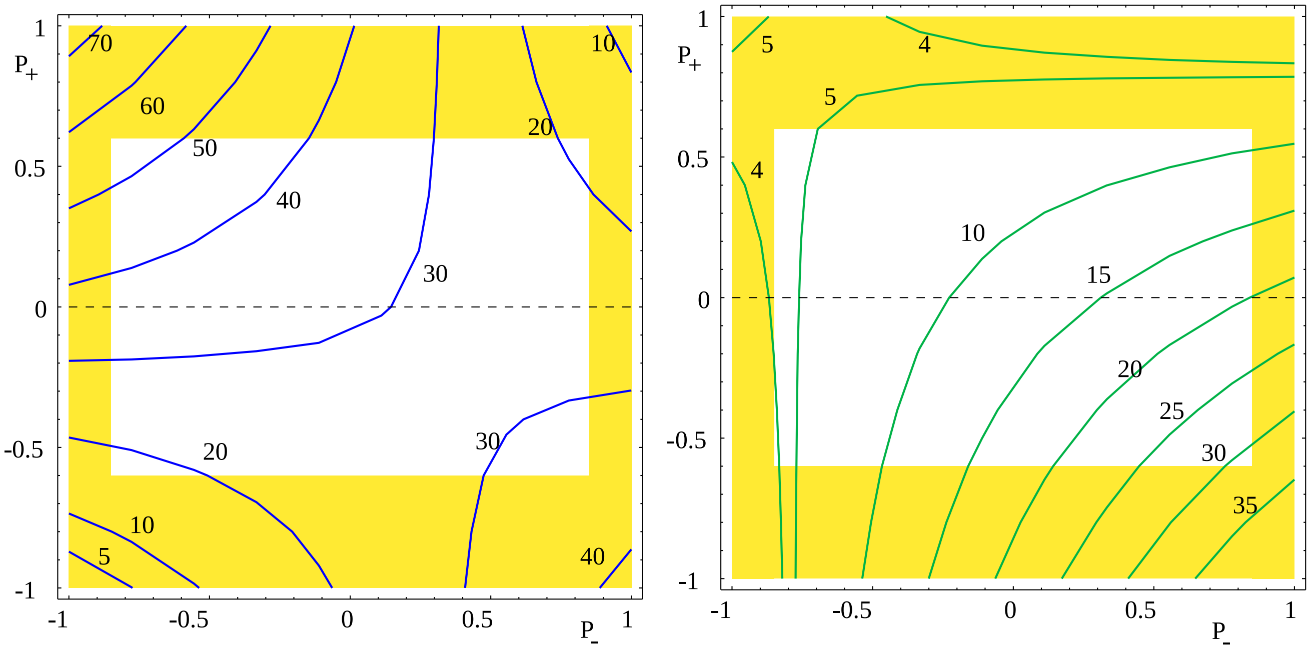


$\Rightarrow M_2$, μ and $\tan\beta$ can be determined from $m_{\chi_1^\pm}$ and $\chi_1^+\chi_1^-$ polarized cross sections

Neutralino production

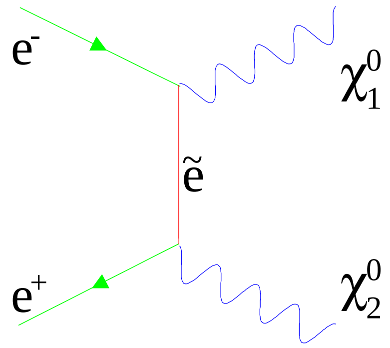
- situation more complicated
  - 4×4 mixing matrix
  - $\chi_1^0 \chi_1^0$ channel not accessible if R-parity conserved
- access to M_1
- since t-channel exchange of $\tilde{e}_L$ and $\tilde{e}_R$ compete with s-channel, positron polarization gives really independent information

