Light Quarks with Twisted Mass Fermions

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ETMC Collaboration

- **Towards precise simulations of Maximally Twisted Mass Quarks**
  - Recap of quenched situation
  - Breakthrough in Simulation Algorithm
  - Understanding the Phase Structure of Lattice QCD
  - Setup of Simulations

    $\longrightarrow$ talk by Enno Scholz and Carsten Urbach

ETMC Collaboration:

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Wilson (Frezzotti, Rossi) twisted mass QCD (Frezzotti, Grassi, Sint, Weisz)

$$D_{\mathrm{tm}} = m_q + i\mu\tau_3\gamma_5 + \tfrac{1}{2}\gamma_\mu\left[\nabla_\mu + \nabla^*_\mu\right] - a\tfrac{1}{2}\nabla^*_\mu\nabla_\mu$$

quark mass parameter m_q , twisted mass parameter μ

- $m_q = m_{\mathrm{crit}} \rightarrow$ O(a) improvement for *hadron masses, matrix elements, form factors, decay constants* without need of tuning additional improvement coefficients

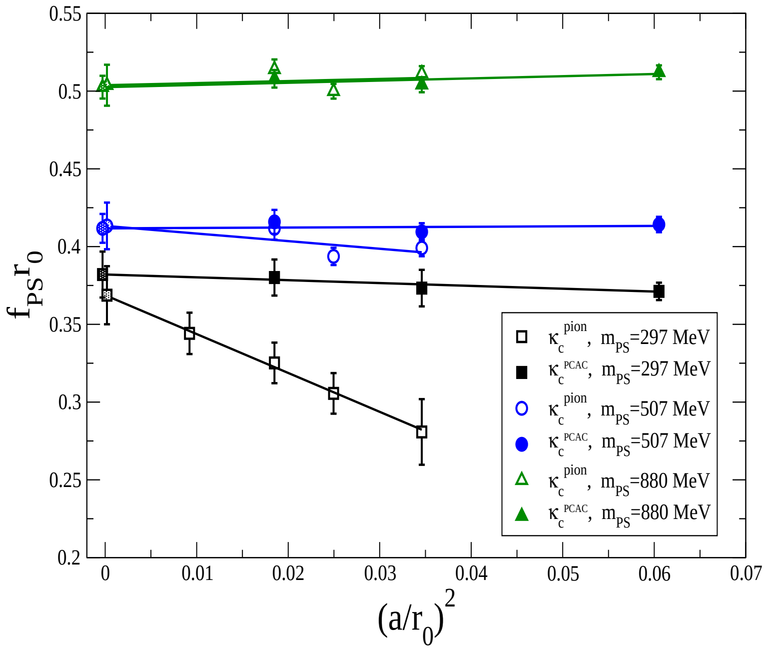
- $\det[D_{\mathrm{tm}}] = \det[D^2_{\mathrm{Wilson}} + \mu^2]$
  $\Rightarrow$ protection against small eigenvalues

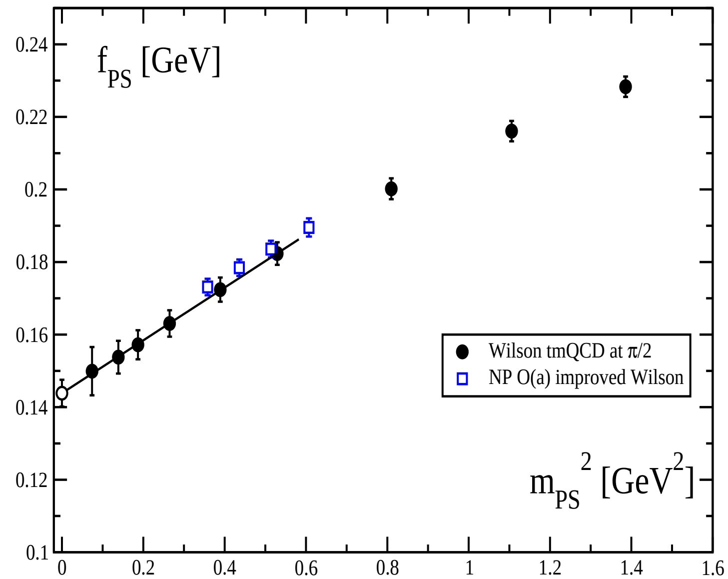
- computational cost comparable to staggered

