# QCD Precision Tests in Deeply Inelastic Scattering

Johannes Blümlein
DESY

- Introduction and Method
- QCD Analysis of Unpolarized Structure Functions
- $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$
- What would we like to know ?

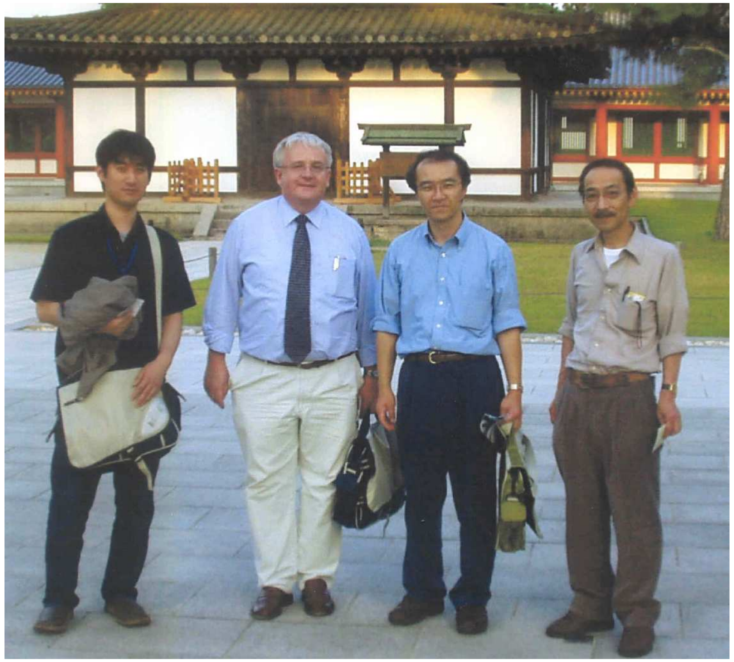


Dedicated to the memory of Jiro Kodaira

# Meeting with Jiro

LL2000 Bastei: Germany,

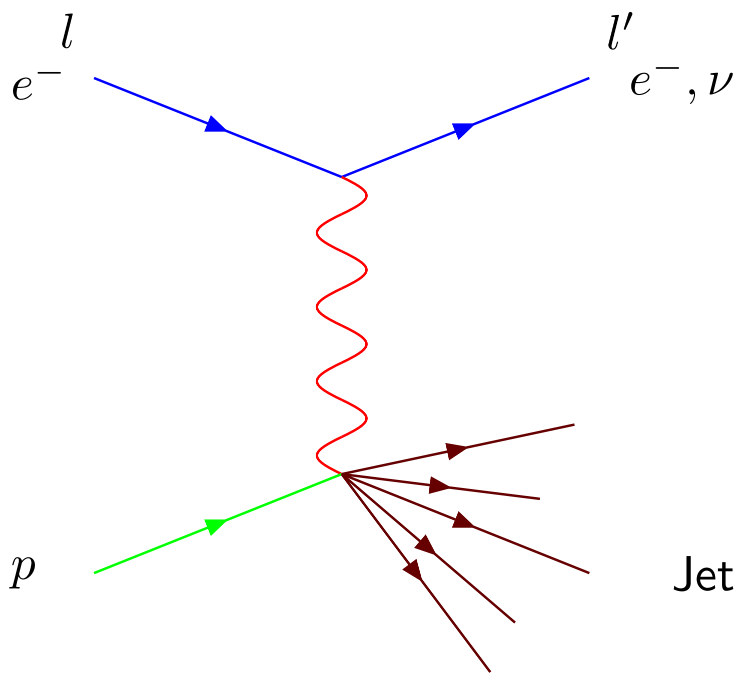
PANIC 02, Nara



He visited DESY Zeuthen first in 1997 and then very regularly.
We had a very good time with him, always interesting scientific discussions and good private conversations, which promoted many long-term ties between Japan & Germany.

We have lost a very good friend.