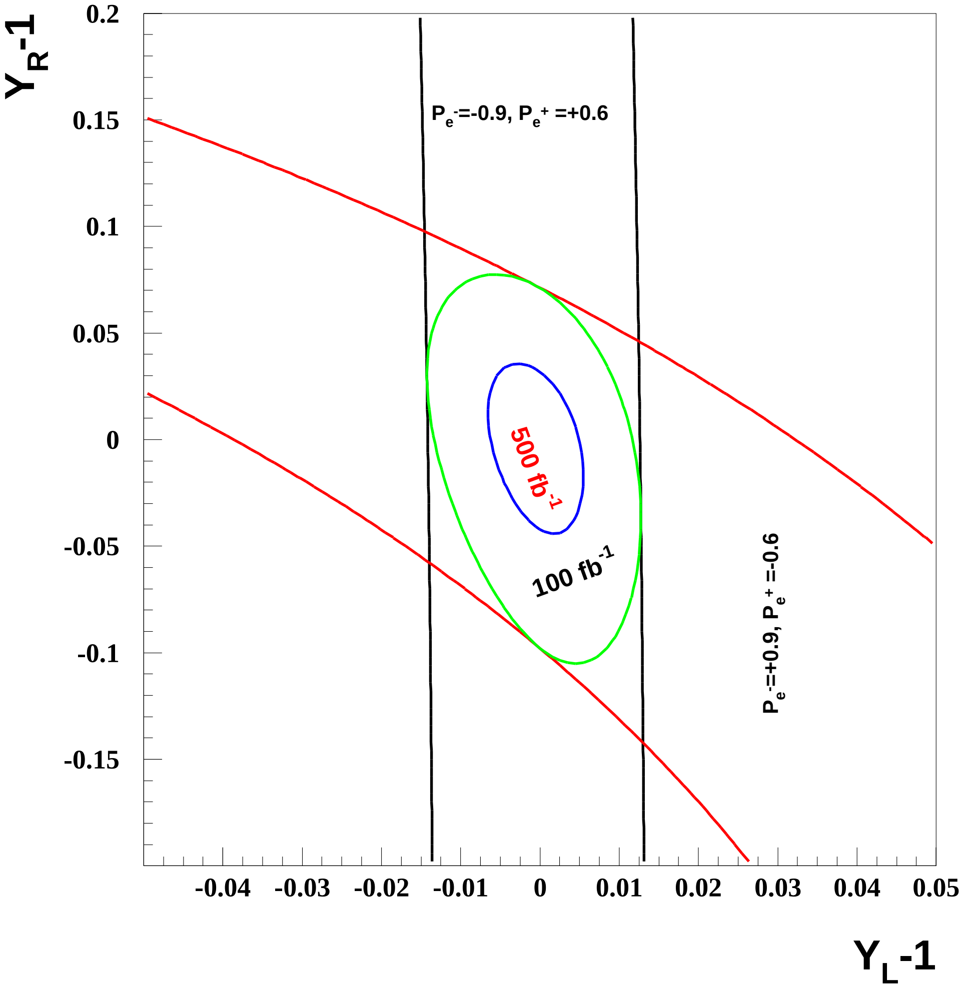
$\chi_1^0 \chi_2^0$ cross section for different SUSY parameters



To prove that the new particles are really SUSY it has to be shown that the couplings amongst the superpartners are the same as for corresponding SM particles

This can be done on the percent level e.g. with $\chi^0_1\chi^0_2$-pair production once M_1, M_2, μ are known from the gaugino masses and chargino cross sections





Reconstruction of SUSY parameters

- measure masses and cross sections of SUSY particles and Higgses
- fit SUSY parameters at the weak scale
- extrapolate to GUT scale using RGEs
- bottom up approach that needs no model assumptions
- get model independent prediction at high scales
- example: SUGRA with $\tan\beta$=3, m_0=100 GeV, $m_{1/2}$=200 GeV, A_0=0 GeV, sign$(\mu) = -$ (excluded now by LEP, but general features should not change)

Accessible particles and mass error for the simulated point

Particle	Mass (GeV)	Error(GeV)
$\tilde{e}_L$	173.0	0.18
$\tilde{e}_R$,$\tilde{\mu}_R$	131.6	0.09
$\tilde{\nu}_e$	157.5	0.07
$\tilde{\mu}_L$	173.0	0.3
$\tilde{\nu}_\mu$	157.5	0.2
$\tilde{\tau}_1$	130.8	0.6
$\tilde{\tau}_2$	173.5	0.6
$\tilde{\nu}_\tau$	157.5	0.6
χ_1^0	76.6	0.05
χ_2^0	142.8	0.07
χ_3^0	343.8	0.3
χ_4^0	349.9	0.6
$\chi_1^\pm$	142.9	0.035
$\chi_2^\pm$	352.6	0.25
h^0	97.7	0.05
H^0	466.7	1.5
A^0	466.7	1.5
H^+	473.3	1.5
$\tilde{t}_1$	353.9	0.6
$\tilde{q}$	$\sim$ 450	1.0
$\tilde{g}$ (LHC)	486.5	10.0