- simplifies mixing problems for renormalization

$\star$ **strong claims $\Rightarrow$ needs a check**

Quenched Experiment: Scaling of and F_{PS}





$\longrightarrow$ O(a^2) scaling for two realizations of O(a)-improvement

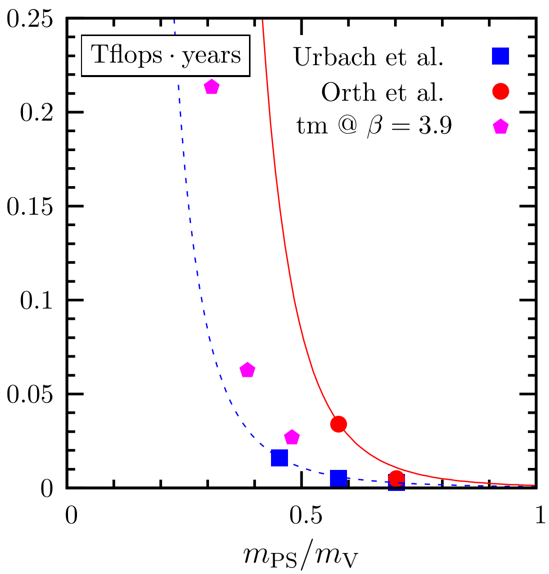
$\longrightarrow$ κ_c^{PCAC} very small O(a^2) effects $\Rightarrow$ THE choice to tune to full twist

$\longrightarrow$ bridging the gap towards the physical point

Shift the Berlin Wall and Twist

New variant of HMC algorithm (Urbach, Shindler, Wenger, K.J.)
see also SAP (Lüscher) and RHMC (Clark and Kennedy) algorithms)

- even/odd preconditioning
- (twisted) mass-shift (Hasenbusch trick)
- multiple time steps



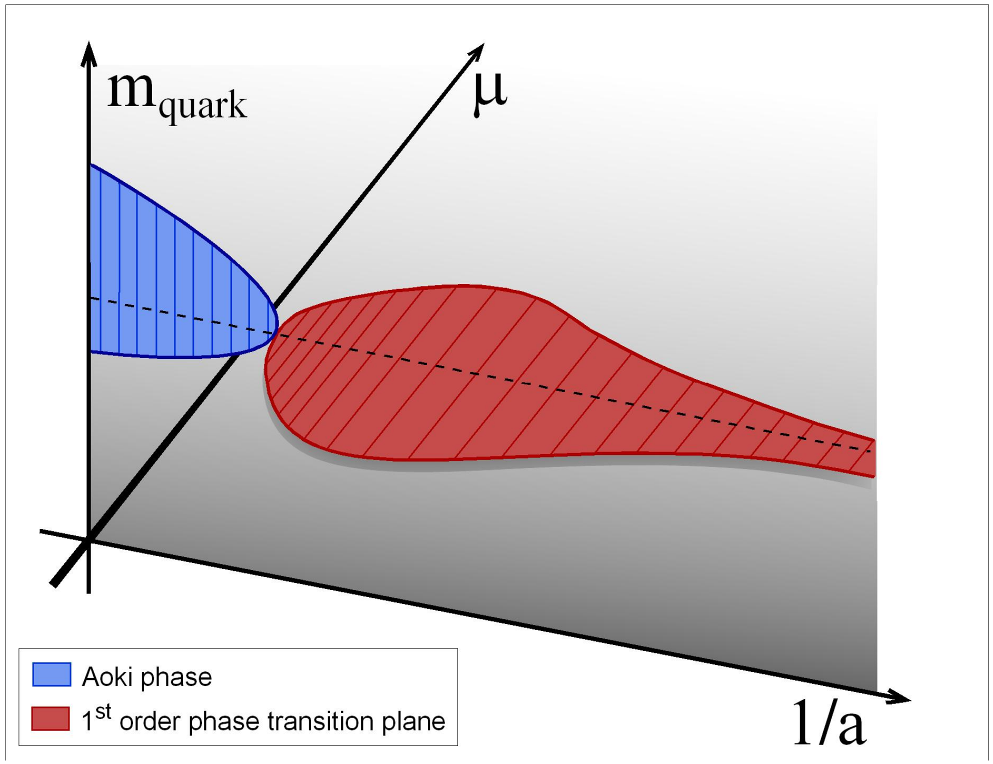
– twisted mass at much smaller $m_{\mathrm{PS}}/m_{\mathrm{V}}$