# Deeply Inelastic Scattering




$$\textbf{space} - \textbf{like process} \;:\quad q^2 = (l - l')^2 = -Q^2 < 0 \qquad W^2 = (p+q)^2 \geq M_p^2$$

$$x = \frac{Q^2}{2p.q}, \qquad\qquad y = \frac{p.q}{p.l} \qquad\qquad 0 \leq x, y \leq 1$$

# DIS: Microscopy of the Nucleon

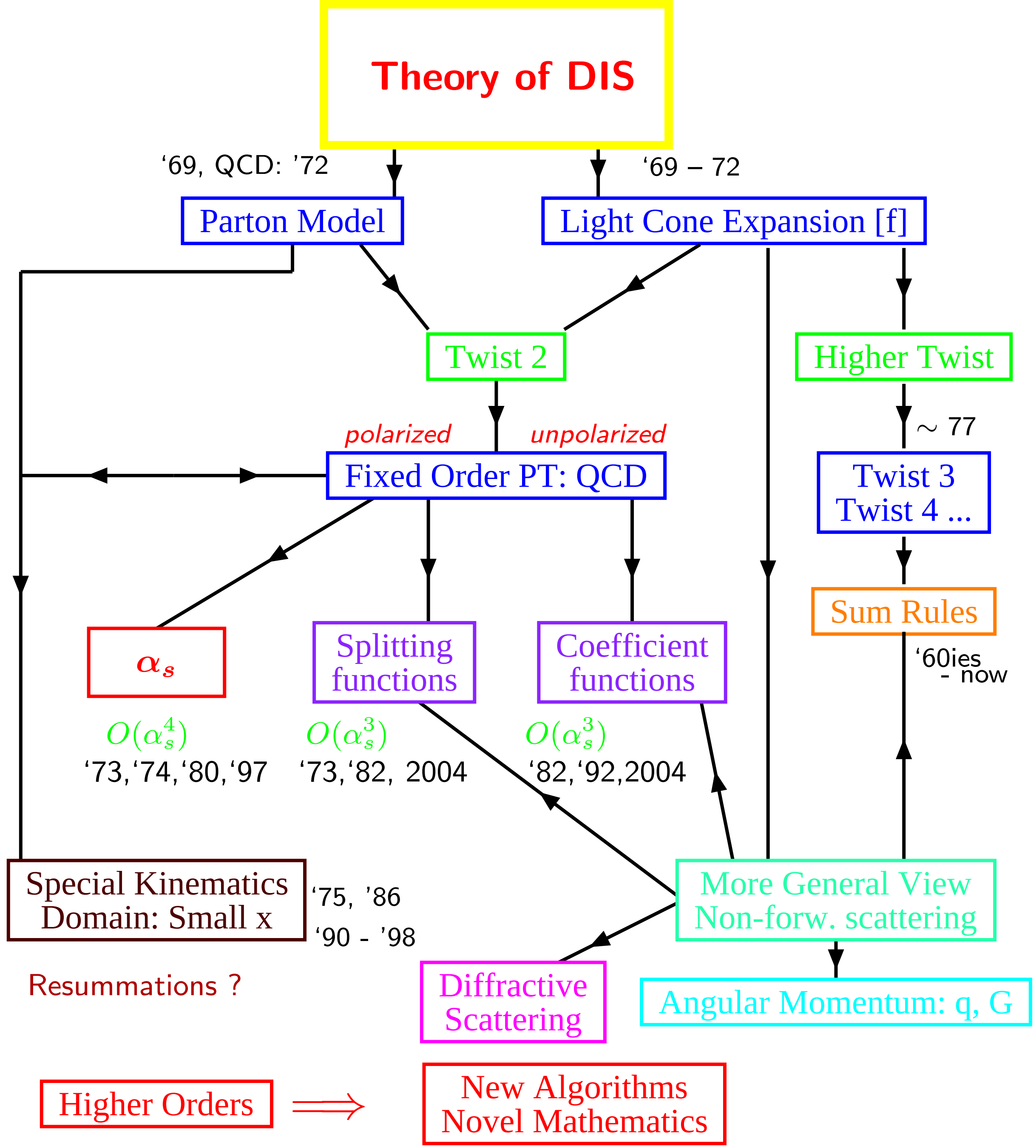
- determination of all quark densities and the gluon distribution
- determination of all polarized parton densities

# DIS: Fundamental Tests of QCD

- precision measurement of $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$
- Thorough verification of the prediction of the light cone expansion: to higher twist
- Test of linear and non-linear resummations

