Anhang A

Astrophysikalische Konstanten

2. ASTROPHYSICAL CONSTANTS AND PARAMETERS

Table 2.1. Revised May 2006 by M.A. Dobbs (McGill U), D.E. Groom (LBNL), and D. Scott (UBC). The figures in parentheses after some values give the one-standard deviation uncertainties in the last digit(s). Physical constants are from Ref. 1. While every effort has been made to obtain the most accurate current values of the listed quantities, the table does not represent a critical review or adjustment of the constants, and is not intended as a primary reference. The values and uncertainties for the cosmological parameters depend on the exact data sets, priors, and basis parameters used in the fit. Many of the parameters reported in this table are derived parameters or have non-Gaussian likelihoods. Their error bars may be highly correlated with other parameters and care must be taken when extrapolating to higher significance levels. In most cases we report the best fit of a spatially-flat ΛCDM cosmology with a power-law initial spectrum to WMAP3 data alone [2]. For more information see Ref. 3 and the original papers.

Quantity	Symbol, equation	Value	Reference, footnote
speed of light	c	$299\,792\,458$ m s^{-1}	defined[4]
Newtonian gravitational constant	G_N	$6.6742(10) \times 10^{-11}$ m^3 kg^{-1} s^{-2}	[1, 5]
astronomical unit (mean Earth-Sun distance)	AU	$149\,597\,870\,660(20)$ m	[6, 7]
tropical year (equinox to equinox) (2005.0)	yr	$31\,556\,925.2$ s	[6]
sidereal year (fixed star to fixed star) (2005.0)		$31\,558\,149.8$ s	[6]
mean sidereal day (2005.0)		$23^{\rm h}\,56^{\rm m}\,04\overset{\rm s}{.}090\,53$	[6]
Jansky	Jy	10^{-26} W m^{-2} Hz^{-1}	
Planck mass	$\sqrt{\hbar c/G_N}$	$1.22090(9) \times 10^{19}$ GeV$/c^2$	[1]
		$= 2.17645(16) \times 10^{-8}$ kg	
Planck length	$\sqrt{\hbar G_N/c^3}$	$1.61624(12) \times 10^{-35}$ m	[1]
Hubble length	c/H_0	$\sim 1.2 \times 10^{26}$ m	[8]
parsec (1 AU/1 arc sec)	pc	$3.085\,677\,580\,7(4) \times 10^{16}$ m $= 3.262\ldots$ ly	[9]
light year (deprecated unit)	ly	$0.306\,6\ldots$ pc $= 0.946\,1\ldots \times 10^{16}$ m	
Schwarzschild radius of the Sun	$2G_N M_\odot/c^2$	$2.953\,250\,08$ km	[10]
Solar mass	$M_\odot$	$1.988\,44(30) \times 10^{30}$ kg	[11]
Solar equatorial radius	$R_\odot$	6.961×10^8 m	[6]
Solar luminosity	$L_\odot$	$(3.846 \pm 0.008) \times 10^{26}$ W	[12]
Schwarzschild radius of the Earth	$2G_N M_\oplus/c^2$	$8.870\,056\,22$ mm	[13]
Earth mass	$M_\oplus$	$5.972\,3(9) \times 10^{24}$ kg	[14]
Earth mean equatorial radius	$R_\oplus$	$6.378\,140 \times 10^6$ m	[6]
luminosity conversion	L	$3.02 \times 10^{28} \times 10^{-0.4\,M_{\rm bol}}$ W	[15]
		($M_{\rm bol}$ = absolute bolometric magnitude	
		= bolometric magnitude at 10 pc)	
flux conversion	$\mathscr{F}$	$2.52 \times 10^{-8} \times 10^{-0.4\,m_{\rm bol}}$ W m^{-2}	from above
		($m_{\rm bol}$ = apparent bolometric magnitude)	
Solar velocity around center of Galaxy	$\Theta_\circ$	$220(20)$ km s^{-1}	[16]
Solar distance from Galactic center	$R_\circ$	$8.0(5)$ kpc	[17]
local disk density	$\rho_{\rm disk}$	3–12 $\times 10^{-24}$ g cm^{-3} $\approx$ 2–7 GeV$/c^2$ cm^{-3}	[18]
local halo density	$\rho_{\rm halo}$	2–13 $\times 10^{-25}$ g cm^{-3} $\approx$ 0.1–0.7 GeV$/c^2$cm^{-3}	[19]
present day CBR temperature	T_0	2.725 ± 0.001 K	[20]
present day CBR dipole amplitude		3.346 ± 0.017 mK	[21]
Solar velocity with respect to CBR		369 ± 2 km/s	[21,22]
		towards $(\ell, b) = (263.86° \pm 0.04°, 48.24° \pm 0.10°)$	
local group velocity with respect to CBR	$v_{\rm LG}$	627 ± 22 km s^{-1}	[23]
		towards $(\ell, b) = (276° \pm 3°, 30° \pm 3°)$	
entropy density/Boltzmann constant	s/k	$2\,889.2\,(T/2.725)^3$ cm^{-3}	[15]
number density of CMB photons	n_γ	(410.5 ± 0.5) cm^{-3}	[24]
present day Hubble expansion rate	H_0	$100\,h$ km s^{-1} Mpc^{-1}	
		$= h \times (9.778\,13$ Gyr$)^{-1}$	[25]
present day normalized Hubble expansion rate‡	h	$0.73^{+0.04}_{-0.03}$	[2]
scale factor for cosmological constant	$c^2/3H_0^2$	$2.853 \times 10^{51}\,h^{-2}$ m^2	
critical density of the Universe	$\rho_c = 3H_0^2/8\pi G_N$	$2.775\,366\,27 \times 10^{11}\,h^2\,M_\odot$Mpc^{-3}	
		$= 1.878\,37(28) \times 10^{-29}\,h^2$ g cm^{-3}	
		$= 1.053\,69(16) \times 10^{-5}\,h^2$ (GeV$/c^2$) cm^{-3}	derived
pressureless matter density of the Universe‡	$\Omega_{\rm m} = \rho_{\rm m}/\rho_c$	$0.127^{+0.007}_{-0.009}\,h^{-2} \Rightarrow 0.24^{+0.03}_{-0.04}$	[2]
baryon density of the Universe‡	$\Omega_{\rm b} = \rho_{\rm b}/\rho_c$	$0.0223^{+0.0007}_{-0.0009}\,h^{-2} \Rightarrow 0.042^{+0.003}_{-0.005}$	[2]
dark matter density of the Universe‡	$\Omega_{\rm dm} = \Omega_{\rm m} - \Omega_{\rm b}$	$0.105^{+0.007}_{-0.010}\,h^{-2} \Rightarrow 0.20^{+0.02}_{-0.04}$	
radiation density of the Universe‡	$\Omega_\gamma = \rho_\gamma/\rho_c$	$(2.471 \pm 0.004) \times 10^{-5}\,h^{-2} \Rightarrow (4.6 \pm 0.5) \times 10^{-5}$	[26]
neutrino density of the Universe‡	Ω_ν	$< 0.007\,h^{-2} \Rightarrow < 0.014$ (95% CL)	[27]
dark energy density‡	Ω_Λ	$0.76^{+0.04}_{-0.06}$	[28]