– compatible with (our own) Wilson
– compatible with staggered

– compatible with RHMC

$\Rightarrow$ 3 algorithms to drive Wilson fermions towards the physical point

Revealing the generic phase structure of lattice QCD

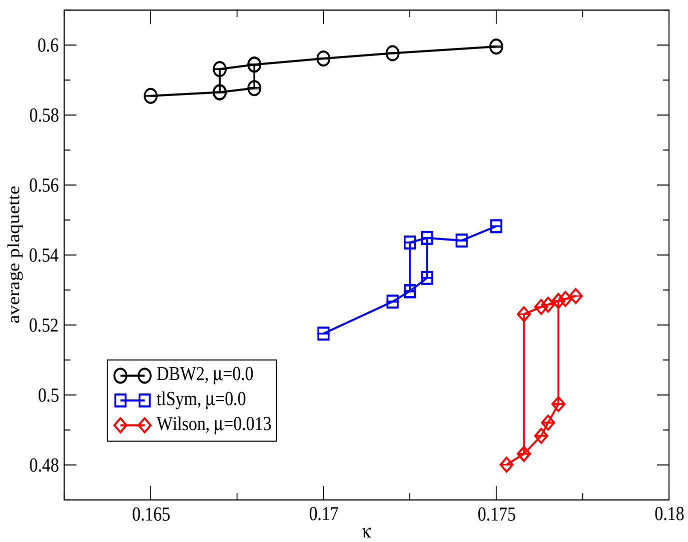


$\longrightarrow$ Knowledge of phase structure for a particular formulation of lattice QCD: pre-requisite for numerical simulation

$\longrightarrow$ Additional criterion to eigenvalue distribution

Choice of Gauge Action

Dependence on strength of first order phase transition on gauge action



$\Rightarrow$ Tree-level Symanzik improved gauge action avoids/compromise

- strong first order phase transition of Wilson plaquette gauge action
- doubts on DBW2 gauge action

UpCharmStrangeDown

$$D_{\mathrm{tm}}^{\mathrm{s,c}} = m_q + \mu_\delta \tau^3 + i\mu_\sigma \gamma_5 \tau^1 + \tfrac{1}{2}\gamma_\mu \left[\nabla_\mu + \nabla_\mu^*\right] - a\tfrac{1}{2}\nabla_\mu^* \nabla_\mu$$

at full twist, $m_q = m_{\mathrm{crit}}$ quark masses are $m_{s,c} = (\mu_\sigma \pm \mu_\delta)$

the fully renomalized currents $\hat{V}$ are related to

$$\mathcal{V} = \begin{pmatrix} Z_P P_{K^+} \\ Z_P P_{D^0} \\ Z_S S_{K^+} \\ Z_S S_{D^0} \end{pmatrix} \text{ by } \hat{V} = \mathcal{M}\mathcal{V} \text{ with } \mathcal{M}(\omega_l, \omega_h)$$

