Physics with next generation Linear Colliders

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❶ Introduction
❷ Projects
❸ Top-quark physics
❹ Higgs physics
❺ Physics of gauge bosons
❻ Supersymmetry
❼ Alternative theories
❽ Precision measurements at lower energies

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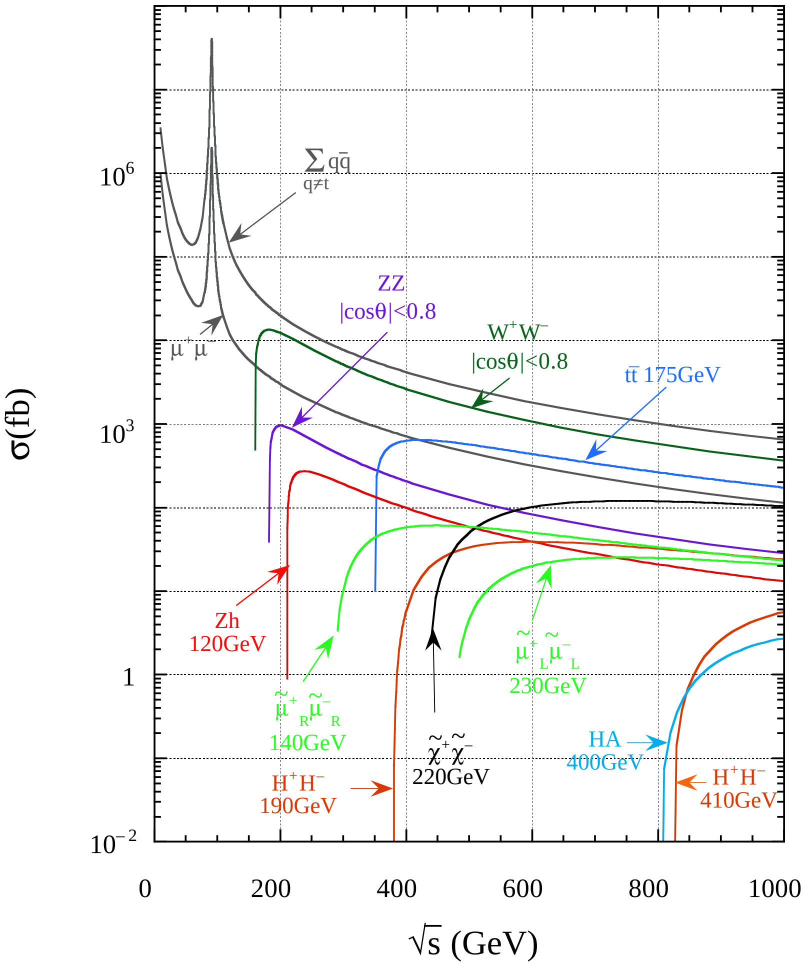
❶ Introduction

- Want to reach energy from LEP2 to ∼ 1 TeV
  ⇒ circular machines no longer possible
- Cross sections in range few fb to few pb



⇒ need luminosities of 10s to 100s fb^{-1}

LC (TESLA) parameters:

- energy range: 1st stage: $\sqrt{s} \leq 500\,\mathrm{GeV}$
  2nd stage: $\sqrt{s} \sim 1\,\mathrm{TeV}$
- Luminosity: 50(91 GeV)−500(800 GeV) fb^{-1}/year
- start data taking ≥ 2012
- electron polarization $\sim 80\%$
- positron polarization of $40 - 60\%$ possible
- any LC can also be used as a $\gamma\gamma$-collider

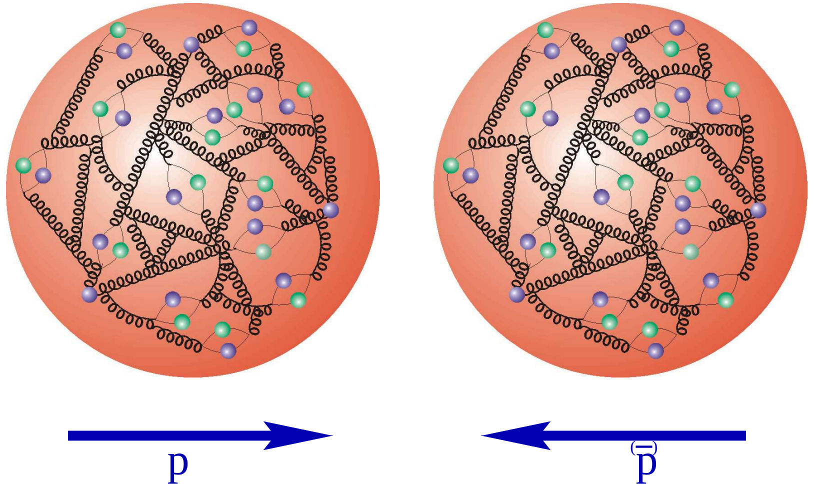
This means:

- few$\cdot 10^4$ $\mathrm{e^+e^-} \rightarrow HZ$/year
  at $\sqrt{s} \approx 350\,\mathrm{GeV}$ ($m_{\mathrm{H}} \approx 120\,\mathrm{GeV}$)
- 10^5 $\mathrm{e^+e^-} \rightarrow \mathrm{t\bar{t}}$/year
  at $\sqrt{s} \approx 350\,\mathrm{GeV}$
- $5 \cdot 10^5$ $\mathrm{e^+e^-} \rightarrow \mathrm{q\bar{q}}$/year
  at $\sqrt{s} \approx 500\,\mathrm{GeV}$ (no rad. ret)
- 10^5 $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-$/year
  at $\sqrt{s} \approx 500\,\mathrm{GeV}$ (no rad. ret)
- 10^6 $\mathrm{e^+e^-} \rightarrow \mathrm{W^+W^-}$/year
  at $\sqrt{s} = 500 - 1000\,\mathrm{GeV}$
- 10^9 $\mathrm{e^+e^-} \rightarrow Z$/year
  at $\sqrt{s} \approx 91\,\mathrm{GeV}$

The most probable scene at the high energy frontier at the startup of a linear collider will be:

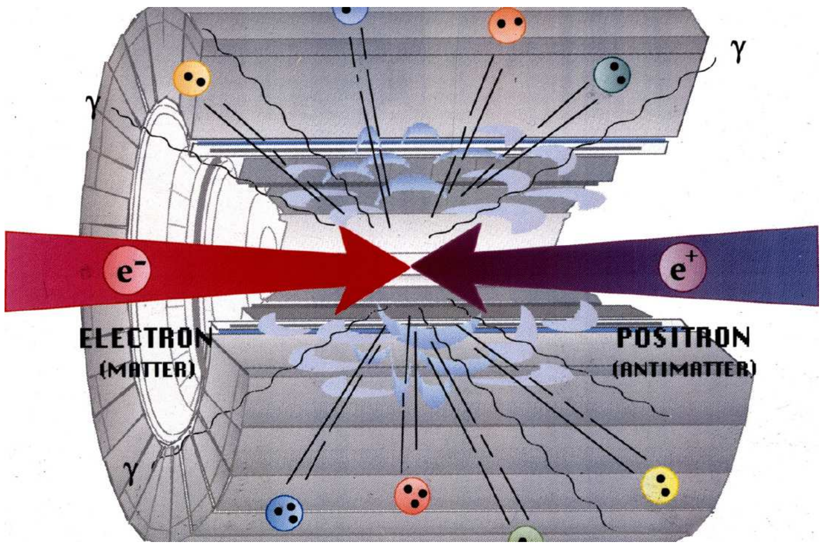
- LEP completed
- TEVATRON run II completed
- LHC has taken several years of data

Hadron collider



- Because of the high proton mass heigh energies are reachable
- however protons are composite particles:
  - parton energies are much lower than proton energy
  - interaction on the parton level is unknown
  - proton remnant disappears in beam-pipe
    ⇒ kinematics must be reconstructed from the decay products
- protons have strong interactions
  - high background
  - not all processes can be reconstructed
- hadron collider are “discovery machines”

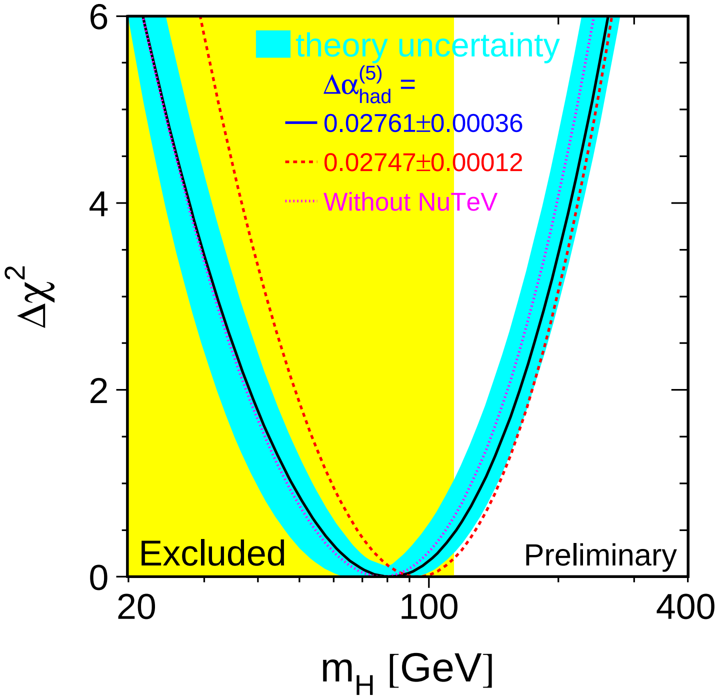
Lepton collider



- Because of the smaller e-mass it is more difficult to reach high energies (synchrotron radiation)
- electrons are point like
  - interaction energy = e^+e^--energy
  - energy-momentum conservation can be used to reconstruct the event kinematics
- electrons have no strong interactions
  - low backgrounds
  - all events can be reconstructed
- lepton-collider are “precision machines”

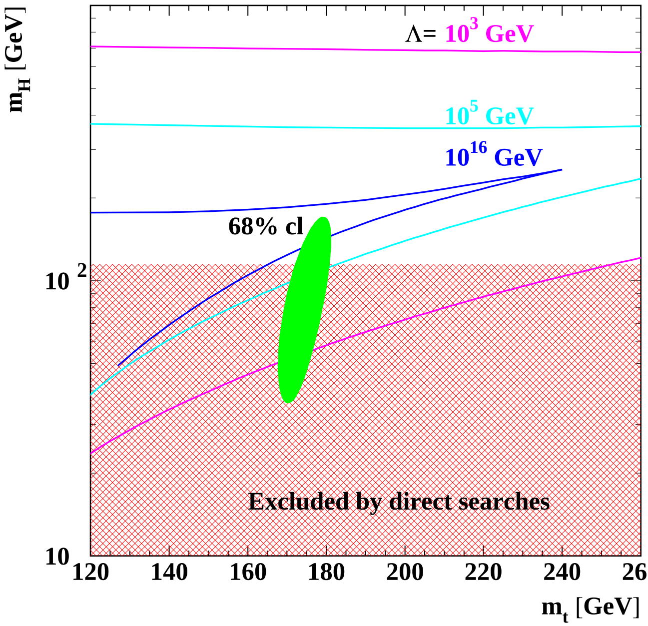
The physics possibilities:

- The Standard Model is the final theory:
  - LEP,SLD,TEVATRON indicate that the Higgs is light



$\Rightarrow m_{\mathrm{H}} < 200\,\mathrm{GeV} \quad (95\%\mathrm{c.l.})$

...which is perfectly consistent with the SM being the final theory:



➧ At least LHC should have seen the Higgs

➨ The Higgs is in the reach of the LC phase 1 and the LC can determine the Higgs properties in detail

- The world is supersymmetric:
  - at least the light Higgs (h) has been seen by at least the LHC
  - probably some supersymmetric particles (squarks) are seen by LHC
  - at least the h has to be in the LC range

  - there is a high chance that (some) sleptons and gauginos are seen by the LC as well

➡ (Some) SUSY parameters can be measured at the LC with good precision

- The gauge group is larger than $SU(3) \times SU(2) \times U(1)$
  - LHC can directly see Z',W' until few TeV
  - LC has a comparable reach by precision measurements via Z'-Z-, Z'-γ-interference
  - if LHC measures the Z' mass, LC can measure its couplings
- Symmetry breaking is realized by a strongly interacting scenario:
  - no Higgs is seen at any machine
  - new resonances (if they exist) might be outside the reach for LHC and LC
  - both machines have a chance to see effects in triple/quartic gauge-boson couplings
- Whatever happens the LC is the first machine to do a precise exploration of the top-threshold

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In general:

Whatever the scenario is, the LHC is the ideal machine to discover it, but has problems to measure its detailed properties

On the contrary an e^+e^- collider is the best machine to do precision measurements, especially if it is known, where to look

In these lectures I would like to convince you that we need the combination LHC-LC to really understand the physics at the TeV scale

Useful Web pages

- DESY/ECFA workshop on linear colliders:
  `http://www.desy.de/conferences/ecfa-desy-lcext.htm`
- TESLA TDR
  `http://tesla.desy.de/tdr`
- Linear Collider Physics Resource Book for Snowmass 2001:
  `http://www.slac.stanford.edu/grp/th/LCBook/`
- Snowmass 2001 “The future of particle physics”
  `http:http://www.slac.stanford.edu/econf/C010630/pr`
- This lecture
  `http://www.ifh.de/www_users/zeus`
  `/moenig/academic_training/`

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❷ Projects and Detectors

Projects for the next generation of lepton colliders:

- NLC: USA (SLAC)
- JLC: Asia (KEK)
- TESLA: international collaboration at DESY

Gross parameters of all projects:

- first phase: $\sqrt{s} \leq 500\,\mathrm{GeV}$
- upgrade: $\sqrt{s} \approx 1\,\mathrm{TeV}$
- tunnel length $\sim$ 30km
- physics start $\sim$ 2012

NLC/JLC:

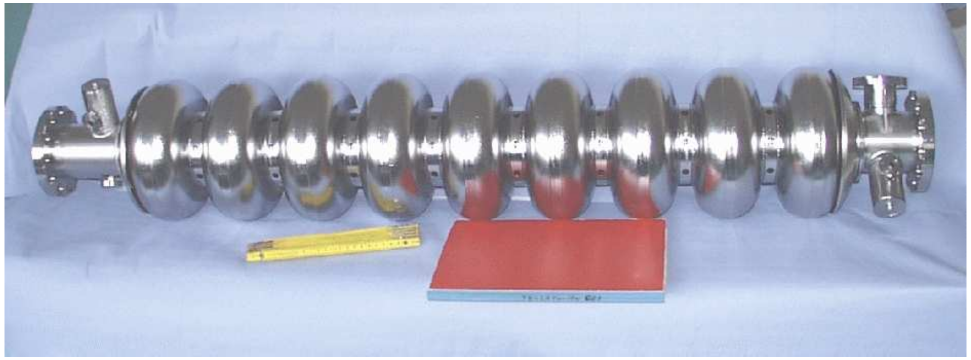
- normal conducting machines
- Luminosity $\mathcal{L} \approx 7 \cdot 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- bunch trains of $\sim$ 100 bunches with $\sim$ 3ns bunch spacing
- repetition rate 120Hz
- small crossing angle at IP
- maximal energy $1 - 1.5\,\mathrm{TeV}$, limited by klystrons

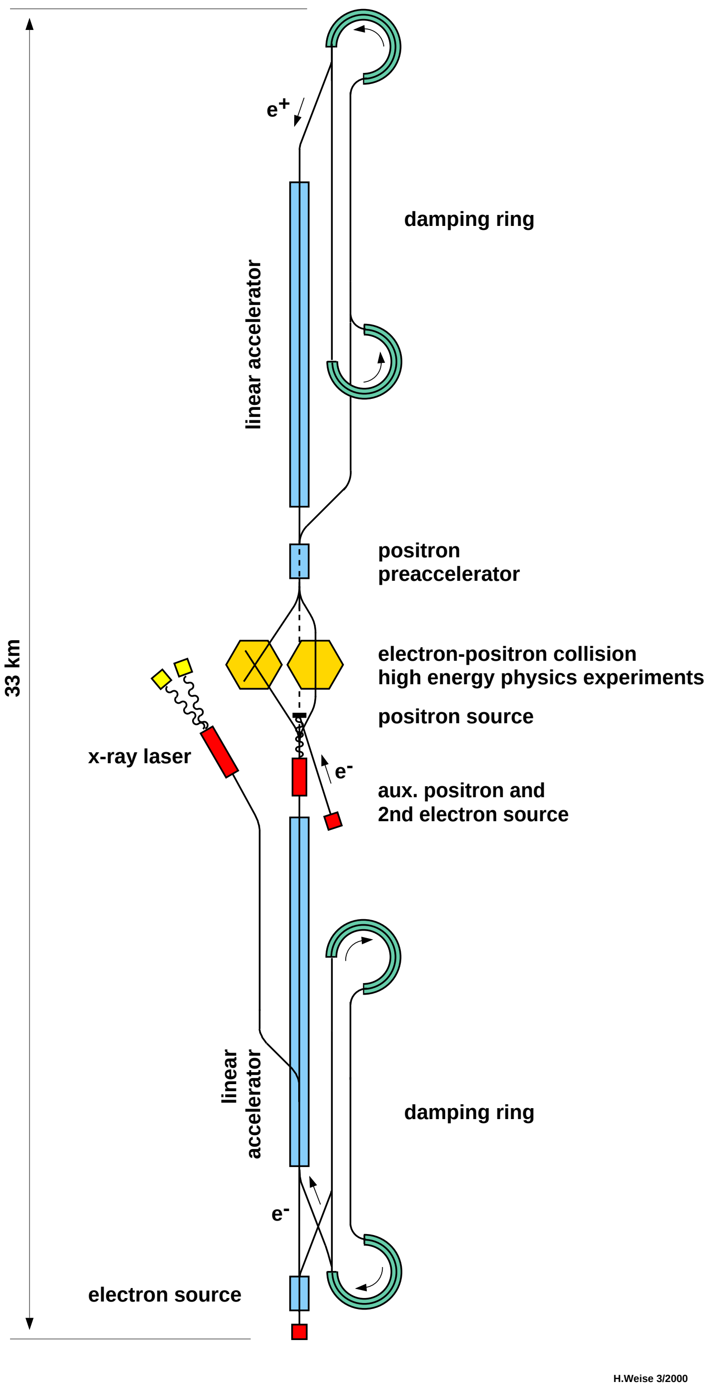
TESLA:→ plot

- superconducting machine
- Luminosity $\mathcal{L} \approx 3 - 5 \cdot 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
  $\Rightarrow \sim 300 - 500\,\mathrm{fb}^{-1}/\mathrm{year}$
- bunch trains of $\sim$ 2800 bunches with $\sim$ 300ns bunch spacing
- repetition rate 5Hz
- head on collisions

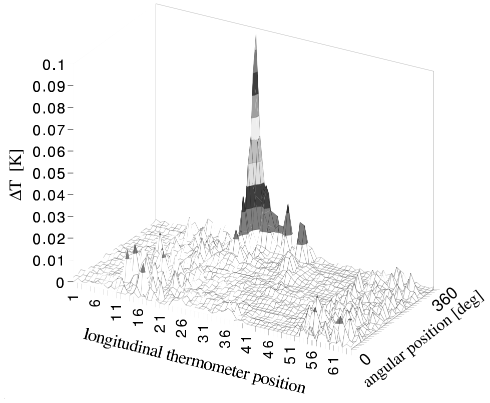
Challenge: Have to increase accelerating field at an affordable cost

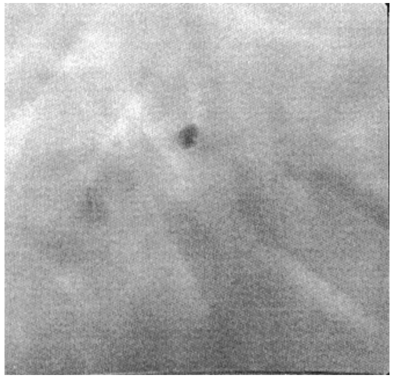
Basic structure: 9-cell niobium cavities



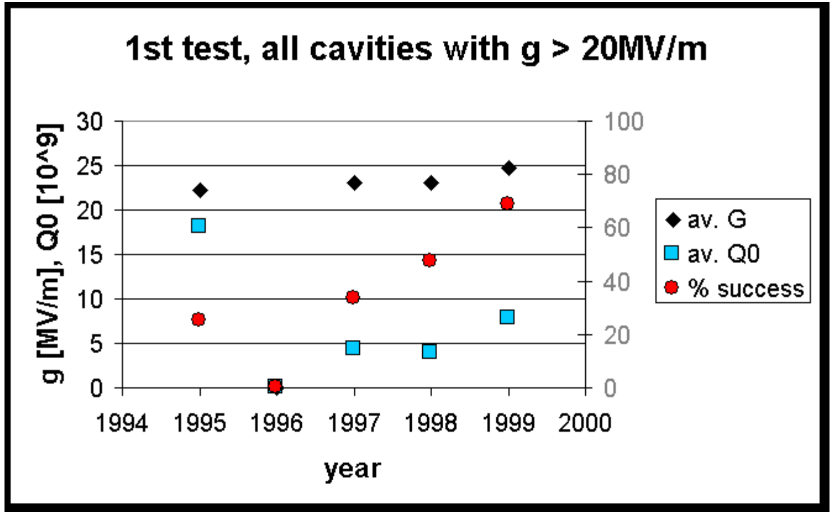


- Limit from critical field of niobium:
  $E < 50\,\mathrm{MeV/m}$
- Practical limit: local impurities:





- solved with special cleaning processes



25 MeV/m is reached routinely

➠ sufficient for $\sqrt{s} = 500\,\mathrm{GeV}$

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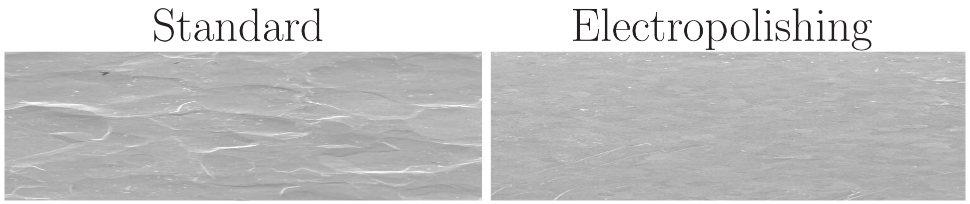
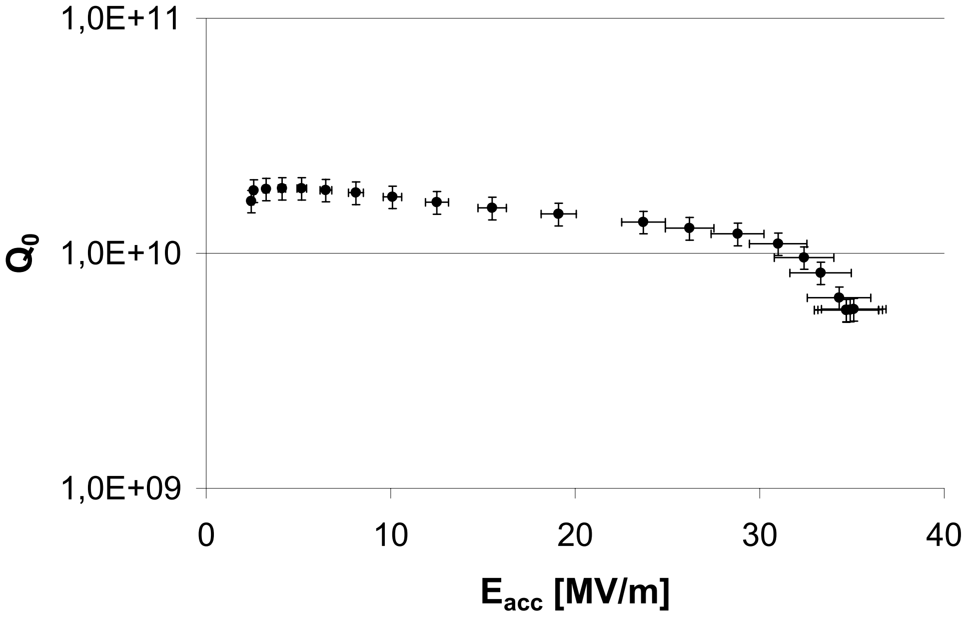
40 MeV/m reached for some single cell modules with electro-polishing


Standard Electropolishing

35 MeV/m reached for the first multi-cell module



➠ $\sqrt{s} = 800\,\mathrm{GeV}$ in reach

Comparison of machine types

Machine parameters at $\sqrt{s} = 500\,\text{GeV}$

	TESLA	X-band
frequency [GHz]	1.3	11.4
gradient [MeV/m]	22	57
AC power [MW]	95	99
$\eta_{\text{AC-to-beam}}$ [%]	23	8.8
Beamstrahlung δ_b [%]	3	4
σ_y at IP [nm]	5	5
Norm $\varepsilon_{x,y}$ at IP [10^{-6}m]	10,0.03	5,0.1
Luminosity [10^{33}]	31	7
Alignment tolerances:		
acc. structures [μm]	500	10
BPM resolution [μm]	10	1
quad pos. drift [μm]	0.5	0.01

- Higher gradient in normal conducting machines may allow higher energy
- Higher efficiency in superconducting machined allows higher luminosity
- Smaller wakefields for lower frequencies relax alignment tolerances
- X-band luminosities can be brought close to TESLA by increasing power and reducing tolerances

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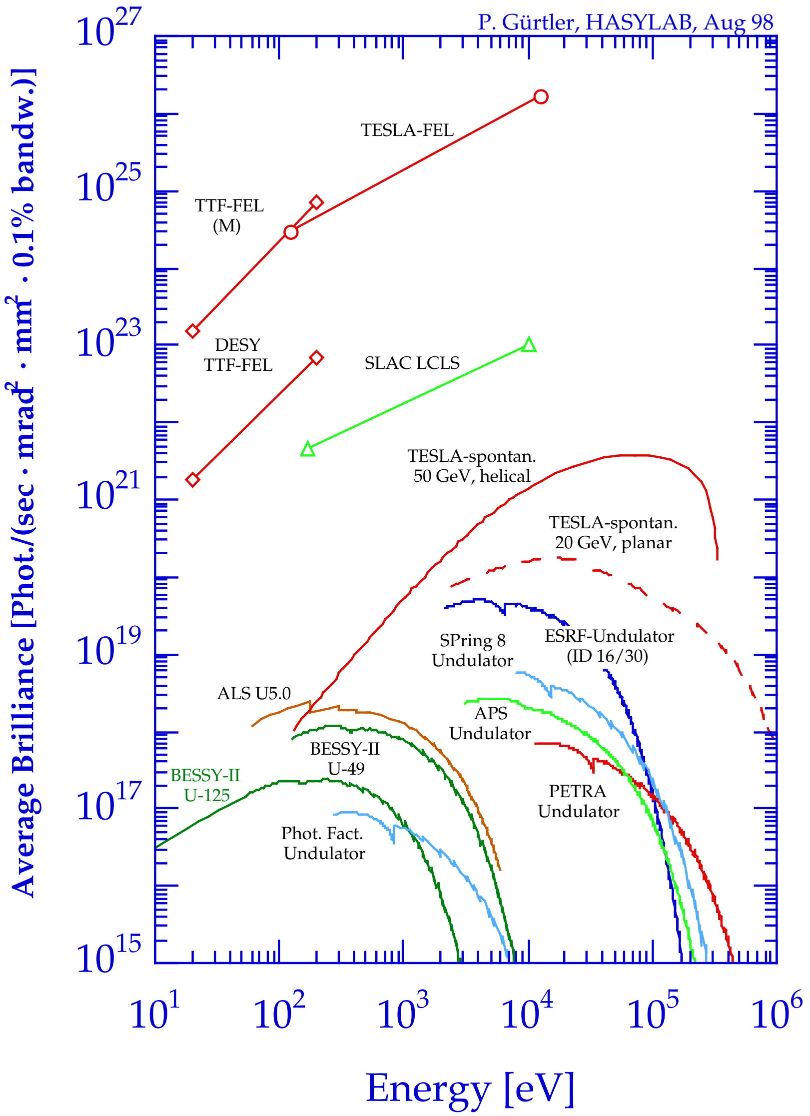
The TESLA design contains an integrated free electron laser with few nanometer wavelength to enlarge the user community (solid state physics, chemistry, biology etc.)



Current TESLA Reference Parameter Set

	500 GeV	**800 GeV**	
repetition rate	5	3	Hz
no. of bunches per pulse	2820	4500	
pulse length	950	850	usec
bunch spacing	337	189	nsec
bunch charge	$2.0\text{x}10^{10}$	$1.4\text{x}10^{10}$	1/e
pulse current	9.5	11.9	mA
AC power (2 linacs)	95	132	MW
normalised IP emittance (x,y)	10, 0.03	8, 0.01	$\text{x}10^{-6}$ m
IP beta-function (x,y)	15, 0.4	15, 0.3	mm
IP beam sizes (x,y)	553, 5	391, 2	nm
IP bunch length	0.4	0.3	mm
beamstrahlung dP/P	2.8	4.7	%
vertical disruption D_y	33	39	
luminosity	$\mathbf{3.1\text{x}10^{34}}$	$\mathbf{5.0\text{x}10^{34}}$	$\mathbf{cm^{-2}s^{-1}}$

Beam polarization

- electrons should be polarizable to $\sim 80\%$ with the same technology as at SLC
- positron polarization:
  - positrons are made by sending the high energy electrons through a wiggler to produce photons which are shot on a target to produce positrons
  - if a helical undulator is used before the IP positron polarization of $50-60\%$ should be possible

Advantage of electron polarization:

- only e^-_L couple to $W^\pm$

➠ cross sections can be enhanced and backgrounds can be suppressed (e.g. W-pair production)

- in the unbroken symmetry only e^-_L couple to the W^0 while both helicities couple to the B

➠ in many channels completely different couplings are probed

Advantage of positron polarization:

- the effective polarization gets increased (e.g. for Z exchange: ($\mathcal{P}_{\rm eff} = \frac{\mathcal{P}_+ + \mathcal{P}_-}{1+\mathcal{P}_+\mathcal{P}_-}$) $\mathcal{P}_+ = 50\%, \mathcal{P}_- = 80\% \Rightarrow \mathcal{P}_{\rm eff} = 93\%$) and the error gets reduced (factor 3 for case above)
- the polarization can be measured with the Blondel scheme
- some backgrounds (e.g. single W) can only be suppressed with both beams polarized
- some analyzes (s-channel $\tilde{\nu}$-exchange, neutralino-production) profit from both beams being polarized

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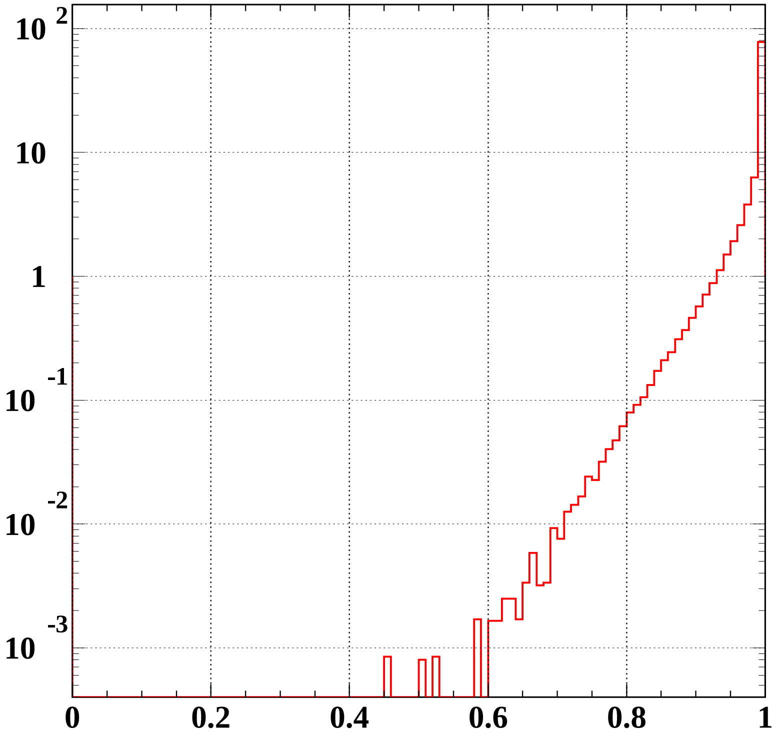
Common problem: beamstrahlung

Beams at IP are extremely collimated with many electrons/bunch

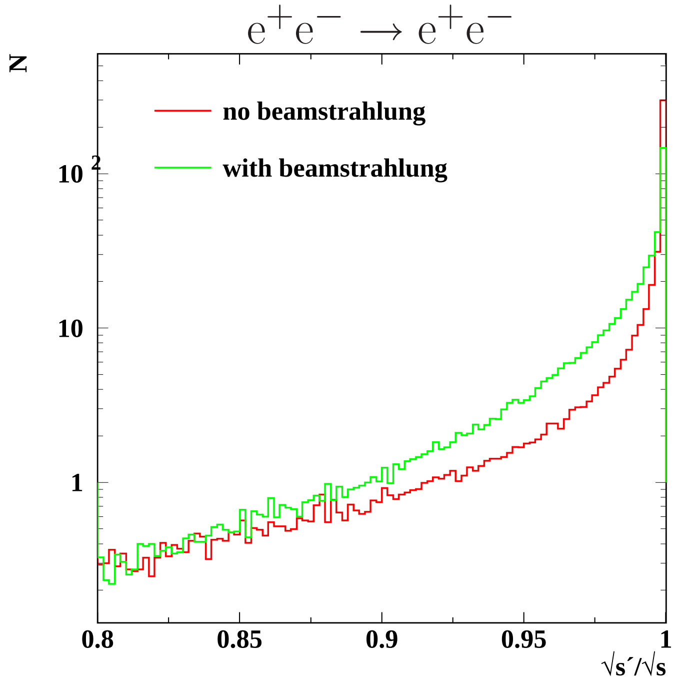
→ very high charge density

⇒ Electrons of one bunch radiate against the coherent field of the other bunch (Beamstrahlung)

Average energy loss for colliding e^+e^--pairs at 500 GeV: $\sim 1.5\%$



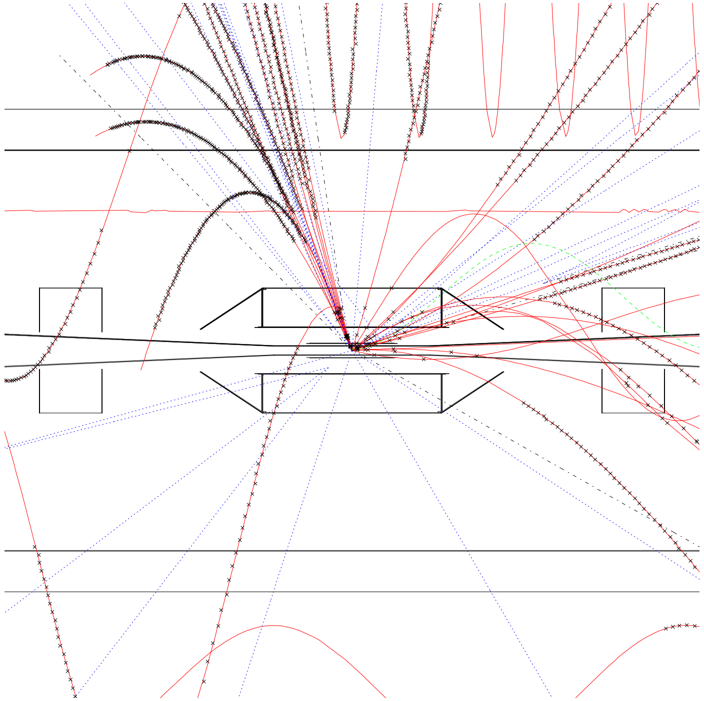
- For continuum processes beamstrahlung comparable to ISR, however with shorter tails



- beamstrahl spectrum can be measured on the 10^{-4} level from the acolinearity of Bhabha-events in the forward $(7° - 25°)$ region
- in general beamstrahlung is not a problem in the analyzes

$\gamma\gamma$-background

- at the LC $\gamma\gamma$-background originates from the usual e^+e^--process and from beamstrahlung
- at TESLA luminosities the overlap probability for a $\gamma\gamma$-event with a physics event is on the few % level

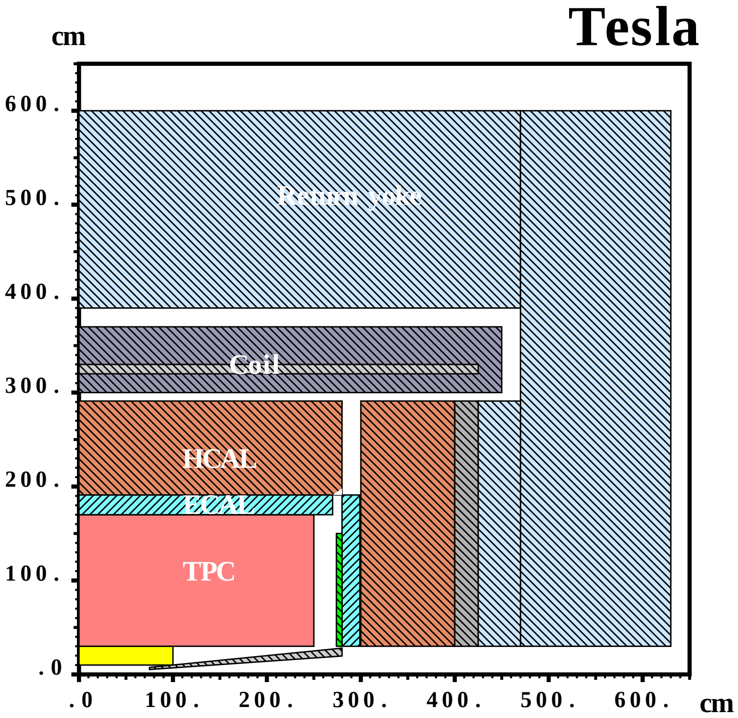


- events with $\gamma\gamma$-overlap can be tagged by a displaced vertex in z and by topological variables
- first studies indicate that they will be no serious problem for physics

A possible Detector for TESLA

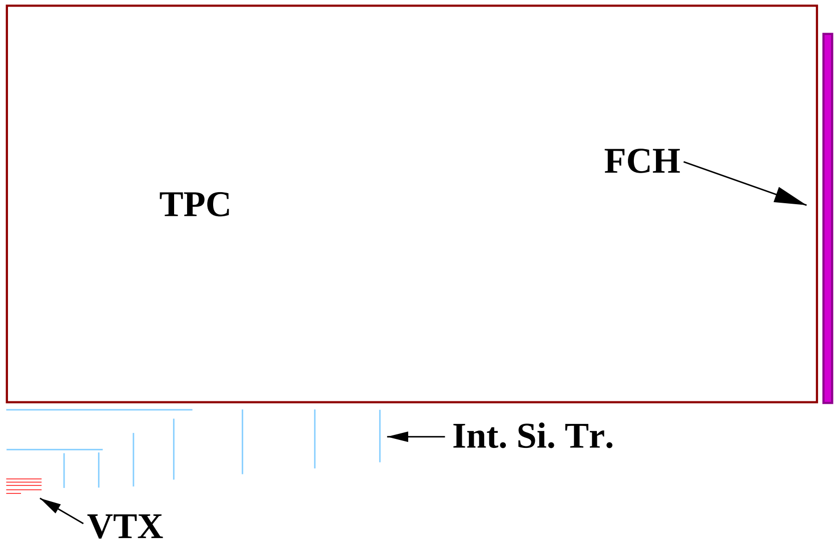
- For the TESLA TDR a possible detector has been designed
- This is meant as a proof that the required detector can be built with the (almost) available technology and with an affordable cost
- The US and Asian detectors are very similar, so only the TESLA detector will be described

Global detector concept



Relatively similar to a LEP detector

Tracking



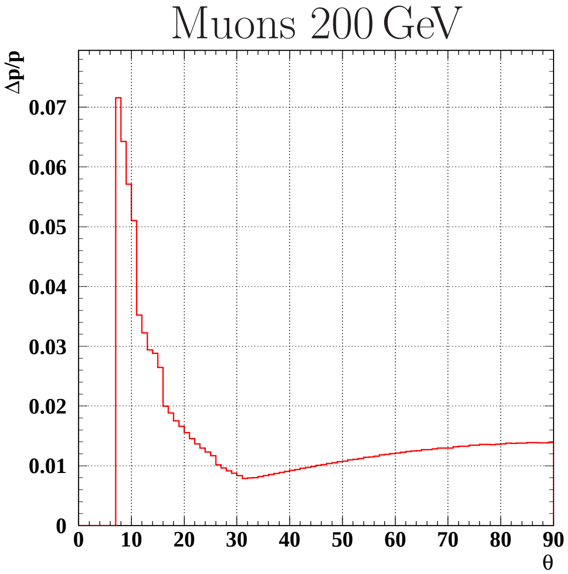
- Superconducting solenoid with $B = 3 - 4T$
- Vertex detector → later
- Main tracker: TPC
- silicon tracker inside TPC consisting of barrel cylinders and forward discs
- forward chamber behind TPC

R&D issues for the main tracker

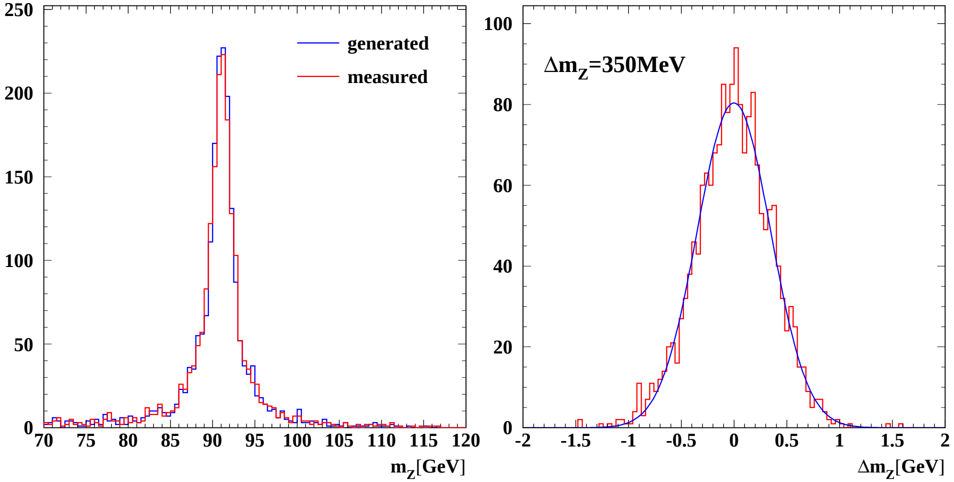
Mainly TPC:

- to cope with larger backgrounds many more pad rows than at LEP (> 150) are needed
- alternative readout schemes like GEMs under study:
  - charge cloud doesn't spread over several pads, how to get good point resolution?
  - how to avoid ion flow back into the sensitive volume?
- dense packing of electronics
- design of a very thin field cages and endplate

Tracking system gives excellent momentum resolution for $\theta > 7°$



E.g. Z-mass resolution for $e^+e^- \to HZ, Z \to \mu^+\mu^-$ totally dominated by intrinsic Z-width



Calorimetry:

- To improve resolution main part of hadron-calorimeter will be inside coil
- Energy flow will be calculated à la LEP ($E_{\text{tot}} = E_{\text{charged}} + E_\gamma + E_{n,K_L^0}$)
  $\Rightarrow$ Spatial resolution is more important than energy resolution
- Aim for
  - ECAL: $\frac{\Delta E}{E} = \frac{0.10}{\sqrt{E}} \oplus 0.01$
  - HCAL: $\frac{\Delta E}{E} = \frac{0.50}{\sqrt{E}} \oplus 0.04$
- several technologies under study
  - shashlik
  - scintillating tiles
  - Si-W (EM only) clearly the best option, if affordable
  - small cells ($1 \times 1\text{cm}^2$) with binary readout (hadron only) might be superior to scintillator because of better separation of nearby showers

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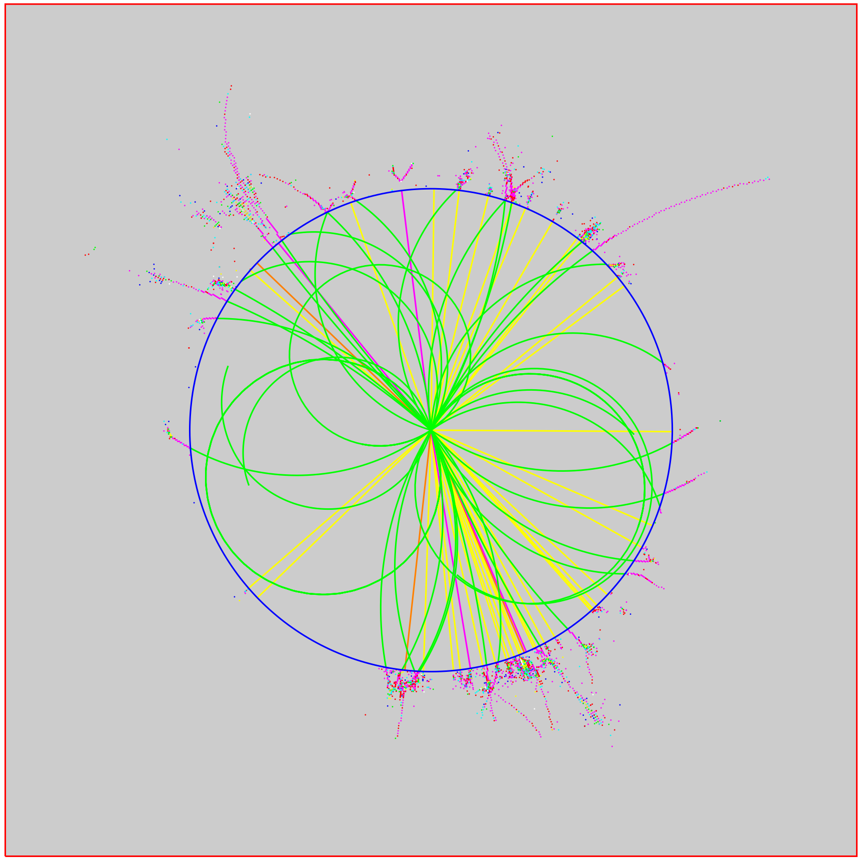
Hadronic event in the SiW-Calorimeter



R&D issues in for the calorimeter

General:

- Energy flow concept requires very sophisticated reconstruction algorithms

SiW:

- minimization of silicon cost
- dense packing of channels
- fabrication of homogeneous tungsten surfaces

Scintillating tiles:

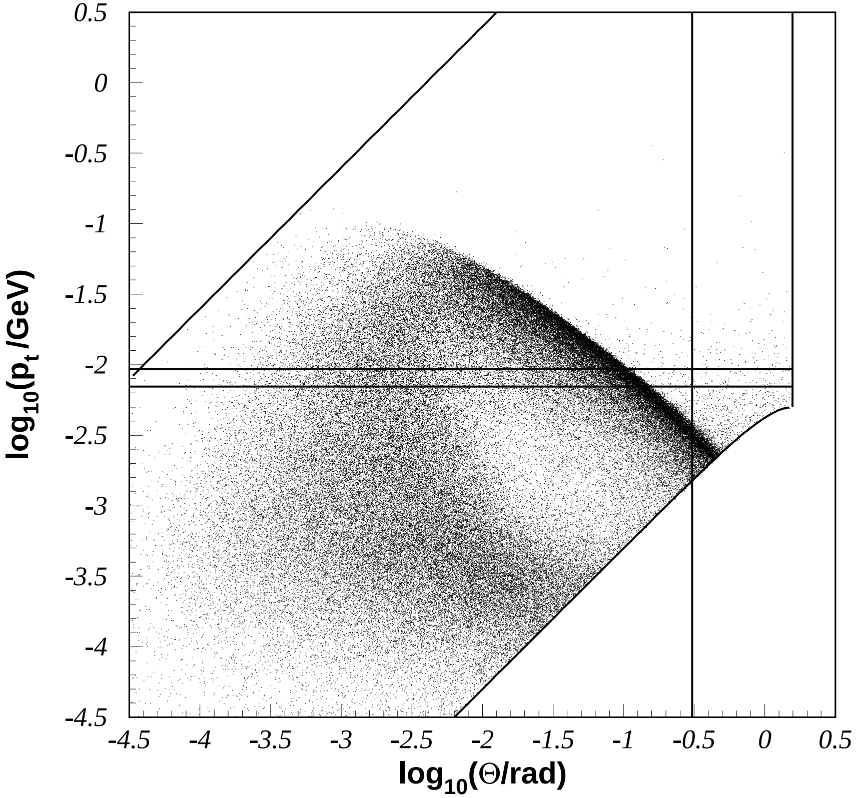
- minimization of tiles (fiber coupling)
- cheap fiber readout

Digital calorimeter:

- cost minimization of readout channel ($5 \cdot 10^7$ channels!!!)