Abbildung A.1: Astrophysikalische Konstanten (aus [14])

Quantity	Symbol, equation	Value	Reference, footnote
total energy density‡	$\Omega_{\rm tot} = \Omega_{\rm m} + \ldots + \Omega_\Lambda$	$1.003^{+0.013}_{-0.017}$	[2]
baryon-to-photon ratio‡	$\eta = n_{\rm b}/n_\gamma$	$4.7 \times 10^{-10} < \eta < 6.5 \times 10^{-10}$ (95% CL)	[29]
number density of baryons‡	$n_{\rm b}$	$(1.9 \times 10^{-7} < n_{\rm b} < 2.7 \times 10^{-7})\,{\rm cm}^{-3}$ (95% CL)	from η
dark energy equation of state parameter‡	w	$-0.97^{+0.07}_{-0.09}$	[2, 30]
fluctuation amplitude at $8h^{-1}$ Mpc scale‡	σ_8	$0.74^{+0.05}_{-0.06}$	[2]
scalar spectral index from power-law fit to data‡	n_s	$0.951^{+0.015}_{-0.019}$	[2]
running spectral index slope at $k_0 = 0.05$ Mpc^{-1} ‡	$dn_s/d\ln k$	$-0.055^{+0.029}_{-0.035}$	[2,31]
tensor-to-scalar field perturbations ratio at $k_0 = 0.002$ Mpc^{-1} ‡	$r = T/S$	< 0.55 at 95% C.L.	[2]
reionization optical depth‡	τ	0.09 ± 0.03	[2]
age of the Universe‡	t_0	$13.7^{+0.1}_{-0.2}$ Gyr	[2]

‡ See caption for caveats.