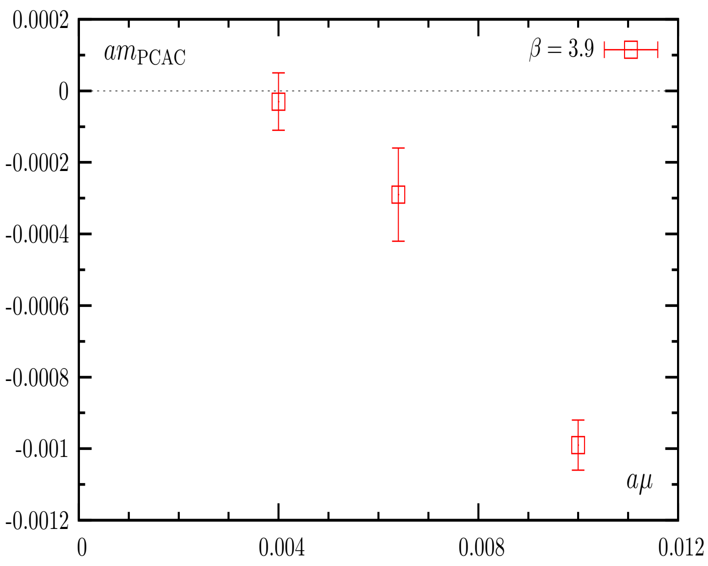
$$\begin{pmatrix} \cos\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & -\sin\frac{\omega_l}{2}\sin\frac{\omega_h}{2} & i\sin\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & i\sin\frac{\omega_h}{2}\cos\frac{\omega_l}{2} \\ -\sin\frac{\omega_l}{2}\sin\frac{\omega_h}{2} & \cos\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & i\sin\frac{\omega_h}{2}\cos\frac{\omega_l}{2} & i\sin\frac{\omega_l}{2}\cos\frac{\omega_h}{2} \\ i\sin\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & i\sin\frac{\omega_h}{2}\cos\frac{\omega_l}{2} & \cos\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & -\sin\frac{\omega_l}{2}\sin\frac{\omega_h}{2} \\ i\sin\frac{\omega_h}{2}\cos\frac{\omega_l}{2} & i\sin\frac{\omega_l}{2}\cos\frac{\omega_h}{2} & -\sin\frac{\omega_l}{2}\sin\frac{\omega_h}{2} & \cos\frac{\omega_l}{2}\cos\frac{\omega_h}{2} \end{pmatrix}$$

$\Rightarrow$ looks complicated but $\omega_{l,h}$ can be tuned to maximal twist by
only tuning the light bare quark mass

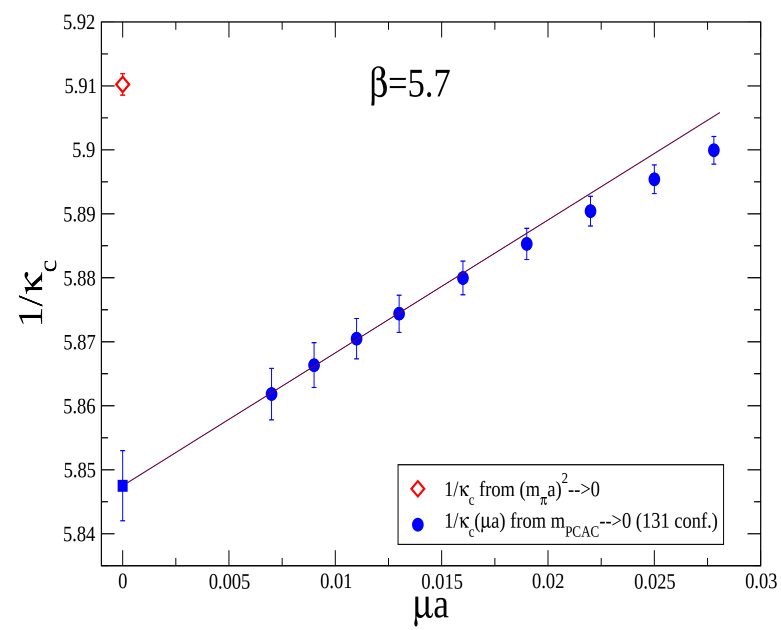
$$0 = am_{\chi l}^{PCAC} \equiv \frac{\langle \partial_\mu^* A_{l,x\mu}^+ P_{l,y}^- \rangle}{2\langle P_{l,x}^+ P_{l,y}^- \rangle}$$ $\Rightarrow$ only tune one additional parameter