Vertex detector

Main issue: e^+e^- pairs from beamstrahlung

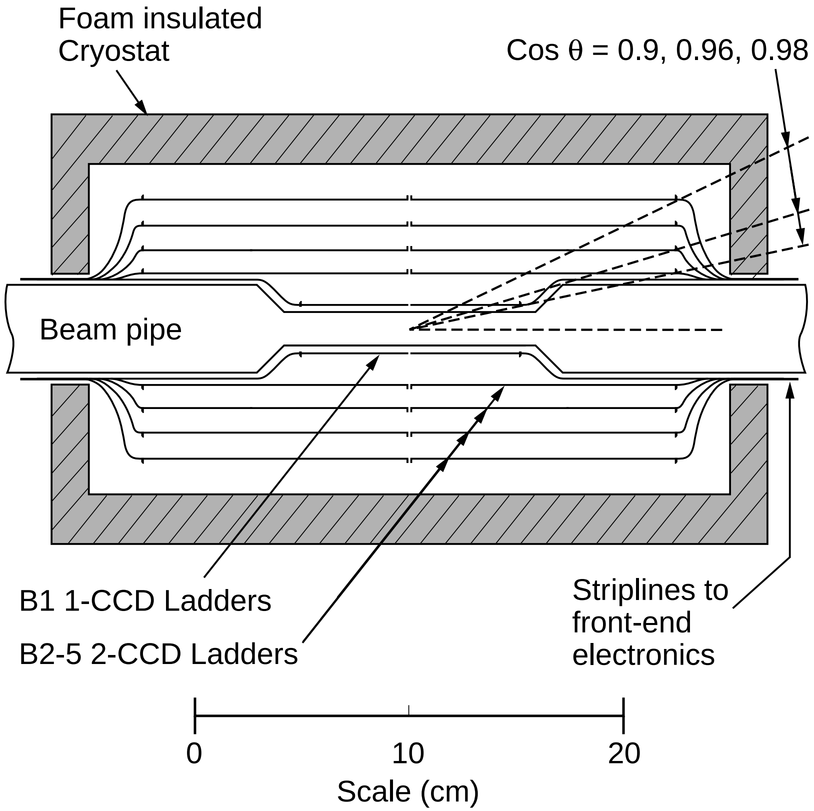


For a given B field p_t translates into a maximum radius
At TESLA $B = 3T$, $r = 1.5$cm corresponds to 0.03 hit/mm^2/BX

Technologies:

- Pixels à la ATLAS
- CMOS-Pixels (very attractive idea)
- CCDs (pioneered at SLD)

CCDs



- point resolution: $\sigma_p \approx 3.5\mu\text{m}$
- layer thickness: $\sim 0.12\% X_0$

$\Rightarrow$ IP resolution $\sigma_{IP} \sim 2\mu\text{m} \oplus \frac{10\mu\text{m}}{p \sin^{3/2}\theta}$

R&D issues for the vertex detector

CCDs:

- currently the readout time is very long accumulating background over many bunch crossings
  - column parallel readout
  - readout frequency 50 MHz
  - readout detector continuously

  $\Rightarrow$ One complete readout extends over $\sim$ 100 bunch crossings ➡ 3 hits/mm^2/BX
- thinning of layers

Pixels:

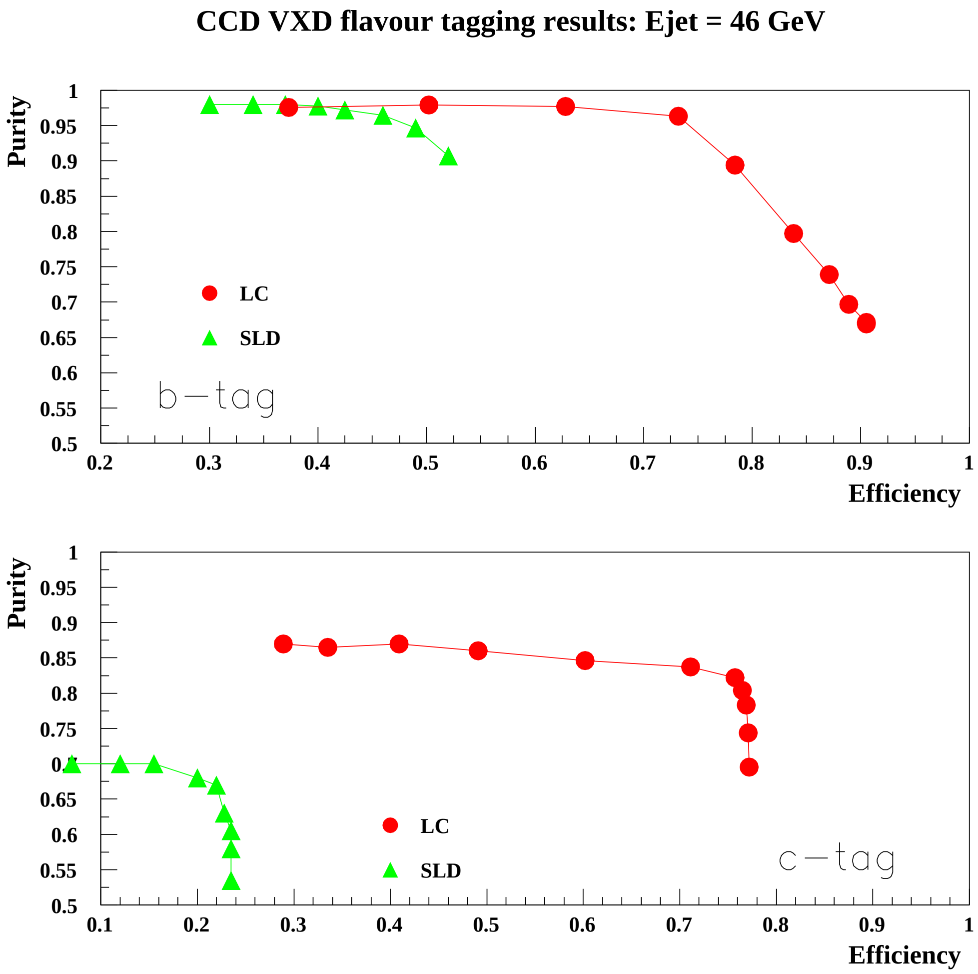
- need to make detector much thinner
- need to improve resolution (floating pixels)

CMOS:

(same technology as in cheap video cameras, potential to be fast and cheap)

- first promising results on small test chips but no running large scale system yet

B-tagging:
Very good results with SLD-like algorithm

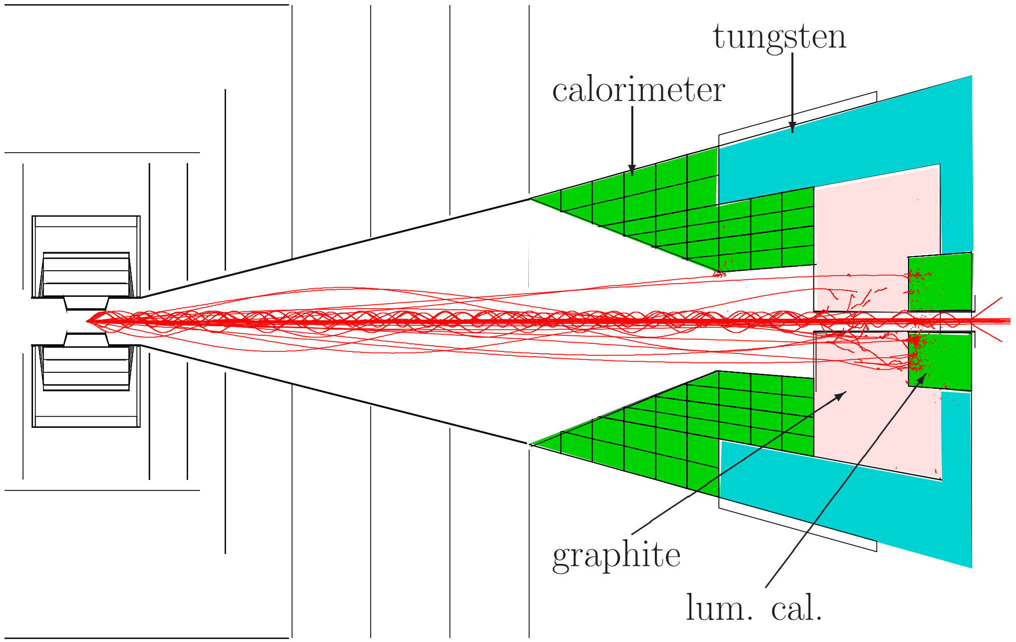


- Very high b-efficiency ⇒ important for multi-b final states with low σ (ZHH,$t\bar{t}$H)
- Good c-efficiency/purity ⇒ important for BR(H $\rightarrow c\bar{c}$)

Very forward region:

- Pairs give large background in very forward region
- Also lots of neutrons in this region

➠ Need mask at $\theta < 5^\circ$



- Environment clean enough above $\theta = 1.5^\circ$ to install hermeticity calorimeters for searches and precision luminosity
- Below 1.5° only luminosity calorimeter for machine tuning and limited tagging for searches
- R&D needed for radiation hardness of LCAL

Trigger

- detector is designed without any hardware trigger
- a full bunch train is read out and send to a PC farm
- bandwidth not larger than level 2 at LHC
- the system is completely deadtime free
- the full detector information can be used to select complicated new physics channels
- no need to include fast detectors for triggering

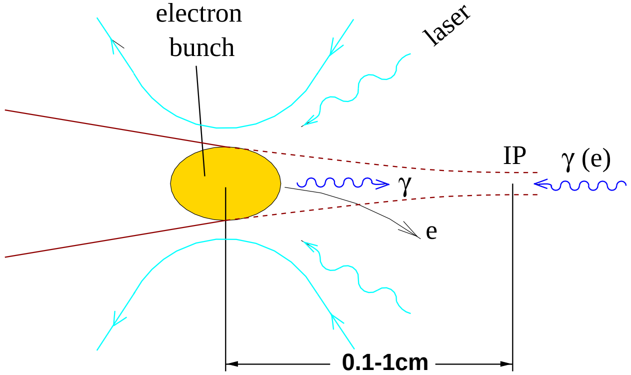
Other collider modes

e^-e^--collider:

- to run an e^+e^--collider in e^-e^- mode should be a relatively simple modification
- since the pinch-effect turns into an anti-pinch-effect luminosity can be about an order of magnitude lower
- the interaction region can stay the same
- physics interest:
  - precision measurement of Møller-scattering
  - precision measurement of the selectron mass (β instead of β^3 suppression due to χ^0 t-channel production)
  - access to the $I = 2$ amplitude in WW-scattering
  - some exotic models, e.g. with doubly charged leptons

$\gamma\gamma$ and $e\gamma$ collider

- high energy γs can be produced by Compton backscattering with laser light close to the IP



- Maximal photon energy ω_m:

$$\omega_m = \frac{x}{x+1}E_0$$
$$x = \frac{4E_0\omega_0}{m^2c^4}$$
$$\simeq 15.3\left[\frac{E_0}{\mathrm{TeV}}\right]\left[\frac{\omega_0}{eV}\right]$$

$(\omega_0(E_0) =$ laser (beam) energy)

have to keep $x < 4.8$ to avoid $\gamma\gamma \to \mathrm{e^+e^-}$ in the laser interaction region $\Rightarrow$ $\omega_m \approx 0.8E_0$

- γ energy spectrum depends on product of electron and laser polarization

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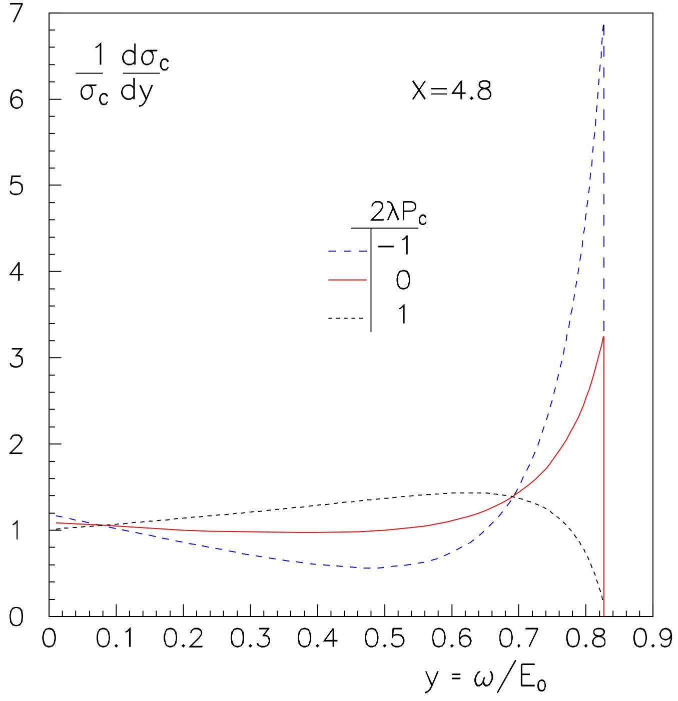
γ-energy spectrum for different polarization products



- luminosity for $\gamma\gamma$, $e\gamma$-colliders factor 5-10 lower than for $\mathrm{e^+e^-}$-colliders for identical beam parameters (can be brought to 40% by optimizing parameters)
- to separate used photon beam from incoming beam a crossing angle is needed
- background situation is similar to $\mathrm{e^+e^-}$-mode
- however $\sim$ 1.5 underlying events from low energy $\gamma\gamma$ collisions

Physics interest:

- some cross sections involving gauge bosons are larger than in e^+e^- $\longrightarrow$ plot
- coupling to photons can be measured without ambiguities from Z-couplings
- a $\gamma\gamma$-collider can measure cleanly the partial width $H \to \gamma\gamma$ $\longrightarrow$ later
- the mass reach for some particles (Higgses, SUSY-particles, W') can be higher than in e^+e^-
- an $e\gamma$-collider is an ideal place to measure the photon structure

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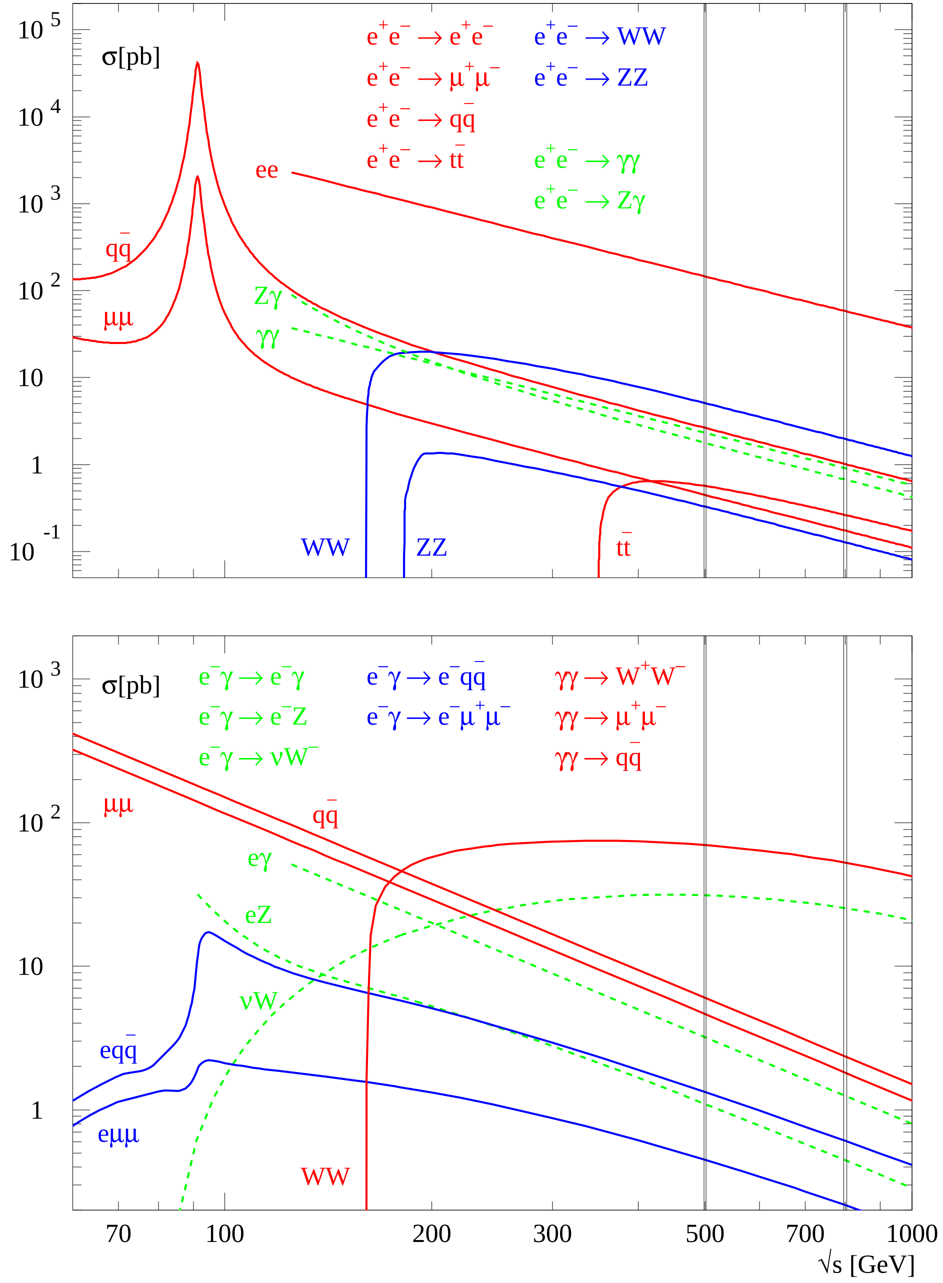
SM cross sections is e^+e^-, $e\gamma$, $\gamma\gamma$



A possible roadmap to an LC (TESLA)

- All regions agree that we need one TeV-class LC in the world as the highest priority project in HEP
- Wherever it will be build we will all collaborate
- the TESLA "Technical Design Report" has been submitted in march 2001
- the project has reviewed by the German science council
- a first reaction from the German government exists
- The projected cost of TESLA is:

– 500 GeV linear collider:	3.1 GEURO
– addition for FEL:	0.5 GEURO
– HEP detector:	0.2 GEURO

- A realistic estimate of the German contribution is $\mathcal{O}(50\%)$.
- The rest has to come as international contribution
- TESLA will be organized as a temporary international organization.
- total construction time 8 years
- we could start data taking in 2012

Recent press release by the German government

A new free electron laser is to be built at the DESY research centre in Hamburg. In view of the locational advantage, Germany is prepared to cover half of the investment costs amounting to 673 million Euro. Talks on European cooperation will soon start so that it will be possible to take a decision on construction within about two years. The construction period will be approximately six years.

No German site is at present proposed for the TESLA linear accelerator. The reason is that the accelerator project will be an international collaboration. International developments must therefore be taken into account. An independent initiative by Germany concerning the site of the accelerator is neither appropriate nor necessary. DESY will, however, be able to continue its international research work so that German participation in a future global project will be possible.

❸ Top-quark-physics

- Introduction
- Measurement of the top-mass
- Top-quark couplings
- Top-Higgs Yukawa coupling → Higgs section
- Conclusions

Introduction

- The top quark is the heaviest fermion ($m_\mathrm{t} \approx 175\,\mathrm{GeV} \sim v$)
- In the SM it is just the isospin partner of the b-quark
- however in some models it plays a special role in electroweak symmetry breaking

➧ it is very important to study the top properties

- since $m_\mathrm{t}^2 \gg m_\mathrm{W}^2$ the top width is very large

$$\Gamma_t \approx \frac{G_\mathrm{F} m_\mathrm{t}^3}{8\sqrt{2}\pi} \approx 1.7\,\mathrm{GeV} \gg \lambda_{QCD} \;\Rightarrow$$

  - there exist no toponium resonances at threshold
  - the top decays before it fragments
    ⇒ the top-polarization gets preserved to the decay, like the τ at LEP

LHC:

- $\sigma(\mathrm{t\bar{t}}) \sim 1\,\mathrm{nb}$ ⇒ huge data samples should allow very precise studies of top decays, especially rare decays
- $\mathrm{t\bar{t}}$ production by strong interaction ⇒ no interesting information on $\mathrm{t\bar{t}}Z$ coupling
- $\mathrm{t\bar{t}}$ production in continuum ⇒ no threshold scan possible

LC:

- $\sigma(\mathrm{t\bar{t}}) \sim 1\,\mathrm{pb} \Rightarrow \sim 10^5$ $\mathrm{t\bar{t}}$ events allow for precise studies
- $\mathrm{t\bar{t}}$ production via γ, Z exchange ⇒ $\mathrm{t\bar{t}}Z$ couplings can be measured
- $\sqrt{s}$ can be adjusted at will ⇒ threshold scan possible

Measurement of the top-mass

Why do we want to know the top-mass as accurate as possible?

- a future theory of flavor hopefully predicts fermion masses or mass/ratios
  $\rightarrow$ The mass of the heaviest quark should be known as close as possible to the precision of the τ-mass
- in precision tests of the SM m_{t} enters quadratically:
  - $\Delta m_{\mathrm{W}}/\Delta m_{\mathrm{t}} = 0.006$
    ultimately: $\Delta m_{\mathrm{W}} = 6\,\mathrm{MeV}$
  - $\Delta \sin^2\theta^{\ell}_{\mathrm{eff}}/\Delta m_{\mathrm{t}} = 0.00003/\,\mathrm{GeV}$
    ultimately: $\Delta \sin^2\theta^{\ell}_{\mathrm{eff}} = 0.00002$

  $\Rightarrow$ need $\Delta m_{\mathrm{t}} < 1\,\mathrm{GeV}$
- In SUSY models radiative corrections to light Higgs (h) mass: $\Delta m_h/\Delta m_{\mathrm{t}} \approx 1$
  $\Rightarrow$ aim for $\Delta m_{\mathrm{t}} \approx \Delta m_h \approx 50\,\mathrm{MeV}$

What is the top mass?

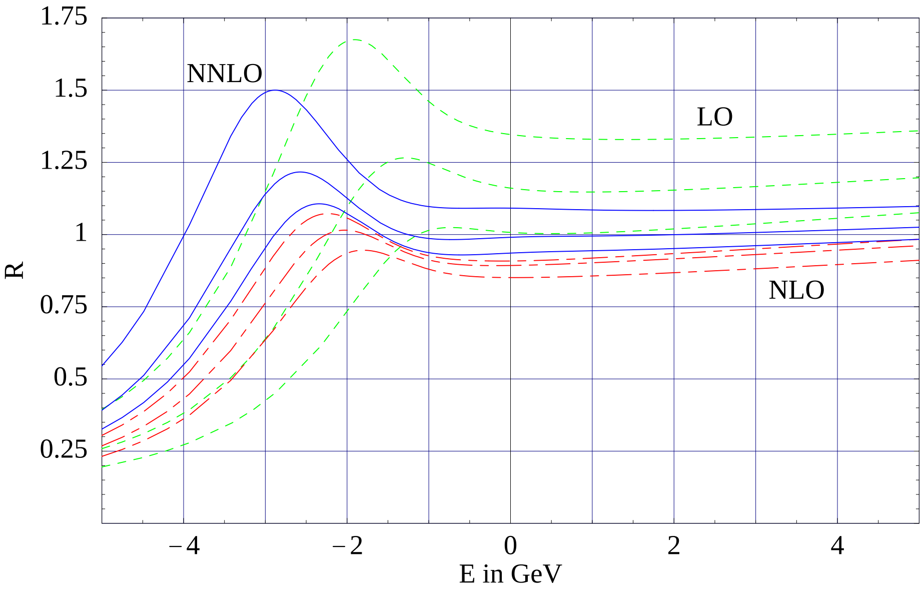
- Quarks are not free particles $\Rightarrow$ their mass is not unambiguously defined
- pole mass: pole of propagator, natural definition if top-decays are reconstructed
- $\overline{\mathrm{MS}}$-mass: running mass in QCD (like coupling constant), needed in radiative corrections
- conversion pole mass $\rightarrow$ $\overline{\mathrm{MS}}$-mass has theoretical uncertainties of $\mathcal{O}(1\,\mathrm{GeV})$
- limit of all top-reconstruction methods
- additional ambiguity of same order for reconstruction methods since only color neutral objects can be reconstructed
- threshold scans: no natural mass definition, can do calculations in several ones

Most promising method: threshold scan

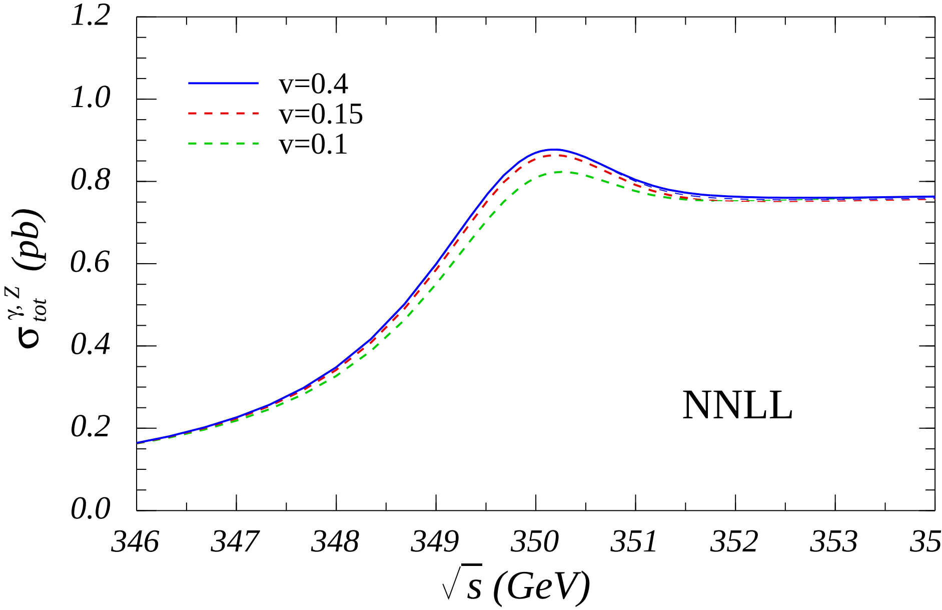
- Due to large top mass and the corresponding short lifetime no toponium resonances are existing any more
- However still large corrections due to Coulomb-like QCD potential:

$$E_{\rm tot}(r) = 2m_{\rm t} + V(r)$$
$$V(r) \propto \frac{\alpha_s(1/r)}{r}$$

- QCD corrections known to 3rd order (pole mass)



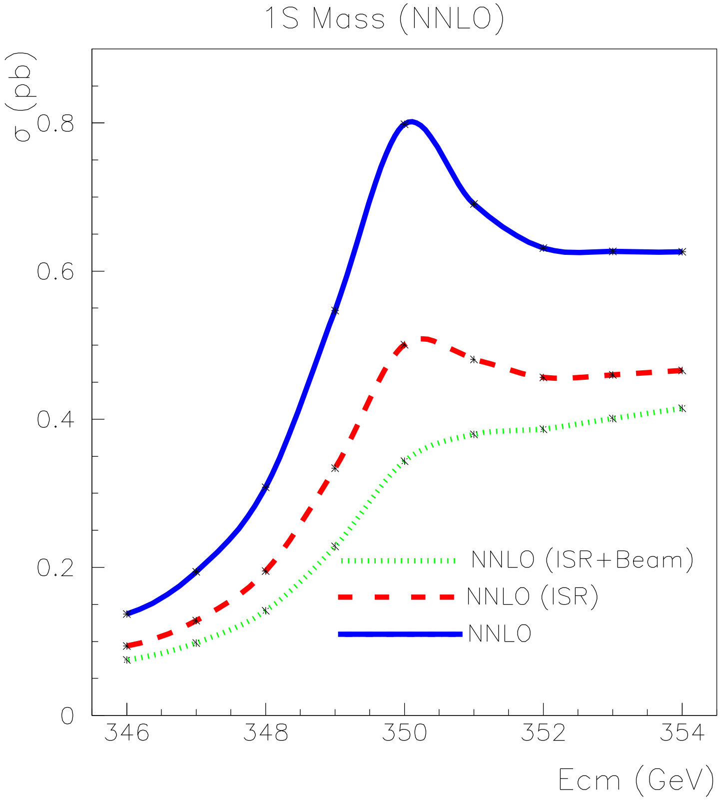
- large scale dependence and huge shift of the peak from order to order using pole mass
- theoretical error on $m_{\rm t}$ unclear
- in addition the same uncertainty appears going from $m_{\rm pole}$ to $m_{\overline{\rm MS}}$
- both problems can be solved by redefining the mass, shifting part of the potential to the mass definition



(v=top velocity parameter, should be > 0.15)

Threshold cross section now very well under control

- In the QCD-corrected cross section some remnant of the 1S peak remains visible



- This peak is completely washed out by ISR, beamstrahlung and beamenergy-spread
- However uncertainties in beam parameters do not effect precision of m_t measurement

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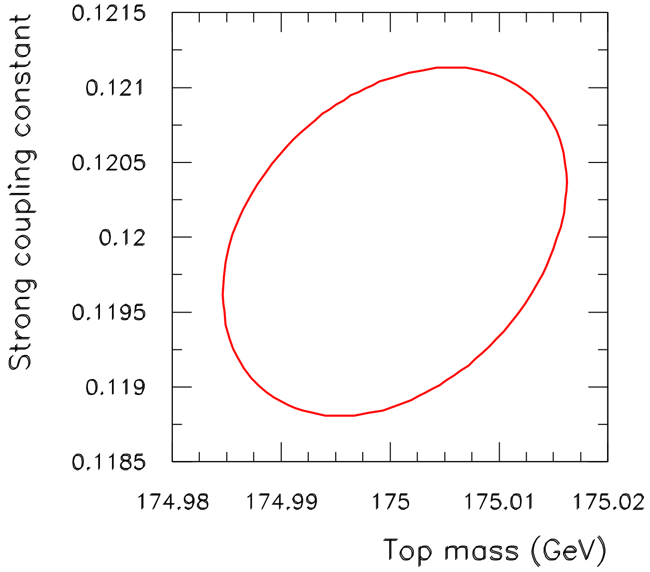
Additional information:

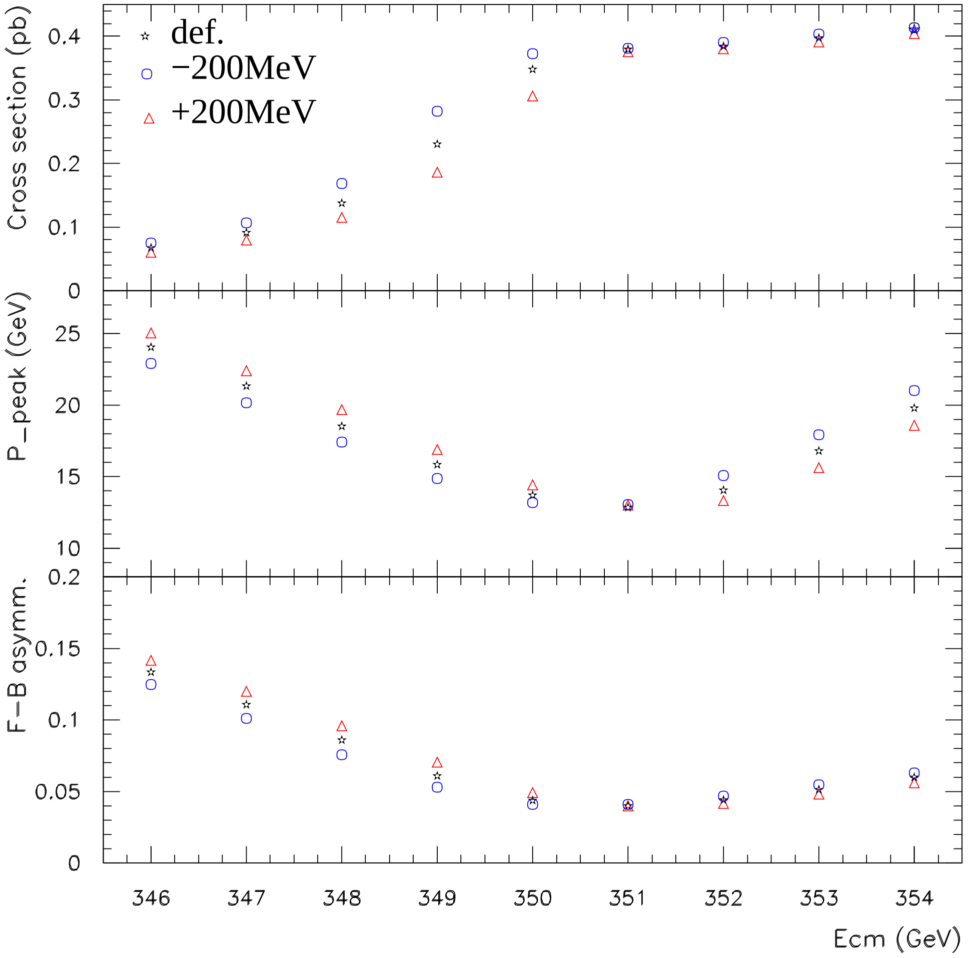
- absolute value of the $\mathrm{t\bar{t}}$ cross section: sensitive to α_s, Γ_t
- Momentum distribution of top quarks near threshold sensitive to m_t
- Forward backward asymmetry: sensitive to Γ_t
- Can try multi-parameter fits

Results (10 scan points with $\mathcal{L} = 30\,\mathrm{fb}^{-1}$ each):

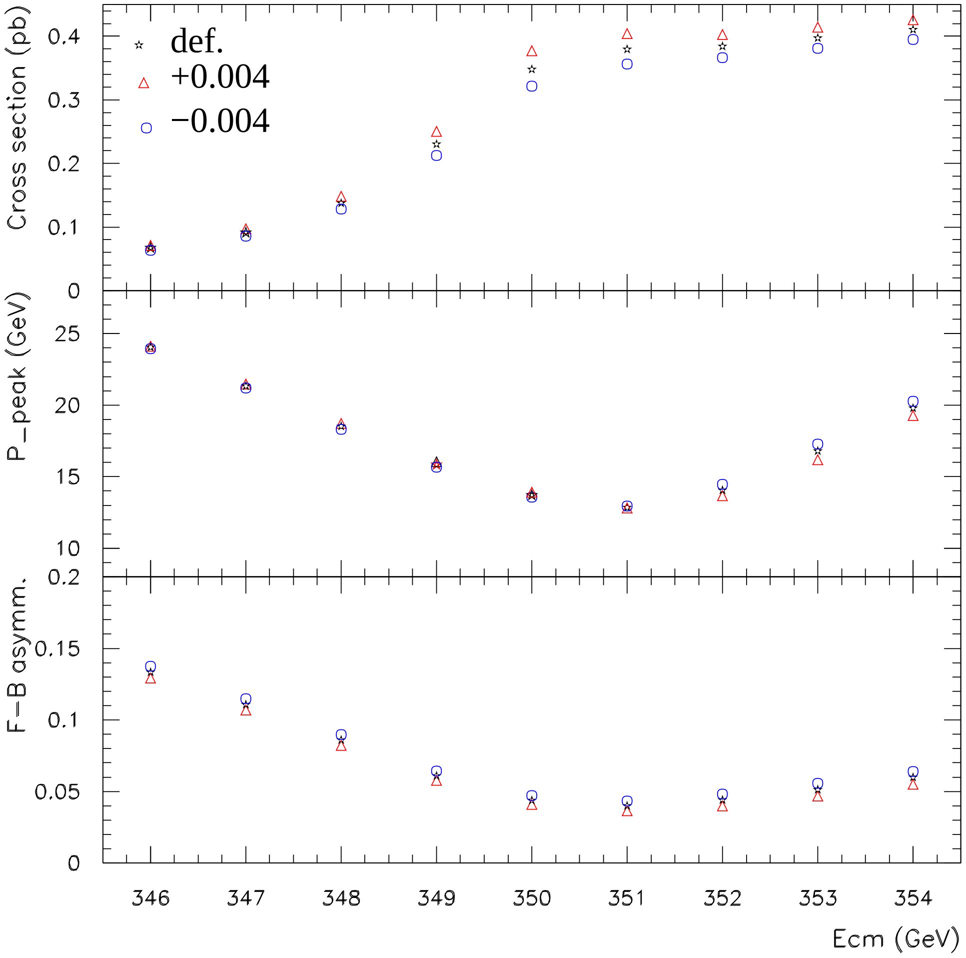
$$\begin{aligned} \Delta m_\mathrm{t} &= 34\,\mathrm{MeV} \\ \Delta \Gamma_\mathrm{t} &= 42\,\mathrm{MeV} \\ \Delta \alpha_s &= 0.0023 \end{aligned}$$