References:

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2. D.N. Spergel *et al.*, "Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology," `astro-ph/0603449`.
3. O. Lahav, A.R. Liddle, "The Cosmological Parameters," this *Review*.
4. B.W. Petley, Nature **303**, 373 (1983).
5. In the context of the scale dependence of field theoretic quantities, it should be remarked that absolute lab measurements of G_N have been performed on scales of 0.01–1.0 m.
6. *The Astronomical Almanac for the year 2005*, U.S. Government Printing Office, Washington, and Her Majesty's Stationary Office, London (2003).
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8. Derived from H_0 [2].
9. 1 AU divided by $\pi/648\,000$; quoted error is from the JPL Planetary Ephemerides value of the AU [7].
10. Product of $2/c^2$ and the heliocentric gravitational constant [6]. The given 9-place accuracy seems consistent with uncertainties in defining Earth's orbital parameters.
11. Obtained from the heliocentric gravitational constant [6] and G_N [1]. The error is the 150 ppm standard deviation of G_N.
12. 1996 mean total solar irradiance (TSI) $= 1367.5 \pm 2.7$ [32]; the solar luminosity is $4\pi \times (1\ {\rm AU})^2$ times this quantity. This value increased by 0.036% between the minima of solar cycles 21 and 22. It was modulated with an amplitude of 0.039% during solar cycle 21 [33].
   Sackmann *et al.* [34] use TSI $= 1370 \pm 2$ W m^{-2}, but conclude that the solar luminosity ($L_\odot = 3.853 \times 10^{26}$ J s^{-1}) has an uncertainty of 1.5%. Their value comes from three 1977–83 papers, and they comment that the error is based on scatter among the reported values, which is substantially in excess of that expected from the individual quoted errors.
   The conclusion of the 1971 review by Thekaekara and Drummond [35] ($1353 \pm 1\%$ W m^{-2}) is often quoted [36]. The conversion to luminosity is not given in the Thekaekara and Drummond paper, and we cannot exactly reproduce the solar luminosity given in Ref. 36.
   Finally, a value based on the 1954 spectral curve due to Johnson [37] ($1395 \pm 1\%$ W m^{-2}, or $L_\odot = 3.92 \times 10^{26}$ J s^{-1}) has been used widely, and may be the basis for the higher value of the solar luminosity and the corresponding lower value of the solar absolute bolometric magnitude (4.72) still common in the literature [15].
13. Product of $2/c^2$, the heliocentric gravitational constant from Ref. 6, and the Earth/Sun mass ratio, also from Ref. 6. The given 9-place accuracy appears to be consistent with uncertainties in actually defining the earth's orbital parameters.
14. Obtained from the geocentric gravitational constant [6] and G_N [1]. The error is the 150 ppm standard deviation of G_N.
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   The value 0.3 GeV/c^2 has been taken as "standard" in several papers setting limits on WIMP mass limits, *e.g.* in M. Mori *et al.*, Phys. Lett. **B289**, 463 (1992).
20. J. Mather *et al.*, Astrophys. J. **512**, 511 (1999). This paper gives $T_0 = 2.725 \pm 0.002$K at 95%CL. We take 0.001 as the one-standard deviation uncertainty.
21. C.L. Bennett *et al.*, Astrophys. J. Supp. **148**, 1 (2003).
22. G. Hinshaw, "Three-year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Temperature Analysis," `astro-ph/0603451`.
23. D. Scott and G.F. Smoot, "Cosmic Microwave Background," this *Review*.
24. $n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_BT}{\hbar c}\right)^3$, using T_0 from Ref. 20.
25. Conversion using length of tropical year.
26. $\rho_\gamma = \frac{\pi^2}{15}\frac{(k_BT)^4}{(\hbar c)^3}$, using T_0 from Ref. 20.
27. Based on $\Omega_\nu h^2 = \sum m_{\nu_i}/93$ eV, with $\sum m_{\nu_i} = 0.7$ eV from CMB + LSS + SN data set, Table 10 in Ref. 2.
28. WMAP + $h = 0.72 \pm 0.08$, Table 11 in Ref. 2. Uses different h than tabulated here.
29. B.D. Fields, S. Sarkar, "Big-Bang Nucleosynthesis," this *Review*.
30. WMAP[2] + Supernova Legacy Survey in a flat Universe.
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Abbildung A.2: Astrophysikalische Konstanten (aus [14])