# Challenges for Theory:
perturbative and non-perturbative

- higher order precision calculations and data analysis
- Lattice gauge theory results for $\Lambda_{\rm QCD}$ and hadronic ME


Theory of DIS
'69, QCD: '72
'69 – 72
Parton Model
Light Cone Expansion [f]
Twist 2
Higher Twist
polarized
unpolarized
~ 77
Fixed Order PT: QCD
Twist 3
Twist 4 ...
Sum Rules
$\alpha_s$
Splitting functions
Coefficient functions
'60ies - now
$O(\alpha_s^4)$
'73,'74,'80,'97
$O(\alpha_s^3)$
'73,'82, 2004
$O(\alpha_s^3)$
'82,'92,2004
Special Kinematics Domain: Small x
'75, '86
'90 - '98
More General View Non-forw. scattering
Resummations ?
Diffractive Scattering
Angular Momentum: q, G
Higher Orders ⟹
New Algorithms Novel Mathematics

1

# Highest order corrections of HO QCD in DIS

- Running $\alpha_s$: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$ Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; $\Delta P_{NS}^{qq}, \Delta P_{qG}$: $O(\alpha_s^3)$ Mertig, van Neerven, 1995; Vogelsang 1995; Moch, Vermaseren, Vogt
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994 $O(\alpha_s^3)$ to come
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike; Kumano, Miyama; Vogelsang; 1997; Gracey 2006
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993 Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$, Watson 1982
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_L$: $O(\alpha_s^3)$ Blümlein, De Freitas, van Neerven, S. Klein 2005

# DIS Structure Functions @ Twist 2

$$F_j(x,Q^2) = \hat{f}_i(x,\mu^2) \otimes \sigma_j^i\left(\alpha_s, \frac{Q^2}{\mu^2}, x\right)$$

$$\uparrow \text{bare pdf} \qquad \uparrow \text{sub}-\text{system cross}-\text{sect.}$$

$$= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i\left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\text{finite pdf}\equiv f_k}$$

$$\otimes \underbrace{C_j^k\left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\text{finite Wilson coefficient}}$$

**Move to Mellin space :**

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

# Evolution Equations

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-2\gamma_\psi(g)\right]F_i(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\gamma_\kappa^N(g)-2\gamma_\psi(g)\right]f_k(N)=0$$

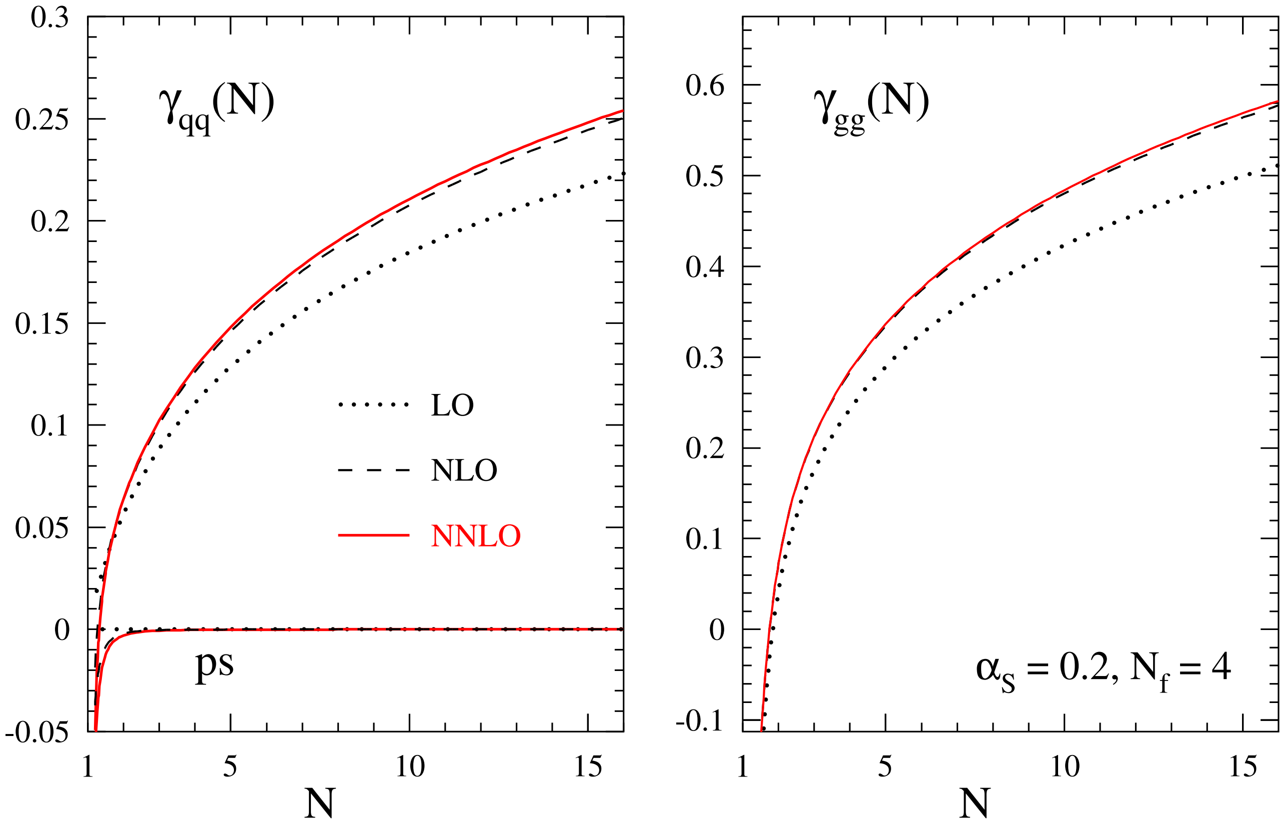
$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-\gamma_\kappa^N(g)\right]C_j^k(N)=0$$