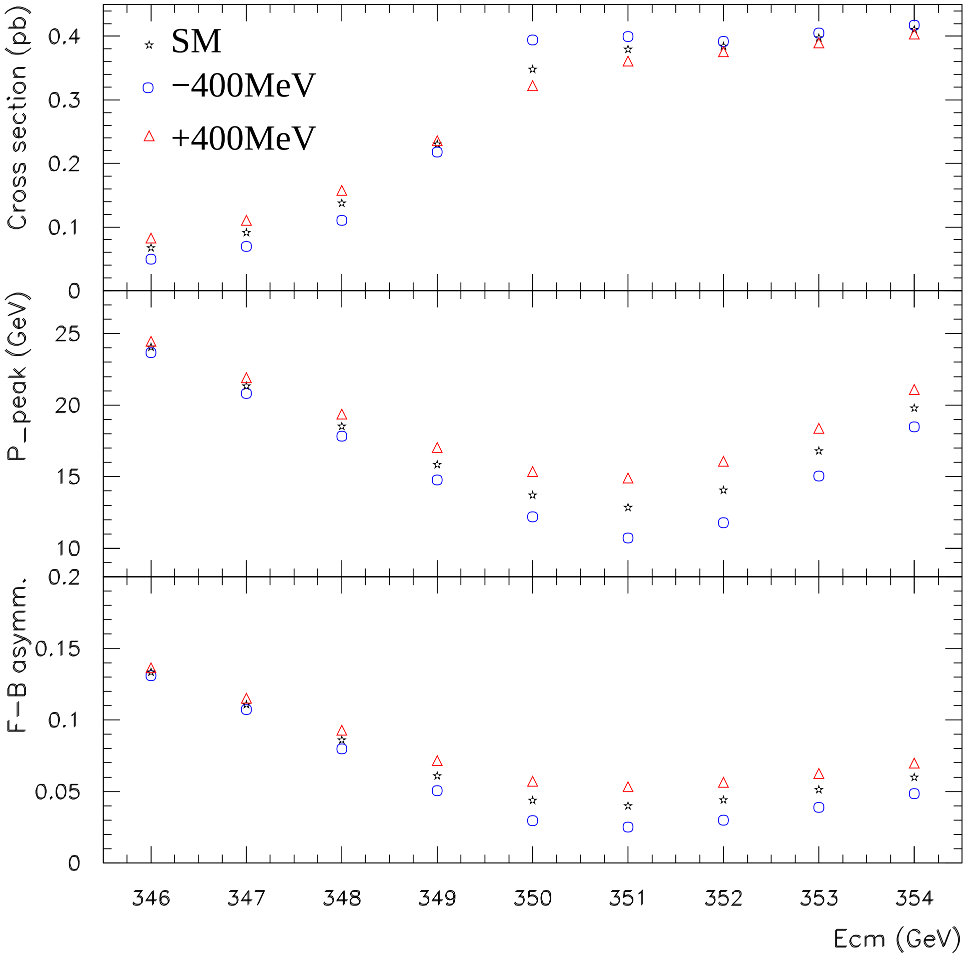
Sensitivity to m_t



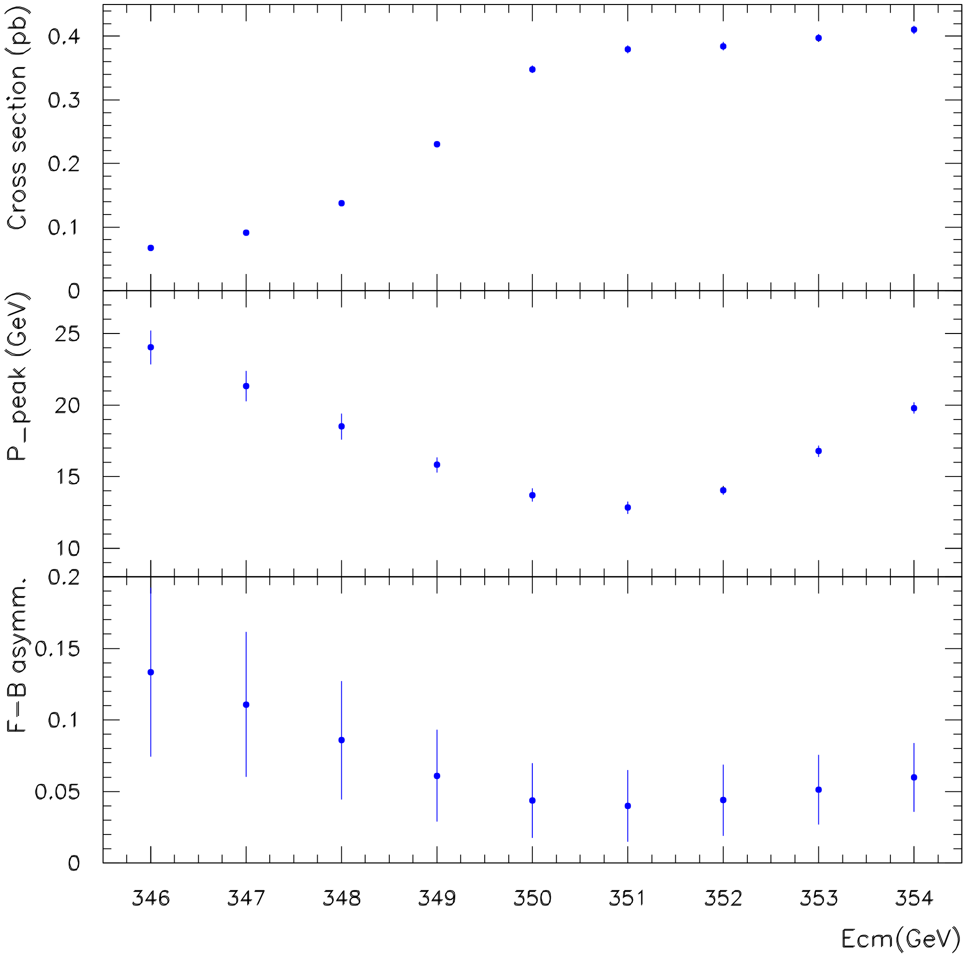
Sensitivity to α_s



Sensitivity to Γ_t



Expected experimental scan results



One step further:

The absolute value of the cross section is also sensitive to the Ht Yukawa coupling (y_t)

$\Rightarrow$ can take α_s from other measurements and fit y_t instead

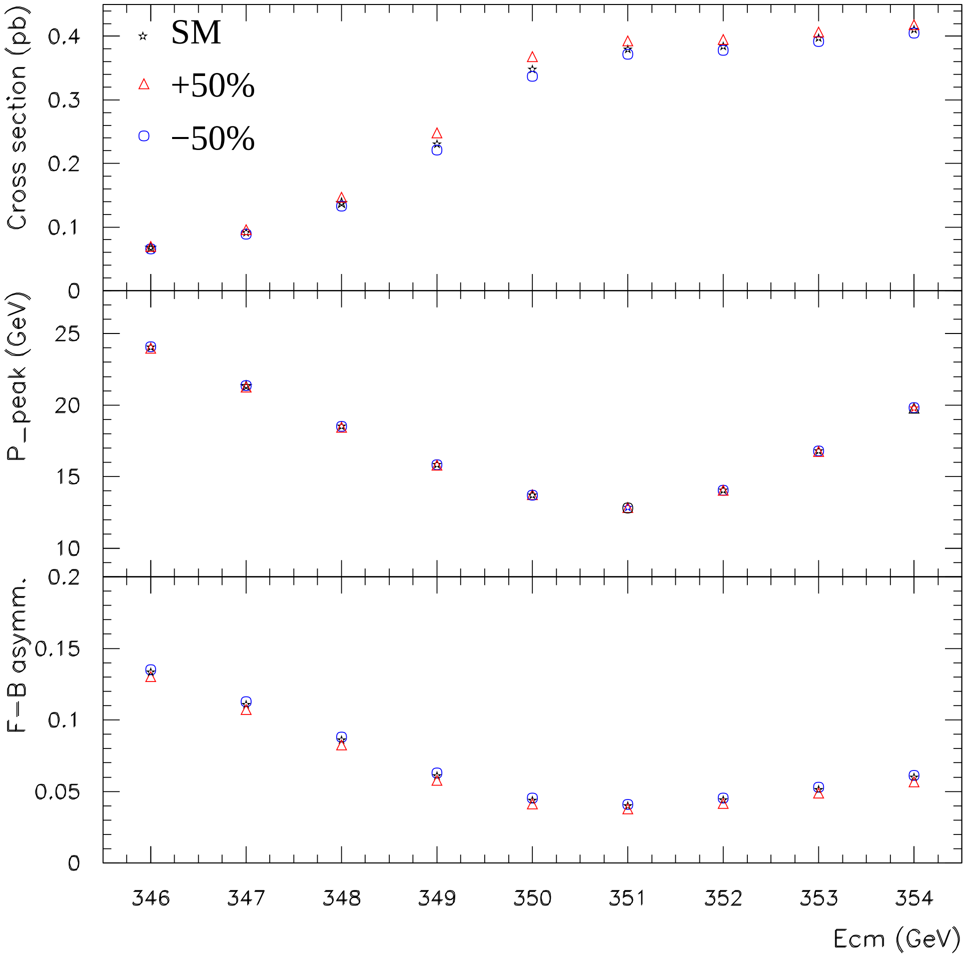
Result:

$$\frac{\Delta y_t}{y_t} = {}^{+0.35}_{-0.65}$$

for a 3-parameter fit

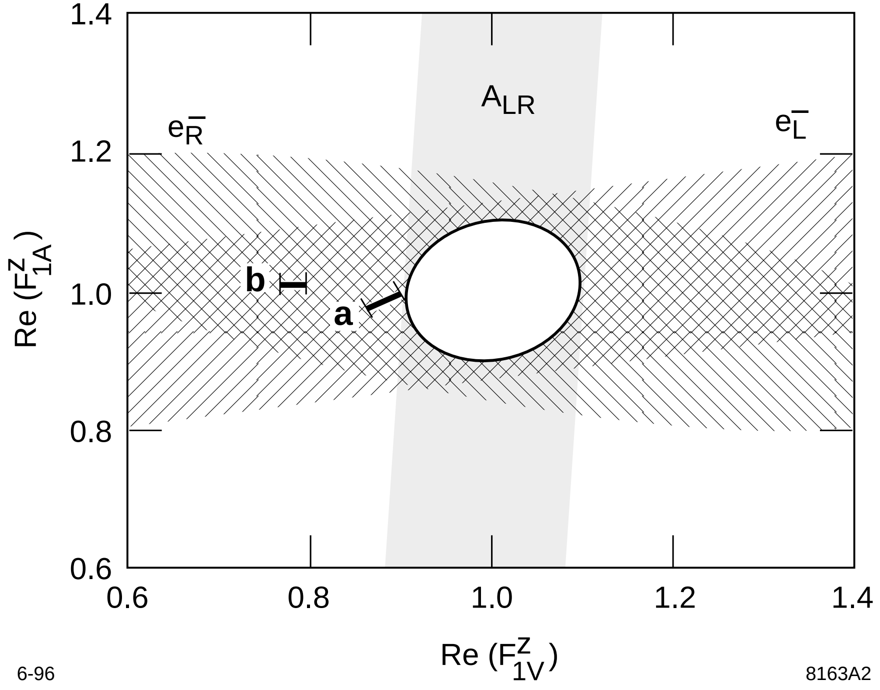
(LHC: $\Delta m_{\mathrm{t}} \approx \pm 1.5\,\mathrm{GeV}$)

Sensitivity to y_t



t$\bar{\text{t}}$Z couplings

- the top-couplings to the Z can be obtained from t$\bar{\text{t}}$-production in the continuum
- due to the interference between Z and γ exchange the total cross section and the left-right asymmetry are sensitive to the Z-couplings
- a very conservative analysis at $\sqrt{s} = 400\,\text{GeV}$ gives 90% c.l. limits on the 10% level for anomalous couplings



- this precision is sensitive to some ETC-models

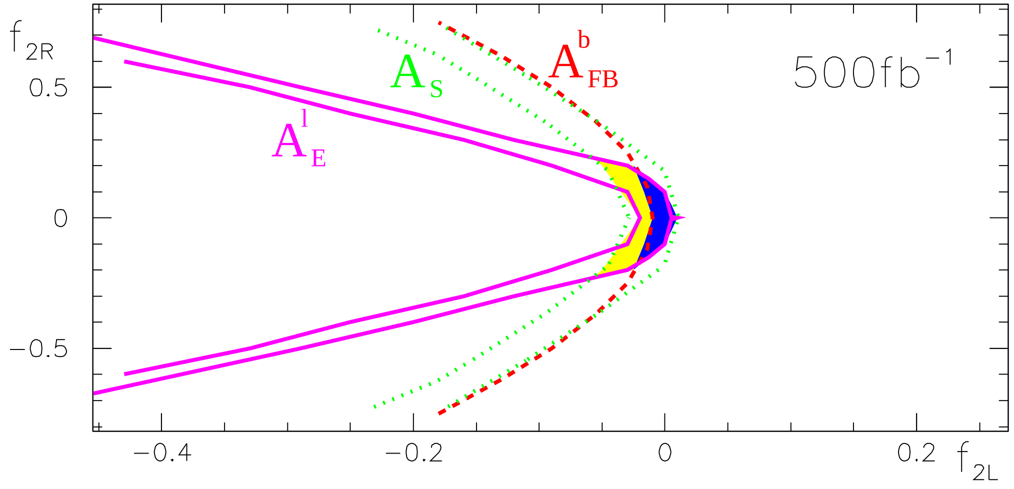
Wtb couplings

- effective Lagrangian:

$$\mathcal{L} = \frac{g}{\sqrt{2}}\Big[W^-_\mu \bar{b}(\gamma_\mu f_{1L} P_- + \gamma_\mu f_{1R} P_+)t - \frac{1}{2M_W} W_{\mu\nu}\bar{b}\sigma^{\mu\nu}(f_{2R}P_- + f_{2L}P_+)t\Big] + \text{h.c.}$$

  ($P_\pm = 1/2(1 \pm \gamma_5)$ SM: $f_{1L} = 1$, rest=0

- Present data put tight constraints on f_{1L}, f_{1R}, so try to measure f_{2L}, f_{2R}
- use $\text{e}^+\text{e}^- \rightarrow \text{t}\bar{\text{t}} \rightarrow X\ell\nu$ at $\sqrt{s} = 500\,\text{GeV}$ and assume t$\bar{\text{t}}$Z-vertex to be standard
- analyze $\text{A}^{\text{b}}_{\text{FB}}$, $\text{A}^{\ell}_{\text{FB}}$ and lepton energy in top rest frame
- results: $\Delta f_{2L} \approx 0.02$, $\Delta f_{2R} \approx 0.2$



Conclusions on top-quark physics

- The $\mathrm{t\bar{t}}$ threshold seems theoretically well under control
- The top quark mass can be measured to $\sim 50\,\mathrm{MeV}$ which is more than one order of magnitude better than what LHC can do
- the top width can be measured on the 3% level in the threshold scan
- the LC is the unique place to test $\mathrm{t\bar{t}Z}$-couplings and can do that with a precision to better than 10%
- for t-decay physics probably the LHC is better due to much higher statistics

❹ Higgs-physics

- The Higgs-mechanism is the only way we know to give masses to particles in the SM
- Up to now we have no direct evidence for any Higgs-particle
- If the Higgs exists, at least the LHC should have found a particle compatible with it,
- The LC has then to prove that this is really the particle responsible for mass generation

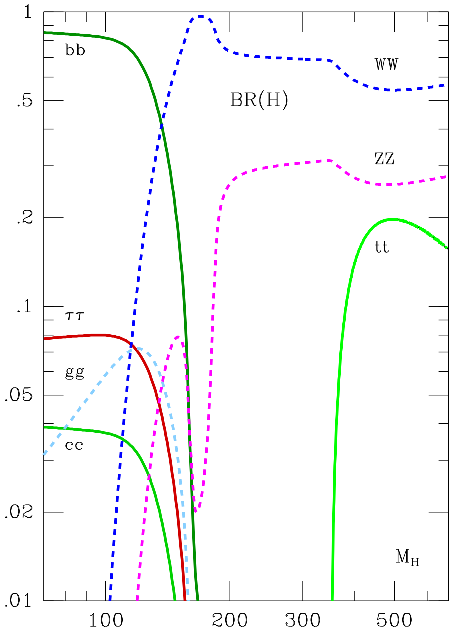
Predictions for the Higgs

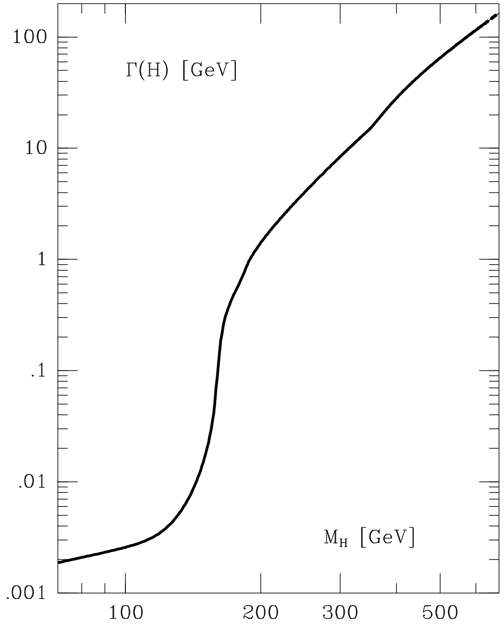
Standard Model:

- One complex Higgs doublet $\begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$ with vacuum expectation value $\begin{pmatrix} 0 \\ v \end{pmatrix}$, $v = 246\,\mathrm{GeV}$.
- Higgs potential $V(\Phi) = \lambda(\Phi^*\Phi - v^2/2)^2$
- Higgs mass $m_{\mathrm{H}}^2 = 2\lambda v^2$

- Partial widths:

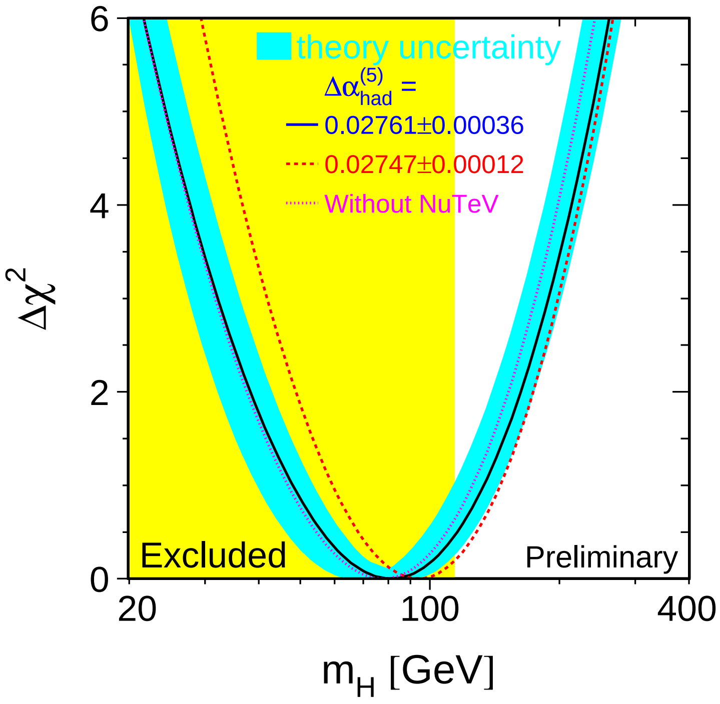
$$\begin{aligned}\Gamma(H\rightarrow \mathrm{f}\bar{\mathrm{f}}) &= \frac{N_c^{(f)} G_\mu}{4\sqrt{2}\pi} m_f^2(m_H) m_H (1+\delta_{QCD}^{(f)})\\ \Gamma(H\rightarrow VV) &= \frac{3G_\mu^2 m_\mathrm{Z}^4}{16\pi^3} m_\mathrm{H} R_V(m_V^2/m_\mathrm{H}^2)\\ &\rightarrow 2(1)\frac{\sqrt{2}G_\mu}{32\pi} m_\mathrm{H}^3 \quad [V=W(Z)]\end{aligned}$$





Limits on m_H

- direct searches at LEP: $m_\mathrm{H} > 114\,\mathrm{GeV}$
- hint of a signal at $m_\mathrm{H} \approx 115\,\mathrm{GeV}$
- electroweak precision data

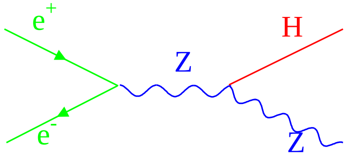


$\Rightarrow m_\mathrm{H} < 200\,\mathrm{GeV} \quad (95\%\mathrm{c.l.})$

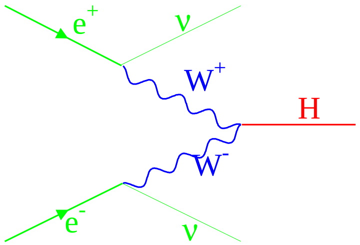
- perturbativity and vacuum stability if SM valid up to M_{pl}: $m_\mathrm{H} \sim 120-180\,\mathrm{GeV}$

Higgs production

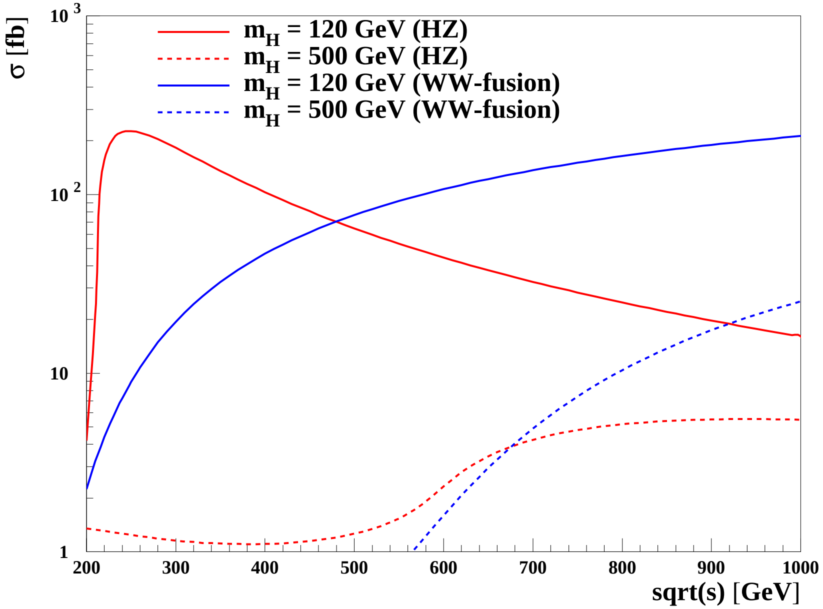
Higgsstrahlung



W-fusion



Cross section:



- both channels accessible at LC
- cross section $\sim 100(\sim 10)$ fb for $m_{\mathrm{H}} = 120(500)$ GeV

➧ few$\times 10^4(10^3)$ Higgses per year

MSSM:

SUSY needs two Higgs-doublets (H_1, H_2) to generate masses of down- and up-type particles

Physical particles:

$$\begin{aligned} h &= H_2 \cos\alpha - H_1 \sin\alpha \\ H &= H_2 \sin\alpha + H_1 \cos\alpha \\ A & \quad \mathrm{CP-odd} \\ H^\pm & \quad \text{charged Higgses} \end{aligned}$$

Define $\tan\beta = \frac{v_2}{v_1}$ = ratio of expectation values $(v_1^2 + v_2^2 = v_{SM}^2)$

Born Formulae:

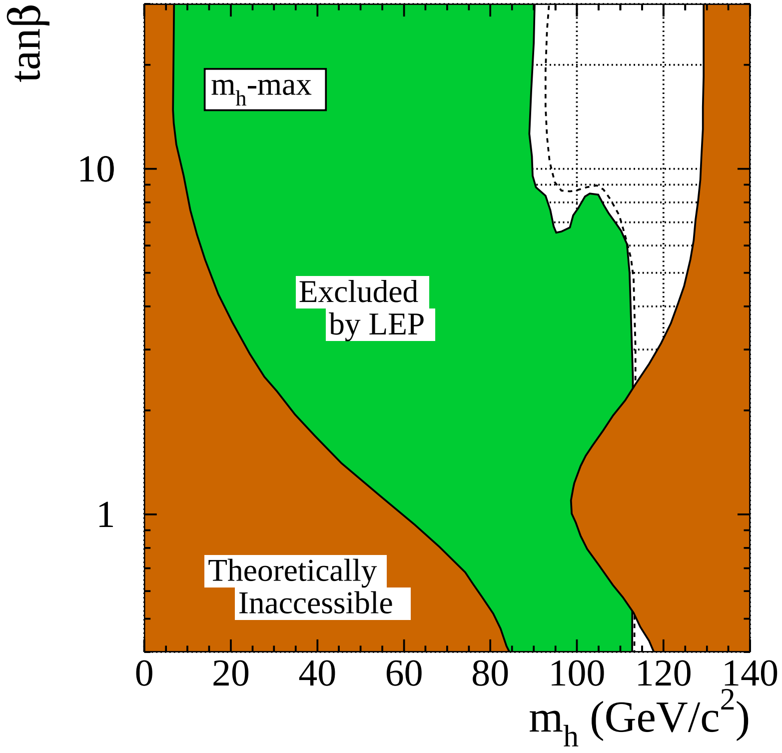
$$\begin{aligned} m^2_{h,H} &= \frac{1}{2}\Big[m_A^2 + m_Z^2 \mp \\ & \qquad \sqrt{\left(m_A^2 + m_Z^2\right)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\Big] \\ m_h &< m_Z \\ m_H &> m_Z \\ m^2_{H^\pm} &= m_A^2 + m_{\mathrm{W}}^2 \\ \tan 2\alpha &= \tan 2\beta \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} \; \left(-\frac{\pi}{2} < \alpha < 0 < \beta < \frac{\pi}{2}\right) \end{aligned}$$

Higgs sector described by two free parameters

However large radiative corrections:

- shift of m_h up to $\sim 130\,\text{GeV}$
- prediction gets dependent on other SUSY parameters, especially on mixing in stop sector
- strong dependence on top mass: $\Delta m_h / \Delta m_{\text{t}} \approx 1$

Currently allowed region:



$\tan\beta > 2$ preferred!

Complementarity of cross sections:

$$\sigma(e^+e^- \to Zh) = \sin^2(\beta-\alpha)\sigma_{SM}$$
$$\sigma(e^+e^- \to Ah) = \cos^2(\beta-\alpha)\bar{\lambda}\sigma_{SM}$$

($\bar{\lambda}$: P-wave suppression)

If m_A large:

- $\beta - \alpha = \pi/2 \;\Rightarrow\; \sigma(e^+e^- \to Zh) = \sigma_{SM}$
- $m_H \approx m_{H^\pm} \approx m_A$

➠ Only one SM-like Higgs can be seen

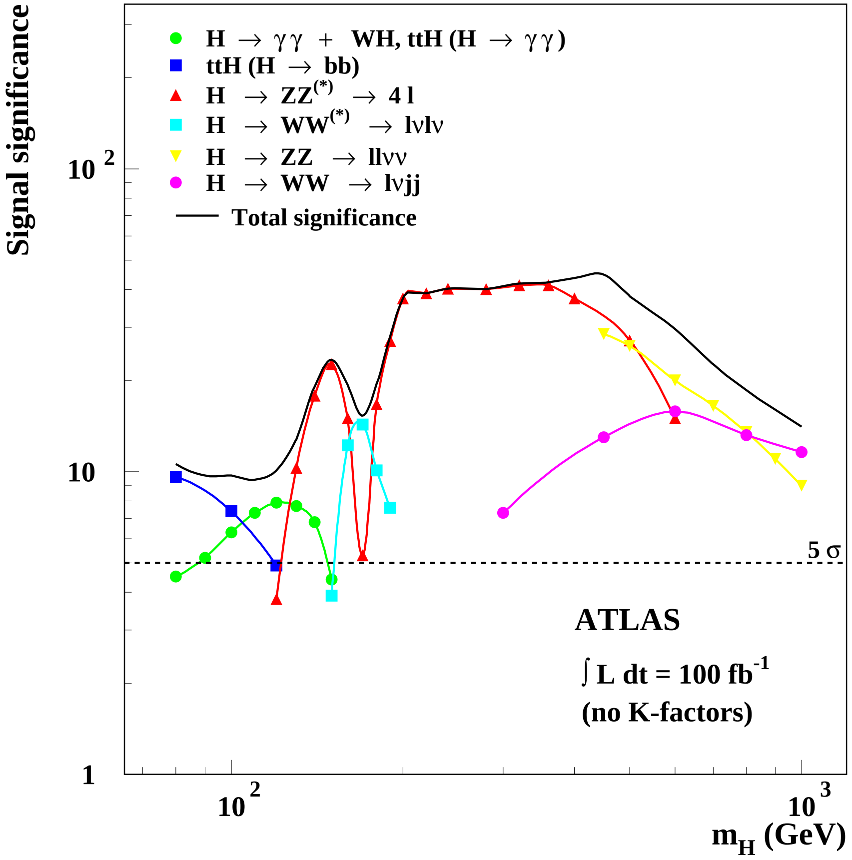
Branching ratios:

$$\Gamma(h \to U\overline{U}) = \frac{\cos^2\alpha}{\sin^2\beta}\Gamma_{\text{SM}}(h \to U\overline{U})$$
$$\Gamma(h \to D\overline{D}) = \frac{\sin^2\alpha}{\cos^2\beta}\Gamma_{\text{SM}}(h \to D\overline{D})$$

- For m_A large also branching ratios become SM like
- however, it turns out that some sensitivity remains in regions where no other Higgs than h can be seen

LHC discovery of the Higgs

A SM-like Higgs cannot be missed by the LHC

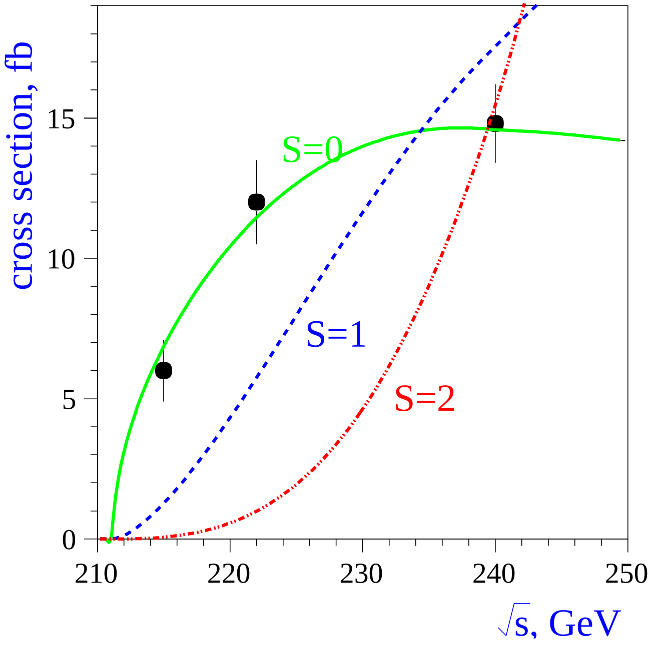


The task of the LC is then precision measurements

Measurement of the H quantum numbers

After the H has been discovered it has to be proven that its quantum numbers are really 0^+

At the LC this can be done with a threshold scan of $e^+e^- \to ZH$:



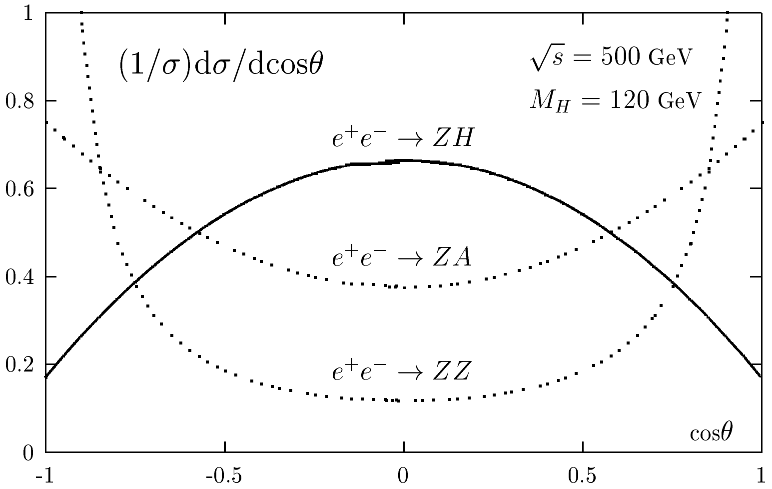
- Large sensitivity to the different states
- The few remaining ambiguities can be resolved from angular dependences and the observation of $H \to \gamma\gamma$
- Alternatively spin/parity can be measured in transverse/longitudinally polarized $\gamma\gamma$-collisions

What can the LHC do on J,P?

- $H \to \gamma\gamma$ excludes J=1
- if $H \to ZZ$ is visible S should be measurable from spin correlations

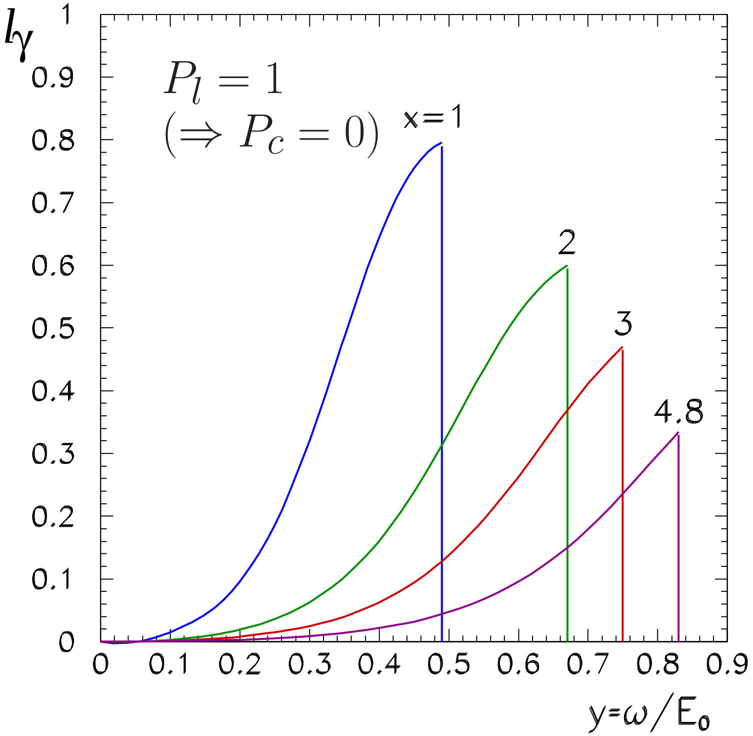
The Higgs CP quantum numbers

- Angular distributions give admixture of CP odd Higgs $|\eta|$
  LC: 3%
  LHC: 30%



- However CP-odd Higgs doesn't couple to vector boson pairs directly
  → η =mixing angle × loop factor
  ➡ might not be visible

- Alternative: $\gamma\gamma$ collisions:
  - Use linear beam polarization $\vec{\varepsilon_1}$, $\vec{\varepsilon_2}$
  - CP-even Higgs: $\sigma \propto \vec{\varepsilon_1} \cdot \vec{\varepsilon_2}$
  - CP-odd Higgs: $\sigma \propto [\vec{\varepsilon_1} \times \vec{\varepsilon_2}] \cdot \vec{k_\gamma}$
  - Coupling strength roughly equal
  - Asymmetry measures CP-even - CP-odd mixture
  - Problem: transverse beam polarization large for small x → small $\sqrt{s}$



⇒ fine for small m_H, difficult for large m_H (heavy SUSY Higgses)

- $f_{CP} < 0.2$ at 95% C.L. might be possible for $m_H = 120\,\mathrm{GeV}$

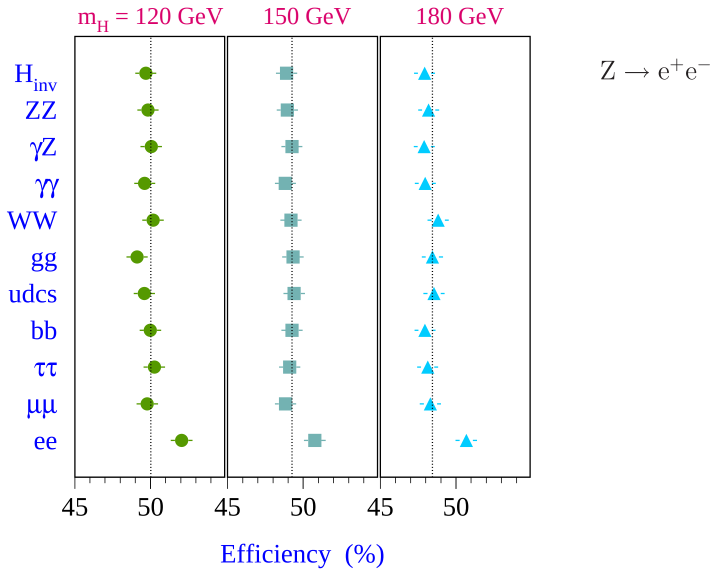
Measurement of the $e^+e^- \to HZ$ cross section

Need a measurement of the total cross section $\sigma(e^+e^- \to HZ)$ independent of the H decay mode:

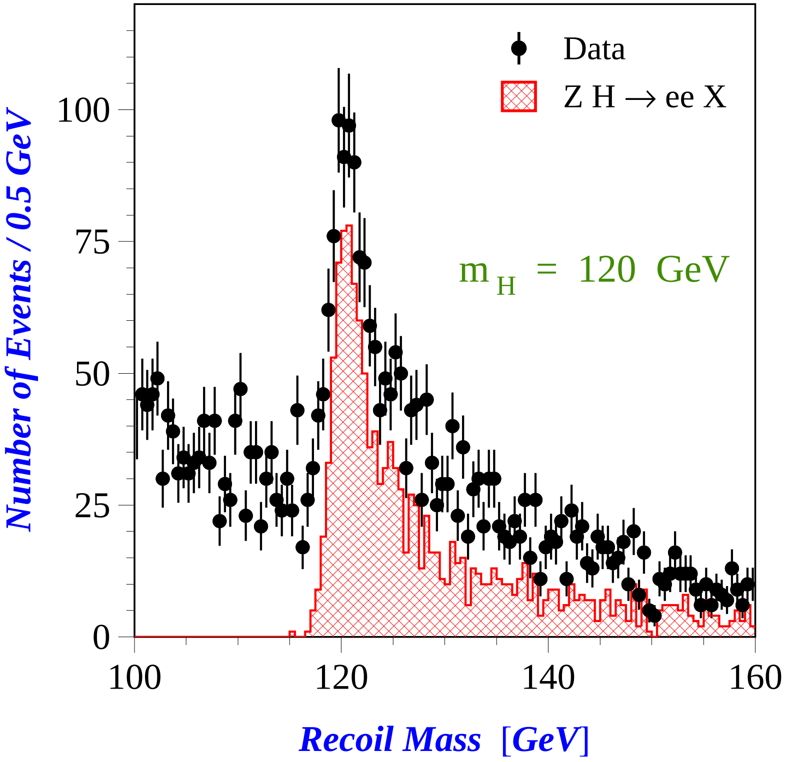
- $\sigma(e^+e^- \to HZ)$ measures $\Gamma(H \to ZZ)$
- absolute normalization for H-branching ratio measurements

Method

- select HZ events with $Z \to e^+e^-, \mu^+\mu^-$ only by looking at the leptons cutting on $m_{\ell\ell} \sim m_Z$
- efficiency (almost) independent of H-decay mode



- fit recoil mass distribution
- Higgs signal clearly visible with some tails from ISR and beamstrahlung



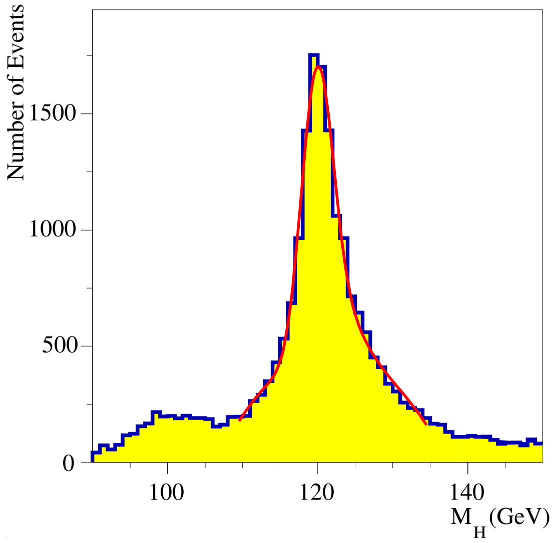
- Results:
  $(\sqrt{s} = 350\,\text{GeV},\ \mathcal{L} = 500\,\text{fb}^{-1},\ \text{m}_\text{H} \sim 120\,\text{GeV})$

$$\Delta\sigma(\text{e}^+\text{e}^- \to \text{HZ}) \approx 2.4\%$$

$$\Delta m_\text{H} \approx 140\,\text{MeV}$$

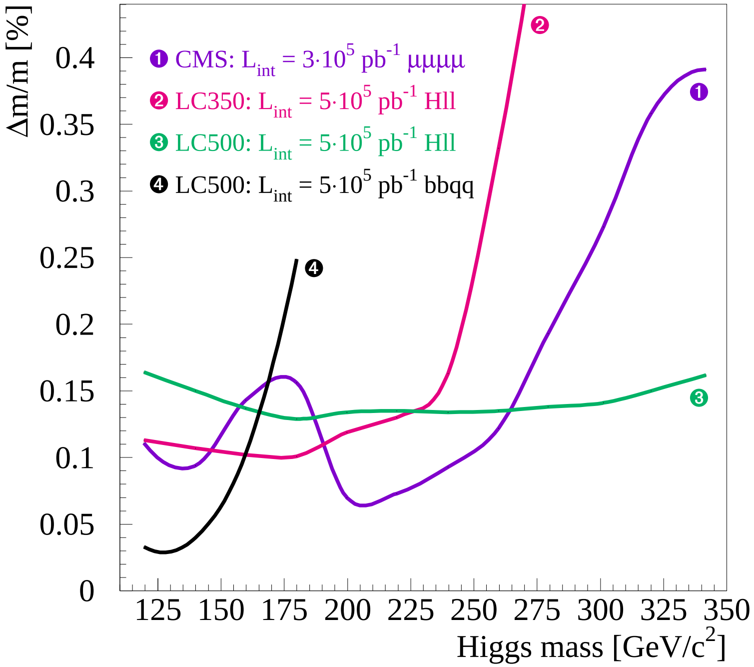
Measurement of m_H

- from Z recoil mass: $\Delta m_\text{H} \approx 140\,\text{MeV}$
- alternative: constrained fit similar to m_W at LEP:
  - Analysis with $m_\text{H} = 120\,\text{GeV}$, $\mathcal{L} = 500\,\text{fb}^{-1}$
  - Select $\text{e}^+\text{e}^- \longrightarrow HZ$-events
  - perform constrained fit imposing energy/momentum conservation and taking into account ISR/beamstrahlung



  - $\Delta m_\text{H} \approx 50\,\text{MeV}$ for $\mathcal{L} = 500\,\text{fb}^{-1}$
    combined with $\text{HZ} \to \ell\ell\text{b}\bar{\text{b}}$: $\Delta\text{m}_\text{H} \approx 40\,\text{MeV}$
  - For larger m_H precision stays at 0.05% level using recoil mass and fit to $\text{ZH} \to \text{q}\bar{\text{q}}\text{W}^+\text{W}^-$ mass distribution

Comparison of Higgs-mass determination at LC and LHC



- recoil-mass method similar to LHC over the full mass range
- direct reconstruction with $H \to \mathrm{b\bar{b}}$ superior at low m_H
  needs to be tried with $H \to WW, ZZ$ at higher masses
- threshold scan not yet explored

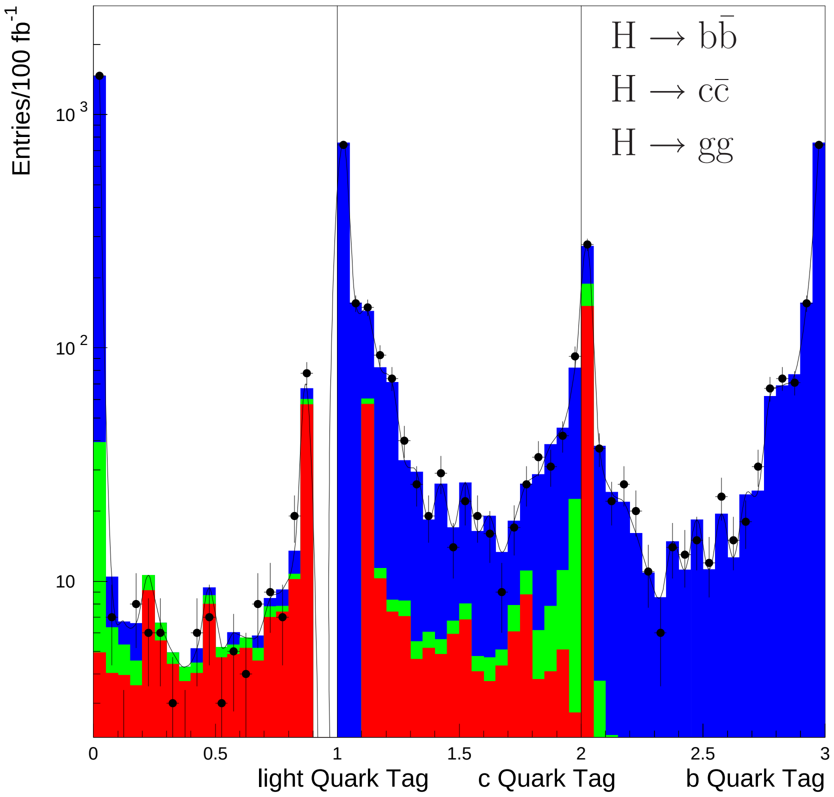
How well do we need to know m_H

- SM: dependence of precision observables on m_H only logarithmic
  $\Rightarrow \Delta m_\mathrm{H} \sim 1\,\mathrm{GeV}$ largely sufficient
- Beyond SM, e.g. SUSY: m_H connected with fundamental parameters of the theory
  $\Rightarrow$ need m_H as good as possible
  However:
  - large radiative corrections from top-sector ($\delta m_\mathrm{H}/\delta m_\mathrm{t} \approx 1$)

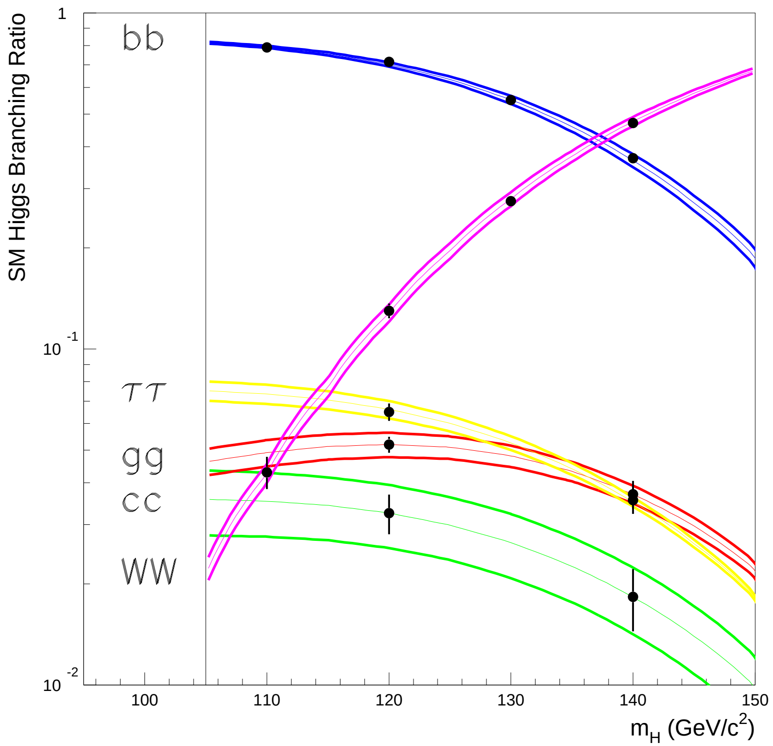
➠ Top mass error might be limiting factor

Measurement of the Higgs branching ratios

- absolute branching ratios can be measured from the $Z \to \ell\ell$ sample
- ratios of branching ratios can also be obtained from other channels
- different 2-jet modes can be separated by b-tagging



Results: $(\sqrt{s} = 350\,\mathrm{GeV},\ \mathcal{L} = 500\,\mathrm{pb}^{-1})$

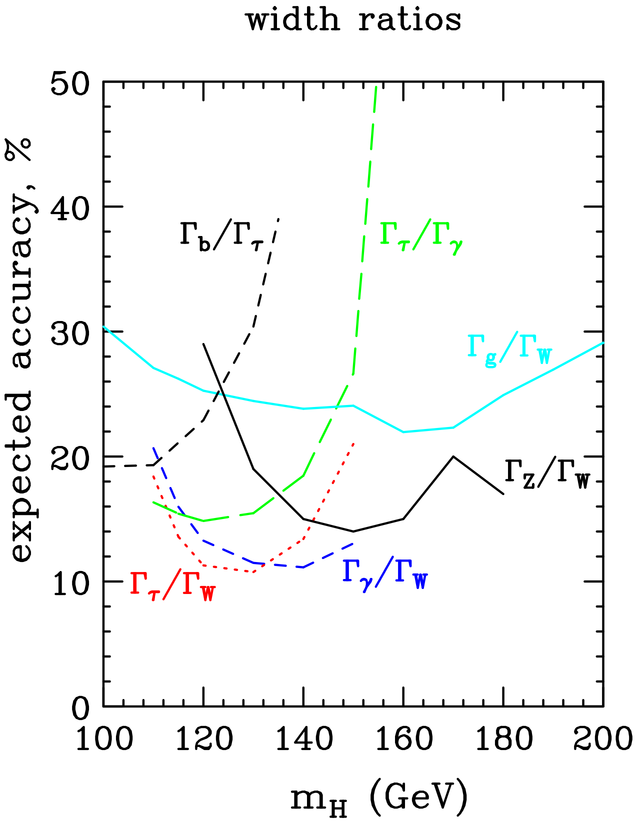


$m_{\mathrm{H}} = 120\,\mathrm{GeV}$:

Channel	$\delta(BR(H \to X)/BR$
$H^0/h^0 \to \mathrm{b\bar{b}}$	±0.024
$H^0/h^0 \to \mathrm{c\bar{c}}$	±0.083
$H^0/h^0 \to gg$	±0.055
$H^0/h^0 \to \tau^+\tau^-$	±0.050
$H^0/h^0 \to WW^*$	±0.051

LHC results on branching ratios

LHC can measure Higgs decays into several channels
$\Rightarrow$ direct measurement of ratios of partial widths



To get partial width the LHC always needs assumptions ($b-\tau$ universality!!)