Anhang B

Ergänzungen

B.1 Zustandsgleichungen

Im Folgenden soll der Zusammenhang zwischen Druck und Energiedichte für Materie und Strahlung für einige wichtige Spezialfälle näher betrachtet werden. Bei der Ableitung der allgemeinen Gasgleichung benutzt man:

$$p = \frac{1}{3} n \, \langle \pi \, v \rangle \tag{B.1}$$

Dabei ist p der Druck, der von Teilchen mit einer Dichte n, mittlerem Impuls π und Geschwindigkeit v auf die Wand eines Gefäßes ausgeübt wird.

Im **nicht-relativistischen Fall**, $v \ll c$, ist $\pi = m\,v$ (m bedeutet immer die Ruhemasse) und (B.1) wird:

$$p = \frac{1}{3} n \, m \langle v^2 \rangle = \frac{1}{3} n \, m \, c^2 \left\langle \frac{v^2}{c^2} \right\rangle = \frac{1}{3} \rho_m \, c^2 \left\langle \frac{v^2}{c^2} \right\rangle , \tag{B.2}$$

wobei $\rho_m \, c^2$ die Energiedichte aufgrund der Ruhemassen ist, die im nicht-relativistischen Fall dominiert. In diesem Fall wird der Druck auch sehr klein:

$$p \;\rightarrow\; 0 \quad \text{für } v \ll c. \tag{B.3}$$

Im **relativistischen Fall**, $v \approx c$, geht man von den relativistischen Formeln für Impuls und Energie aus:

$$\pi = \gamma \, m \, v \qquad \text{und} \qquad E = \gamma \, m \, c^2 \tag{B.4}$$

Tabelle B.1: Zustandsgleichung, Energiedichte und Skalenparameter, der die Ausdehnung des Universums beschreibt, jeweils für die Dominanz einer Energieform in einer Entwicklungsphase des Universums.

Dominante Energieform	Zustandsgleichung	Energiedichte	Skalenparameter
Strahlung	$p = \frac{1}{3}\rho_s$	$\rho_s \sim R^{-4}$	$R \sim t^{1/2}$
Materie	$p = \frac{1}{3}\rho_m \, c^2 \langle \frac{v^2}{c^2} \rangle \overset{v \ll c}{\rightarrow} 0$	$\rho_m \sim R^{-3}$	$R \sim t^{2/3}$
Vakuum	$p = -\rho_v$	$\rho_v = const$	$R \sim \exp(\alpha t)$

Damit ergibt sich:

$$\lim_{v \to c} \pi \to E/c \tag{B.5}$$

Diese Gleichung entspricht natürlich dem Zusammenhang zwischen Energie und Impuls eines Photons:

$$\pi = \frac{h}{\lambda}; \quad E = h\,\nu = \frac{h\,c}{\lambda} = \pi\,c \tag{B.6}$$

Damit ergibt (B.1) im relativistischen Fall, $v \approx c$:

$$p = \frac{1}{3} n \,\langle \pi\, c \rangle = \frac{1}{3} n \,\langle E \rangle = \frac{1}{3} \rho_s \tag{B.7}$$

Quantenfluktuationen im Vakuum führen zu einer **Vakuumenergie** ρ_v, die negativen Druck ausübt:

$$p = -\rho_v\, c^2 \tag{B.8}$$

Der negative Druck lässt sich dadurch erklären, dass die Energie proportional dem Volumen zunimmt, weil mit wachsendem Phasenraum mehr Schwingungsmoden möglich werden (entspricht dem Casimir-Effekt).

In Tabelle B.1 ist zusammengestellt, wie sich das Universum jeweils entwickelt, wenn ein bestimmter Zustand dominiert. Im allgemeinen ist die Energiedichte eine Summe aus den Beiträgen von Strahlung, Materie und Vakuumenergie. Die normierte Energiedichte ist dann:

$$\Omega_{tot} = \frac{\rho}{\rho_c} = \frac{\rho_s}{\rho_c} + \frac{\rho_m}{\rho_c} + \frac{\rho_v}{\rho_c} = \Omega_s + \Omega_m + \Omega_v \tag{B.9}$$