Used cross sections and errors

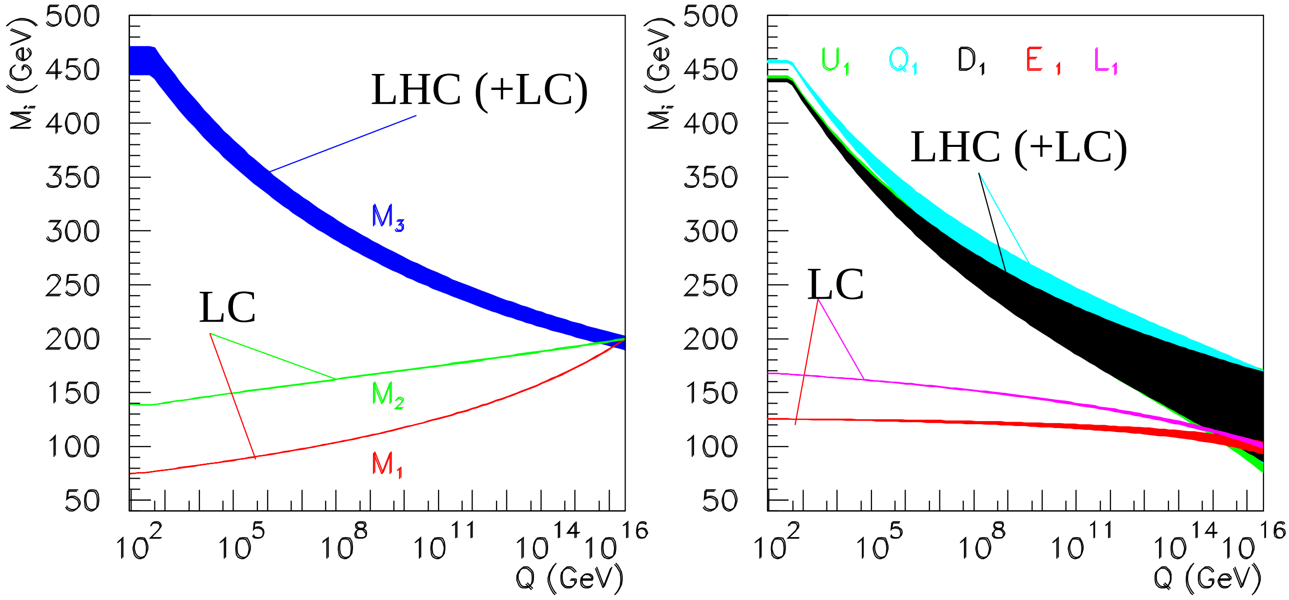
Process	$\sigma(e^-_L e^+_R)(fb)$	Error (fb)	$\sigma(e^-_R e^+_L)$(fb)	Error (fb)
$\tilde{t}_1\tilde{t}_1$	16.7	0.41	21.1	0.46
$\tilde{t}_1\tilde{t}_2$	4.55	0.21	3.41	0.18
$\tilde{t}_2\tilde{t}_2$	3.80	0.19	0.64	0.08
$\tilde{b}_1\tilde{b}_1$	18.6	0.43	1.42	0.12
$\tilde{b}_1\tilde{b}_2$	0.21	0.05	0.16	0.04
$\tilde{b}_2\tilde{b}_2$	0.69	0.08	2.28	0.15
$\tilde{\tau}_1\tilde{\tau}_1$	18.9	0.43	66.37	0.81
$\tilde{\tau}_1\tilde{\tau}_2$	0.67	0.08	0.50	0.07
$\tilde{\tau}_2\tilde{\tau}_2$	77.64	0.88	19.13	0.44
$\tilde{\nu}_\tau\tilde{\nu}_\tau$	20.0	0.45	15.0	0.39
H^+H^-	2.10	0.15	9.73	0.31
A^0h^0	1.22	0.11	0.91	0.10
A^0H^0	0.52	0.07	0.39	0.06
Z^0h^0	2.41	0.16	1.81	0.13
Z^0H^0	2.14	0.15	1.60	0.13