Even with these assumptions it is about a factor 4 worse than LC

The total width of the Higgs

For $m_\mathrm{H} < 2m_\mathrm{W}$

$$\begin{aligned} BR(H\to X\bar{X}) &= \Gamma(H\to X\bar{X})/\Gamma_\mathrm{H} \\ \sigma(\mathrm{e^+e^-}\to \mathrm{HZ}) &\propto \Gamma(H\to ZZ) \\ \sigma(\mathrm{W^+W^-}\to \mathrm{H}) &\propto \Gamma(\mathrm{H}\to \mathrm{W^+W^-}) \end{aligned}$$

Assuming SU(2) invariance for the Higgs couplings:

$$\Gamma_\mathrm{H} \propto \frac{\sigma(\mathrm{e^+e^-}\to \mathrm{HZ})}{BR(\mathrm{H}\to \mathrm{W^+W^-})}$$

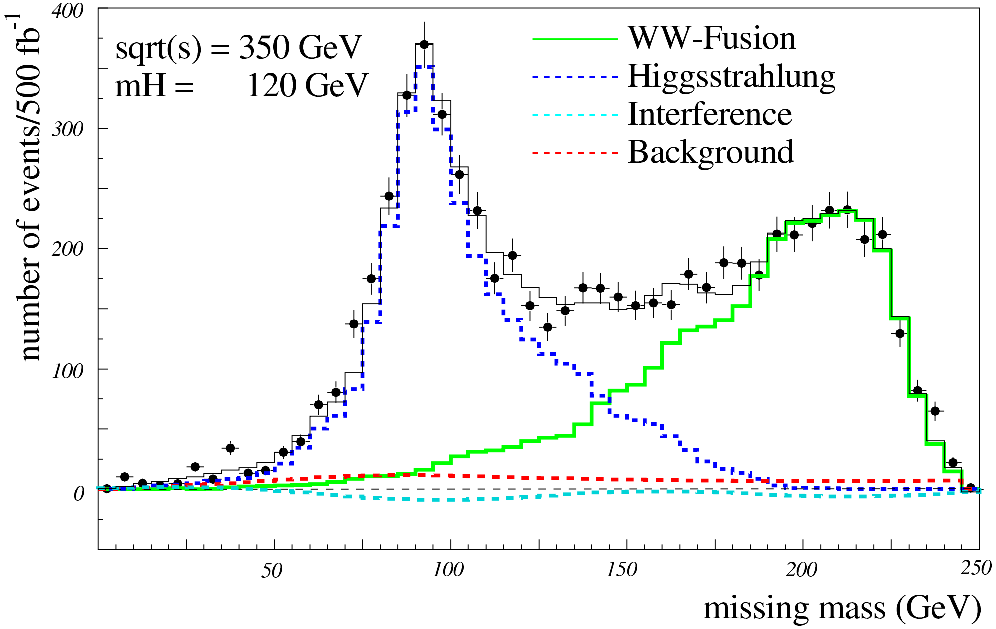
➠ Can obtain Higgs width with $\Delta\Gamma_\mathrm{H}/\Gamma_\mathrm{H} < 6\%$ up to $m_\mathrm{H} \sim 180\,\mathrm{GeV}$

Drop assumption of SU(2) invariance

➨ Have to measure Higgs-fusion cross section

Measurement of $e^+e^- \rightarrow \nu\nu H \rightarrow \nu\nu b\bar{b}$

- $e^+e^- \rightarrow \nu\nu H \rightarrow \nu\nu b\bar{b}$ events are selected using b-tag, $m_{\rm rec}$, $m_{\rm miss}$ and $E_{\rm miss}$



- $e^+e^- \rightarrow$ ZH with Z $\rightarrow \nu\nu$ and WW $\rightarrow$ H are separated by a fit to the missing mass distribution
- for $m_{\rm H} < 140\,\rm GeV$ $\Gamma_{\rm H}$ can be determined with similar accuracy without any assumptions
- for $m_{\rm H} > 140\,\rm GeV$ the necessary analysis of of $e^+e^- \rightarrow \nu\nu H \rightarrow \nu\nu WW$ is not yet done

---

Indirect $\Gamma_{\rm H}$ at LHC:

- LHC can do an indirect measurement of $\Gamma_{\rm H}$ with 20% precision
- however several assumptions are needed for that
  - b-τ universality
  - W-Z universality
  - no unexpected H-decays

The Higgs width for $m_{\rm H} > 2m_{\rm W}$

- For $m_{\rm H} > 2m_{\rm W}$ the Higgs becomes very wide ($\Gamma_{\rm H} \propto m_{\rm H}^3$)

➠ $\Gamma_{\rm H}$ can be fitted from the resonance curve

- example $m_{\rm H} = 240\,\rm GeV$
  - LHC: $\Delta\Gamma_{\rm H}/\Gamma_{\rm H} = 25\%$
  - LC : $\Delta\Gamma_{\rm H}/\Gamma_{\rm H} = 10\%$

  improving with $m_{\rm H}$

Interpretation in the MSSM:

$m_A \gg m_Z \Rightarrow \beta - \alpha = \pi/2 - \eta$ with

$$\eta = \frac{m_Z^2 |\cos 2\beta|}{m_A^2} \sin 2\beta$$

$$\Rightarrow \frac{\sin^2 \alpha}{\cos^2 \beta} = 1 - 2\eta \tan \beta$$

$$\sin^2(\beta - \alpha) = 1 - \eta^2$$

$$\frac{\cos^2 \alpha}{\sin^2 \beta} = 1 + 2\eta / \tan \beta$$

In addition for large m_A:

$$\eta \tan \beta = -\frac{m_Z^2 |\cos 2\beta| + m_h^2}{m_A^2}$$

For $\tan \beta > 2$ (suggested by LEP) $|\cos 2\beta| \approx 1$

$$\Rightarrow \eta \tan \beta = -\frac{m_Z^2 + m_h^2}{m_A^2} \text{ independent of } \tan \beta$$

- $BR(h \to \mathrm{b\bar{b}})/BR(h \to \mathrm{W^+W^-})$ sensitive to m_A
- Effects on $BR(h \to \mathrm{c\bar{c}})$ suppressed by $1/\tan \beta$ and knowledge of m_c

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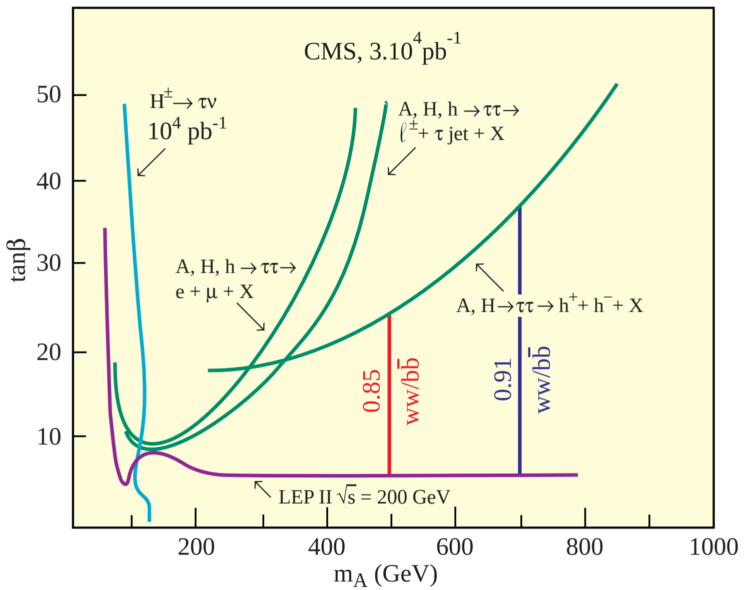
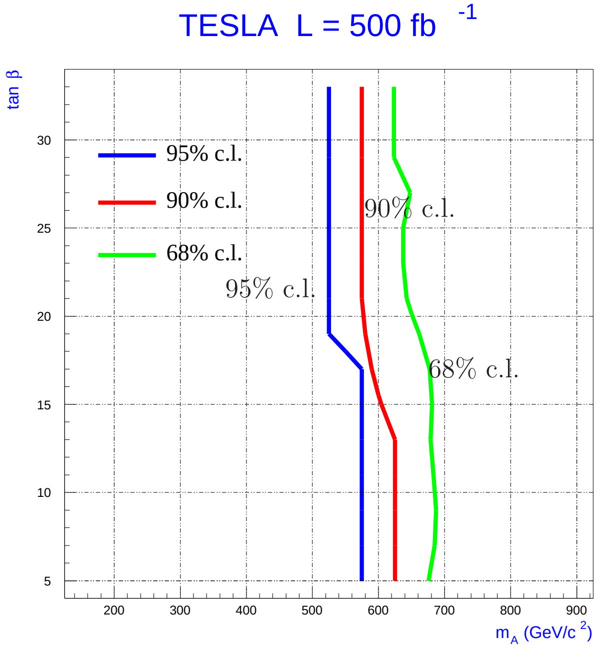
Quantitatively:



Exclusion limits



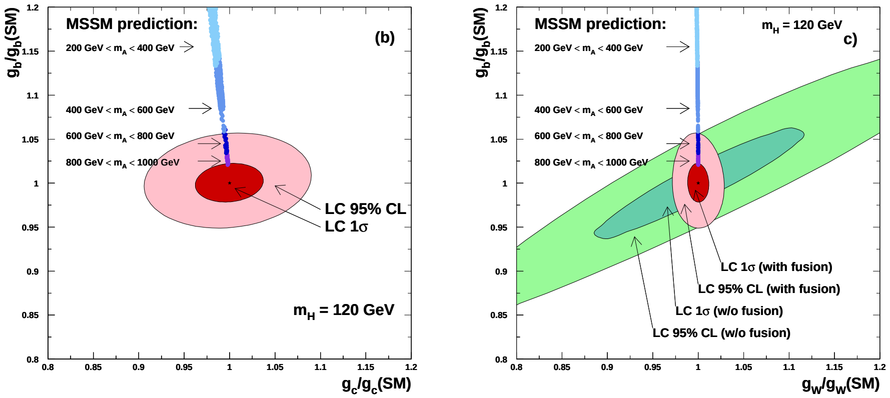
Determination of Higgs couplings

Measurement of Higgs BRs and total width allows determination of Higgs couplings:

$$\begin{aligned}\Gamma(\mathrm{H} \to \mathrm{X\overline{X}}) &= \mathrm{BR}(\mathrm{H} \to \mathrm{X\overline{X}}) \cdot \Gamma_{\mathrm{H}} \\ &\propto g^2_{\mathrm{H}\to\mathrm{X\overline{X}}}\end{aligned}$$

Couplings are obtained from a fit to all related measurements

Model independent Higgs couplings can be compared to model predictions



Measurement of $\Gamma(H \to \gamma\gamma)$

- $H \to \gamma\gamma$ is loop induced process sensitive to couplings of heavy particles to the Higgs (e.g. stop heavier than 250GeV can give effects of $> 10\%$)
- $BR(H \to \gamma\gamma)$ can be measured to $\sim 10 - 15\%$ for $m_{\mathrm{H}} = 120\,\mathrm{GeV}$, rapidly getting worse when Γ_H increases

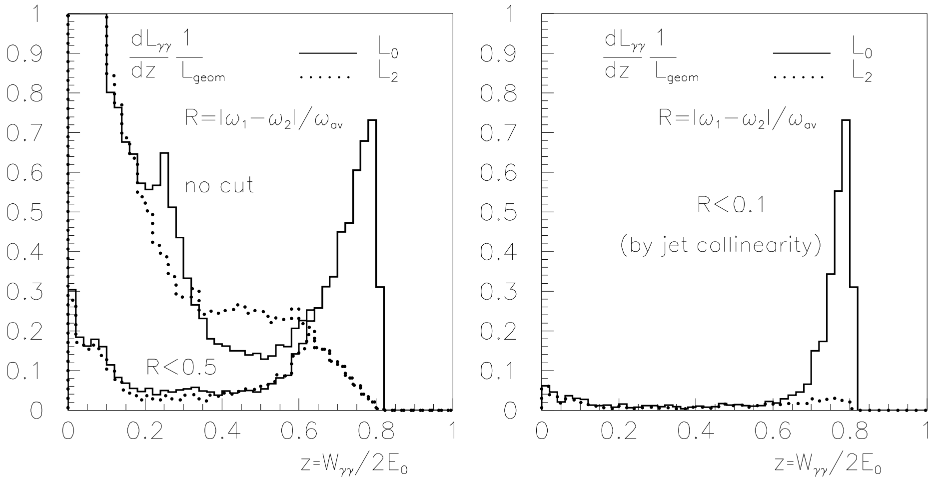
Alternative: measure $\sigma(\gamma\gamma \to H)$ in photon-collider

- cross section for $\sqrt{s_{\gamma\gamma}} = m_{\mathrm{H}}$:

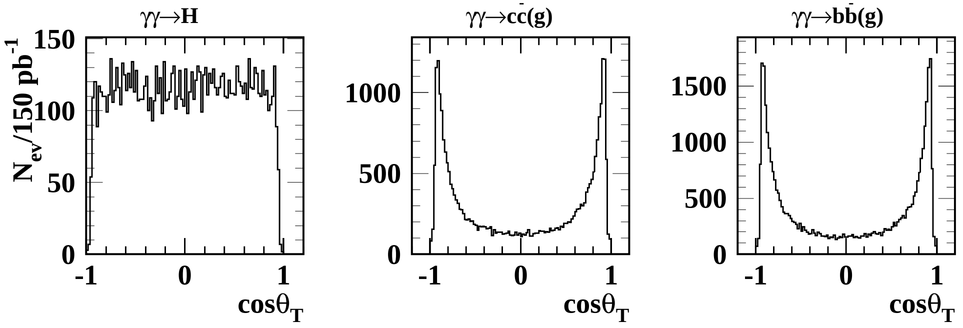
  $$\sigma(\gamma\gamma \to \mathrm{H} \to \mathrm{X}) = \frac{4\pi^2}{\mathrm{m}^3_{\mathrm{H}}} \Gamma(\mathrm{H} \to \gamma\gamma) \cdot \mathrm{BR}(\mathrm{H} \to \mathrm{X})(1+\lambda_1\lambda_2)$$

  (λ_i = helicity of photon i)
- m_{H} is already known when measurement is done $\Rightarrow$ can tune $\gamma\gamma$ energy (peak of dist.) to m_{H}
- analysis up to now done for light Higgs with $\mathrm{H} \to \mathrm{b\bar{b}}$

Can adjust polarization to be mainly $J_z = 0$

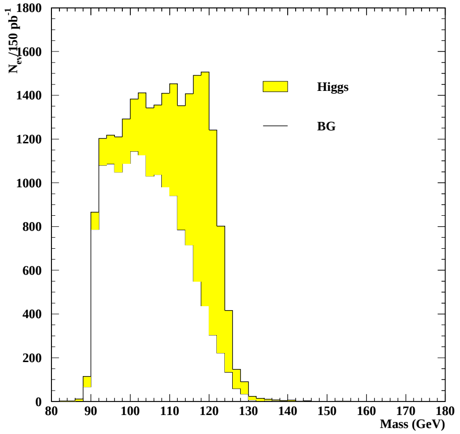


- signal cross section $\sim 0.1\,\mathrm{pb}$
- background: QED $\gamma\gamma \rightarrow q\bar{q}$
  - cross section $\propto Q_q^4 \Rightarrow$ b's suppressed
  - $J_z = 0$ cross section suppressed by m_q^2/s, however $\sim 100\%$ QCD corrections
  - total background from $J_z = 0, 2 \sim$ equal
  - background strongly forward peaked



cut on $|\cos\theta| < 0.7$

- background more concentrated at lower masses



apply mass cuts

- suppress light quarks completely and $c\bar{c}$ by factor 20 using b-tagging

- final purity $\sim 40\%$ with $b\bar{b}$- and $c\bar{c}$-background about equal
- for $\mathcal{L}_{\gamma\gamma}(0 < z < z_{\max}) = 150\,\mathrm{fb}^{-1}$ corresponding to $\mathcal{L}_{\gamma\gamma}(0.65 < z < z_{\max}) = 43\,\mathrm{fb}^{-1}$ corresponding to $\mathcal{L}_{ee} = 200\,\mathrm{fb}^{-1}$ about 8000 signal events are selected

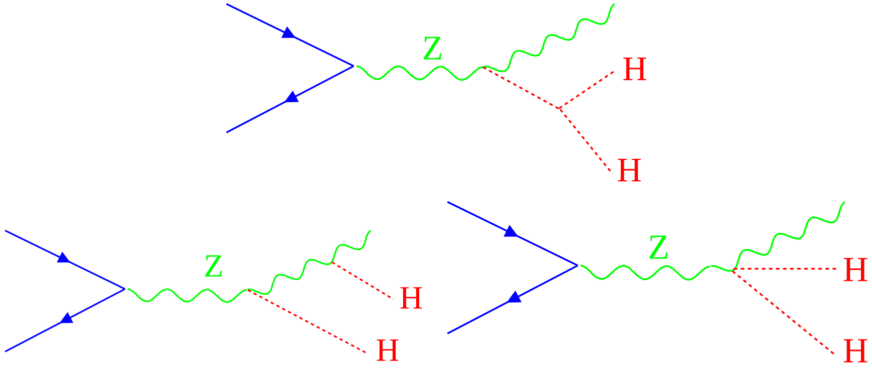
➨ $\frac{\Delta\Gamma(H\rightarrow\gamma\gamma)BR(H\rightarrow b\bar{b})}{\Gamma(H\rightarrow\gamma\gamma)BR(H\rightarrow b\bar{b})} \approx 2\%$

➨ with $\Delta BR(H \rightarrow b\bar{b}) = 2.4\%$: $\frac{\Delta\Gamma(H\rightarrow\gamma\gamma)}{\Gamma(H\rightarrow\gamma\gamma)} \approx 3\%$

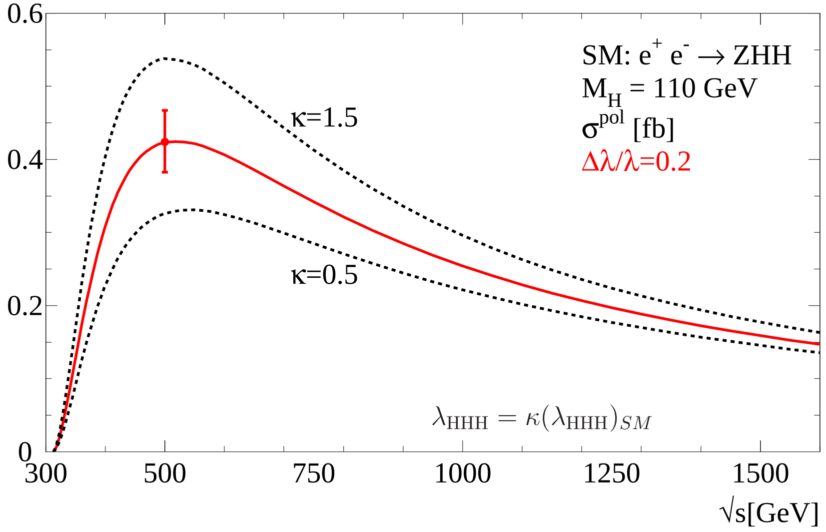
Measurement of the Higgs self-couplings

- Higgs potential $V(\Phi) = \lambda(\Phi^*\Phi - v^2/2)^2$
- Inside the SM completely known once m_H is measured
- Have to reconstruct the Higgs potential as much as possible to prove that the Higgs is really responsible for electroweak symmetry breaking
- trilinear Higgs coupling:
  $\lambda_\mathrm{HHH} = 3m_\mathrm{H}^2/m_\mathrm{Z}^2\lambda_0$, $\lambda_0 = m_\mathrm{Z}^2/v$
- quadrilinear Higgs coupling:
  $\lambda_\mathrm{HHHH} = 3m_\mathrm{H}^2/m_\mathrm{Z}^4\lambda_0$
- trilinear coupling can be seen at LC, quadrilinear coupling too small

Processes for $\mathrm{e^+e^-} \rightarrow ZHH$:



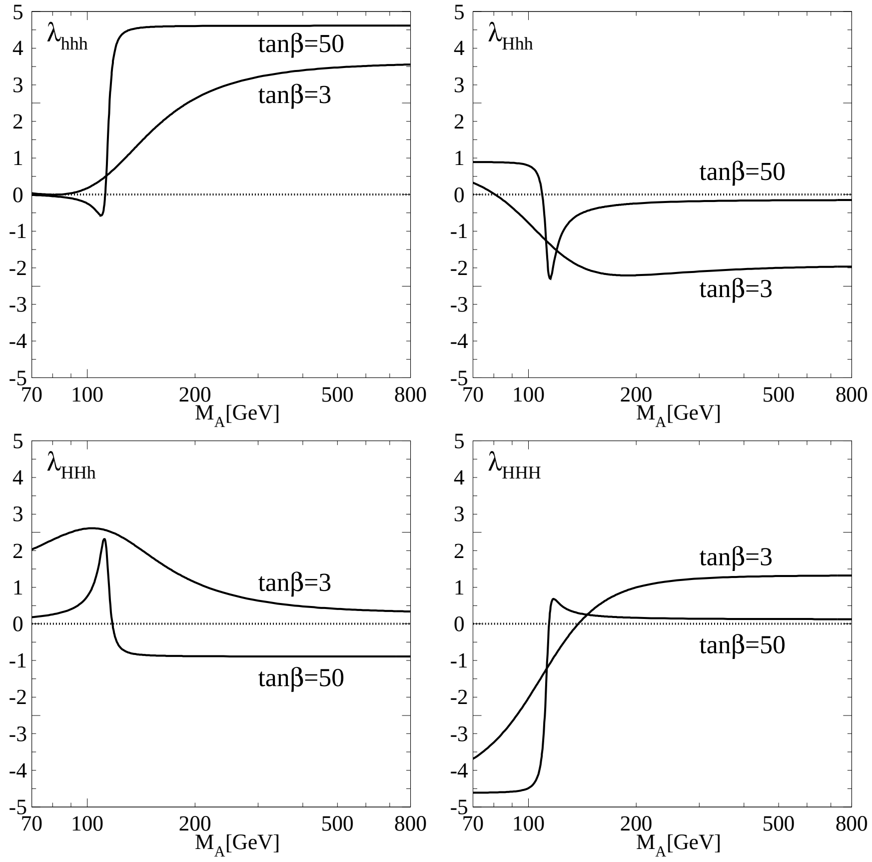
Cross section and sensitivity to λ_HHH:



For a light Higgs it should be possible to establish Higgs-self-coupling with $\sqrt{s} = 500\,\mathrm{GeV}$ and several hundred fb^{-1} luminosity

For heavier Higgses WW fusion can take over

Situation more complicated in SUSY

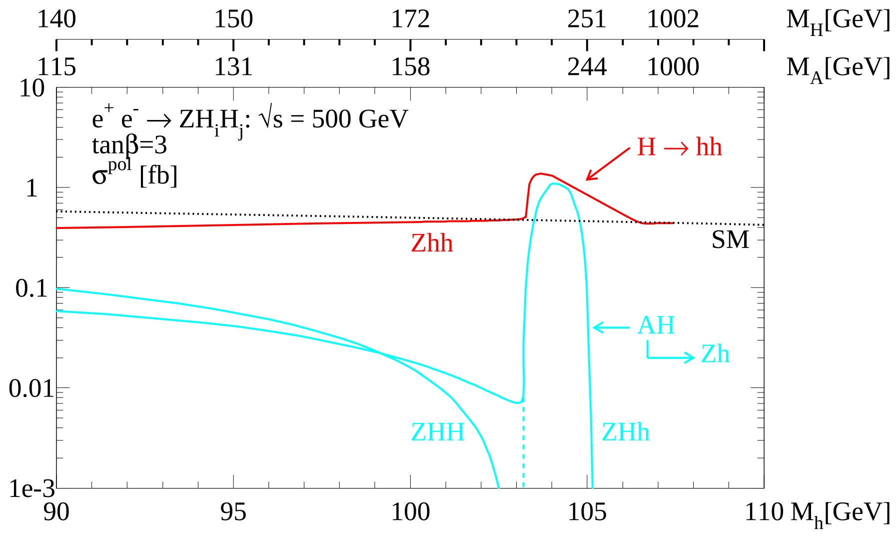


(hAA, HAA couplings generally small)

Has to be folded with Zhh (ZHH) coupling

(SM: $\lambda \approx 5$)

Some effects should remain visible



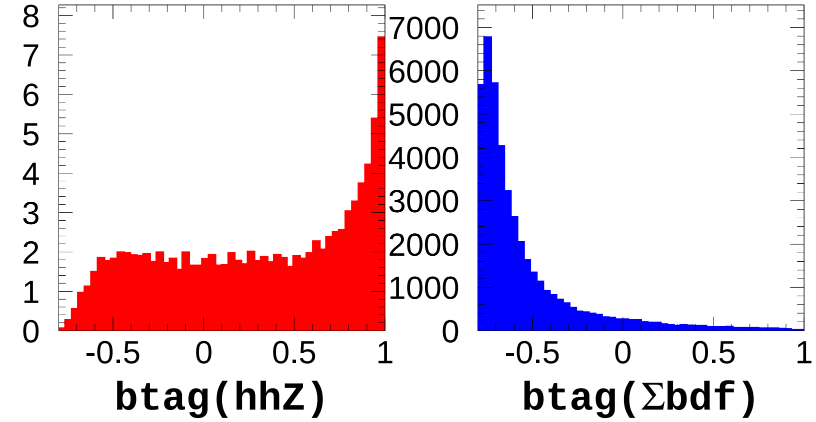
- Experimental SM analysis exists
- SUSY analysis to be done

Experimental analysis of HHH-coupling

- Assume $\sqrt{s} = 500\,\mathrm{GeV}$, $\mathcal{L} = 500\,\mathrm{fb}^{-1}$, $m_\mathrm{H} = 100\,\mathrm{GeV}$
- Signal $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZHH} \to \mathrm{b\bar{b}b\bar{b}f\bar{f}}$ $\sigma \sim 0.5\,\mathrm{fb}$
- Background: after preselection $\sim 500\times$signal (WW, Zγ, ZZ, WWZ, ZZZ, hZ)

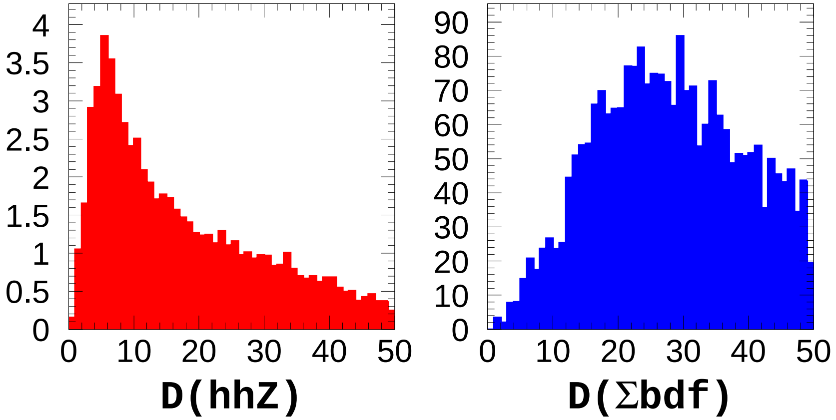
- Key: b-tagging

Combined b-tagging variable (from 6 jets)



- plus topological cuts after constrained fit

$$D = \sqrt{(m_{12} - m_{\mathrm{H}})^2 + (m_{34} - m_{\mathrm{H}})^2 + (m_{56} - m_{\mathrm{Z}})^2}$$

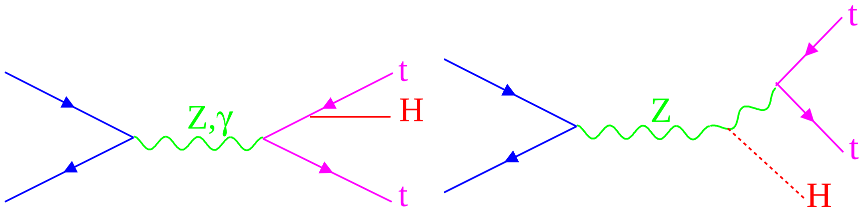


- final efficiency $\varepsilon = 15\%$ with $\mathrm{S/B} \sim 1$
- final backgrounds mainly Z(γ), WW, ZZ, ZZZ; 75% with one $\mathrm{Z} \to \mathrm{t\bar{t}}$ or $\mathrm{W} \to \mathrm{tb}$

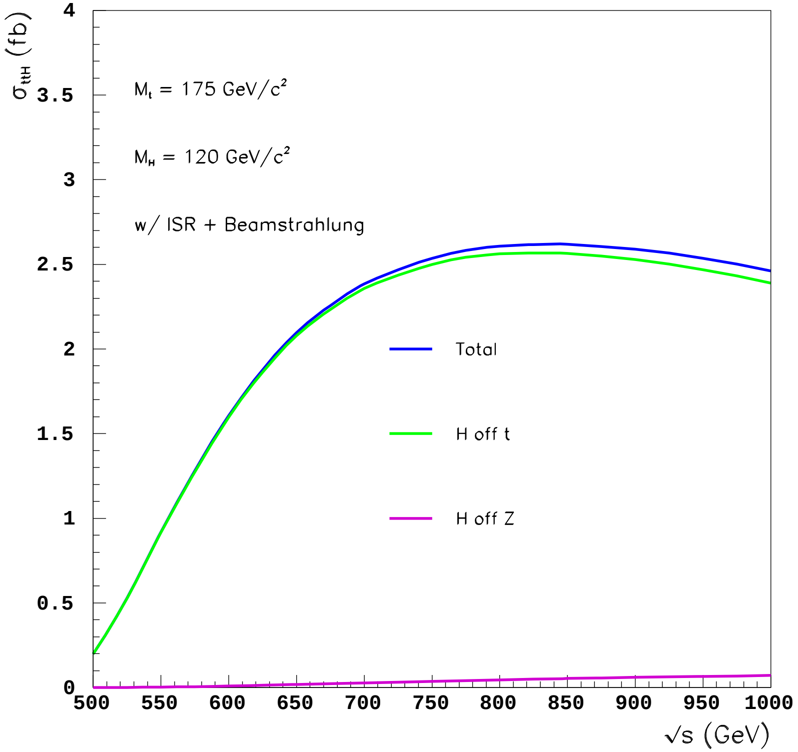
➟ $\Delta\lambda/\lambda \approx 0.2$ is possible

The H$\mathrm{t\bar{t}}$-Yukawa coupling

- If the Higgs is responsible for mass generation its couplings should be proportional to the particle mass
- The couplings HZZ, HWW are known from the cross sections $\mathrm{e^+e^-} \to \mathrm{ZH}$ and $\mathrm{WW} \to \mathrm{H}$
- The Yukawa couplings $\mathrm{Hb\bar{b}}$, $\mathrm{Hc\bar{c}}$, $\mathrm{H}\tau^+\tau^-$ can be obtained from the partial decay widths
- The top-Yukawa coupling is especially interesting since $g_{\mathrm{ttH}} \sim 1$ and the top-quark plays a special role in some theories
- A $\sim 35\%$ estimate of the top-Yukawa coupling can be obtained from the $\mathrm{t\bar{t}}$-threshold scan
- The top-Yukawa coupling can be measured from $\mathrm{t\bar{t}H}$- events



Cross section:



Event signatures:
$t\bar{t}H \to WbWbb\bar{b} \to$ 4q4b, 2q$\ell\nu$4b
(2($\ell\nu$) events and H decays not to $b\bar{b}$ are not considered)

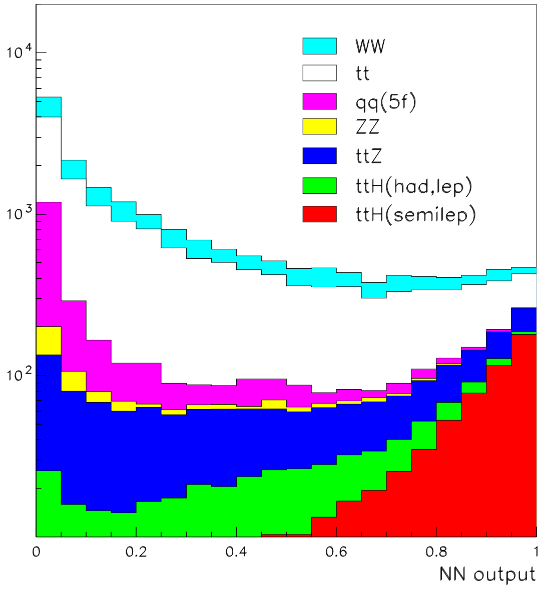
Assumptions:
$\sqrt{s} = 800\,\mathrm{GeV}$, $\mathcal{L} = 1000\,\mathrm{fb}^{-1}$, $m_H = 120\,\mathrm{GeV}$

Example: 2q$\ell\nu$4b analysis

- start with preselection cuts, mainly to separate "round" from "jetty" events
- after preselection

Signal ($\varepsilon = 54\%$)	0.61 fb
Most dangerous backgrounds:	
$t\bar{t}$	10.97 fb
WW	4.05 fb
Total background:	17.59 fb

Process events with neural network including event shapes, b-tagging, lepton-id

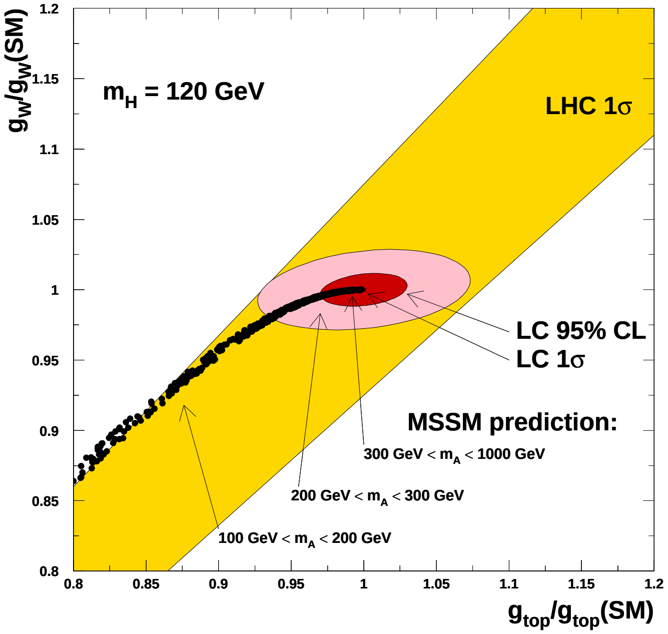


Results:

- Can achieve $S/B = 0.5$ with $\varepsilon = 27\%$

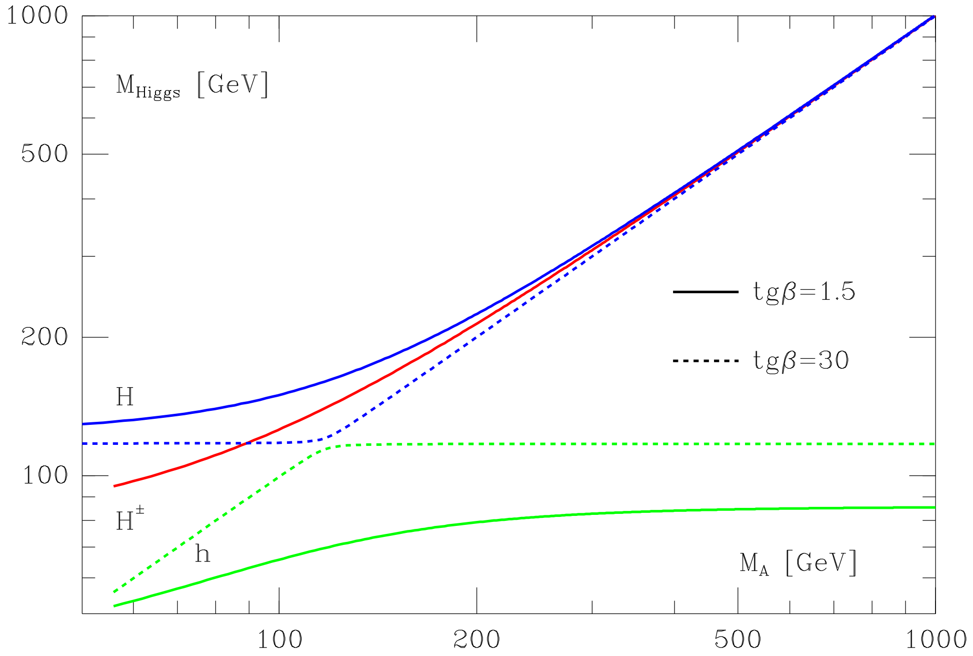
➠ $\Delta g_{\mathrm{ttH}} = \pm 5.1\%(\mathrm{stat}) \pm 3.8\%(\mathrm{syst})$ for 5% error on background normalization

- slightly worse results in fully hadronic channel
- total error of $\Delta g_{\mathrm{ttH}} = \pm 5.5\%$ seems possible
- $\sim$ factor 3 better than LHC



Other SUSY Higgses

Masses of Higgs bosons:



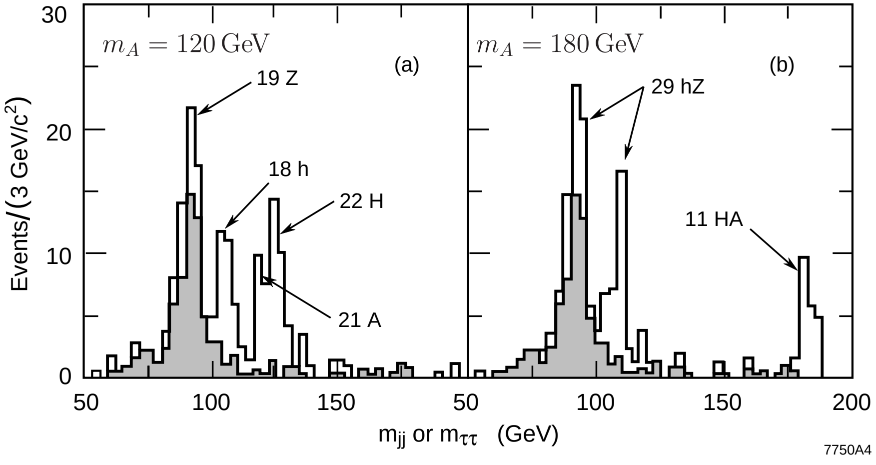
If A heavy ($m_A > 200\,\mathrm{GeV}$):

- $\sin^2(\beta - \alpha) \approx 1 \Rightarrow$
  - h is SM like
  - H produced mainly in $e^+e^- \to HA$
- $H, A, H^\pm$ almost degenerate in mass

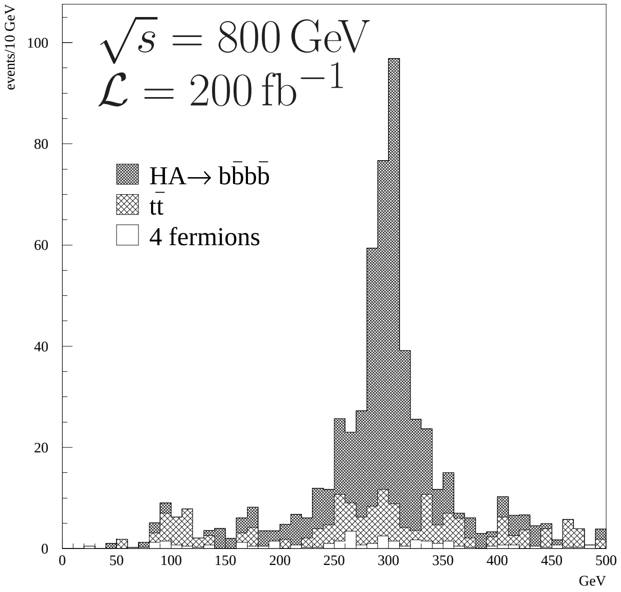
➠ if $m_A > \sqrt{s}/2$ only h can be seen
if $m_A < \sqrt{s}/2$ full spectrum in reach