CALLAN–SYMANNZIK equations for mass factorization
$\equiv$ ALTARELLI–PARISI evolution equations
**x-space :**

$$\frac{d}{d\log(\mu^2)}\begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s)\otimes\begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2) \end{pmatrix}$$

$$\boldsymbol{P}(x,\alpha_s)=\boldsymbol{P}^{(0)}(x)+\frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x)+\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{P}^{(2)}(x)+\ldots$$

# Anomalous Dimensions and Wilson Coefficients




Vermaseren, Moch, Vogt 2004

# The Basic Functions of massless QCD to w=5:$\equiv$ 3 Loops

Representative : $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight w=3 :

$$F_1(N) = \mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N)$$

$$F_2(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N), \qquad F_3(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_2(x)}{1-x}\right)_+\right](N)$$

Yndurain et al., 1981: $F_2(N)$

Weight w=4 :

$$F_4(N) = \mathbf{M}\left[\frac{S_{1,2}(x)}{1+x}\right](N), \qquad F_5(N) := \mathbf{M}\left[\left(\frac{S_{1,2}(x)}{1-x}\right)_+\right](N)$$

$F_3(N) - F_5(N)$: J.B., S. Moch, 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_4(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_8(N) = \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N),$$

$$F_{9,10}(N) = \mathbf{M}\left[\left(\frac{S_{2,2}(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_{11}(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N),$$

$$F_{12,13}(N) := \mathbf{M}\left[\left(\frac{S_{2,2}(-x) - \mathrm{Li}_2^2(-x)/2}{1 \pm x}\right)_{(+)}\right](N)$$

$F_6(N) - F_{13}(N)$ : J.B., S. Moch, 2004.

**Massless QCD to 3 Loops depends on 14 Functions.**

$\Rightarrow$ Representation for 3 Loop Wilson Coefficients under way.