Reconstructed parameters at the weak scale

Parameter	True Value	Fit Error
M_1	74.64	0.15
M_2	138.65	0.10
M_3	467.55	12.1
A_τ	-128.7	43
A_b	-586.5	41
A_t	-358.7	2.5
$\mathrm{m}^2(H_1)$	27 646	601
$\mathrm{m}^2(H_2)$	-100 750	146
$\mathrm{m}^2(e_R)$	15 785	17
$\mathrm{m}^2(e_L)$	28 140	19
$\mathrm{m}^2(d_R)$	193 876	624
$\mathrm{m}^2(u_R)$	195 779	624
$\mathrm{m}^2(q_L)$	209 047	457
$\mathrm{m}^2(\tau_R)$	15 745	156
$\mathrm{m}^2(\tau_L)$	28 120	139
$\mathrm{m}^2(b_R)$	193 547	806
$\mathrm{m}^2(t_R)$	120 582	657
$\mathrm{m}^2(Q_{3L})$	171 616	513
$\tan\beta$	3.0	0.01
μ	335.7	1.3

Extrapolation to the GUT scale

- calculate low energy parameters (gaugino masses, sfermion masses, trilinear couplings) and extrapolate to GUT scale using RGEs
- check for unification at GUT scale



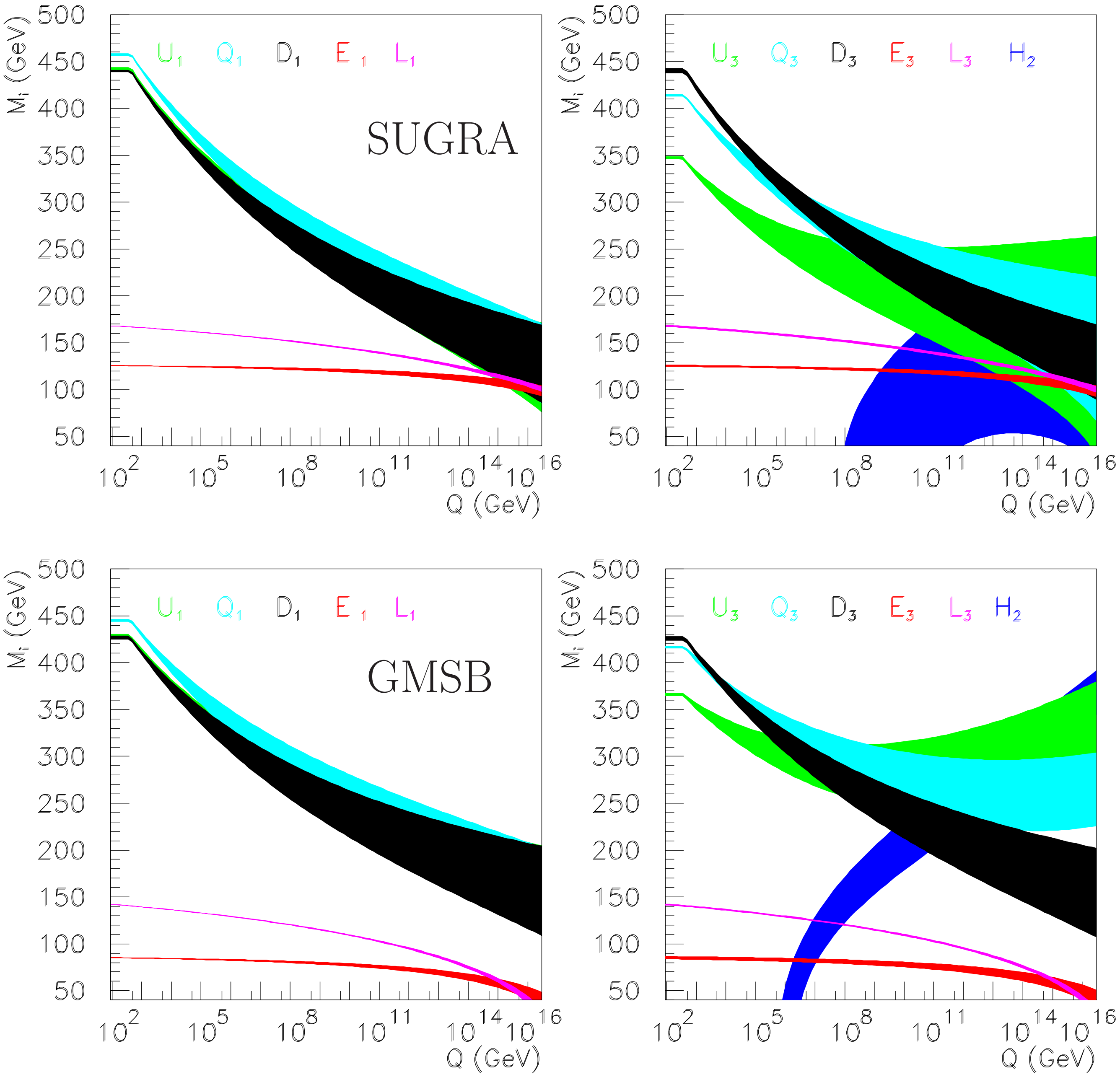
Need very small errors at weak scale to get useful results!