Tuning in the case of $N_f = 2$ Dynamical Quarks

- tlSym gauge action plus Wilson twisted mass
- use PCAC quark mass $m_{\rm PCAC} = 0$ to tune to full twist
- $\kappa_{\rm crit}$ determined and fixed at smallest value of twisted mass parameter $\mu = 0.004$ used in the simulations



Dynamical



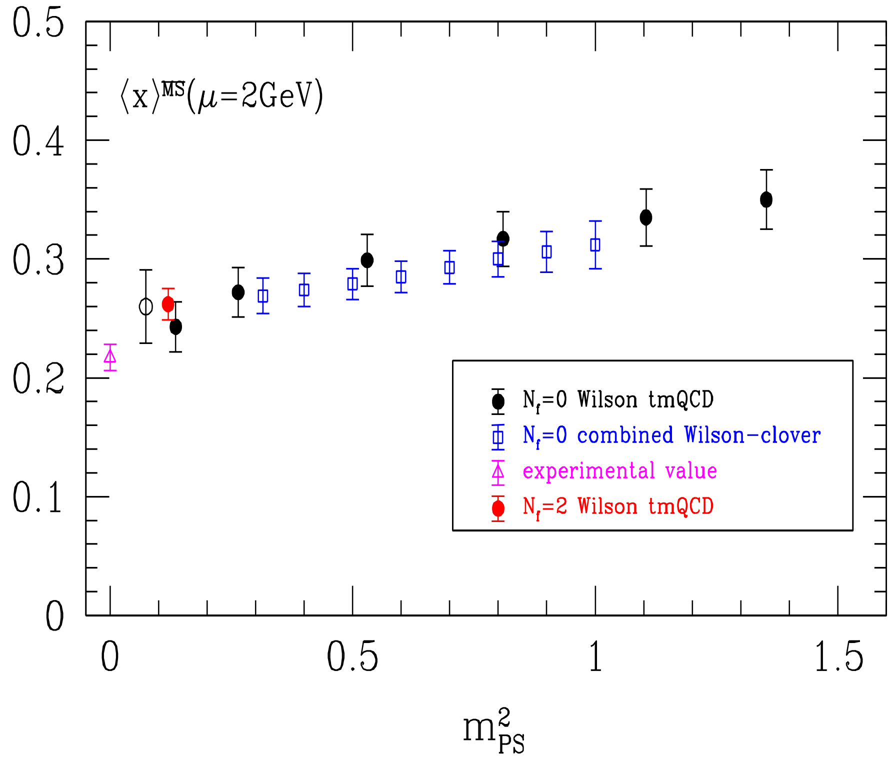
Quenched

Setup for $N_f = 2$ maximally twisted Dynamical Quarks

- $\beta = 3.9$, 5000 thermalized trajectories
- first step: set scale by $r_0 = 0.5$fm
- ongoing runs at a smaller and a larger lattice spacing at matched pion masses and volumes
- test scaling and perform continuum limit

$L^3 \cdot T$	β	κ_{crit}	$a\mu$	a[fm]	m_π[MeV]
$24^3 \cdot 48$(*)	3.90	0.160856	0.0040	≈0.095	280
$24^3 \cdot 48$(*)	3.90	0.160856	0.0064	≈0.095	350
$24^3 \cdot 48$(*)	3.90	0.160856	0.0100	≈0.095	430
$24^3 \cdot 48$(*)	3.90	0.160856	0.0150	≈0.095	510

Example: Lowest Moment of Non-singlet, Pion Parton Distribution Function $\langle x \rangle$