Modest m_A: no problem to see $\mathrm{e^+e^-} \rightarrow \mathrm{Zh}$, $\mathrm{e^+e^-} \rightarrow \mathrm{ZH}$ and $\mathrm{e^+e^-} \rightarrow \mathrm{HA}$

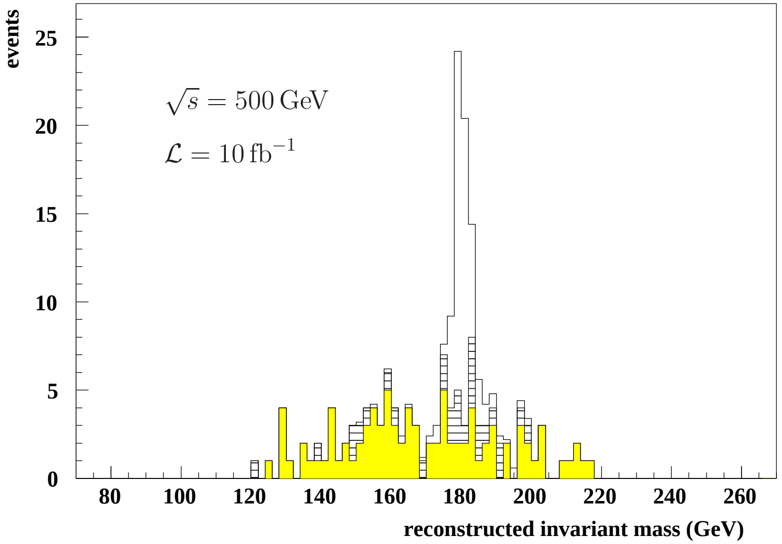
$\sqrt{s} = 400\,\mathrm{GeV}$, $\mathcal{L} = 10\,\mathrm{fb}^{-1}$



Large m_A: For $\sqrt{s} = 800\,\mathrm{GeV}$ can see $\mathrm{e^+e^-} \rightarrow \mathrm{HA}$ up to $m_A \sim 350\,\mathrm{GeV}$



Charged Higgses can be detected, independent of the decay mode up to $\sim 80\%\sqrt{s}/2$ with low luminosity:

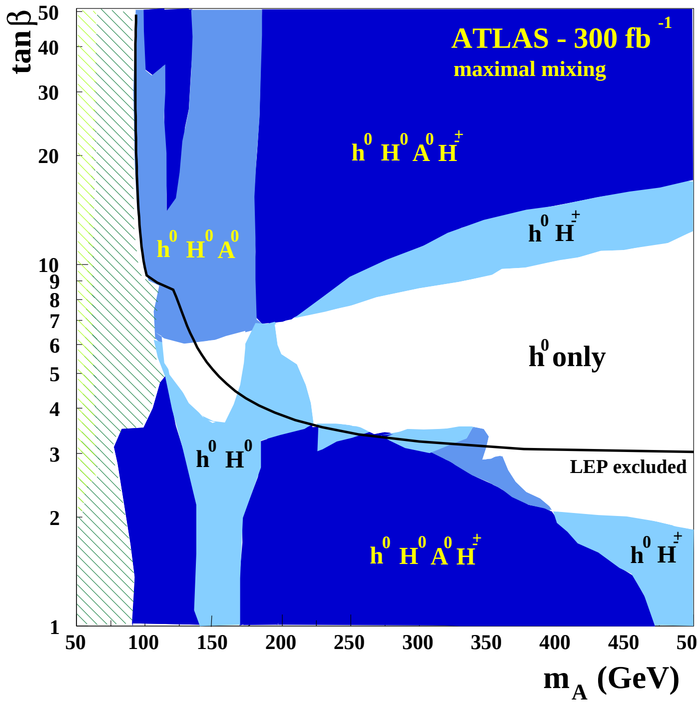


$\gamma\gamma$ collider

- Higgses are produced singly
- $\sqrt{s}_{max} \approx 0.8\sqrt{s}_{ee}$
- can see H,A up to $650\,\mathrm{GeV}$ for $\sqrt{s}_{ee} = 800\,\mathrm{GeV}$

Heavy SUSY Higgses at LHC

- LHC results are very dependent on $\tan\beta$
- $\tan\beta$ small: (almost) excluded by LEP → ignore
- $\tan\beta$ large: H,A-Strahlung off b-quark largely enhanced
  ➟ can see H,A in $b\bar{b}\tau^+\tau^-$ events up to fairly high masses
- $\tan\beta$ moderate: “wedge region” no heavy Higgses seen (however there are chances if the Higgses decay into SUSY particles)



Summary Higgs physics

- A SM-like Higgs definitely will be discovered at LHC
- If a Higgs exists in the LC energy range, it will be seen
- The task of the LC will be to measure the properties of the Higgs and to show that it is really responsible for electroweak symmetry breaking.
- The present analyzes mostly assume a light Higgs, for a heavier Higgs they have to be redone replacing a $b\bar{b}$-pair by a W-pair.
- The Higgs-mass can be measured to ≈ 50 MeV
- The Higgs couplings to heavier fermions and to gauge bosons can be measured at the few percent level
- The trilinear Higgs-coupling can be established on the 20% level
- Not covered here: One can construct exotic models, where the LHC doesn’t see the Higgs, but the LC still can

❺ Electroweak Gauge-Bosons

- Introduction
- Measurement of the W-mass → later
- Triple gauge-couplings
- Strong interaction of electroweak gauge bosons

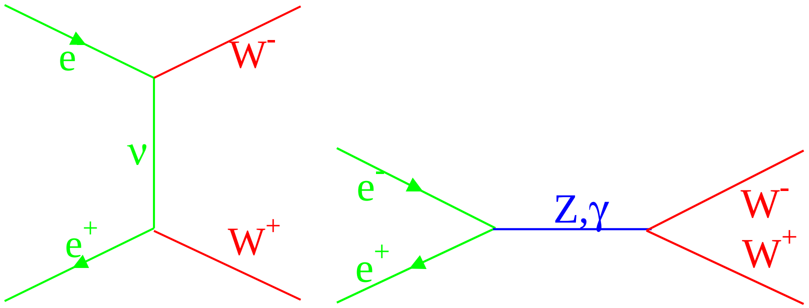
Introduction

- Self-interactions among gauge-bosons are directly given by structure of gauge group

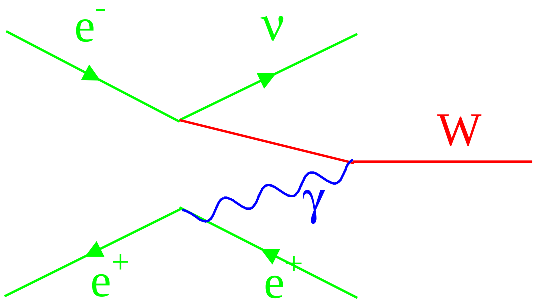
➠ study of gauge-boson interactions can show details of the gauge group.

Two main classes of processes:

Pair production (e.g. W pairs):



Fusion processes (e.g. single W production):



Or combination of both for quartic couplings

Pair production:

- Cross sections fall like $1/s$
- The scale of the interesting interaction is $\sqrt{s}$
- the events are fully contained in the detector

Fusion processes:

- the total cross section rises with energy
- the scale of the interesting interaction remains low
- particles from the incoming fermion are often lost in the beampipe or as neutrinos

Triple gauge couplings

Usual parameterization for WWV (V=Z,γ) couplings:

$$
\begin{aligned}
i\mathcal{L}_{eff}^{WWV} = g_{WWV}\cdot [& \\
& g_1^V V^\mu \left(W^-_{\mu\nu}W^{+\nu} - W^+_{\mu\nu}W^{-\nu}\right) + \\
& \kappa_v W^+_\mu W^-_\nu V^{\mu\nu} + \\
& \frac{\lambda_V}{m_W^2} V^{\mu\nu} W^{+\rho}_\nu W^-_{\rho\mu} + \\
& i g_5^V \epsilon_{\mu\nu\rho\sigma} \left(\left(\partial^\rho W^{-\mu}\right) W^{+\nu} - \right. \\
& \qquad \left. W^{-\mu}\left(\partial^\rho W^{+\nu}\right)\right) V^\sigma + \\
& i g_4^V W^-_\mu W^+_\nu \left(\partial^\mu V^\nu + \partial^\nu V^\mu\right) - \\
& \frac{\tilde{\kappa}_V}{2} W^-_\mu W^+_\nu \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \\
& \frac{\tilde{\lambda}_V}{2m_W^2} W^-_{\rho\mu} W^{+\mu}{}_\nu \epsilon^{\nu\rho\alpha\beta} V_{\alpha\beta}]
\end{aligned}
$$

With $V = \gamma,\, Z,\ g_{WW\gamma} = e,\ g_{WWZ} = e\cot\theta_W$ and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$

Gauge invariance: $g_1^\gamma(q^2=0)=1,\ g_5^\gamma(q^2=0)=0$

SM: $g_1^V = \kappa_V = 1$ all other couplings $= 0$

Static quantities:

- magn. dipole-moment: $\mu_W = \frac{e}{2m_W}(1+\kappa_\gamma+\lambda_\gamma)$
- elec. quadr.-moment: $q_W = -\frac{e}{m_W^2}(\kappa_\gamma-\lambda_\gamma)$

Symmetries:

- $g_1,\ \kappa,\ \lambda$ C,P-conserving
- g_5 C,P-violating, CP-conserving
- $g_4,\ \tilde{\kappa},\ \tilde{\lambda}$ CP-violating

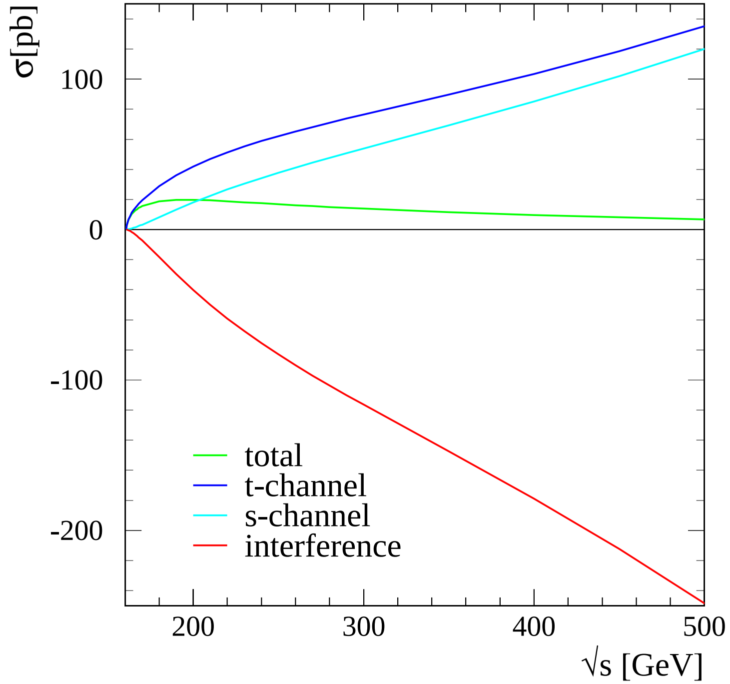
Expect largest experimental sensitivity and largest deviations in C,P-conserving couplings

➟ mainly studied up to now

However construction of C,CP-violating observables measures the other couplings independent from the C,P-conserving ones

Gauge cancellations:

- W-pair production via t-channel ν-exchange and s-channel Z,γ-exchange violates unitarity individually
- unitarity gets restored by s-t interference



➠ sensitivity increases with energy

➡ anomalous couplings have to vanish for $\sqrt{s} \to \infty$

LC:

- main sensitivity from W-pair production

➠ measurement of TGCs at fixed scale ($= \sqrt{s}$)

➠ take energy dependence into account in interpretation of results

LHC:

- main sensitivity in Wγ and WZ pair production
- $\sqrt{s}$ varies event by event due to PDFs

➠ have to take energy dependence into account in analysis

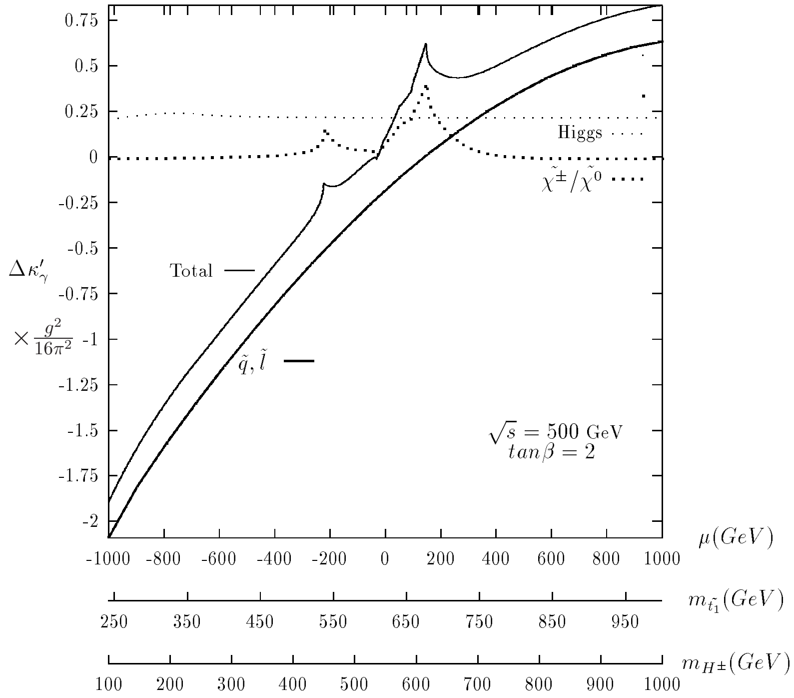
- typically regularize coupling by form factor: $x' = \frac{x}{(1+s/\Lambda^2)^n}$, $n > 0.5$ for $\Delta\kappa$, $n > 1$ for λ
- Λ can be viewed as scale where new physics sets in, so it makes sense to compare experiments for very high Λ
- in case anomalous couplings are found, have to measure detailed shape with $\sqrt{s}$ (LHC+LC!)

Theoretical expectations:

Triple gauge couplings should be modified on 1-loop level

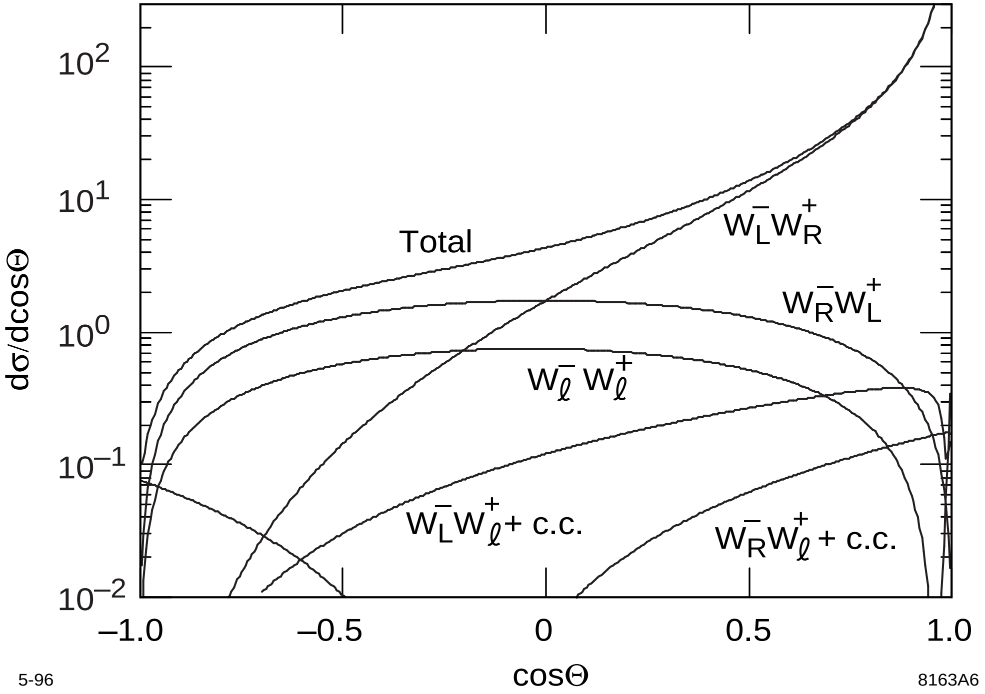
$\Rightarrow$ expect deviations of order $g^2/16\pi^2 \approx 2.7 \cdot 10^{-3}$

E.g. MSSM contributions to $\Delta\kappa_\gamma$:



Experimental analyzes (all very similar to LEP II)

- sensitive quantities
  - cross section
  - W-production angle
  - W polarization → W-decay angles

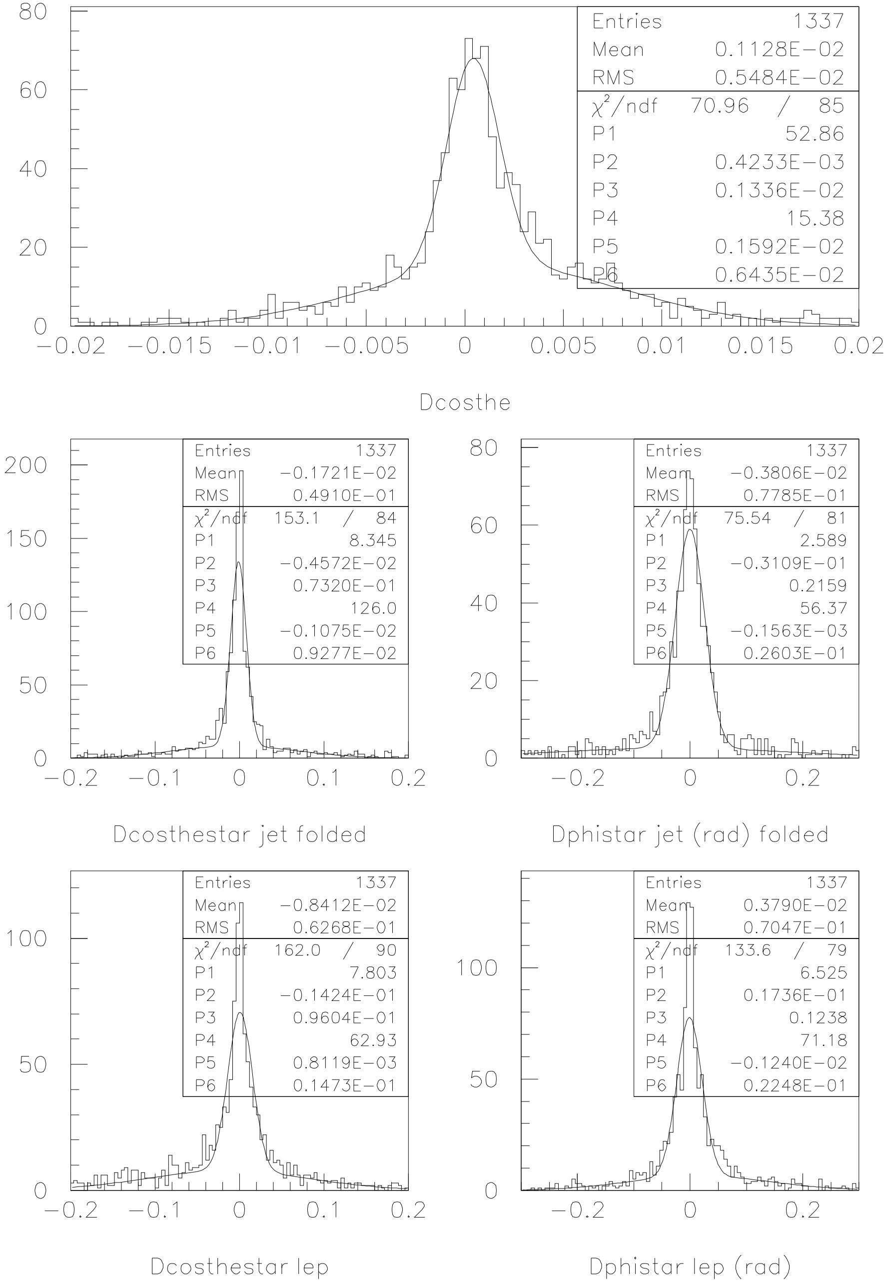


- huge peak in forward region,insensitive to anomalous coupling
  $\Rightarrow$ cross section dependence contained in angular dependence

- Ws have much larger boost than at LEP:
  - the two Ws are well separated
  - the resolution in the production angle is better than at LEP → plot
  - the resolution in the decay angles is somewhat worse → plot
  - however detector resolution does not effect the analysis strongly
- up to now only mixed decays WW → $\ell\nu q\bar{q}'$
  - about ∼ 40% of the statistics
  - W^+, W^- can be separated without ambiguity
  - decay angles of leptonically decaying W can be measured without ambiguity
  - decay angles of hadronically decaying W can be measured with twofold ambiguity
- analysis methods similar as at LEP:
  optimal observables, spin density matrix, maximum likelihood fits
- expect factor 100 smaller errors
  - factor 10 from sensitivity → applies also to systematics
  - factor 10 from luminosity → have to improve systematics by that amount

---

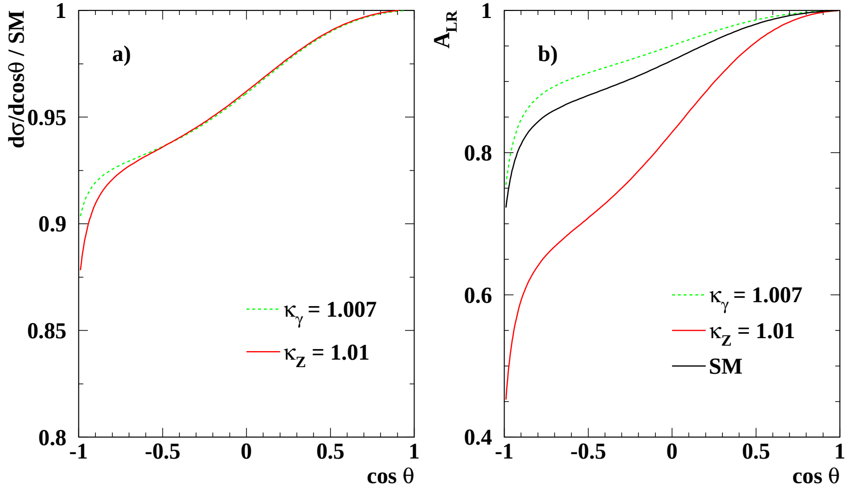
Angular resolutions for $\theta_W > 20°$



Separation of WWγ and WWZ couplings

- for the W-pairs WWγ and WWZ couplings cannot be separated from the event information
- however initial state $e^+e^-\gamma$ and e^+e^-Z couplings are different for different electron polarization

➨ can use beam polarization to separate the two


total cross section

A_{LR}

- (for fits relating the WWγ and WWZ couplings polarization also reduces the error by more than a factor 2)
- in addition single W production, $e\gamma$- and $\gamma\gamma$-collider measure WWγ coupling only

---

Results:

Statistical precision for $\sqrt{s} = 500\,\mathrm{GeV}$, $\mathcal{L} = 500\,\mathrm{fb}^{-1}$, $\mathcal{P}_{e^-} = \pm 80\%$:

$$\Delta\kappa_\gamma, \Delta\kappa_Z, \Delta\lambda_\gamma, \Delta\lambda_Z \approx (3-4)\times 10^{-4}$$
$$\Delta g_1^Z \approx (8-13)\times 10^{-4}$$

depending on the number of fit parameters

$\sqrt{s} = 800\,\mathrm{GeV}$ ~factor 2 better

Systematics:

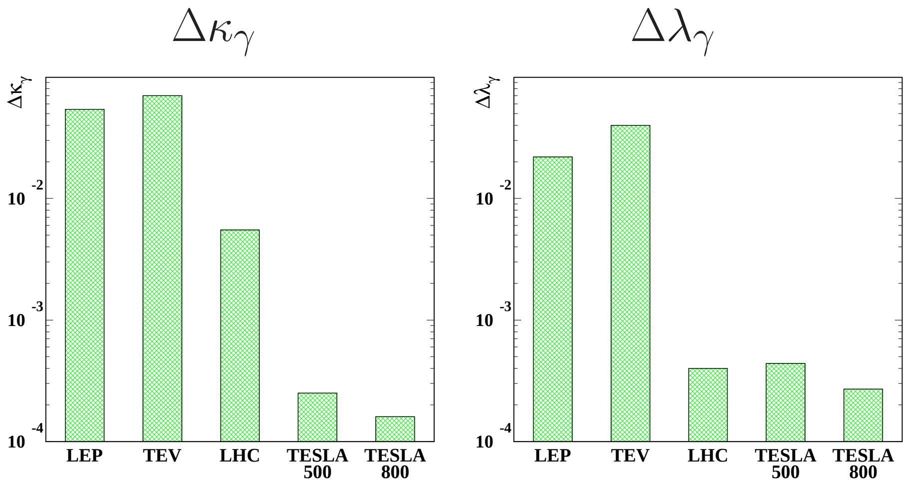
- ISR needs to be known to the 1% level
- beamstrahlung seems no problem
- detector effects should also be under control due to better θ_W resolution
- with the standard parameterization polarization can be obtained from A_{LR} in forward peak

CP-violating couplings

$$\Delta\tilde{\kappa}, \Delta\tilde{\lambda} = (1-2)\times 10^{-2}$$

from CP-odd observables

Comparison with LHC etc.:

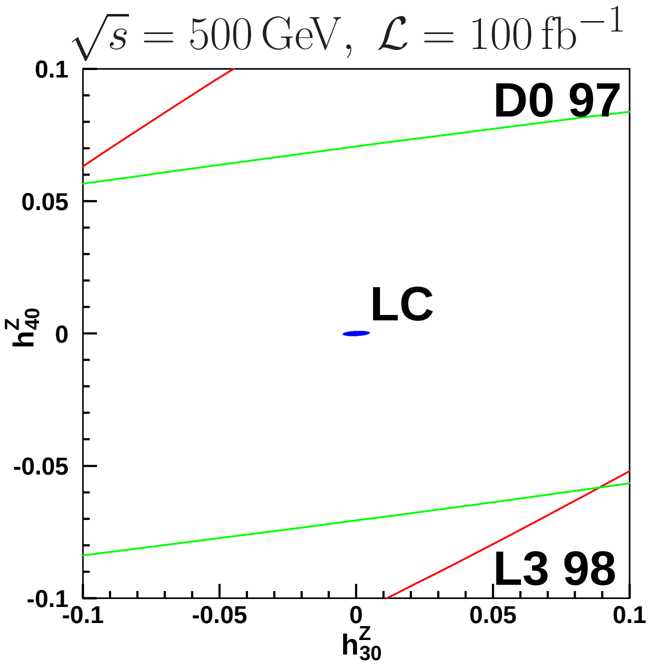

$\Delta\kappa_\gamma$ $\Delta\lambda_\gamma$

- LC much better than LHC for κ, somewhat better for λ
- if new physics scale is high, effects are expected in κ because of lower dimension
  ➠ big advantage for LC
- if new physics scale is low, both couplings can show effects and LHC probes at higher scales where new physics might be visible directly
  ➠ advantage for LHC
- if some effect is found somewhere it is definitely invaluable to have complementary information

---

Measurements of neutral TGCs

- neutral TGCs forbidden in the SM at loop level
- possible anomalous couplings only come in at higher dimensions (8)
- studies exist e.g. for γZZ- and $\gamma\gamma Z$-couplings in $Z\gamma$ events with high p_t photons
- dramatic improvement compared to existing machines


$\sqrt{s} = 500\,\mathrm{GeV},\ \mathcal{L} = 100\,\mathrm{fb}^{-1}$

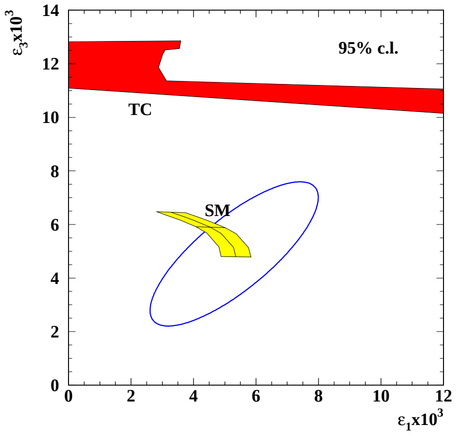
- however still factor 10 worse than SM prediction and LHC

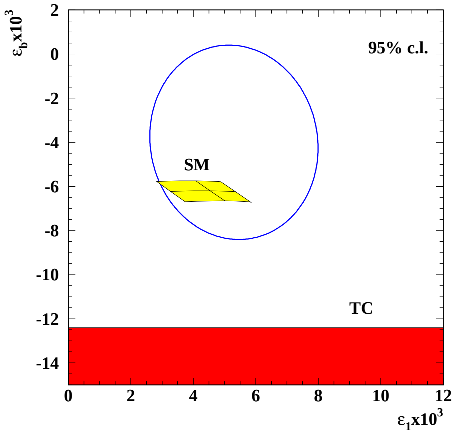
Strong Electroweak Symmetry Breaking

- If no Higgs exists electroweak interactions become strong at high energy and e.g. WW scattering violates unitarity at $\sqrt{s_{WW}} \sim 1.2\,\mathrm{TeV}$.

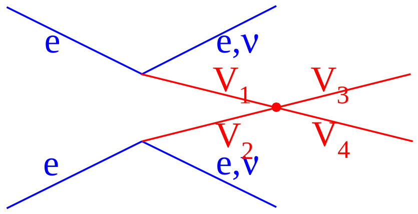
➠ expect new effects at this energy

- Typical models invoke a new strong interaction at the TeV scale (Technicolor)
- The Goldstone-bosons (Pions) of the new theory become the longitudinal degrees of freedom of the vector-bosons
- Warning: simple copy of QCD is excluded by LEP/SLD precision data

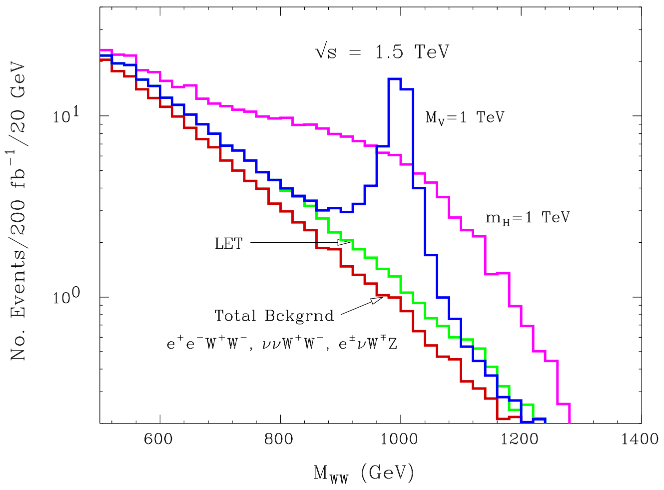




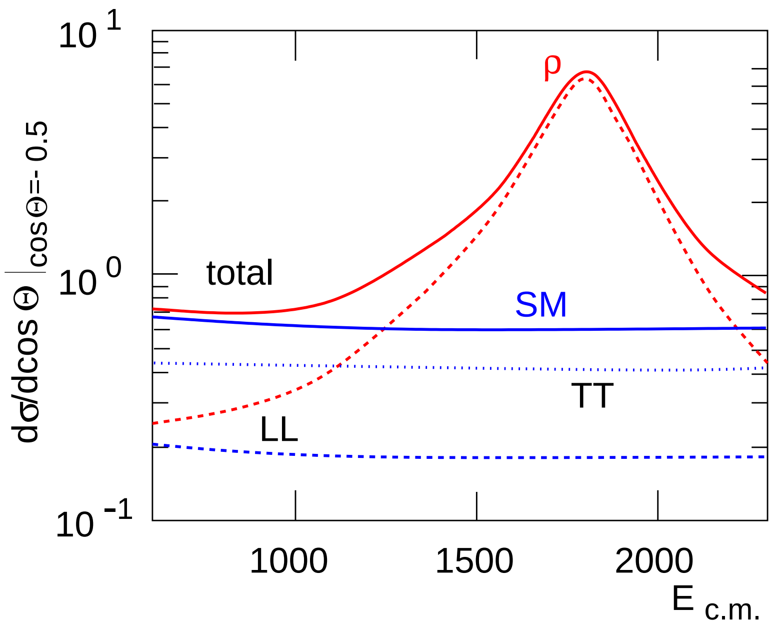
- interpretations within this model are certainly a very useful indication but should not be taken literally, however no other concrete model exists
- most intuitive channel: VV scattering V=W,Z



- ideally find resonances (like ρ, ω, etc.)
- however also if no resonances are found the LET says that longitudinal VV-scattering at high energy behaves like $\pi\pi$-scattering at low energy



also in W-pair production effects from J=1 resonances should be visible (like in $e^+e^- \to \rho \to \pi^+\pi^-$)



with high precision resonance effects remain visible at lower energy

---

Systematic approach: Effective Lagrangian

- symmetry breaking is realized non-linearly
- expand Lagrangian in the dimension of the field operators ($\propto \sqrt{s}$)
- keep lowest order that contains analyzed interaction

Trilinear couplings:

$$\mathcal{L}_{TGC} = \frac{\alpha_1}{16\pi^2}\frac{gg'}{2}B_{\mu\nu}\,\mathrm{tr}(\sigma_3 W^{\mu\nu}) + \frac{\alpha_2}{16\pi^2}\mathrm{i}g' B_{\mu\nu}\,\mathrm{tr}(\sigma_3 V^\mu V^\nu) + \frac{\alpha_3}{16\pi^2}2\mathrm{i}g\,\mathrm{tr}(W_{\mu\nu}V^\mu V^\nu)$$

Strong interaction:

$$\frac{\alpha_i}{16\pi^2} = \left(\frac{v}{\Lambda_i^*}\right)^2$$

Unitarity requires:

$$\Lambda^* \approx 3\,\mathrm{TeV}$$

α's can be expressed in terms of g_1, κ:

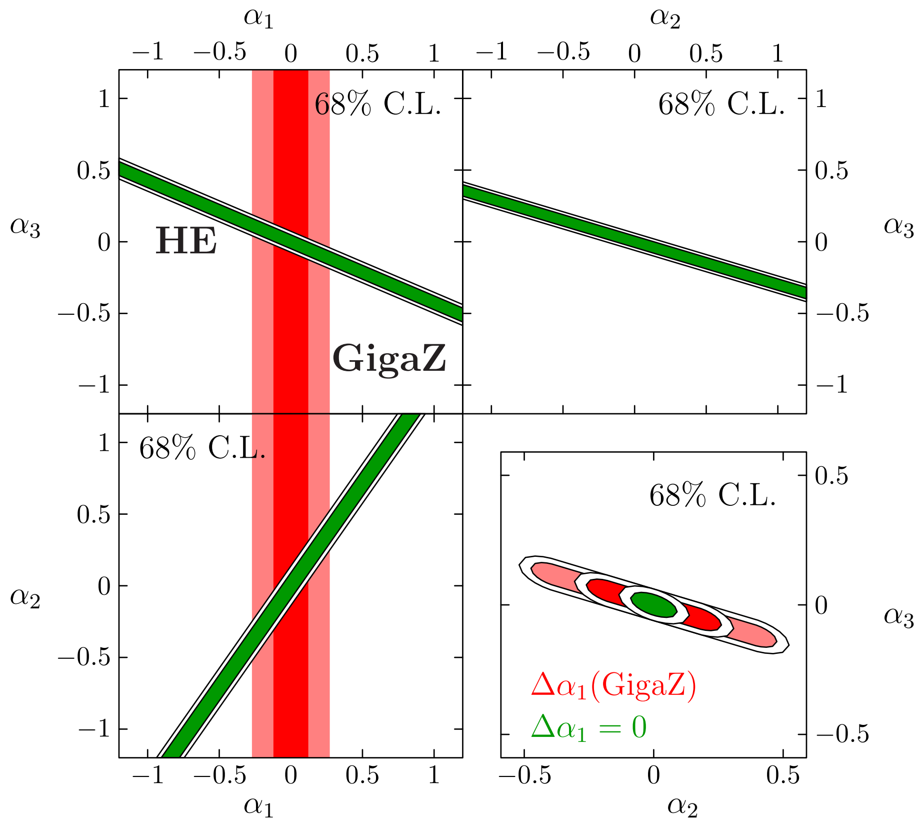
$$\Delta g_1^Z = \frac{e^2}{\cos^2\theta_W(\cos^2\theta_W-\sin^2\theta_W)}\frac{\alpha_1}{16\pi^2} + \frac{e^2}{\sin^2\theta_W\cos^2\theta_W}\frac{\alpha_3}{16\pi^2}$$

$$\Delta\kappa_\gamma = -\frac{e^2}{\sin^2\theta_W}\frac{\alpha_1}{16\pi^2} + \frac{e^2}{\sin^2\theta_W}\frac{\alpha_2}{16\pi^2} + \frac{e^2}{\sin^2\theta_W}\frac{\alpha_3}{16\pi^2}$$

$$\Delta\kappa_Z = \frac{2e^2}{\cos^2\theta_W-\sin^2\theta_W}\frac{\alpha_1}{16\pi^2} - \frac{e^2}{\cos^2\theta_W}\frac{\alpha_2}{16\pi^2} + \frac{e^2}{\sin^2\theta_W}\frac{\alpha_3}{16\pi^2}$$

System is degenerate, but α_1 can be tightly constrained with m_{W} and $\sin^2\theta^l_{eff}$ measurements at GigaZ

Results for $\sqrt{s} = 800\,\mathrm{GeV}$, $\mathcal{L} = 1000\,\mathrm{fb}^{-1}$

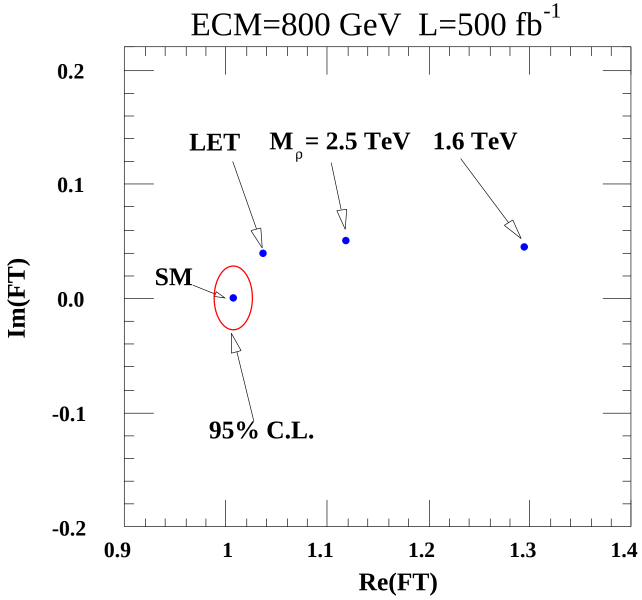


α-limits correspond to $\Lambda^* = \mathcal{O}(10\,\mathrm{TeV}) \gg 3\,\mathrm{TeV}$

SEWSB should be seen in triple gauge couplings at LC

Analysis within technicolor models

- Parameterize $e^+e^- \rightarrow WW$ with a form factor similar to $e^+e^- \rightarrow \pi^+\pi^-$
- can predict form factor as a function of m_ρ
- LET is limit for large m_ρ



- Linear Collider is sensitive to techni-ρ masses up to $\sim$ 2.5 TeV and can distinguish LET from SM
- The LHC has a similar reach
- however the information is very complementary since the LHC measures the mass of a resonance and the LC measures the couplings

---

Quartic couplings:

Luminosity spectrum of "W beam":

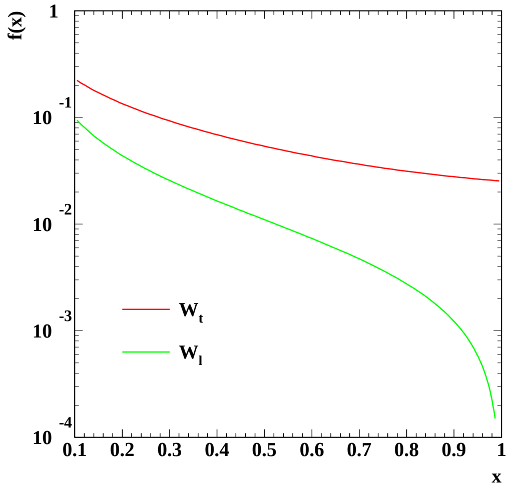
Effective W approximation

- Transversely polarized Ws:

$$f_W^T(x) = \frac{\alpha}{4\pi s_w^2} \frac{1+(1-x)^2}{2x} \ln \frac{xs}{M_W^2}$$

- Longitudinally polarized Ws:

$$f_{W/e}^L(x) = \frac{\alpha}{4\pi s_w^2} \frac{1-x}{x}$$

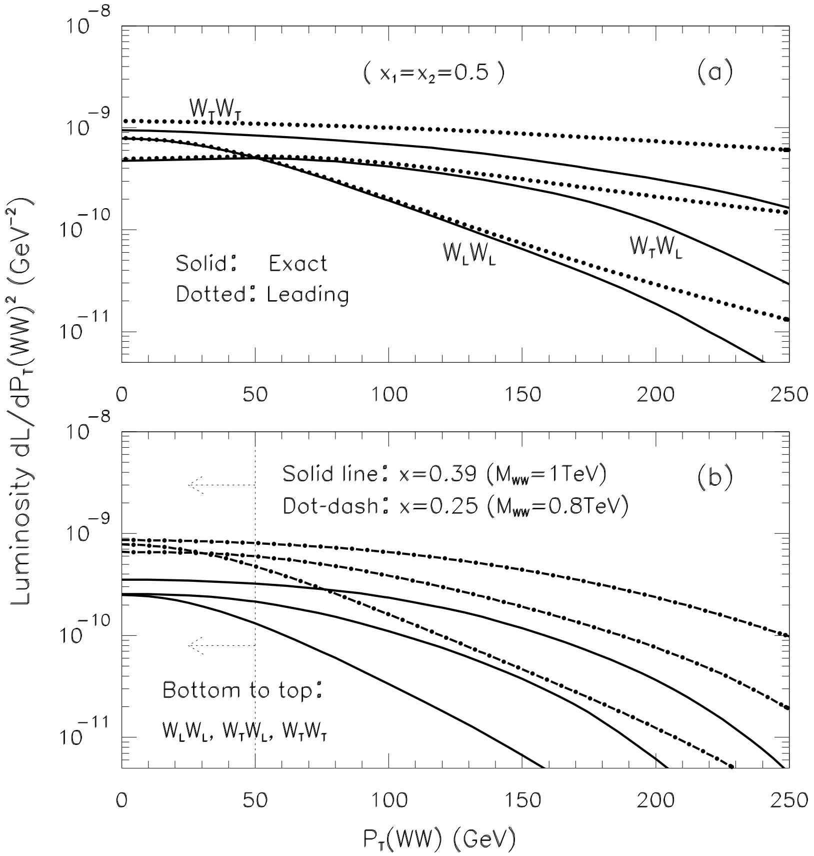


(Calculations use improved W spectra)

Longitudinal Ws are suppressed in the interesting region at large x

Suppression is mainly at large p_t

$$\sqrt{s} = 1.6\,\mathrm{TeV}$$



Have to cut low p_t to reject $\gamma\gamma \rightarrow \mathrm{W^+W^-}$ background

---

Effective Lagrangian:

Two terms not already constrained by TGCs:

$$\begin{aligned}\mathcal{L}_4 &= \frac{\alpha_4}{16\pi^2}\left[\frac{g^4}{2}\left[(W_\mu^+ W^{-\mu})^2 + (W_\mu^+ W^{+\mu})(W_\nu^- W^{-\nu})\right]\right.\\ &\qquad\left. + \frac{g^4}{c_w^2}(W_\mu^+ Z^\mu)(W_\nu^- Z^\nu) + \frac{g^4}{4c_w^4}(Z_\mu Z^\mu)^2\right]\\ \mathcal{L}_5 &= \frac{\alpha_5}{16\pi^2}\left[g^4(W_\mu^+ W^{-\mu})^2 + \frac{g^4}{c_w^2}(W_\mu^+ W^{-\mu})(Z_\nu Z^\nu)\right.\\ &\qquad\left. + \frac{g^4}{4c_w^4}(Z_\mu Z^\mu)^2\right]\end{aligned}$$

Again with $\frac{\alpha_i}{16\pi^2} = \left(\frac{v}{\Lambda_i^*}\right)^2$

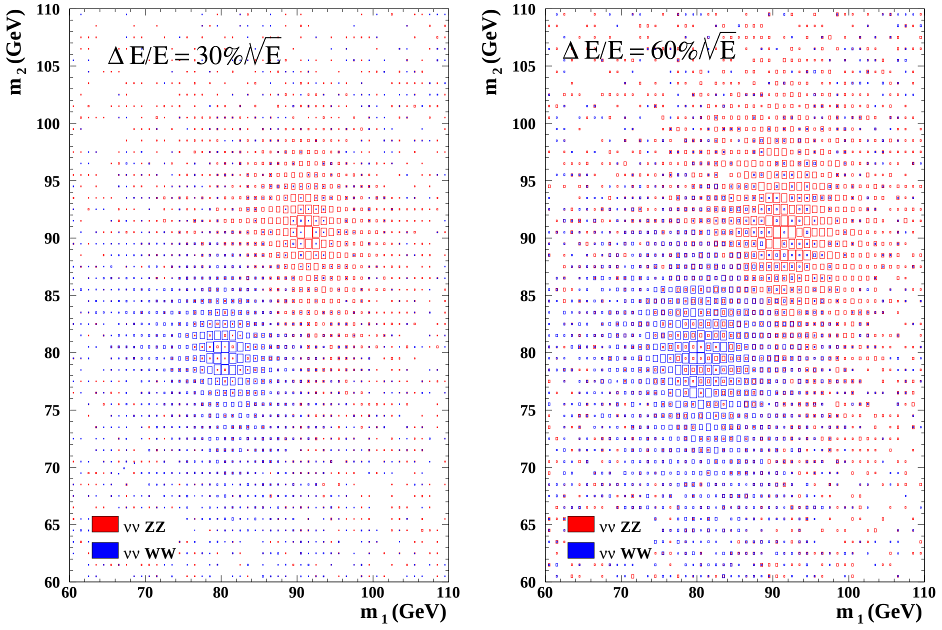
Three sensitive observables for two unknowns:

$$\begin{aligned}\mathrm{e^+e^-} &\rightarrow \nu\nu\mathrm{W^+W^-}\\ \mathrm{e^+e^-} &\rightarrow \nu\nu\mathrm{ZZ}\\ \mathrm{e^-e^-} &\rightarrow \nu\nu\mathrm{W^-W^-}\end{aligned}$$

Analysis:

Select $e^+e^- \rightarrow \nu\nu VV$ events at $\sqrt{s} = 800\,\mathrm{GeV}$

(very good energy flow resolution needed to separate W and Z)

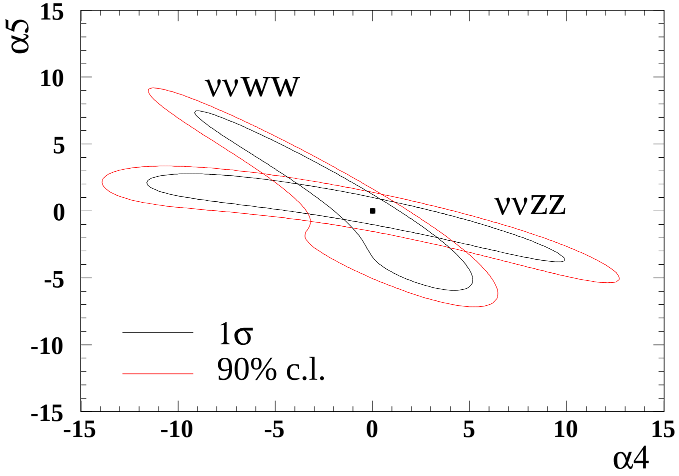


Analyze differential in terms of

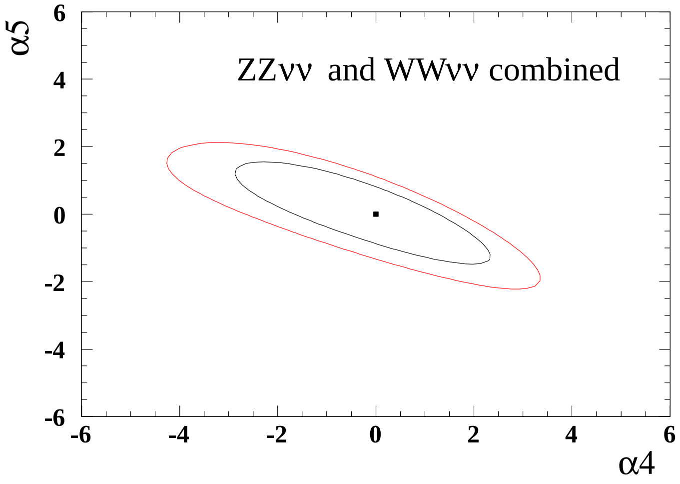
- V-decay angles (to select longitudinal V-polarization)
- V-scattering angle (to select hard scattering)
- VV invariant mass

Results:

Single channels give limits of about $\alpha_i < 10$



Combination of the two channels improves limits to about $\alpha_i < 1$



For single parameter fits one gets

$$\Lambda_4^* > 2.9\,\text{TeV}$$
$$\Lambda_5^* > 4.9\,\text{TeV}$$

- limits ~factor 1.5 better than LHC
- however weak signals, so all possible redundancy needed
- dramatic improvements, if $\sqrt{s}$ can be increased
- LHC can see resonances up to $m \sim 1 - 2\,\text{TeV}$ dependent on width

➠ In such a situation the LC could do a precise measurement of quantum numbers and couplings

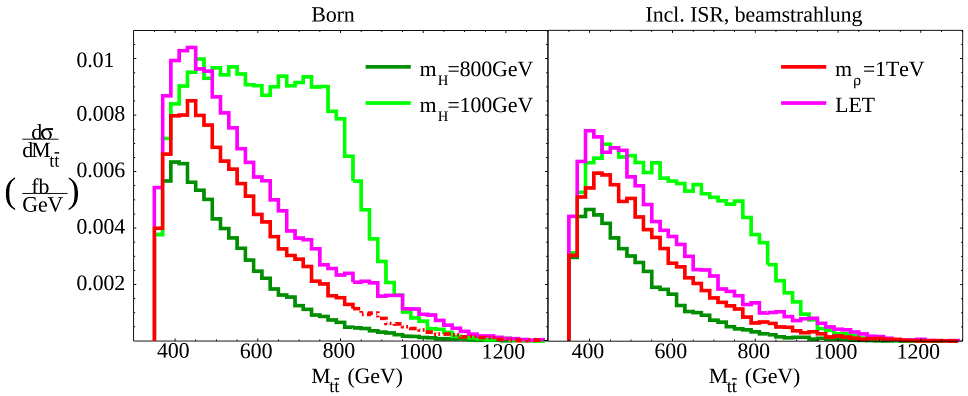
---

Signal in $W^+W^- \to t\bar{t}$

- the mechanism simulating the Higgs must also couple to fermions to produce fermion masses

➠ should see a signal in $W^+W^- \to t\bar{t}$

- 1st analysis exists at $\sqrt{s} = 1.5\,\text{TeV}$



- different models can be separated by $> 10\sigma$ with $\mathcal{L} = 200\,\text{fb}^{-1}$
- additional information in t-polarization, but not yet fully analyzed
- also here the LHC is sensitive to resonances up to $\sim 2\,\text{TeV}$

Conclusions Gauge-Boson-interactions

- For triple gauge couplings involving Ws the LC has a unique sensitivity to loop corrections and to a strongly interacting weak sector.
- For purely neutral couplings the sensitivity is still an order of magnitude worse than the expected effects and than the LHC expectation.
- There is a very high chance that the LC can see effects if electroweak symmetry breaking is realized in a strongly interacting scenario.
  If there are resonances in the LHC region, the LHC is the better machine.
  If there are no resonances the LHC and an 800 GeV LC have comparable statistical power, where the LC-backgrounds should be easier to calculate.

---

❻ Supersymmetry

- In the SM enormous fine-tuning is required to keep m_H in the 100 GeV range
- Way out: couple bosons and fermions to protect m_H → Supersymmetry
- the quadratic divergences of fermion- and sfermion-loops cancel
  ⇒ Higgs remains light

Particle content:

- all known particles
- SUSY needs two Higgs doublets to give masses to up- and down-type particle
  ⇒ 5 Higgs particles → Higgs section
- each fermion has a scalar partner (where left- and right-handed fermions have to be counted separately)
- each boson has a fermionic partner:
  - Two charginos $\chi^\pm_{1,2}$ ($m_{\chi^\pm_1} < m_{\chi^\pm_2}$), partner of $W^\pm, H^\pm$, mixed
  - Four neutralinos $\chi^0_{1,2,3,4}$ ($m_{\chi^0_1} < ... < m_{\chi^0_4}$), partner of γ, Z, h, H, mixed
  - gluinos ($\tilde{g}$), gravitino ($\tilde{G}$)

However $m_{\text{Particle}} \neq m_{\text{Partner}}$ $\Rightarrow$ SUSY is broken

Need $m_{\text{SUSY}} < 1\text{TeV}$ to solve hierarchy-problem

In general > 100 new free parameters $\Rightarrow$ have to make some assumptions how they are correlated

SUSY-breaking parameters in the minimal model (MSSM):

- $U(1), SU(2), SU(3)$ Gaugino-masses $M_{1,2,3}$
- Higgsino mass-parameter μ
- Scalar-masses m_i (or universal m_0)
- Sfermion-Higgs couplings A_i, B_i

R-parity $R = (-1)^{2S+L+3B}$

- SUSY-particles only in pairs
- lightest SUSY particle (LSP) is stable

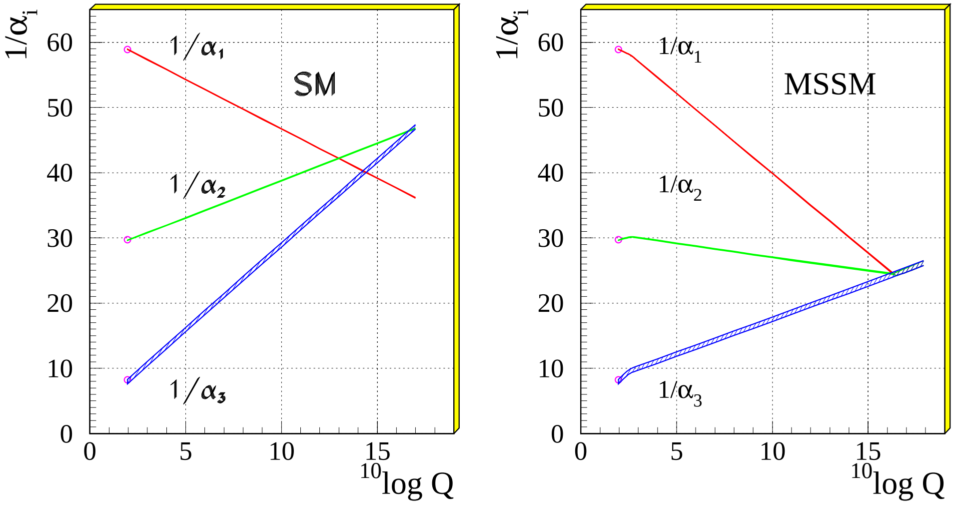
➠ Excellent dark matter candidate

R-parity can also be broken

- very rich phenomenology
- however care has to be taken to avoid proton decay

Other virtues of SUSY:

- SUSY can be a new source of CP-violation
  ➠ may explain the matter/anti-matter asymmetry in the universe
- String theories are the only known way to connect gravity with quantum mechanics
  ➠ all string theories are supersymmetric
- SUSY enables unification of forces at a high scale



SUSY breaking schemes

Gravity mediated SUSY breaking

- SUSY is broken at a high scale by gravitational interaction to a hidden sector
- Gauge coupling unification at the GUT scale ($m_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$) possible

➨ Common gaugino mass $m_{1/2}$ at m_{GUT}
$\Rightarrow \frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}$ at the weak scale

- often also universal scalar mass m_0 assumed
- slepton masses:

$$\begin{aligned} M^2_{\tilde\nu} &= m_0^2 + 0.77 M_2^2 + 0.5 m_Z^2 \cos 2\beta \\ M^2_{\tilde\ell_L} &= m_0^2 + 0.77 M_2^2 - 0.27 m_Z^2 \cos 2\beta \\ M^2_{\tilde\ell_R} &= m_0^2 + 0.22 M_2^2 - 0.27 m_Z^2 \cos 2\beta \end{aligned}$$

- squark masses similar with M_3^2 term
- L-R sfermion mixing $\propto m_f(A_f - \mu\tan\beta)$ only relevant for 3rd generation
- chargino mass matrix

$$\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

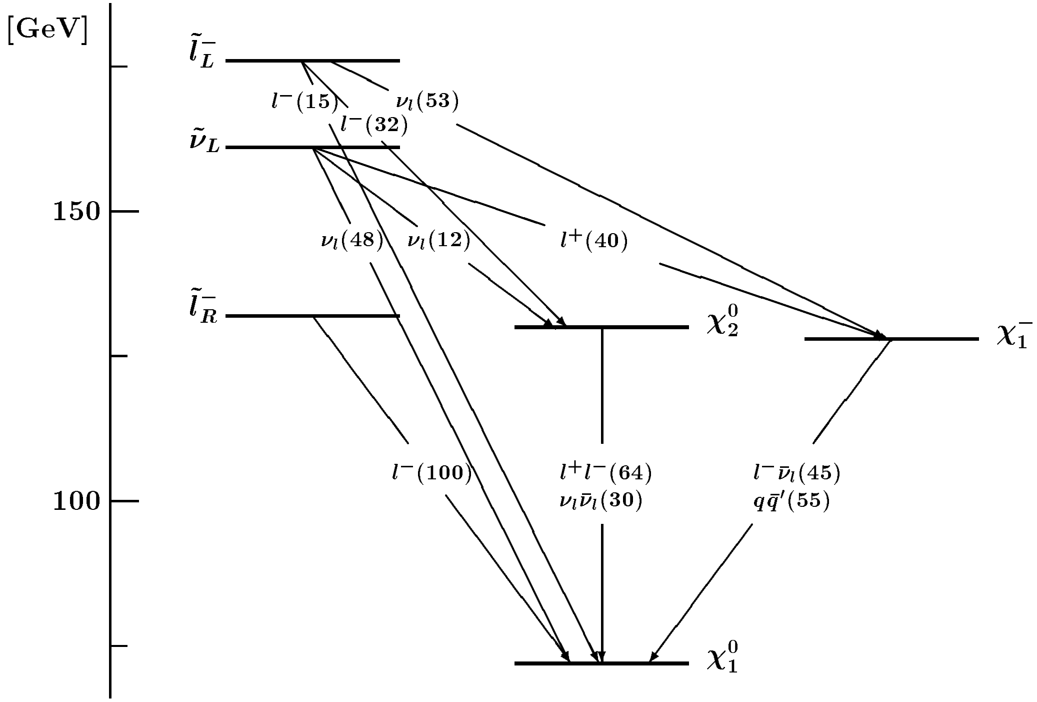
detailed properties of $\chi^\pm_{1,2}$ (gaugino-,Higgsino-like) depend on values of parameters

- neutralinos similar

“Typical” mass spectrum

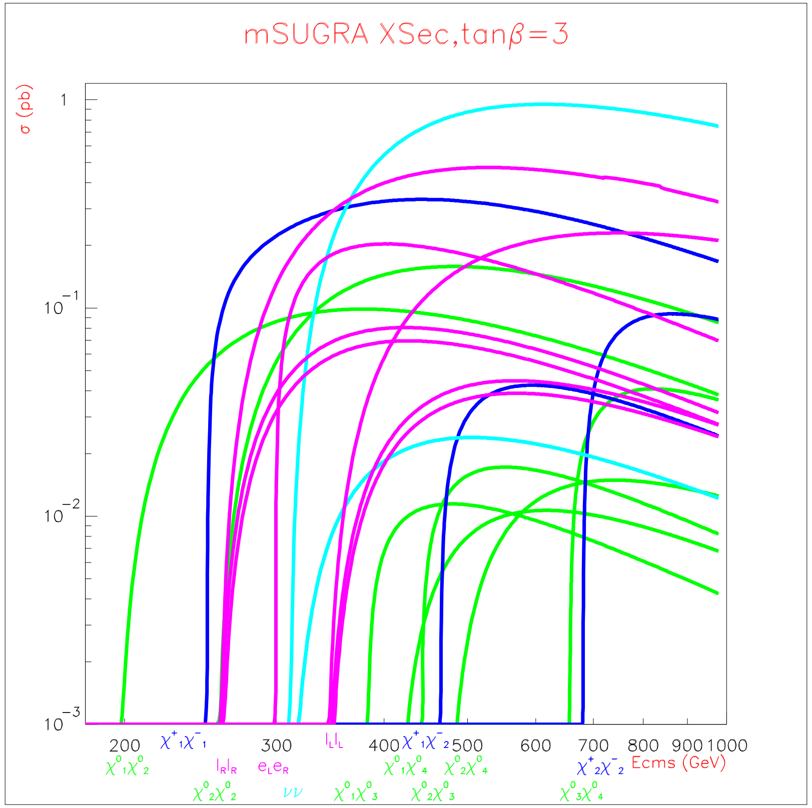
($m_0 = 100\,\mathrm{GeV}, m_{1/2} = 200\,\mathrm{GeV}$)

$$\begin{aligned} m_{\chi^0_1} &\sim 70\,\mathrm{GeV} \\ m_{\chi^\pm_1,\chi^0_2} &\sim 130\,\mathrm{GeV} \\ m_{\chi^\pm_2,\chi^0_{3,4}} &\sim 350\,\mathrm{GeV} \\ m_{\tilde\ell} &\sim 150\,\mathrm{GeV} \\ m_{\tilde q} &\sim 430\,\mathrm{GeV} \end{aligned}$$



- of course all moves with m_0, $m_{1/2}$
- $m_{\tilde t_1}$ can be moved arbitrarily by changing A

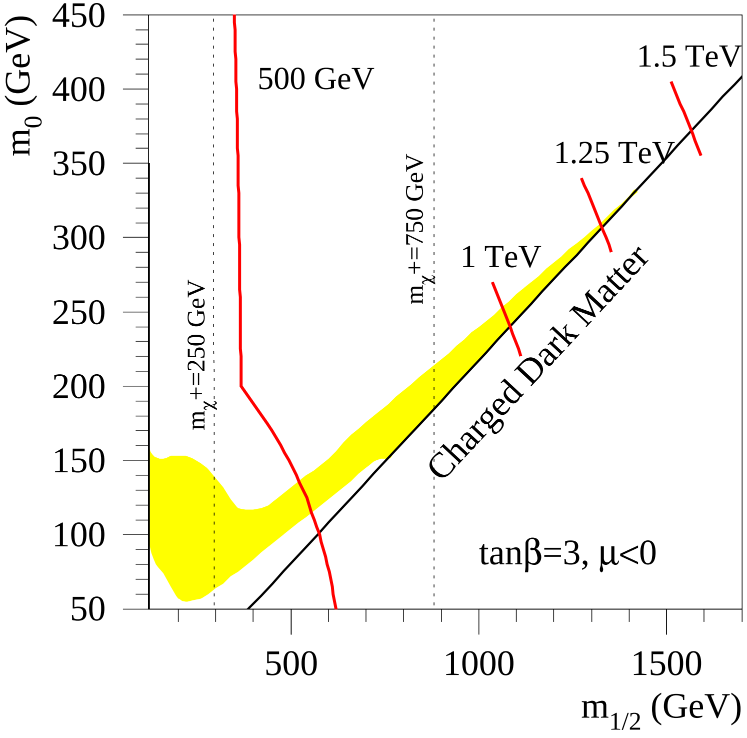
typical cross sections:



- all channels have cross sections $\sim 10 - 1000\,\mathrm{fb}$
- all channels have visible decays of at least 50%

Where do we expect SUGRA?

- naturalness suggests $\tilde{m} < 1\,\mathrm{TeV}$, however only logarithmic dependence
- recent analysis looks into correct neutralino density as dark matter



- “natural” region predicts SUSY well below 500 GeV
- however some tails due to coannihilation
- mostly covered at 1 TeV

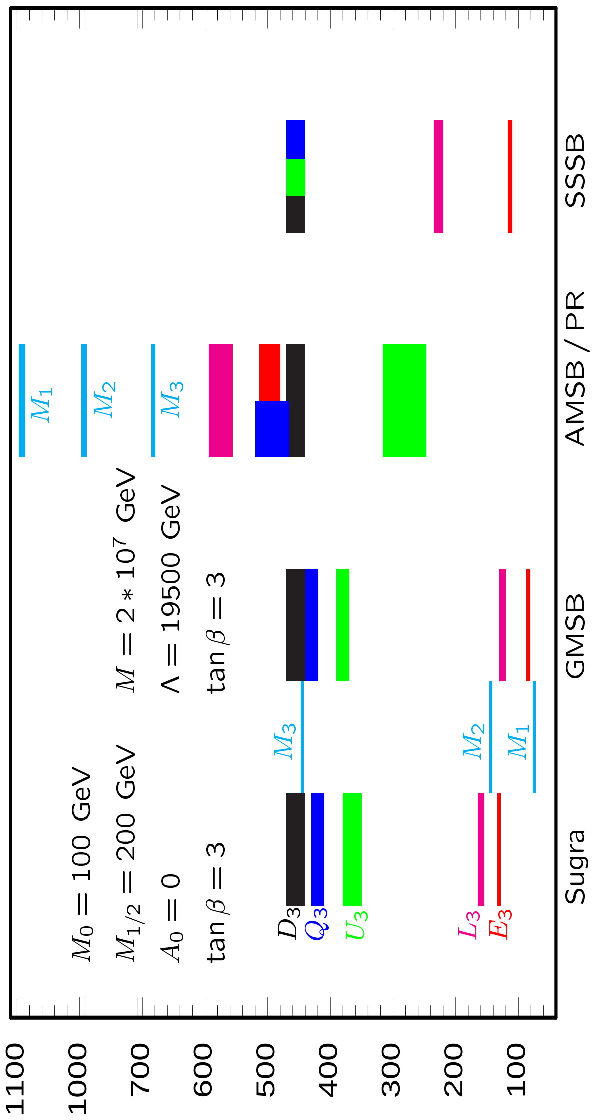
Gauge mediated SUSY breaking

- SUSY is broken at intermediate scales ($10^3 - 10^8$ GeV) by gauge interactions involving messengers between the visible and the hidden sector
- main free parameters:

$M_{\rm mess}$	messenger mass scale
$N_{\rm mess}$	number of messenger generations
Λ	universal soft braking scale
$\tan\beta$	
sign(μ)	

- main differences to SUGRA
  - very light gravitino $\sim$ eV
  - NLSP either χ^0_1 with $\chi^0_1 \to \tilde{G}\gamma$ or $\tilde{\ell}$ with $\tilde{\ell} \to \tilde{G}\ell$ (if mixing is large in 2nd case, $\tilde{\tau}_1$ is NLSP) in both cases NLSP lifetime can be significant
  - sfermion masses $\propto \alpha_i$, i =QED,QCD $\Rightarrow$ larger mass splitting between sleptons and squarks

Gaugino and Sfermion Mass Parameters



The complementarity LC/LHC

LHC:

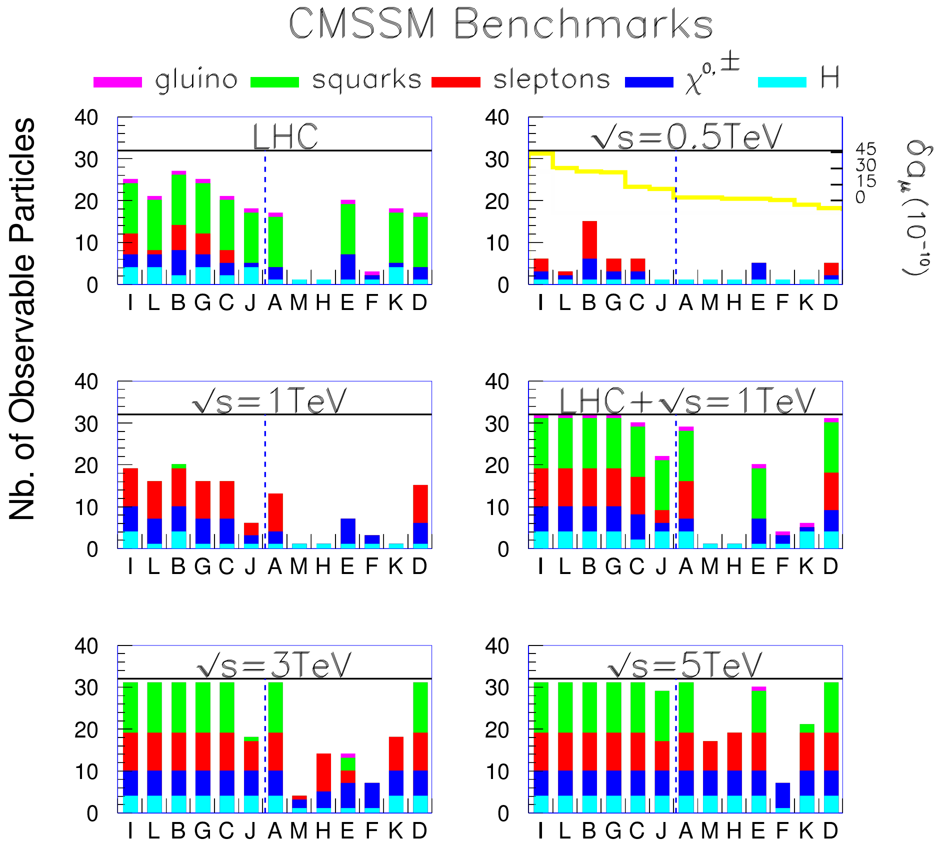
- Mass reach $\mathcal{O}(1\,\text{TeV})$
- Squarks are produced strongly ⇒ huge cross section
- Sleptons and gauginos are produced weakly or in cascades ⇒ maybe difficult to see
- LSP cannot be reconstructed completely due to missing information ⇒ mainly sensitive to mass differences

LC:

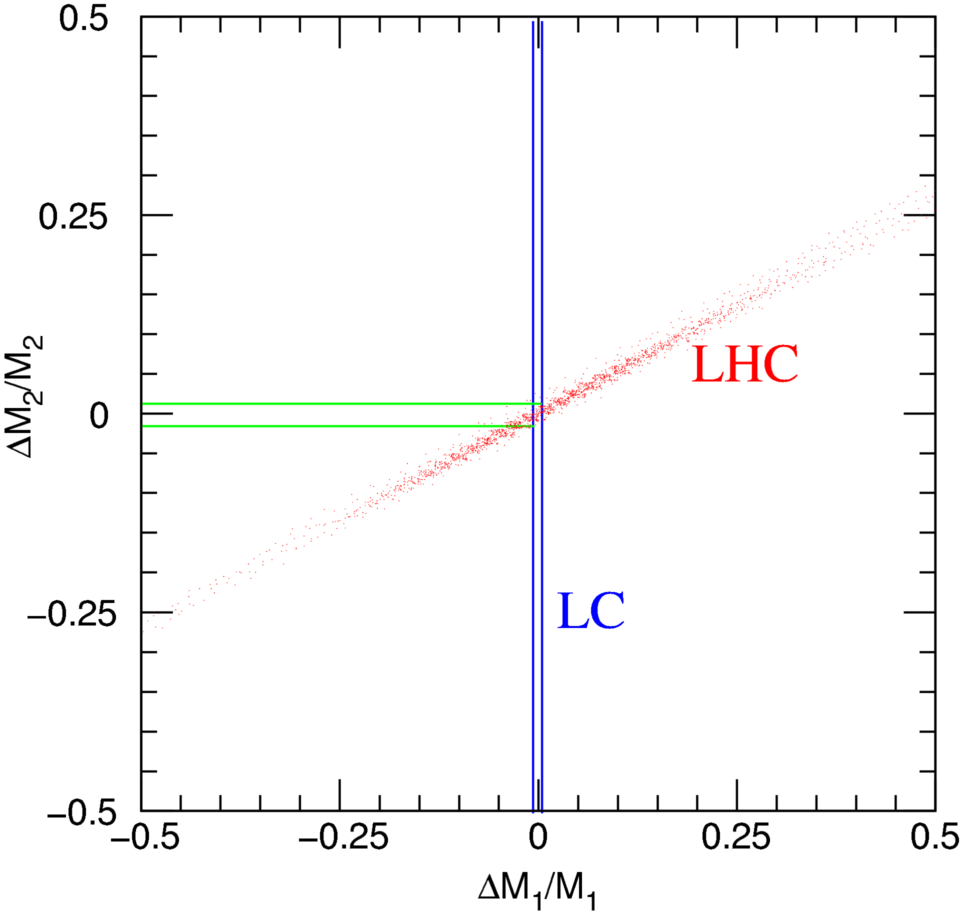
- all SUSY processes within mass reach have similar cross section
  ⇒ all particles can be cleanly reconstructed
- LSP can be reconstructed from kinematic quantities ⇒ all masses can be measured absolute
- all particles are produced in electroweak processes that can be calculated accurately ⇒ particle couplings can be measured
- squarks and gluinos are probably too heavy to be produced at LC

LC+LHC:

If SUSY light enough all masses and the lepton and gaugino couplings can be measured with good precision



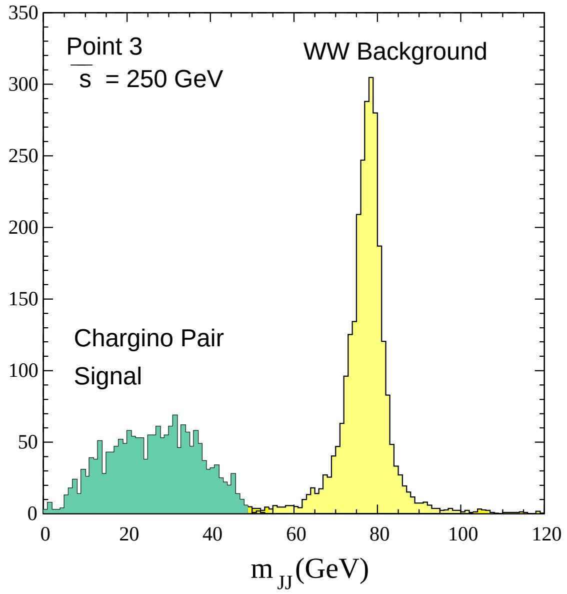
Example: $m(\chi_1^0) - m(\chi_2^0)$ from LHC combined with $m(\chi_1^0)$ from LC



Typical signals in SUSY

SUGRA signals:

- due to stable LSP always missing mass and missing p_t
- most simple decay $\tilde{f} \rightarrow f\chi_1^0$: two identical lept. or jets and missing mass and p_t
- decay $\chi_i \rightarrow f f' \chi_j$: four leptons/jets and missing mass and p_t
- in general cascade decays can have many leptons+jets
- good detector resolution separates SUSY-signals from known physics



GMSB signals:

- if the NLSP lifetime is large: like SUGRA
- due to NLSP decay the missing quantities are smaller
- this is compensated by the additional visible γ/lepton

Signals with R-parity violation:

- The LSP decays into ordinary particles
  - ➠ also LSP pair productions is visible
  - ➠ the LSP needs not to be neutral
- SUSY breaking can be in any scheme
- experimentally the missing mass/energy is replaced by LSP-reconstruction
  $\Rightarrow$ similar efficiencies

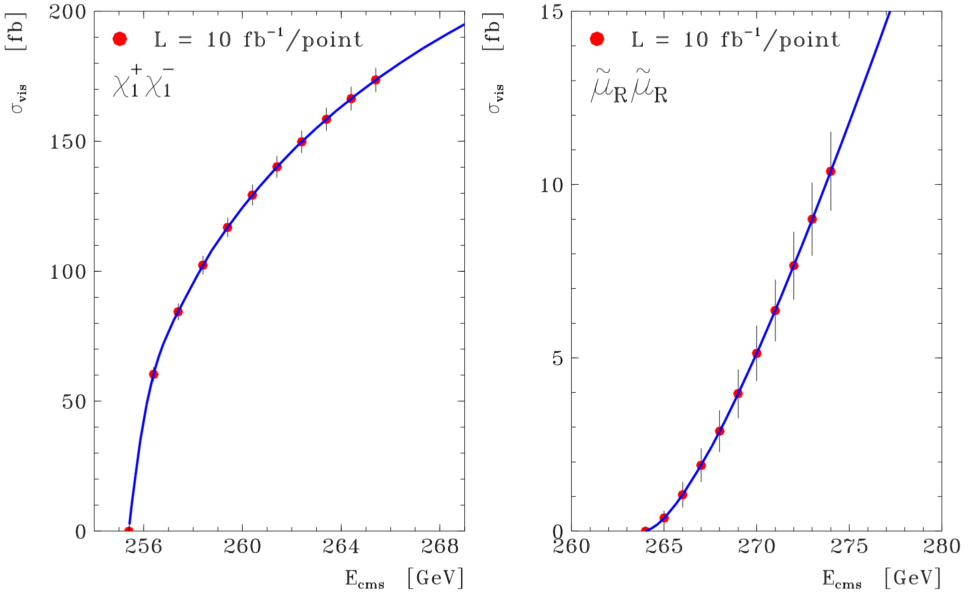
Mass measurements

Two principle methods:

- threshold scan
- reconstruction

Threshold scan:

- gauginos: threshold suppression $\propto \beta$
  $\Rightarrow$ good precision
- sfermions: threshold suppression $\propto \beta^3$
  $\Rightarrow$ precision relatively worse
- $\tilde{e}, \tilde{\nu}_1$: mixture of β^3 from s-channel Z,γ- and β from t-channel χ-exchange $\rightarrow$ model dependent



Reconstruction:

Decay of scalar particle $\tilde{\ell} \to \ell\chi$:

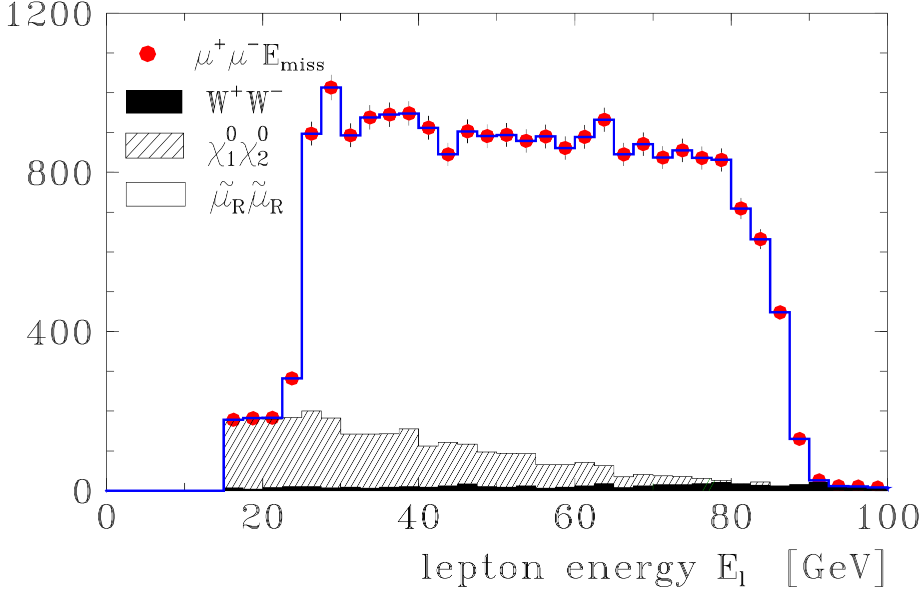
Flat energy distribution of ℓ between

$$\frac{E_\ell}{E_{\rm beam}} = \frac{1}{2}(1 \pm \beta)\left(1 - \frac{m_\chi^2}{m_{\tilde{\ell}}^2}\right)$$

$\Rightarrow m_\chi$ and $m_{\tilde{\ell}}$ can be obtained in a model independent way

e.g. $\mathrm{e}^+\mathrm{e}^- \to \tilde{\mu}_R^+\tilde{\mu}_R^- \to \mu^+\chi_1^0\mu^-\chi_1^0$:

low background sample in $\mu\mu$+missing mass

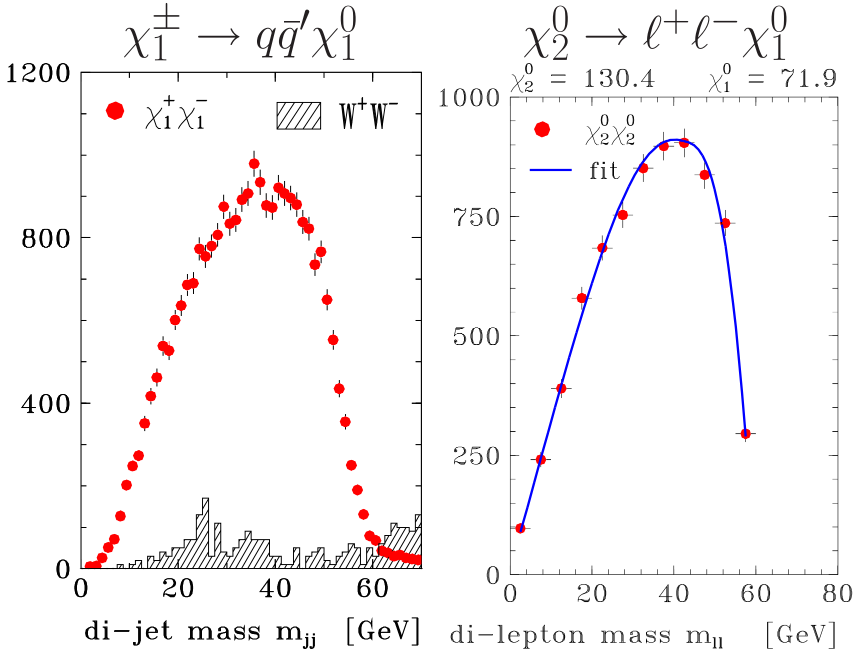


$\Rightarrow$ Can measure $m_{\chi_1^0}$ and $m_{\tilde{\mu}_R}$ to 0.3% with $\sqrt{s} =$ 320 GeV, $\mathcal{L} = 160\,\mathrm{fb}^{-1}$

Gauginos decay in 3-prongs and have spin $\Rightarrow$ mass determination from gaugino production not so easy

However for decay chain $\chi' \to ff'\chi$ $m(ff')$ gives accurate measurement of mass difference $m_{\chi'} - m_\chi$

Measurements can be done with gauginos from direct production and decays



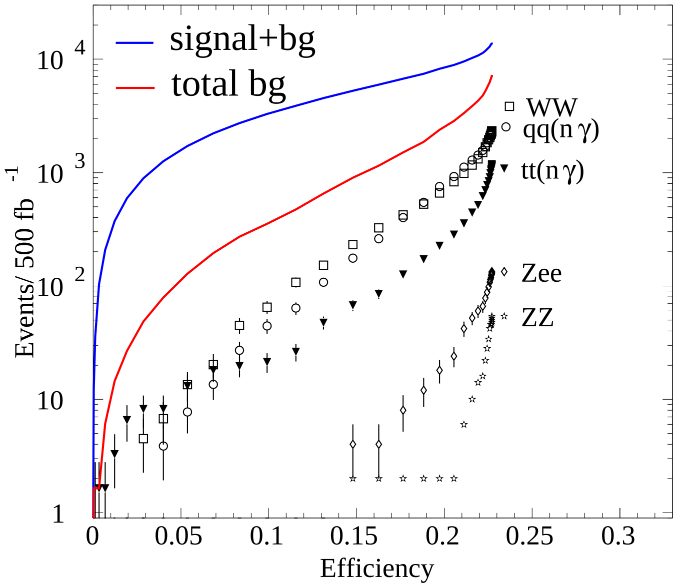
Both mass differences can be measured to 50 MeV

Study of stop production

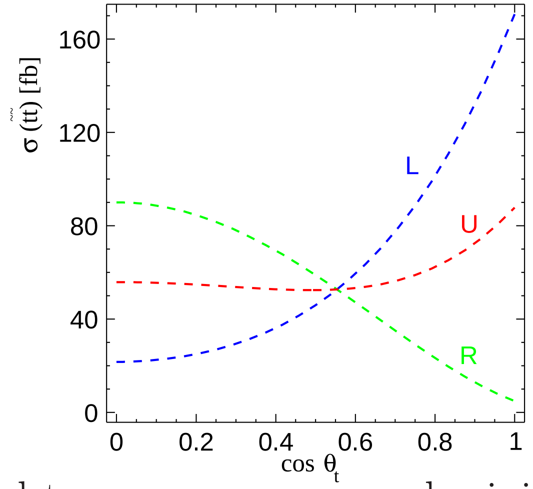
- due to mixing effects $\tilde{t}_1$ can be very light ($\tilde{t}_2$ then very heavy)
- $\tilde{t}_1$ decays into $\chi_1^+ b$ if kinematically allowed, otherwise into $\chi_1^0 c$

Analysis with 180 GeV $\tilde{t}_1 \to \chi_1^0 c$:

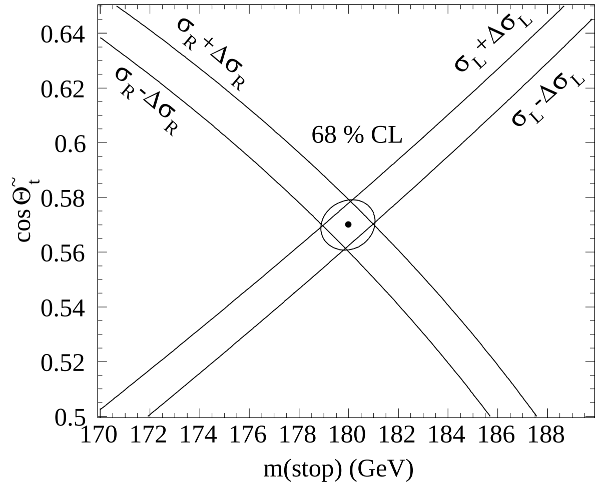
- iterative discriminant analysis using event shapes and jet energies
- $10-20\%$ efficiency with $\sim 90\%$ purity can be achieved



- cross section depends on $\tilde{t}_1$ mass and $\tilde{t}_L - \tilde{t}_R$ mixing angle
- dependence different for different beam polarization



Can be used to measure mass and mixing angle:



$$\delta m_{\tilde{t}_1} \approx 1\,\mathrm{GeV},\ \delta \cos\theta_{\tilde{t}} \approx 0.01$$

Analyzes of charginos and neutralinos

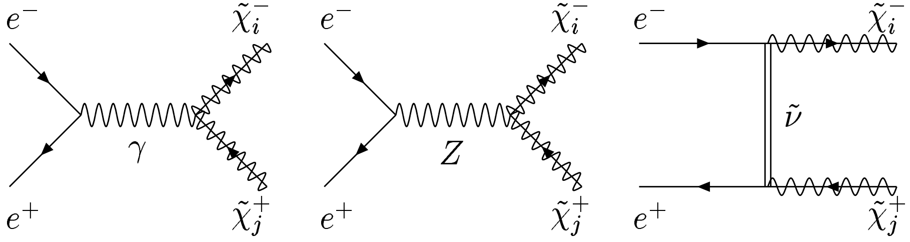
Chargino mass matrix is given by

$$\mathcal{M}_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \cos\beta \\ \sqrt{2} m_W \sin\beta & \mu \end{pmatrix}$$

Matrix not symmetric ⇒ need two mixing angles $\Phi_{L,R}$ for left- and right-handed states