# Complex Analysis of these Functions

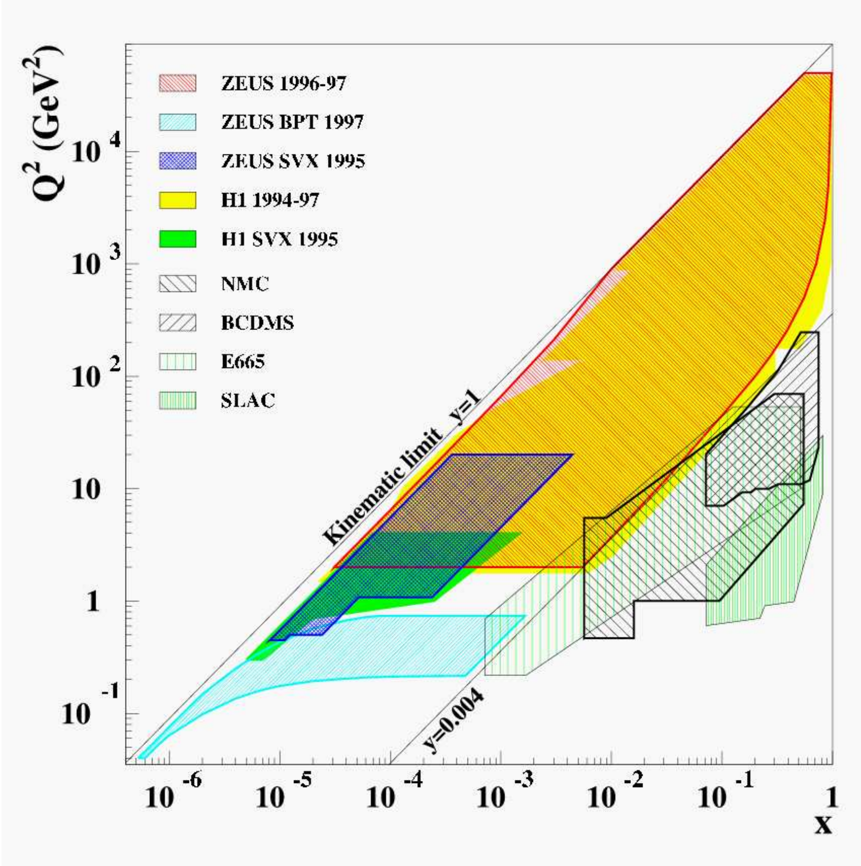
- Construct exact analytic continuations to complex $N$
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \to z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all $Q^2$
- Include the heavy flavor Wilson coefficients in Mellin Space
- Perform a single fast, numerical Mellin inversion
(at high precision)

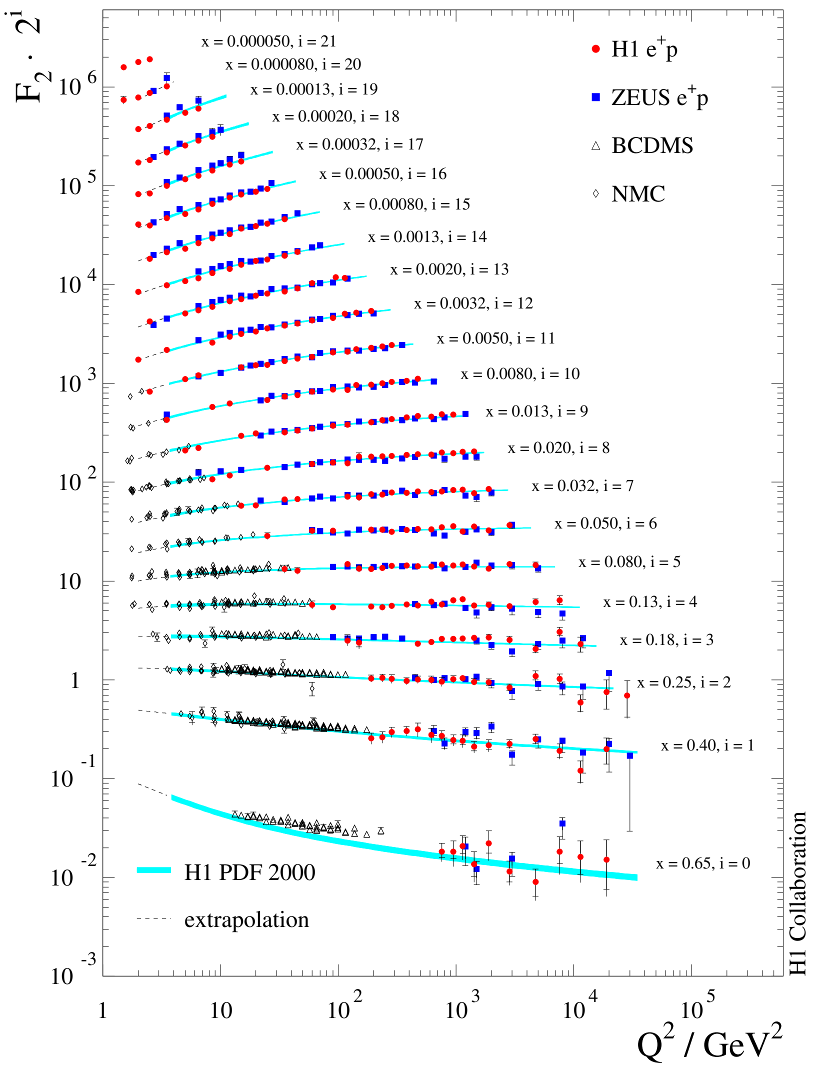
$\Longrightarrow$ **Fastest and most Precise Way of Analysis**