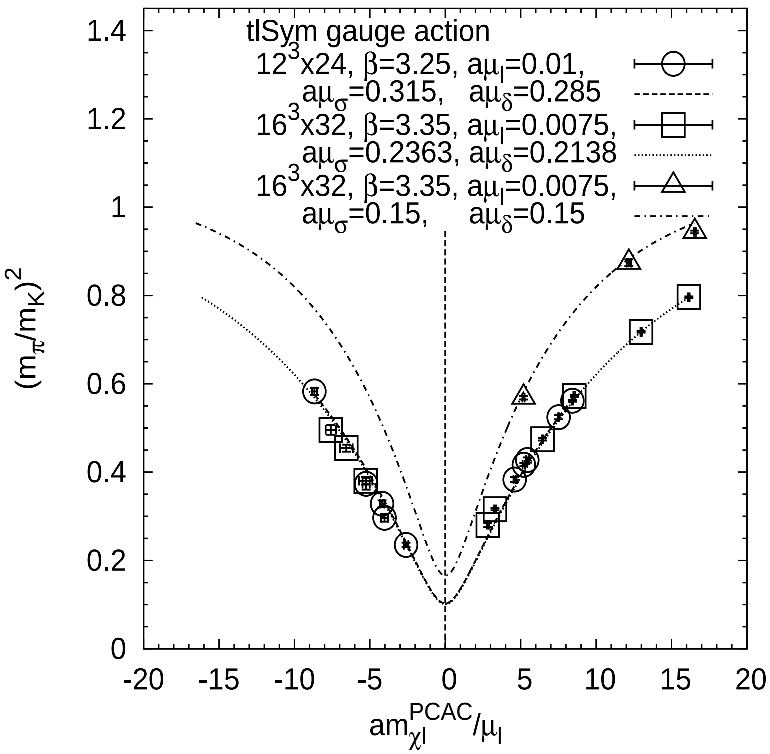


$\longrightarrow$ simulation at small pseudoscalar masses feasible

$\longrightarrow$ dynamical point consistent with quenched (?)

Results for $N_f = 2 + 1 + 1$ Dynamical Quarks

(PHMC algorithms: talk by E. Scholz)



- simulations are perfectly feasible, *no demanding tuning*
- knowledge of phase structure
  $\Rightarrow$ knowledge of parameters for production simulations

Conclusion

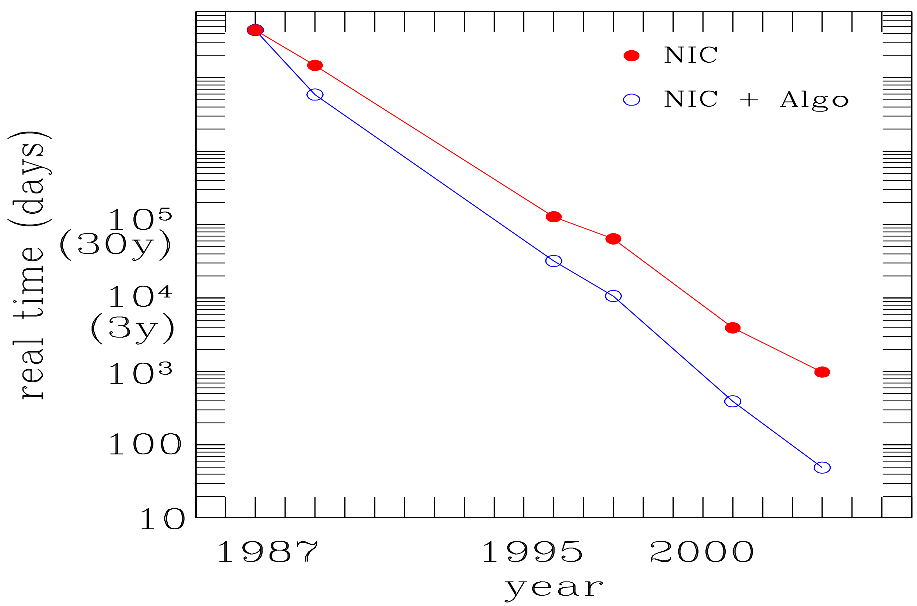
- Fully Wilson Twisted Mass Fermions
  - can be realized in practical simulations
  - provide one way to *solve* QCD

- Cost is comparable to staggered
  $\longrightarrow$ Our algorithm performs as well as Domain decomp. and RHMC

- Reach $M_{\mathrm{PS}} \approx 250\mathrm{MeV}$ while simulations are stable

- Can include s and d quarks

- First physics results
  $\longrightarrow$ following talk by Carsten Urbach



time for dynamical (red) point for $\langle x \rangle$