Mixing angles and and chargino masses are (complicated) functions of M_2, μ and $\tan\beta$

Chargino production via Z,γ s-channel and $\tilde{\nu}_e$ t-channel exchange



Need to know sneutrino mass to calculate cross section

Cross sections of $\mathcal{O}(100\,\mathrm{fb})$ ⇒ expect several$\times 10^4$ events per channel

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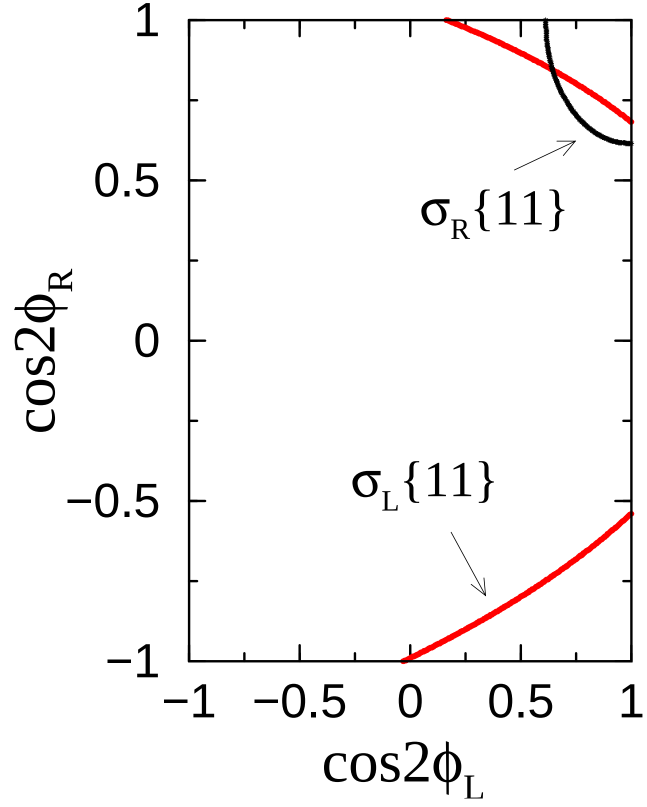
Even if only $\chi_1^+\chi_1^-$ channel is accessible (and $m_{\tilde{\nu}}$ known) can reconstruct mixing angles from polarized cross section

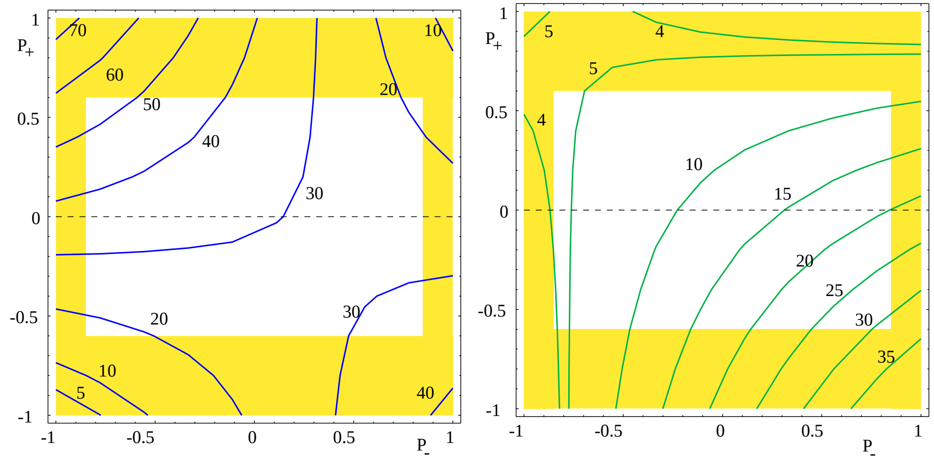


⇒ M_2, μ and $\tan\beta$ can be determined from $m_{\chi_1^\pm}$ and $\chi_1^+\chi_1^-$ polarized cross sections

Neutralino production

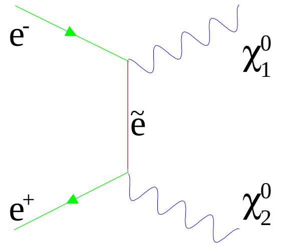
- situation more complicated
  - 4×4 mixing matrix
  - $\chi^0_1\chi^0_1$ channel not accessible if R-parity conserved
- access to M_1
- since t-channel exchange of $\tilde{e}_L$ and $\tilde{e}_R$ compete with s-channel, positron polarization gives really independent information

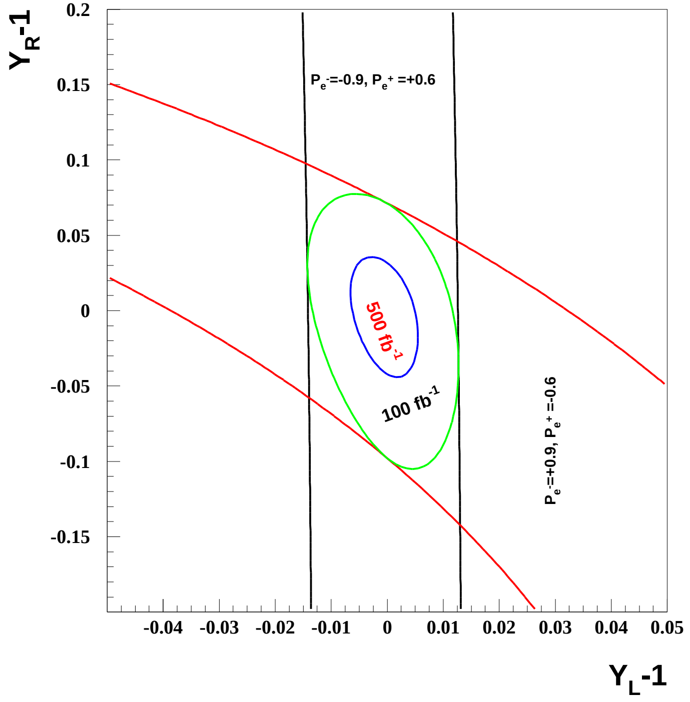
$\chi^0_1\chi^0_2$ cross section for different SUSY parameters



To prove that the new particles are really SUSY it has to be shown that the couplings amongst the superpartners are the same as for corresponding SM particles

This can be done on the percent level e.g. with $\chi^0_1\chi^0_2$-pair production once M_1, M_2, μ are known from the gaugino masses and chargino cross sections





Reconstruction of SUSY parameters

- measure masses and cross sections of SUSY particles and Higgses
- fit SUSY parameters at the weak scale
- extrapolate to GUT scale using RGEs
- bottom up approach that needs no model assumptions
- get model independent prediction at high scales
- example: SUGRA with $\tan\beta$=3, m_0=100 GeV, $m_{1/2}$=200 GeV, A_0=0 GeV, sign(μ) = $-$ (excluded now by LEP, but general features should not change)

Accessible particles and mass error for the simulated point

Particle	Mass (GeV)	Error(GeV)
$\tilde{e}_L$	173.0	0.18
$\tilde{e}_R$,$\tilde{\mu}_R$	131.6	0.09
$\tilde{\nu}_e$	157.5	0.07
$\tilde{\mu}_L$	173.0	0.3
$\tilde{\nu}_\mu$	157.5	0.2
$\tilde{\tau}_1$	130.8	0.6
$\tilde{\tau}_2$	173.5	0.6
$\tilde{\nu}_\tau$	157.5	0.6
χ^0_1	76.6	0.05
χ^0_2	142.8	0.07
χ^0_3	343.8	0.3
χ^0_4	349.9	0.6
$\chi^\pm_1$	142.9	0.035
$\chi^\pm_2$	352.6	0.25
h^0	97.7	0.05
H^0	466.7	1.5
A^0	466.7	1.5
H^+	473.3	1.5
$\tilde{t}_1$	353.9	0.6
$\tilde{q}$	$\sim$ 450	1.0
$\tilde{g}$ (LHC)	486.5	10.0

Used cross sections and errors

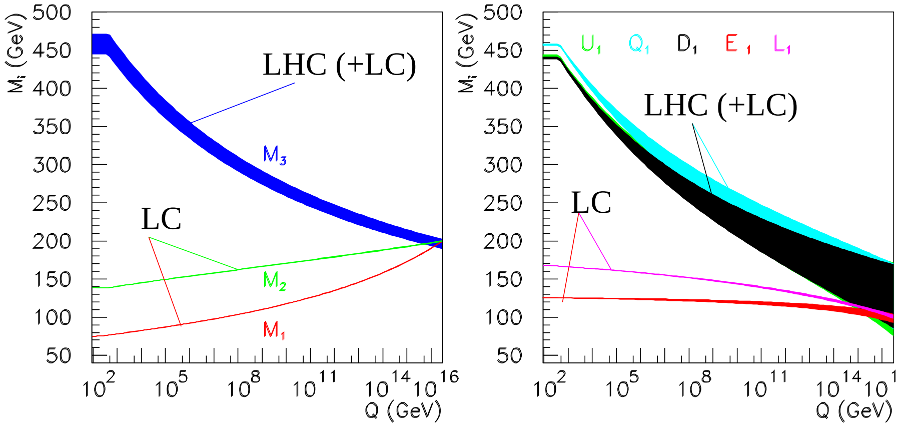
Process	$\sigma(e^-_L e^+_R)(fb)$	Error (fb)	$\sigma(e^-_R e^+_L)$(fb)	Error (fb)
$\tilde{t}_1\tilde{t}_1$	16.7	0.41	21.1	0.46
$\tilde{t}_1\tilde{t}_2$	4.55	0.21	3.41	0.18
$\tilde{t}_2\tilde{t}_2$	3.80	0.19	0.64	0.08
$\tilde{b}_1\tilde{b}_1$	18.6	0.43	1.42	0.12
$\tilde{b}_1\tilde{b}_2$	0.21	0.05	0.16	0.04
$\tilde{b}_2\tilde{b}_2$	0.69	0.08	2.28	0.15
$\tilde{\tau}_1\tilde{\tau}_1$	18.9	0.43	66.37	0.81
$\tilde{\tau}_1\tilde{\tau}_2$	0.67	0.08	0.50	0.07
$\tilde{\tau}_2\tilde{\tau}_2$	77.64	0.88	19.13	0.44
$\tilde{\nu}_\tau\tilde{\nu}_\tau$	20.0	0.45	15.0	0.39
$\mathrm{H}^+\mathrm{H}^-$	2.10	0.15	9.73	0.31
$\mathrm{A}^0\mathrm{h}^0$	1.22	0.11	0.91	0.10
$\mathrm{A}^0\mathrm{H}^0$	0.52	0.07	0.39	0.06
$\mathrm{Z}^0\mathrm{h}^0$	2.41	0.16	1.81	0.13
$\mathrm{Z}^0\mathrm{H}^0$	2.14	0.15	1.60	0.13

Reconstructed parameters at the weak scale

Parameter	True Value	Fit Error
M_1	74.64	0.15
M_2	138.65	0.10
M_3	467.55	12.1
A_τ	-128.7	43
A_b	-586.5	41
A_t	-358.7	2.5
$\mathrm{m}^2(H_1)$	27 646	601
$\mathrm{m}^2(H_2)$	-100 750	146
$\mathrm{m}^2(e_R)$	15 785	17
$\mathrm{m}^2(e_L)$	28 140	19
$\mathrm{m}^2(d_R)$	193 876	624
$\mathrm{m}^2(u_R)$	195 779	624
$\mathrm{m}^2(q_L)$	209 047	457
$\mathrm{m}^2(\tau_R)$	15 745	156
$\mathrm{m}^2(\tau_L)$	28 120	139
$\mathrm{m}^2(b_R)$	193 547	806
$\mathrm{m}^2(t_R)$	120 582	657
$\mathrm{m}^2(Q_{3L})$	171 616	513
$\tan\beta$	3.0	0.01
μ	335.7	1.3

Extrapolation to the GUT scale

- calculate low energy parameters (gaugino masses, sfermion masses, trilinear couplings) and extrapolate to GUT scale using RGEs
- check for unification at GUT scale



Need very small errors at weak scale to get useful results!

The fit is pure bottom up, no a priory assumptions at higher scales

The current fit assumes that all masses are measured, however some useful information (cross sections, forward backward asymmetries) is not used

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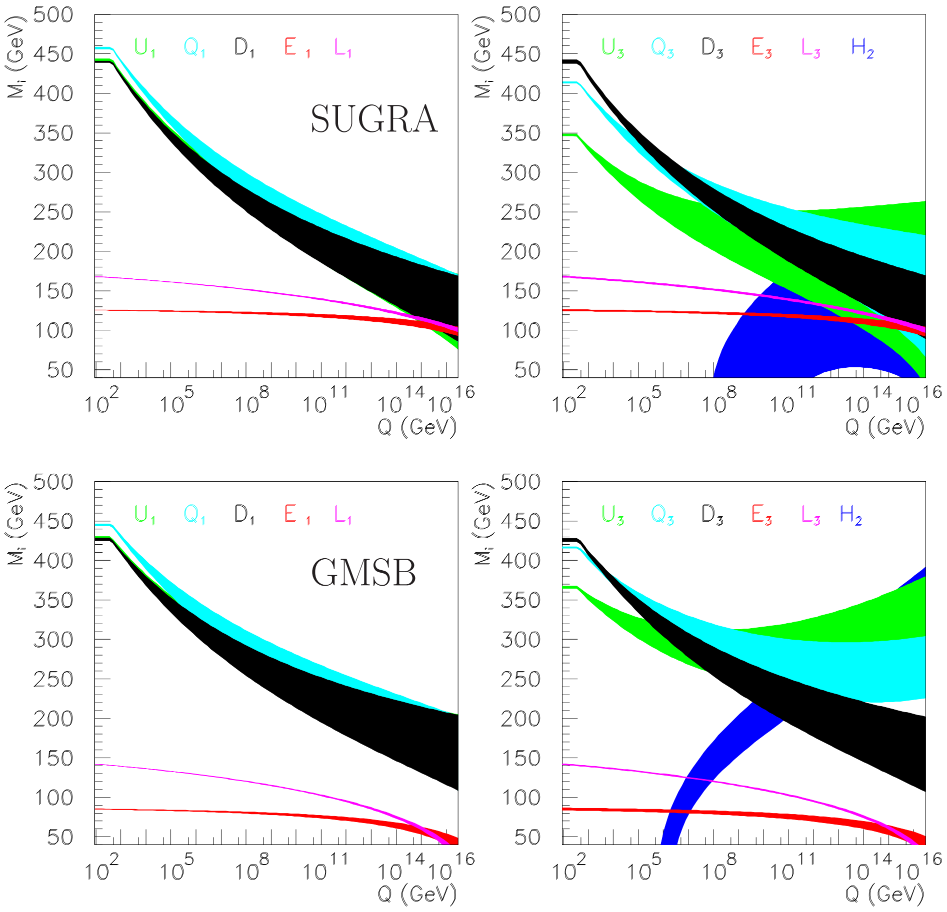
The results at the GUT scale can then be used to test models

E.g. comparison of SUSY and GMSB

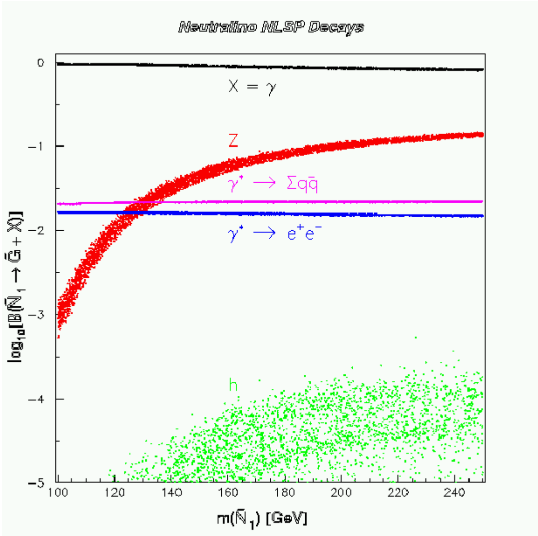


Models can be clearly distinguished

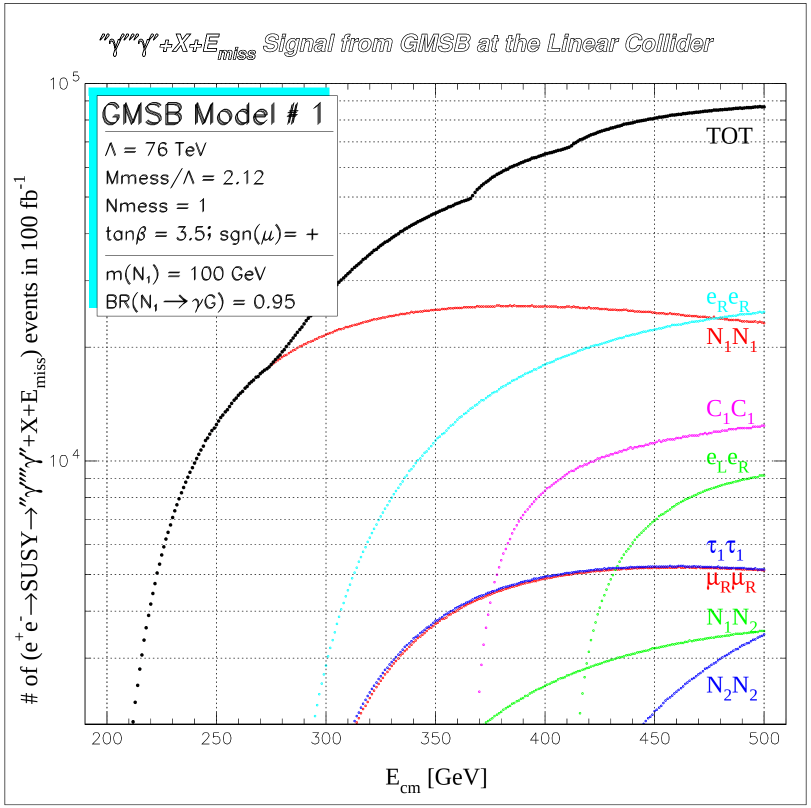
Analyzes in GMSB

- $\tilde{G}$ is very light ($m_{\tilde{G}} \simeq \left(\frac{\sqrt{F}}{100\,\mathrm{TeV}}\right)^2 2.37\mathrm{eV}$) $\Rightarrow$
  - all other SUSY particles are unstable
  - no reason for NLSP to be neutral

  ($\sqrt{F}$: fundamental scale of symmetry breaking $F > \Lambda M_{\mathrm{mess}}$)
- NLSP normally χ_1^0 or $\tilde{\ell}$ ($\tilde{\tau}_1$ in case of significant mixing)
- depending on SUSY breaking scale NLSP can decay between prompt and outside the detector
- interesting decays: $\chi_1^0 \to \tilde{G}\gamma(\gamma^*, Z)$ or (and) $\tilde{\ell} \to \tilde{G}\ell$



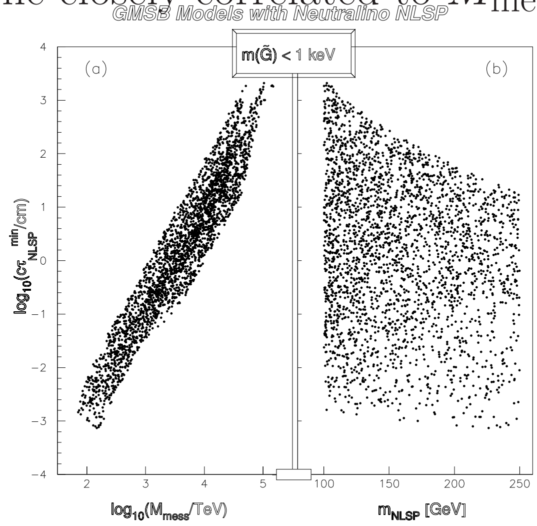
- typical mass spectrum for LC-relevant GMSB-models ($\propto \Lambda$)
  - $\tilde{\ell}_R, \tilde{\tau}_1, \chi_1^0 \sim 100 - 200\,\mathrm{GeV}$
  - $\tilde{\ell}_L, \tilde{\tau}_2, \chi_2^0, \chi_1^\pm \sim 200 - 500\,\mathrm{GeV}$
  - other SUSY-particles $> 500\,\mathrm{GeV}$
- Typical cross sections:



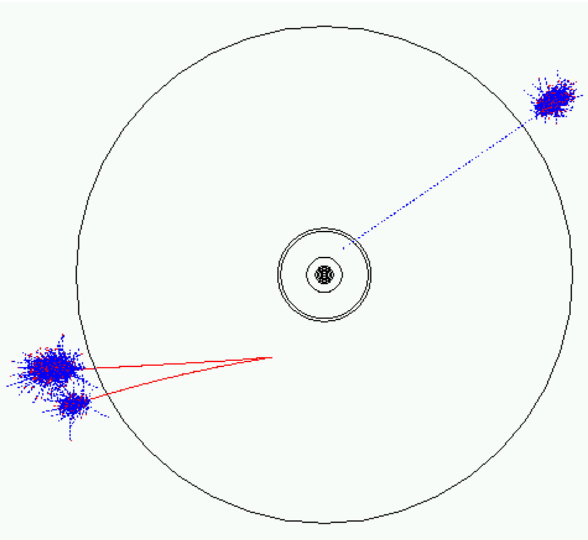
Expect several 10000 events

Detailed analysis for χ_1^0-NLSP scenario exists

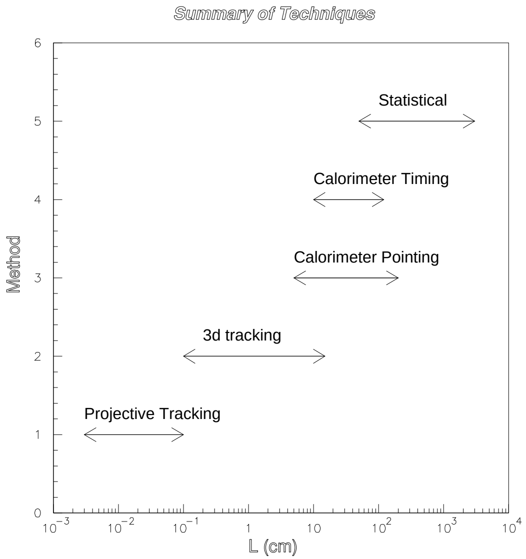
- χ_1^0-mass can be well measured with threshold scan
- χ_1^0-lifetime closely correlated to M_{mess}

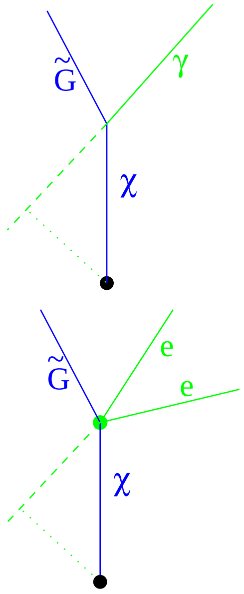


Experimental signatures: non-pointing photons and e^+e^--pairs starting in the detector



Measurement of χ_1^0 lifetime with tracking, calorimeter pointing, calorimeter timing and statistical methods (ratio between two and one photon events)





$c\tau_{\chi_1^0}$ can be measured from $\mathcal{O}(10\mu\mathrm{m})$ up to more then 100m corresponding to $M_{\mathrm{mess}} \sim 100-10^5\,\mathrm{GeV}$

Including mass measurements all model parameters can be measured to the 1% level

Roadmap to SUSY

- Discover SUSY:
  - LHC: $\tilde{q}$, $\tilde{g}$, ($\tilde{\ell}$, $\tilde{\nu}$, χ?)
  - LC: $\tilde{\ell}$, $\tilde{\nu}$, χ
- Prove that coupling(particle)=coupling(partner)
  ➠ LC
- reconstruct SUSY breaking scheme from accurate measurements of masses and couplings
  ➠ LHC+LC
- If SUSY is realized in nature need LHC+LC to understand it

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❼ Alternative Theories

- Contact interactions
- Models with Z's
- Extra dimensions
- Conclusions

Contact Interactions

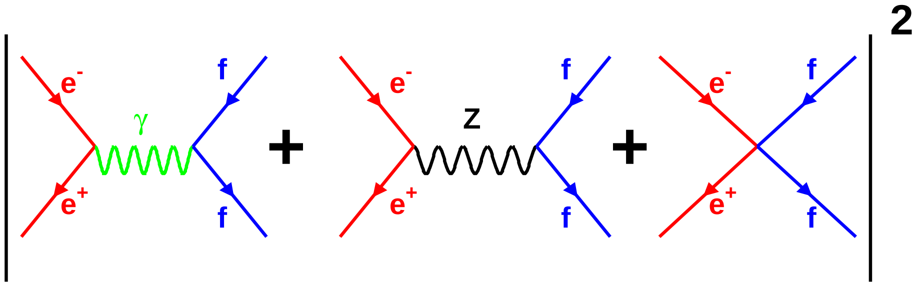
Very heavy exchange-particle: Propagator $\propto \frac{1}{M^2}$
Effective Lagrangian:

$$\mathcal{L}_{eff} = \sum_{i,k=L,R} \lambda_{ik}^2/M^2 \alpha^{ik} (\bar{e}_i \gamma^\mu e_i)(\bar{f}_k \gamma^\mu f_k)$$

with $\alpha^{ik} = \pm 1$

Scale-parameter $\Lambda^2 = \frac{4\pi M^2}{\lambda^2}$

(e.g. μ decay $\Lambda = \left(\sqrt{2} G_\mu\right)^{-1/2} \sim 250\,\mathrm{GeV}$)



$$\frac{d\sigma}{d\cos\theta} = SM(s,t) + C_2(s,t)\frac{1}{\Lambda^2} + C_4(s,t)\frac{1}{\Lambda^4}$$

(Equivalent to t-channel exchange of a heavy scalar with mass M and coupling λ)

Main sensitivity is in interference term, so large dependence on helicity structure

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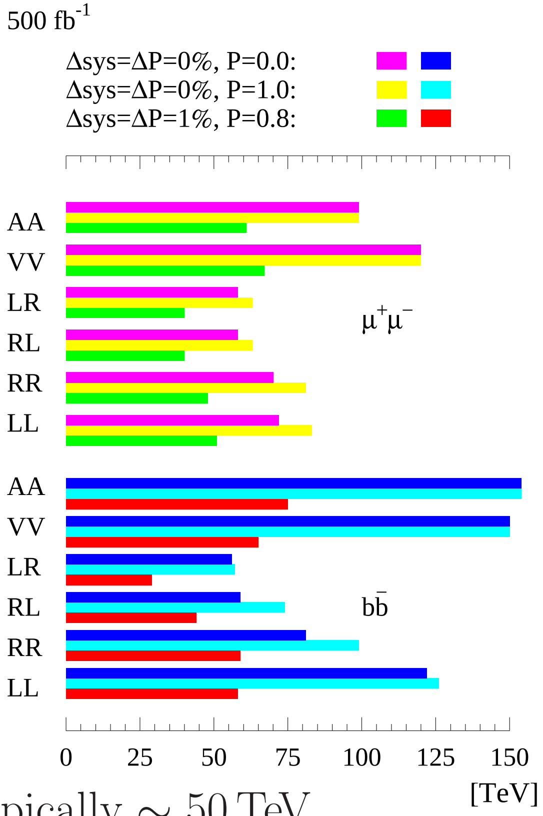
Assumptions

- $\sqrt{s} = 500\,\mathrm{GeV}$, $\mathcal{L} = 500\,\mathrm{fb}^{-1}$
- b-tagging efficiency $\varepsilon_\mathrm{b} = 60\%$
- systematic error 0, 1% (pessimistic)

Results



- limits typically $\sim 50\,\mathrm{TeV}$
- systematics will dominate, otherwise $\Lambda_{\mathrm{lim}} \propto \mathcal{L}^{1/4}$
- polarization helps little
- LHC reach similar but in different channels

Models with Z's

- models with extended gauge groups (left-right-symmetric, E_6) normally require additional Z-bosons
- in principle Z and Z' mix, however $Z - Z'$ mixing angle tightly constrained by Z-precision data
- for direct production LHC reaches much higher Z'-limits than LC ($\sim 3\,\mathrm{TeV}$)
- however for f$\bar{\mathrm{f}}$-production Z'-exchange interferes with Z and γ exchange so that Z'-effects remain visible for $m_{Z'} \gg \sqrt{s}$
  (in the same way PEP and PETRA could measure properties of the Z)
- measurement of cross sections and asymmetries gives access to vector- and axial-vector-couplings separately
- model dependent analyzes:
  - – assume a given model
  - ➠ all couplings are defined
  - – can use leptonic and hadronic events
  - – deviations from SM prediction translate directly into Z'-mass

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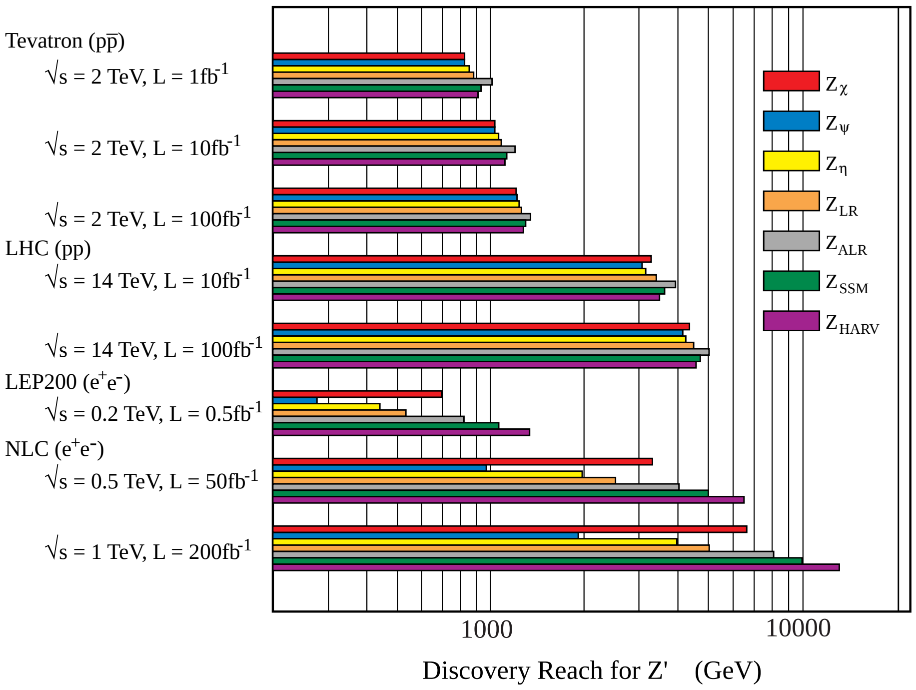


- (very moderate Luminosity assumptions for LC, however statistical scaling only with $\mathcal{L}^{1/4}$ and large contributions from Luminosity systematics)
- on average limits comparable to LHC
- however much larger difference between models, since sensitivity is in interference term
- on the contrary LC is not sensitive to the total width of the Z'

- model independent analyzes:
  - LC sensitive to normalized couplings

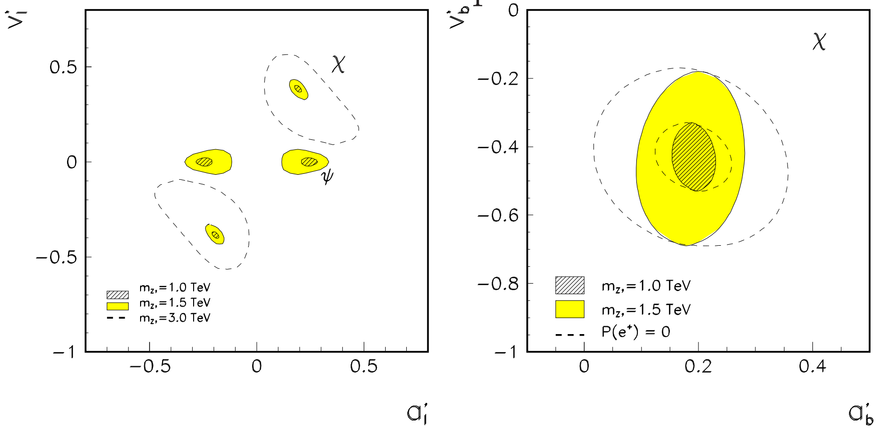
$$a_f^N = a'_f \sqrt{\frac{s}{m_{Z'}^2 - s}}$$
$$v_f^N = v'_f \sqrt{\frac{s}{m_{Z'}^2 - s}}$$

  - for leptons can obtain model independent limits/measurements on normalized couplings
  - all hadronic observables depend on product of leptonic couplings (Z'-production) and hadronic couplings (Z'-decay)

  ➠ can measure hadronic couplings only if leptonic couplings deviate significantly from zero
- experimental assumptions:
  - beam polarizations 90/60% with $\Delta\mathcal{P}/\mathcal{P} = 1\%$
  - luminosity known to 0.5%
  - leptons can be tagged with $\varepsilon = 95 \pm 0.5\%$
  - b quarks can be tagged with $\varepsilon = 60 \pm 0.6\%$
  - measure cross sections, A_{LR} and A_{FB}^{ℓ}

Ideal case: LHC discovers a Z', so mass is known and LC can measure the couplings

95% c.l. contours for $\sqrt{s} = 500\,\mathrm{GeV}$ and
$\mathcal{L} = 500\,\mathrm{pb}^{-1}$



- measure leptonic couplings to few % and b-couplings to $\sim 10\%$ for $m_{Z'} = 1.5\,\mathrm{TeV}$
- limits should roughly stay constant for $m_{Z'}/\sqrt{s} = \mathrm{const}$
- the LC can distinguish the models over basically the full LHC discovery range

Large extra dimensions

Hierarchy-problem:

Why is $m_{\mathrm{H}} \sim 100\,\mathrm{GeV} \ll \mathrm{M_{pl}} \sim 10^{19}\,\mathrm{GeV}$?

Possible answers:

- SUSY (already seen)
- in reality is $\mathrm{M_{pl}} \sim 100\,\mathrm{GeV}$ but it appears so large because gravity lives in $4+n$ dimensions

$$\mathrm{M_{pl}^2} = \mathrm{M_D^{2+n} R^n}$$

R : compactification radius of extra dimensions

$$\Rightarrow R = \mathrm{M_{pl}^{\frac{2}{n}} M_D^{-(\frac{2}{n}+1)}}$$
$$\sim 10^{\frac{30}{n}-17} \left(\frac{1\,\mathrm{TeV}}{\mathrm{M_D}}\right)^{1+\frac{2}{n}} [\mathrm{cm}]$$

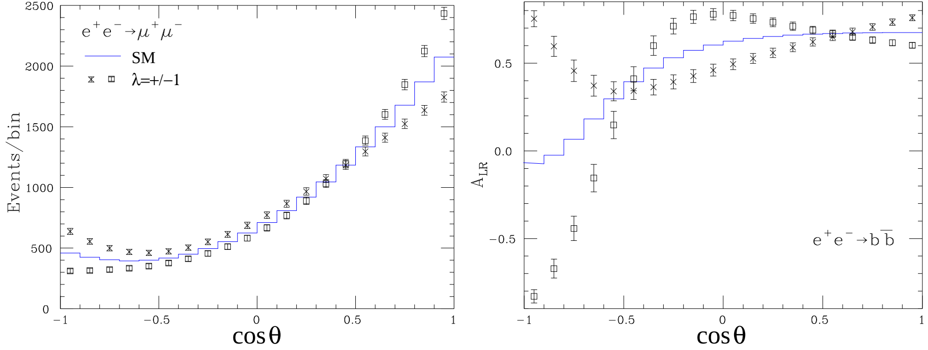
$n=1$	$R = \mathcal{O}(10^{13}\mathrm{cm})$	excluded
$n=2$	$R = \mathcal{O}(1\mathrm{mm})$	~excluded
$n=7$	$R = \mathcal{O}(1\mathrm{fm})$	

Experimental signatures:

- In the bulk of the extra dimensions there live a huge number of graviton states (Kaluza-Klein towers G^*)

➠ Expect effects in single γ production
($\mathrm{e^+e^-} \to \gamma G^*$, G^* invisible)
and fermion pair production
($\mathrm{e^+e^-} \to G^* \to \mathrm{f\bar{f}}$)

e.g. G^*-effects in $\mathrm{e^+e^-} \to \mu^+\mu^-$ and $\mathrm{e^+e^-} \to \mathrm{b\bar{b}}$



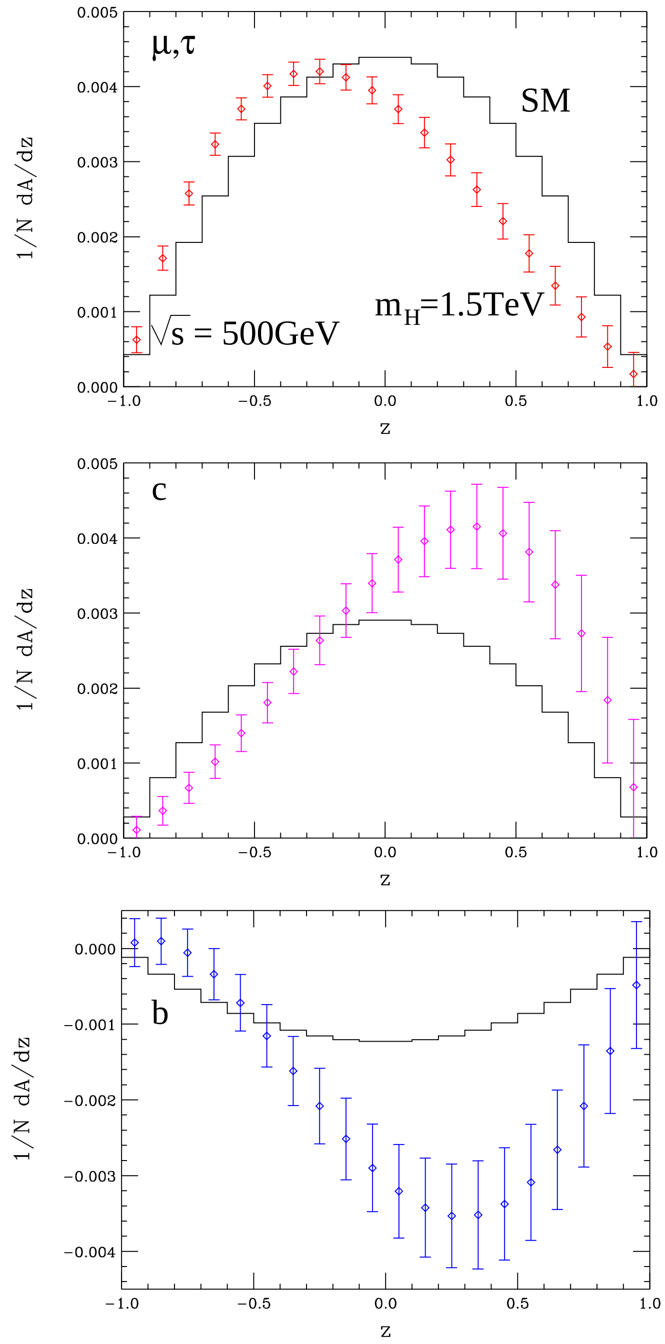
- LC limit $M_D < 4(7)$ TeV for $\sqrt{s} = 0.5(1)$ TeV
- LHC comparable
- $\cos\theta (= z)$ dependence very different from Z'

Additional possibility: transverse polarization

- with transverse beam polarization there exists an azimuthal asymmetry depending on $\cos\theta \rightarrow$ plot
- this asymmetry is symmetric in $\cos\theta$ for vector or scalar particle exchange
- for tensor exchange (gravitons) it receives an asymmetric component

➡ Graviton and Z' exchange can be distinguished up to $M < 10\sqrt{s}$

- extra dimensions can be excluded up to $M_D <$ 10(22) TeV for $\sqrt{s} = 0.5(1)$ TeV
  (highest reach at next generation colliders)



Conclusions on alternatives

- The LC is sensitive to a "General new physics scale" of order 50 TeV
- In concrete models (Z', extra dimensions) this translates into mass scales of few TeV
- LC and LHC have similar reach but are highly complementary
  - The LC is mainly sensitive to $\mathrm{e^+e^-}\ell^+\ell^-$ and $\mathrm{e^+e^-b\bar{b}}$ couplings while LHC is sensitive to $\ell^+\ell^-\mathrm{q\bar{q}}$ (q=u,d)
  - LHC mainly sees the pure new physics while LC sees its interference with the SM
  - The LHC can discover that there is "something new" by seeing a resonance, then the LC can distinguish models by measuring the couplings

❽ Precision measurements at lower energies

- Introduction
- Measurements of electroweak quantities on the Z
- Measurement of m_W
- Theoretical aspects
- Study of CP-violation in the B-sector

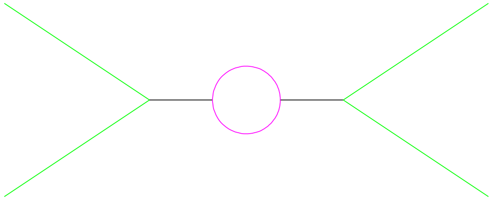
Introduction

Interest in precision measurements

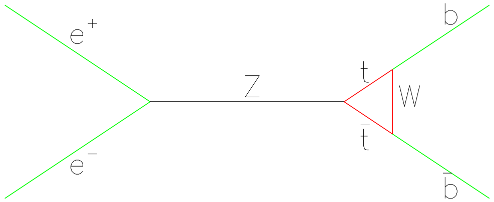
Test consistency of the theory on the loop level

Two types of loop corrections:

- universal corrections to propagator



parameters:
  - $\Delta\rho$: absolute normalization of Z couplings
  - $\Delta\kappa$ ($\sin^2\theta^l_{eff}$): effective weak mixing angle in Z-fermion couplings
  - Δr: Relation $G_\mu \leftrightarrow m_{\rm W}$
- vertex corrections (only interesting fir b-quarks as partner of top)



Contributions to loop corrections

- corrections from isospin masssplitting ($\propto m_{\rm t}^2$ in SM)
- corrections from Higgs sector ($\propto \log(m_{\rm H})$ in SM)

Contributions to vertex corrections for b-quarks

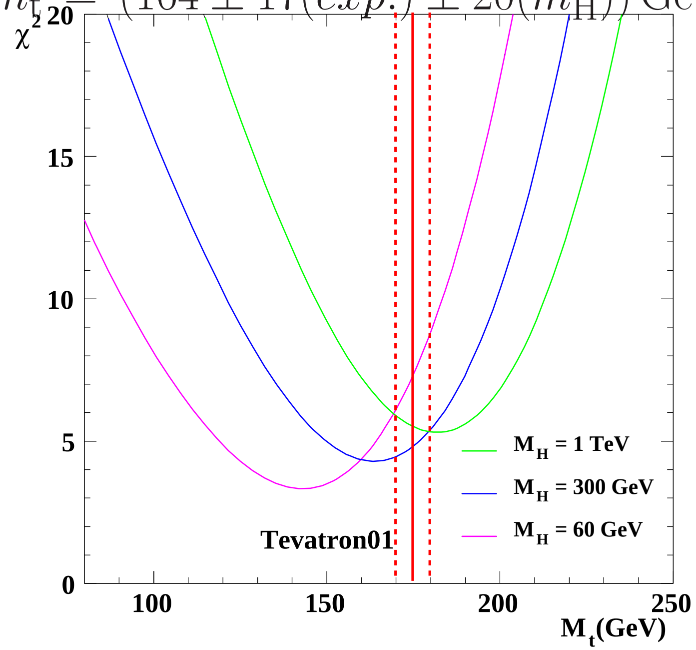
- corrections from b-t masssplitting ($\propto m_{\rm t}^2$)
- corrections from charged Higgs sector and its SUSY partners, if exists
- corrections from special role of top-quark e.g. in technicolor models

Aim: see effects of new physics in precision data

Historical example: Top mass prediction (1993)

Fit to all electroweak precision data gave

$$m_t = (164 \pm 17(exp.) \pm 20(m_H))\,\mathrm{GeV}$$



In 1995 the top-quark was discovered at the TEVATRON with $m_t \sim 175\,\mathrm{GeV}$