The fit is pure bottom up, no a priory assumptions at higher scales

The current fit assumes that all masses are measured, however some useful information (cross sections, forward backward asymmetries) is not used

The results at the GUT scale can then be used to test models

E.g. comparison of SUSY and GMSB

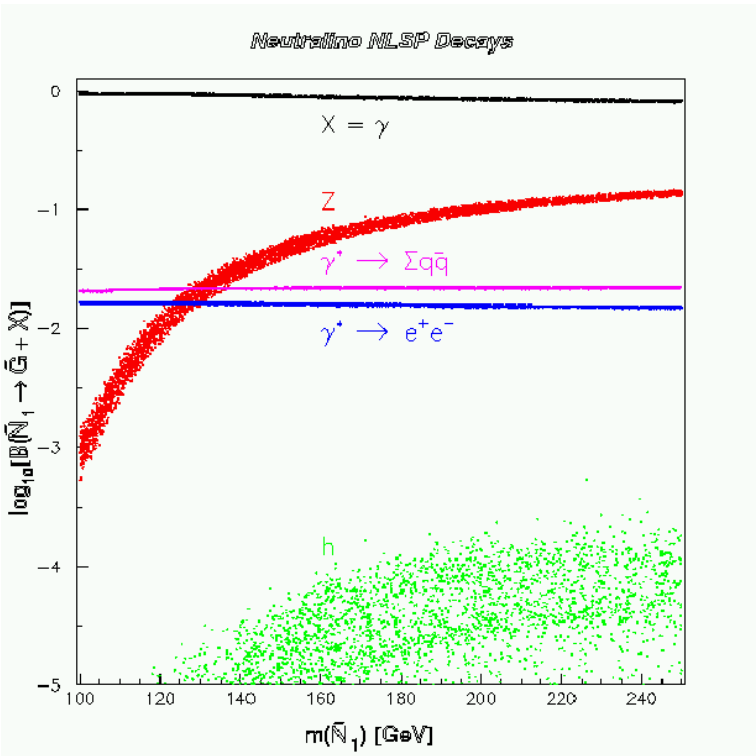


Models can be clearly distinguished

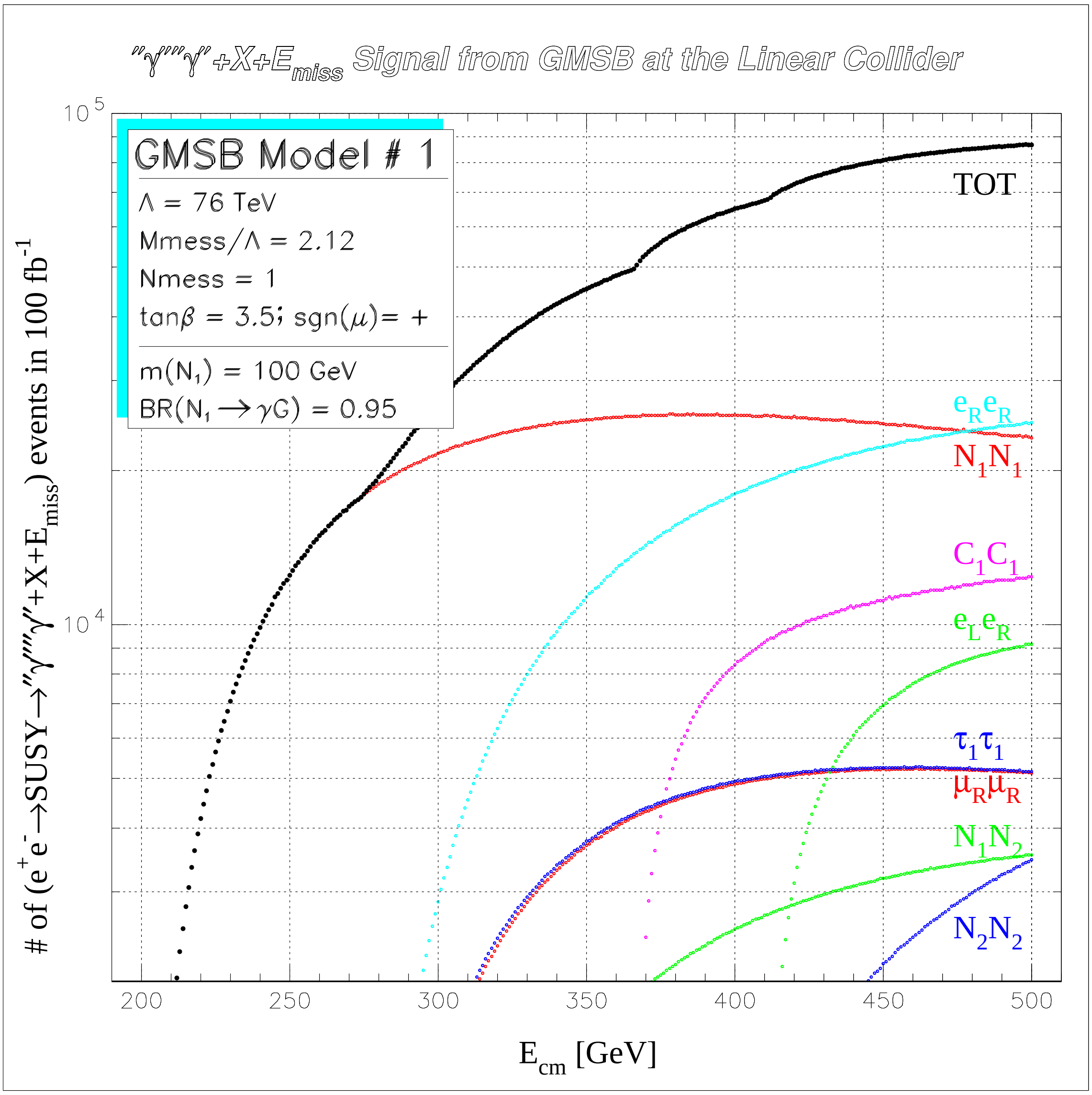
Analyzes in GMSB

- $\tilde{G}$ is very light $\left(m_{\tilde{G}} \simeq \left(\frac{\sqrt{F}}{100\,\mathrm{TeV}}\right)^2 2.37\mathrm{eV}\right) \Rightarrow$
  - all other SUSY particles are unstable
  - no reason for NLSP to be neutral

  ($\sqrt{F}$: fundamental scale of symmetry breaking $F > \Lambda M_{\mathrm{mess}}$)
- NLSP normally χ_1^0 or $\tilde{\ell}$ ($\tilde{\tau}_1$ in case of significant mixing)
- depending on SUSY breaking scale NLSP can decay between prompt and outside the detector
- interesting decays: $\chi_1^0 \rightarrow \tilde{G}\gamma(\gamma^*, Z)$ or (and) $\tilde{\ell} \rightarrow \tilde{G}\ell$



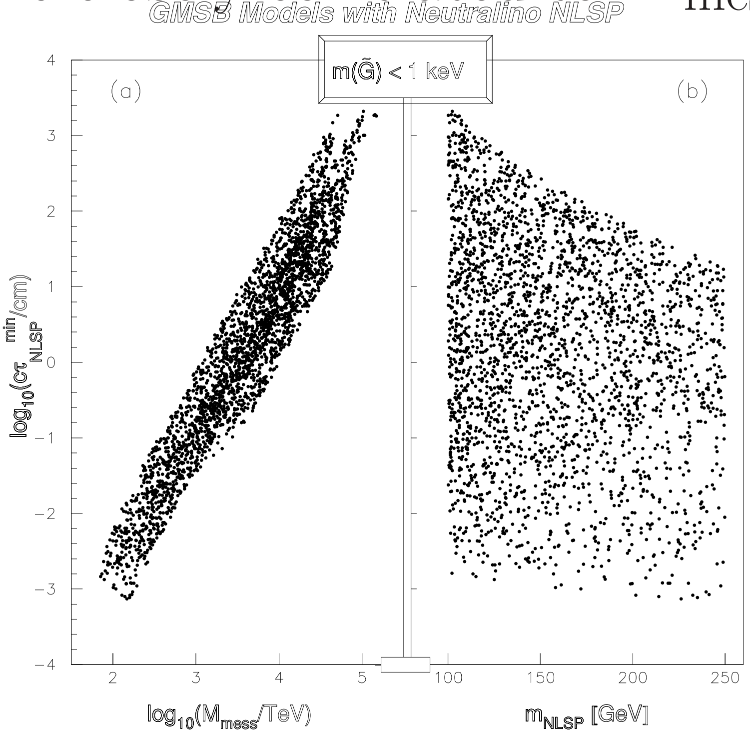
- typical mass spectrum for LC-relevant GMSB-models ($\propto \Lambda$)
  - $\tilde{\ell}_R, \tilde{\tau}_1, \chi^0_1 \sim 100 - 200$ GeV
  - $\tilde{\ell}_L, \tilde{\tau}_2, \chi^0_2, \chi^\pm_1 \sim 200 - 500$ GeV
  - other SUSY-particles > 500 GeV
- Typical cross sections:



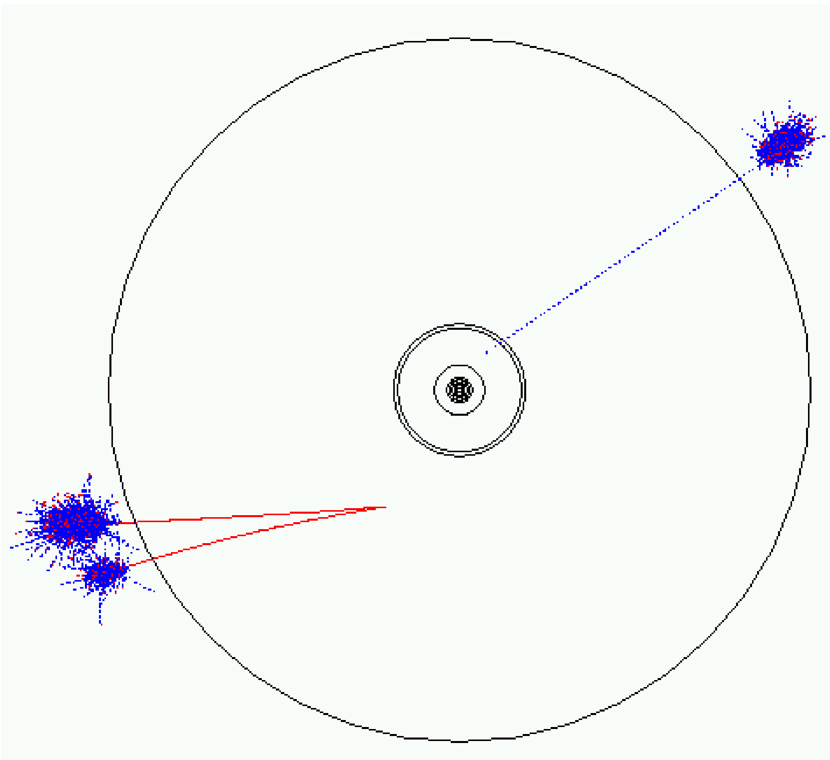
Expect several 10000 events

Detailed analysis for χ_1^0-NLSP scenario exists

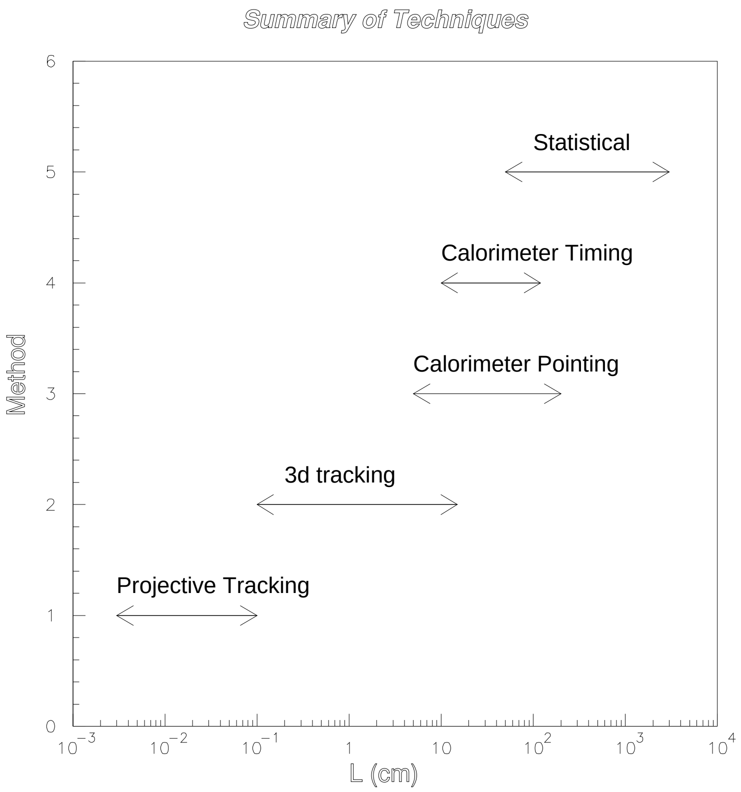
- χ_1^0-mass can be well measured with threshold scan
- χ_1^0-lifetime closely correlated to M_{mess}

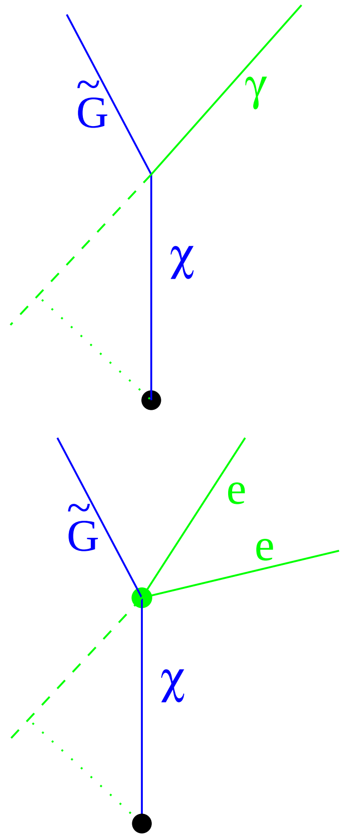


Experimental signatures: non-pointing photons and e^+e^--pairs starting in the detector



Measurement of χ_1^0 lifetime with tracking, calorimeter pointing, calorimeter timing and statistical methods (ratio between two and one photon events)





$c\tau_{\chi_1^0}$ can be measured from $\mathcal{O}(10\mu\mathrm{m})$ up to more then 100m corresponding to $M_{\mathrm{mess}} \sim 100 - 10^5$ GeV

Including mass measurements all model parameters can be measured to the 1% level

Roadmap to SUSY

- Discover SUSY:
  - LHC: $\tilde{q}$, $\tilde{g}$, ($\tilde{\ell}$, $\tilde{\nu}$, χ?)
  - LC: $\tilde{\ell}$, $\tilde{\nu}$, χ

- Prove that coupling(particle)=coupling(partner)
  ➠ LC

- reconstruct SUSY breaking scheme from accurate measurements of masses and couplings
  ➠ LHC+LC

- If SUSY is realized in nature need LHC+LC to understand it