Hope at least to repeat this with the Higgs Boson

LEP+SLD+TEVATRON measure electroweak observables on the permille level

Quantities:

- Z-lineshape: Partial widths of $Z \to f\bar{f}$, $\Delta\rho$, N_ν
- Asymmetries: Weak mixing angle in Z-decays, $\sin^2\theta^\ell_{\mathrm{eff}}$
- b-quark partial width and asymmetries (R_b, $\mathcal{A}_b$) Mass dependent vertex corrections
- W-mass: Δr

Present situation:

- LEP: $\sim 4 \times 4 \cdot 10^6$ Zs with unpolarized beams $\sim 4 \times 500\,\mathrm{pb}^{-1}$ above the W-threshold
- SLD: $\sim 5.5 \cdot 10^5$ Zs with $\mathcal{P} \sim 75\%$ electron polarization

Assumptions

- The linear collider can produce $\sim 10^9$ Zs on resonance
  (corresponds to $\sim 30\,\mathrm{fb}^{-1}$ or 50 days)
  $\mathcal{L} = 7 \cdot 10^{33}\mathrm{cm}^{-2}\mathrm{s}^{-1} \Rightarrow 230\,\mathrm{Hz}$ of $\mathrm{Z} \to \mathrm{q\bar{q}}$
- similar luminosity is possible near the W-threshold
- electrons and positrons can be polarized with $\mathcal{P}_{e^-} = \pm 80\%$, $\mathcal{P}_{e^+} = \pm 60\%$
  (corresponds to an effective polarization of $\frac{\mathcal{P}_{e^+}+\mathcal{P}_{e^-}}{1+\mathcal{P}_{e^+}\mathcal{P}_{e^-}} \sim 95\%$)
- positive and negative polarizations can be switched randomly from bunch to bunch (or train to train) independent for electrons and positrons
- polarimeters are available for relative measurements

Lineshape parameters

Cross section around Z-peak:

$$\sigma_f(s) = \frac{12\pi}{m_Z}\frac{\Gamma_e\Gamma_f s}{\left(s-m_Z^2\right)^2+\left(\frac{s}{m_Z}\right)^2\Gamma_Z^2} + \sigma_{\mathrm{int}} + \sigma_\gamma + \mathrm{rad.\,corr.}$$

$$\begin{aligned}\Gamma_\ell &\approx (1+\Delta_\rho)\Gamma_\ell^{(B)} \\ \Gamma_{\mathrm{had}} &= (1+\alpha_s/\pi+\ldots)\Gamma_{\mathrm{had}}^{(0)}\end{aligned}$$

Minimally correlated observables:

	LEP precision
m_Z	$0.2 \cdot 10^{-4}$
Γ_Z	$0.9 \cdot 10^{-3}$
$\sigma_0^{\mathrm{had}} = \frac{12\pi}{m_\mathrm{Z}^2}\frac{\Gamma_e\Gamma_{\mathrm{had}}}{\Gamma_Z^2}$	$0.9 \cdot 10^{-3}$
$R_\ell = \frac{\Gamma_{\mathrm{had}}}{\Gamma_l}$	$1.2 \cdot 10^{-3}$

⇒ Need to scan

⇒ Need absolute cross sections

Assumptions:

- relative beam energy error around Z-pole: 10^{-5}
  $\Rightarrow \Delta\Gamma_Z/\Gamma_Z = 0.4 \cdot 10^{-3}$
  (Currently under debate if $\Delta E_b = 10^{-5}$ is possible and if beamstrahlung and beamspread are enough under control)
- selection efficiency for μs, τs, hadrons (and exp error on $\mathcal{L}$) improved by a factor three relative to the best LEP experiment
  $\Rightarrow \Delta R_\ell/R_\ell = 0.3 \cdot 10^{-3}$
- theoretical error on luminosity stays at 0.05%
  $\Rightarrow \Delta\sigma_0^{\mathrm{had}}/\sigma_0^{\mathrm{had}} = 0.6 \cdot 10^{-3}$
  (again if beamspread/-strahlung understood)

Improvement on lineshape related quantities:

	LEP	Giga-Z
m_Z	$91.1874 \pm 0.0021\,\mathrm{GeV}$	$\pm 0.0021\,\mathrm{GeV}$
$\alpha_s(m_Z^2)$	0.1183 ± 0.0027	± 0.0009
$\Delta\rho$	$(0.55 \pm 0.10) \cdot 10^{-2}$	$\pm 0.05 \cdot 10^{-2}$
N_ν	2.984 ± 0.008	± 0.004

$\boxed{R_b}$

scale DELPHI analysis:

$$\begin{aligned} R_b = 0.21634 &\pm 0.00075\ (\mathit{stat\,dat} + \mathit{MC}) \\ &\pm 0.00028\ (\mathit{uds} - \mathit{bg}) \\ &\pm 0.00030\ (c - \mathit{bg}) \\ &\pm 0.00027\ (\mathit{hem\ corr}) \end{aligned}$$

DELPHI working point: $\varepsilon_b \approx 30\%$ purity $\approx 98\%$
Possible for TESLA: $\varepsilon_b \approx 40\%$ purity $\approx 99.5\%$

- statistical error down by a factor 20
- c-background down by a factor 4
- uds-background mainly from gluon splitting to $\mathrm{b\bar{b}}$ can be measured much better with TESLA
- hemisphere correlation is mainly QCD
  - detector resolution factor 10 better than LEP
  - losses are mainly due to mass cut (Lorenz invariant)
  - energy dependence should be much smaller
  - also this source should decrease by a factor 4-5
- $\Delta R_b = 0.00014$ should be possible (factor 5 to LEP)

A_{LR}

Definition

$$\sigma = \sigma_u \left[1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(\mathcal{P}_{e^+} - \mathcal{P}_{e^-})\right]$$

with $\mathcal{P}_{e^+}$ ($\mathcal{P}_{e^-}$) longitudinal polarizations of the positrons (electrons)

A_{LR} measures weak mixing angle $\sin^2\theta^{\ell}_{\mathrm{eff}}$:

$$\begin{aligned} A_{\mathrm{LR}} &= \mathcal{A}_\ell \\ \mathcal{A}_\ell &= \frac{2g_{Vl}g_{Al}}{g_{Vl}^2 + g_{Al}^2} \\ \frac{g_{Vl}}{g_{Al}} &= 1 - 4|Q_l|\sin^2\theta^{\ell}_{\mathrm{eff}} \end{aligned}$$

- $\sin^2\theta^{\ell}_{\mathrm{eff}}$ is a very sensitive variable to see loop corrections to the Z-couplings.
- A_{LR} is the variable most sensitive to $\sin^2\theta^{\ell}_{\mathrm{eff}}$

The (extended) Blondel scheme

Four independent measurements:
(4 combinations with positive/negative electron/ positron polarization)

$$\begin{aligned} \sigma_{++} &= \sigma_u \left[1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(\ \ \mathcal{P}_{e^+} - \mathcal{P}_{e^-})\right] \\ \sigma_{-+} &= \sigma_u \left[1 + \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(-\mathcal{P}_{e^+} - \mathcal{P}_{e^-})\right] \\ \sigma_{+-} &= \sigma_u \left[1 + \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(\ \ \mathcal{P}_{e^+} + \mathcal{P}_{e^-})\right] \\ \sigma_{--} &= \sigma_u \left[1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\mathrm{LR}}(-\mathcal{P}_{e^+} + \mathcal{P}_{e^-})\right] \end{aligned}$$

$\Longrightarrow$ A_{LR} can be measured without knowing $\mathcal{P}_{e^+}$, $\mathcal{P}_{e^-}$:

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

About 10% of the statistics is needed on the small cross sections

Only difference between $|\mathcal{P}^+_{e^\pm}|$ and $|\mathcal{P}^-_{e^\pm}|$ needs to be known from polarimetry

Can be brought under control with polarimeters a la SLD

Polarization difference ($\Delta\mathcal{P}_{e^\pm} = |\mathcal{P}^+_{e^\pm}| - |\mathcal{P}^-_{e^\pm}|$):

- Need SLD like polarimeter
- Asymmetry in one polarimeter channel:
  $A_i = a_i \mathcal{P}_e \mathcal{P}_\gamma$ (a_i =analyzing power)
- Laser polarization can be switched pulse to pulse
- Allow for different laser currents dependent on the polarization
- Need two polarimeter channels with different analyzing power
- combined fit of Z-rates and polarimeter rates can get $\Delta\mathcal{P}_{e^\pm}$ and a_i as well
- However need polarimeter counting rates about 10 times the Z rate (ok for SLD)

Statistical precision:

$$\Delta A_{\mathrm{LR}} = 4 \cdot 10^{-5} \cdot \sqrt{\frac{10^9}{N_Z}}$$

Systematic uncertainties

- Beam energy: $\Delta A_{\mathrm{LR}}/\Delta\sqrt{s} \approx 2 \cdot 10^{-2}/\,\mathrm{GeV}$
  $\Rightarrow$ need $\Delta\sqrt{s} \approx 1\,\mathrm{MeV}$ relative to m_{Z}
- Luminosity difference: Only relative precision needed.
  Should be no problem if luminometer inside the mask is possible
- Backgrounds: To be kept below 10^{-4}
  According to LEP experience no problem
- Beamstrahlung: $\Delta A_{\mathrm{LR}} = 9 \cdot 10^{-4}$
  Needs to be known on the few percent level (partially covered by Z-scan)

Assume $\Delta A_{\mathrm{LR}} = 10^{-4} \Rightarrow \Delta\sin^2\theta^\ell_{\mathrm{eff}} = 0.000013$

$\mathcal{A}_\mathrm{b}$

Without polarized beams (LEP) the forward-backward asymmetries can be measured:

$$A^q_{FB} = \frac{\sigma^{(q)}_F - \sigma^{(q)}_B}{\sigma^{(q)}_T} = \frac{3}{4}\mathcal{A}_\mathrm{e}\mathcal{A}_\mathrm{q}$$

With polarized beams (SLD,TESLA) the left-right-forward-backward asymmetries can be measured:

$$A^q_{FB,LR} = \frac{\sigma^{(q)}_{L,F} - \sigma^{(q)}_{L,B} - \sigma^{(q)}_{R,F} + \sigma^{(q)}_{R,B}}{\sigma^{(q)}_L + \sigma^{(q)}_R} = \frac{3}{4}\mathcal{P}\mathcal{A}_\mathrm{q}$$

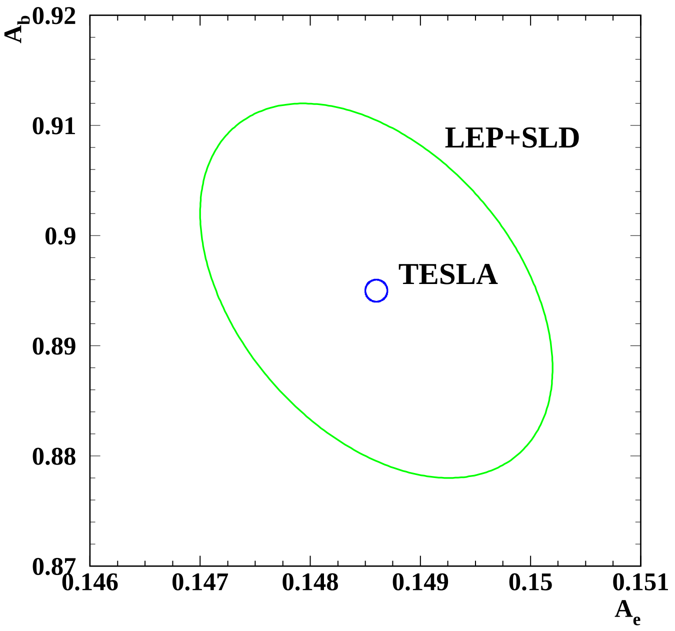
Statistically factor $\mathcal{P}/\mathcal{A}_\mathrm{e} \sim 6$ more sensitive to $\mathcal{A}_\mathrm{b}$

However most systematics scale with the asymmetry

Two main techniques: leptons and jetcharge

- Statistical error $\Delta\mathcal{A}_\mathrm{b} \approx 4 \cdot 10^{-4}$ in both cases
- Light quark systematics can be reduced by a (harder) lifetime tag
- For jetcharge reduce hemisphere correlations by a thrust cut
- leptons will be dominated by $B\overline{B}$-mixing (statistical error!)
- A total error of $\Delta\mathcal{A}_\mathrm{b} = 1 \cdot 10^{-3}$ seems realistic

Similar improvement as for $\mathcal{A}_\mathrm{e}$



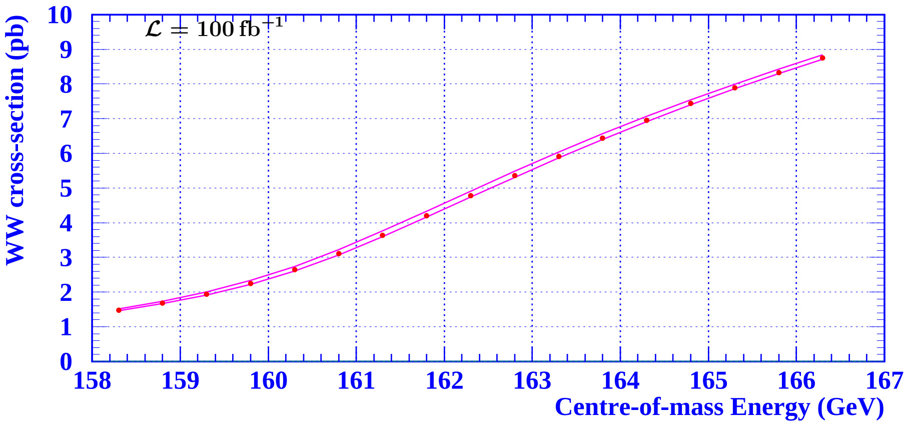
m_{W}

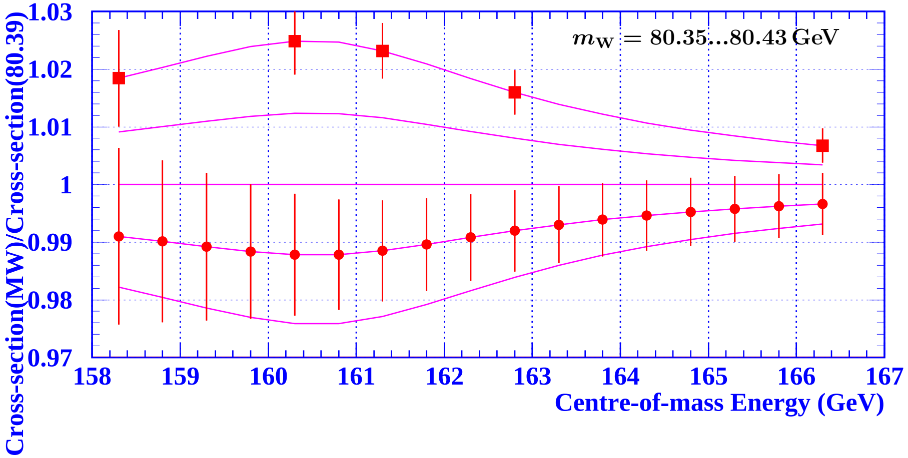
Best possible method: threshold scan

- spend $100\,\mathrm{fb}^{-1}$ at $\sqrt{s} \sim 161\,\mathrm{GeV}$ (1 year!)
- polarization is very useful to enhance cross section or measure background

$$\begin{aligned} \sigma_{\mathrm{WW}} &= 3\sigma_{\mathrm{WW}}^{\mathrm{unpol}} & \mathcal{P}_{e^-} = -0.8,\ \mathcal{P}_{e^+} = 0.6 \\ \sigma_{\mathrm{WW}} &= 0.1\sigma_{\mathrm{WW}}^{\mathrm{unpol}} & \mathcal{P}_{e^-} = 0.8,\ \mathcal{P}_{e^+} = -0.6 \end{aligned}$$

- assume efficiency/background as at LEP
- perform 5-point scan
- assume point to point systematics negligible
- beam energy is known to well below 5 MeV (A relative calibration to the Z-mass is fine)





Results

$$\begin{aligned} &\Delta\varepsilon/\varepsilon = 0.5\%,\ \Delta\mathcal{L}/\mathcal{L} = 0.25\% & \Delta m_{\mathrm{W}} = 6\,\mathrm{MeV} \\ &\Delta\varepsilon/\varepsilon,\ \Delta\mathcal{L}/\mathcal{L}\ \text{fitted} & \Delta m_{\mathrm{W}} = 7\,\mathrm{MeV} \end{aligned}$$

Measurement is statistics limited

Precision data and the LHC

m_{W}

- LHC has infinite statistics for W-production
- two main sources of error:
  - energy scale of the detector
  - parton distribution function
- $\Delta m_{\mathrm{W}} = 15\,\mathrm{MeV}$ might be possible although extremely difficult

$\sin^2\theta^l_{eff}$

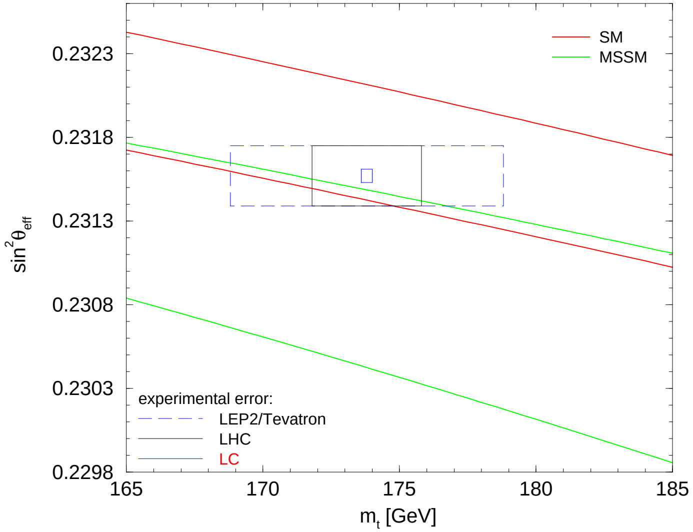
- in principle $\sin^2\theta^l_{eff}$ can be measured from forward backward asymmetry $q\bar{q} \to \ell^+\ell^-$ (At $\sqrt{s} = m_{\mathrm{Z}}$: $A^0_{FB} = \frac{3}{4}\mathcal{A}_i\mathcal{A}_f$)
- select events with $m(\ell^+\ell^- \approx m_{\mathrm{Z}}$ and large boost
- the high energy quark is then on average a valence quark, the low energy one a (sea) antiquark
- possible statistical precision $\Delta\sin^2\theta^l_{eff} = 0.0001$
- unclear if systematics can be brought to this level

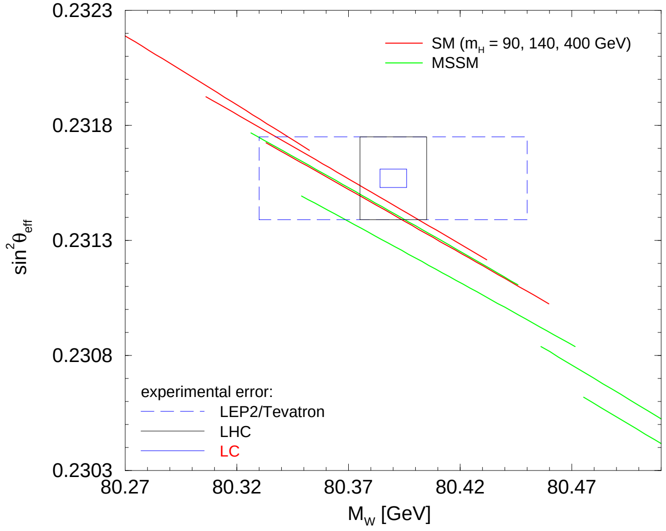
Interpretation of precision measurements

Parametric errors

- largest effect: Running of α ($\alpha(m_{\mathrm{Z}}) = \alpha(0)\frac{1}{1-\Delta\alpha}$)
  - Using data only (without the latest BES results) ($\delta(\Delta\alpha) = 0.00065$): $\Delta\sin^2\theta^\ell_{\mathrm{eff}} = 0.00023$, $\Delta m_{\mathrm{W}} = 12\,\mathrm{MeV}$
  - $\sim$ factor three improvement using perturbative QCD at low energy
  - with $\sigma(\mathrm{e}^+\mathrm{e}^-\!\to\mathrm{had})$ below the Υ to 1% ($\delta(\Delta\alpha) = 0.000046$): $\Delta\sin^2\theta^\ell_{\mathrm{eff}} = 0.000017$, $\Delta m_{\mathrm{W}} < 1\,\mathrm{MeV}$
- 2 MeV error on m_{Z} gives $\Delta\sin^2\theta^\ell_{\mathrm{eff}} = 0.000014$, $\Delta m_{\mathrm{W}} = 1\,\mathrm{MeV}$ (if W-mass calibrated to m_{Z})
- $\Delta m_{\mathrm{t}} = 1\,\mathrm{GeV}$ gives $\Delta\sin^2\theta^\ell_{\mathrm{eff}} = 0.00003$, $\Delta m_{\mathrm{W}} = 6\,\mathrm{MeV}$ $\Rightarrow$ no problem with LC precision of m_{t} ($<$ 200 MeV)

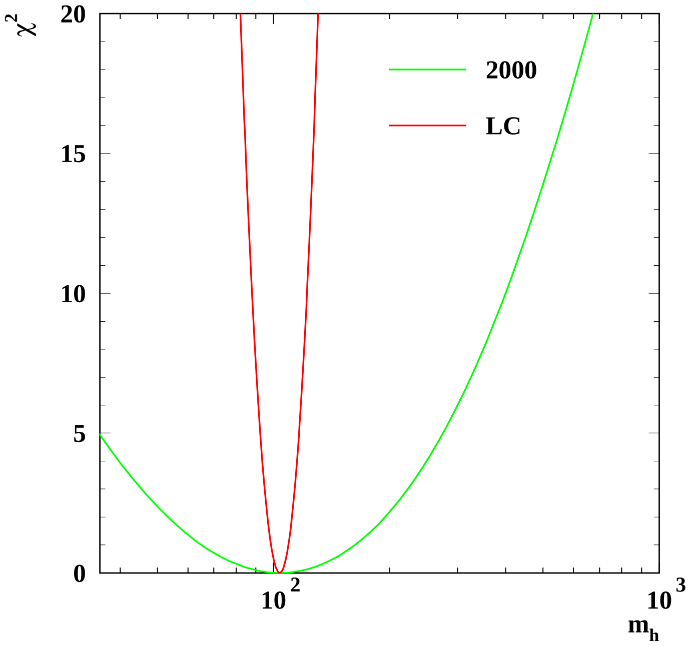
SM and MSSM make accurate predictions for $\sin^2\theta^{\ell}_{\mathrm{eff}}$ and m_{W}





If no new physics found up to then:

Standard Model Higgs can be predicted to 5% accuracy:

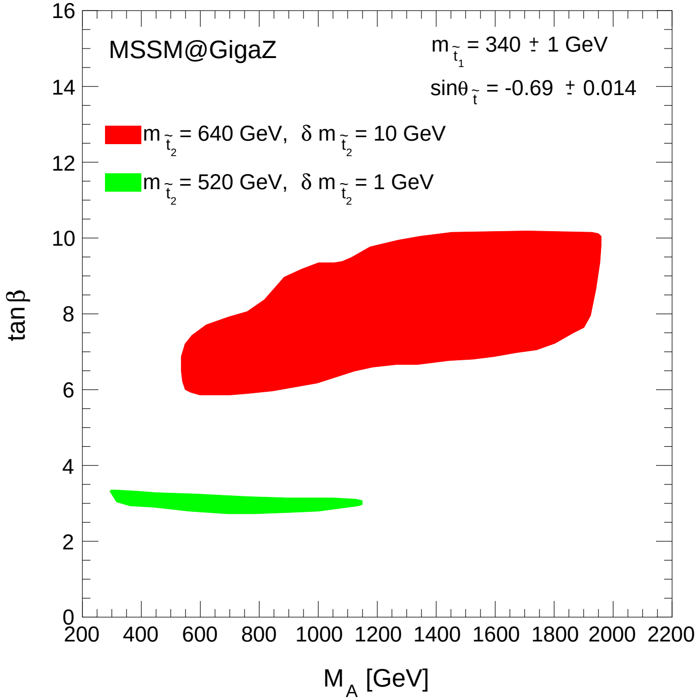


Can test the theory if a Higgs of $m_{\mathrm{H}} \sim 170\,\mathrm{GeV}$ is found

Possible scenario inside the MSSM:

- some SUSY parameters measured at LHC e.g. stop sector
- however some of the parameters still uncertain

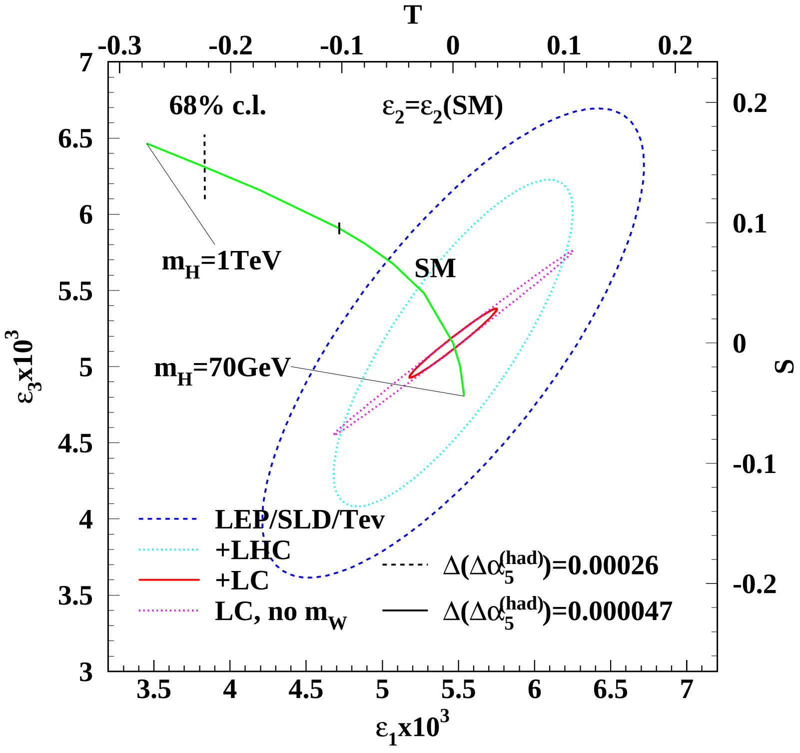
Precision measurements can constrain allowed SUSY parameter range



In this example one can get a fairly good measurement of $\tan\beta$ and some ideas on m_A

Model independent analysis (ε, ST parameters)

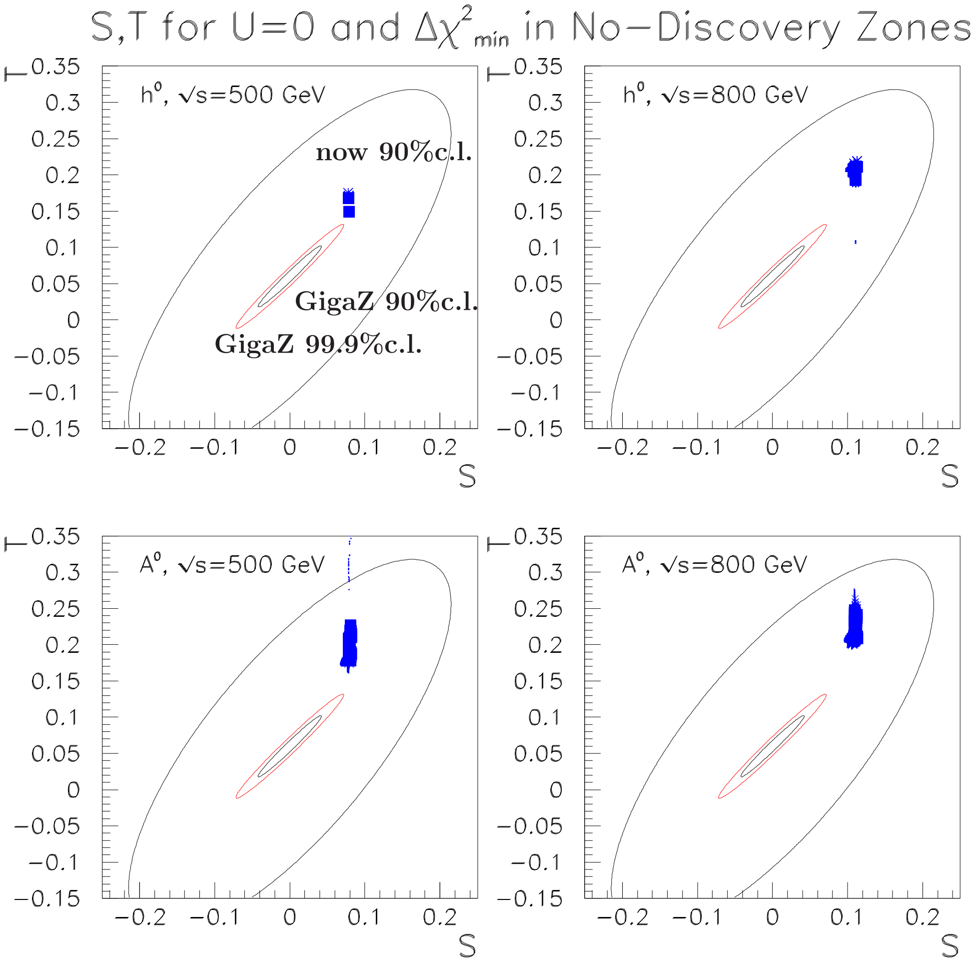
- ε_1 (T): absorbs large isospin splitting corrections
- ε_3 (S): only logarithmic dependencies
- ε_2 (U): additional (small) correctins to m_W



- dramatic improvement in m_H direction
- improvement perpend. to m_H largely due to m_W
- significant Higgs constraint independent of ε_1 (T) possible

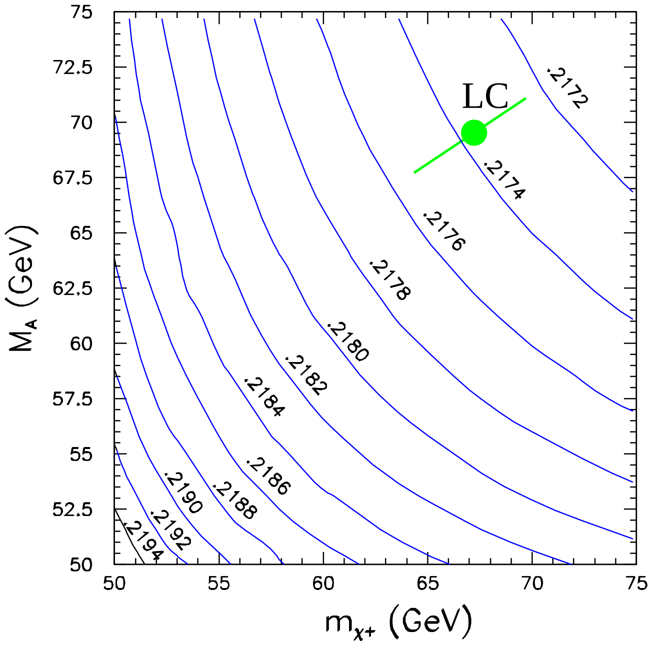
E.g. exclusion of a two Higgs doublet model with a light Higgs

(that cannot be excluded by direct searches)



For these types of exclusions m_{W} is important!

R_{b} is sensitive e.g. to masses within Supersymmetry



CP-violation studies

measure time dependent asymmetries

$$A(t) = \frac{N_{B^0}(t) - N_{\bar{B}^0}(t)}{N_{B^0}(t) + N_{\bar{B}^0}(t)} = a_{\cos} \cos \Delta m t + a_{\sin} \sin \Delta m t$$

mainly two examined decay modes

- $B^0 \rightarrow J/\Psi K_s^0$:
  - $a_{\sin} = -\sin 2\beta,\ a_{\cos} = 0$
- $B^0 \rightarrow \pi^+\pi^-$:
  - $a_{\sin} = -\sin 2\alpha,\ a_{\cos} = 0$ if penguin diagrams can be ignored
  - however $a_{\sin}, a_{\cos}$ modified by penguin contributions, hard to calculate
  - can be disentangled by measuring branching ratios $B^0 \rightarrow \pi^+\pi^-$, $B^0 \rightarrow \pi^0\pi^0$, $B^+ \rightarrow \pi^+\pi^0$

Experimental analysis:

- identify initial state b-charge
- reconstruct decay mode
- measure eigentime to decay (easy in LC environment with fully reconstructed decays)

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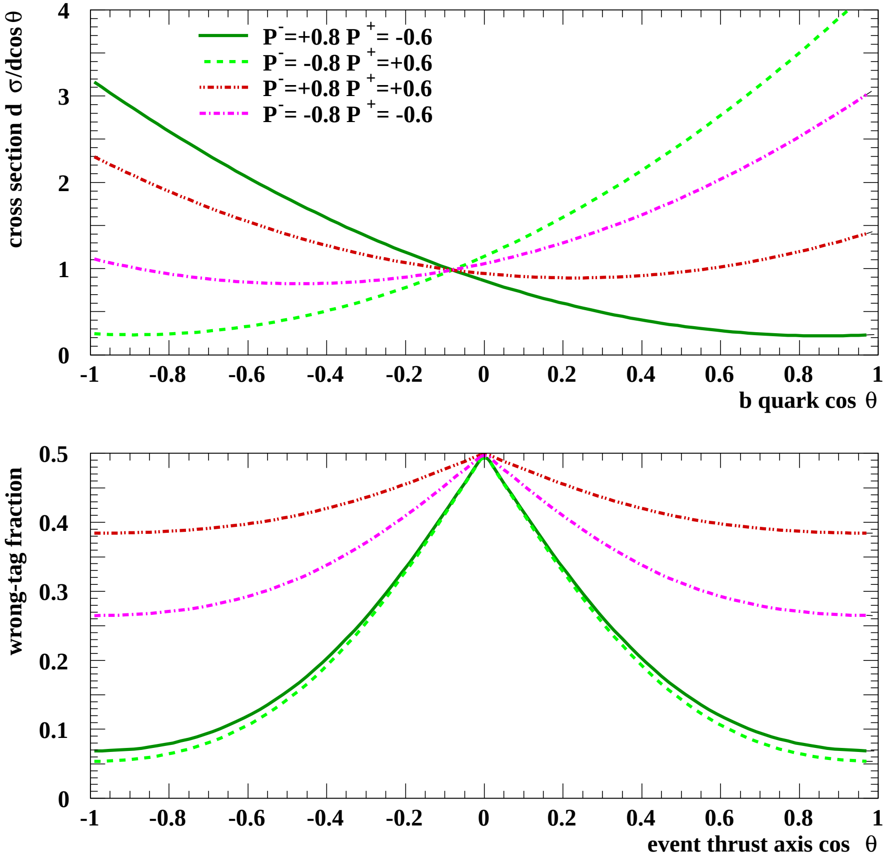
total statistics: $4 \cdot 10^8$ b-hadrons

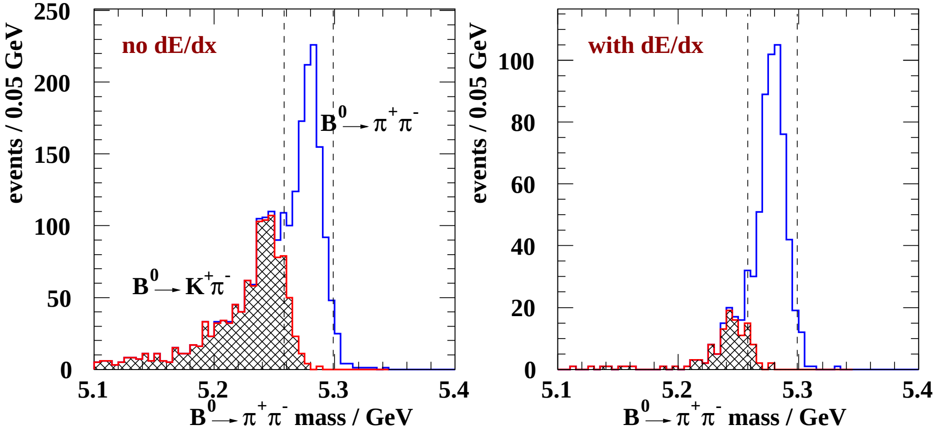
Tagging of primary b-charge:

- Polarization gives primary flavor tagging “for free”



Final state identification:

- Missing particle ID can be replaced by excellent momentum resolution



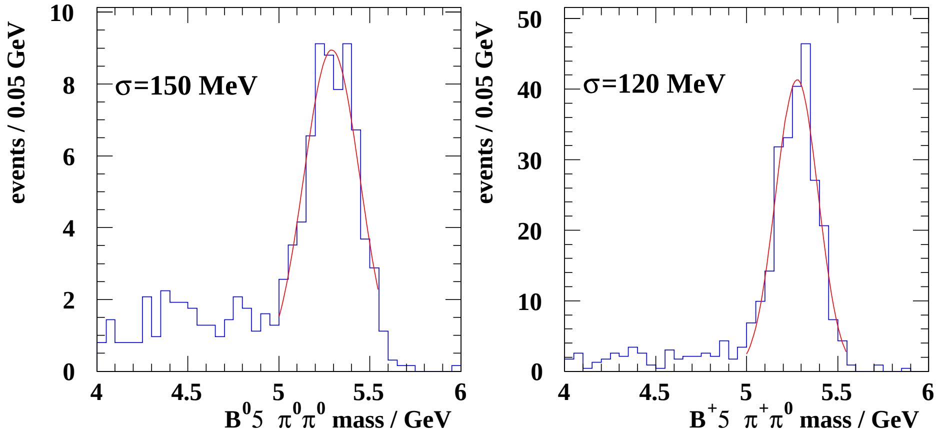
Results

	$\sin 2\beta$	"$\sin 2\alpha$"
BaBar	0.12	0.26
CDF	0.08	0.10
ATLAS	0.02	0.14
LHC-b	0.01	0.05
TESLA	0.04	0.07

Not the best, but interesting cross check!

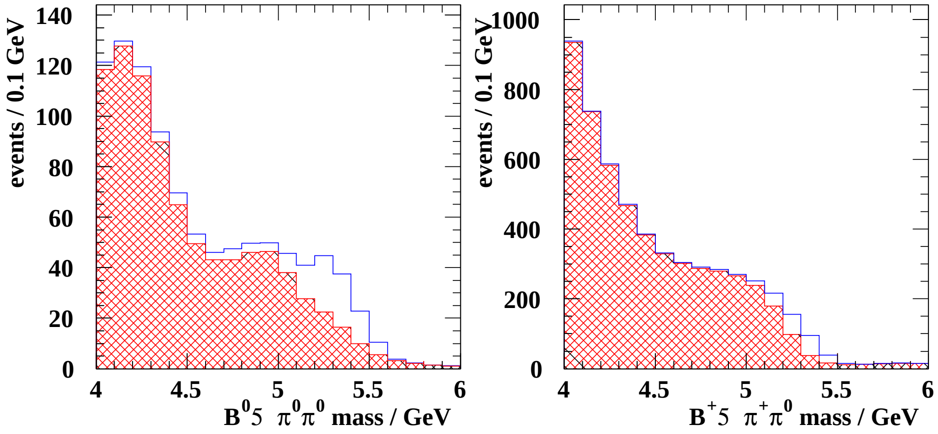
Branching ratios $B^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$

- needed to disentangle direct from penguin contributions in $B^0 \to \pi^+\pi^-$
- only possible in e^+e^--machines
- Needs at a linear collider:
  - b-tagging opposite to signal hemisphere for $b\bar{b}$-selection
  - anti-b-tagging in signal hemisphere to suppress other b-decays
  - good calorimeter resolution (mainly spatial) for mass measurement



(Resolution depends strongly on the calorimeter design)

Finally a signal should be seen above background

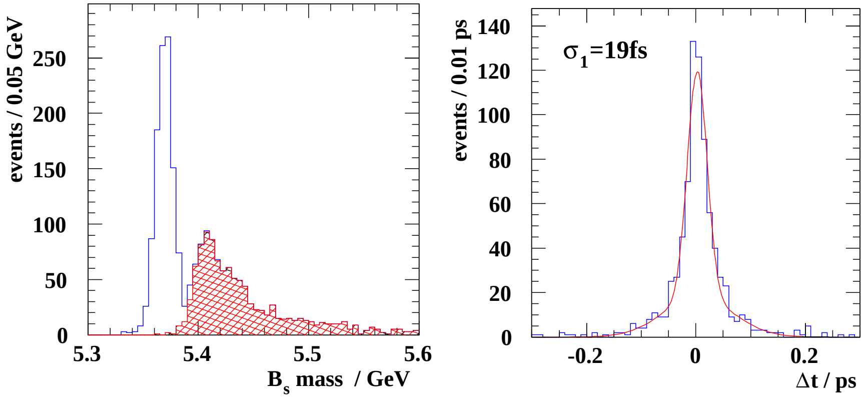


	$\frac{\Delta BR(B^+\to\pi^+\pi^0)}{BR}$ $(5\cdot10^{-6})$	$\frac{\Delta BR(B^0\to\pi^0\pi^0)}{BR}$ $(2\cdot10^{-6})$
BaBar (300 fb^{-1})	11	17
GigaZ (10^9 Zs)	15	24

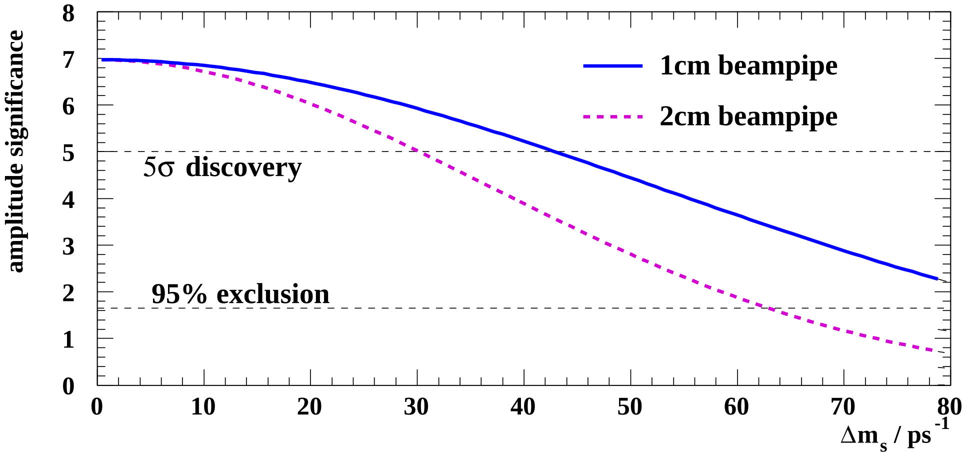
Competitive with 10^9 Zs, leading with 10^{10} Zs

$B_s\overline{B}_s$-oscillations

- "golden" mode: $B_s \to D_s\pi$, $D_s \to \phi\pi, KK$ can be reconstructed almost background free
- proper time res. dominated by vertex res.



- $\Delta m_s \sim 40\text{ps}^{-1}$ possible with 10^9 Zs
- resolution limit around $\Delta m_s \sim 80\text{ps}^{-1}$



Conclusions on lower energy running

- With less than a year of running on the Z huge progress on some important electroweak precision observables can be made
- With an additional year around the W-pair threshold also a significant improvement on m_{W} can be obtained
- It seems that with some effort at Beijing/ Novosibirsk the running of α can be measured to a high enough precision
- Only with the precise data from TESLA the experimental measurements can match the theoretical predictions after the Higgs is found
- Some interesting cross checks in B-physics, however no "golden channel" (yet)

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❾ Conclusions

- A linear collider with an energy range of about 1 TeV can do a lot of precision measurements in
  - top physics,
  - Higgs physics,
  - electroweak gauge bosons,
  - Supersymmetry,
  - extended gauge theories,
  - B-physics.
- In many respects the linear collider is complementary to the LHC and we need both to understand how electroweak symmetry breaking works.
- The motivation we have from the present experimental data is strong enough to build the LC now and not to wait for the findings of LHC.