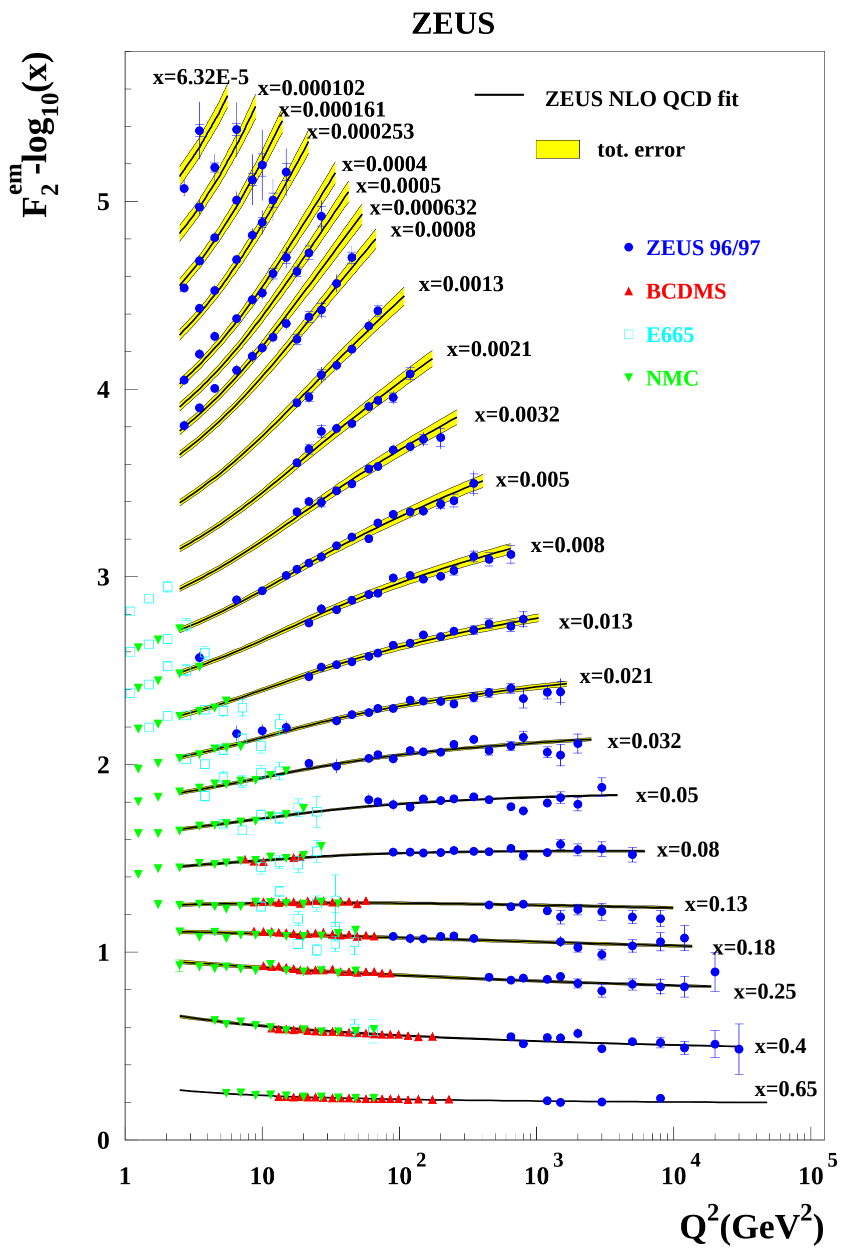
# 2. QCD Analysis of Unpolarized Parton Distributions

DIS range
Nucleon structure:

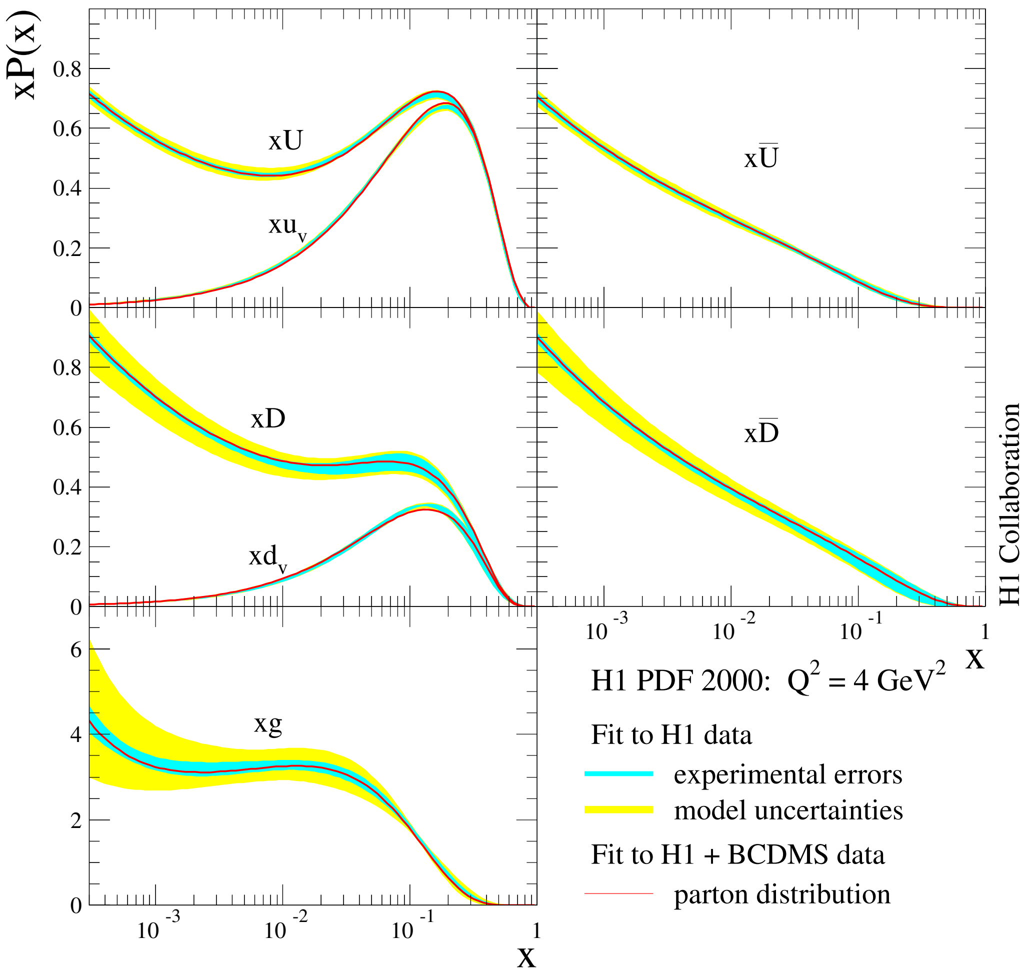
$10^{-5} < x < 0.9,$
$1 < Q^2 < 50.000 \mathrm{GeV}^2$




H1 e+p
ZEUS e+p
BCDMS
NMC
H1 PDF 2000
extrapolation
H1 Collaboration
x = 0.000050, i = 21
x = 0.000080, i = 20
x = 0.00013, i = 19
x = 0.00020, i = 18
x = 0.00032, i = 17
x = 0.00050, i = 16
x = 0.00080, i = 15
x = 0.0013, i = 14
x = 0.0020, i = 13
x = 0.0032, i = 12
x = 0.0050, i = 11
x = 0.0080, i = 10
x = 0.013, i = 9
x = 0.020, i = 8
x = 0.032, i = 7
x = 0.050, i = 6
x = 0.080, i = 5
x = 0.13, i = 4
x = 0.18, i = 3
x = 0.25, i = 2
x = 0.40, i = 1
x = 0.65, i = 0
F2 · 2^i
Q^2 / GeV^2

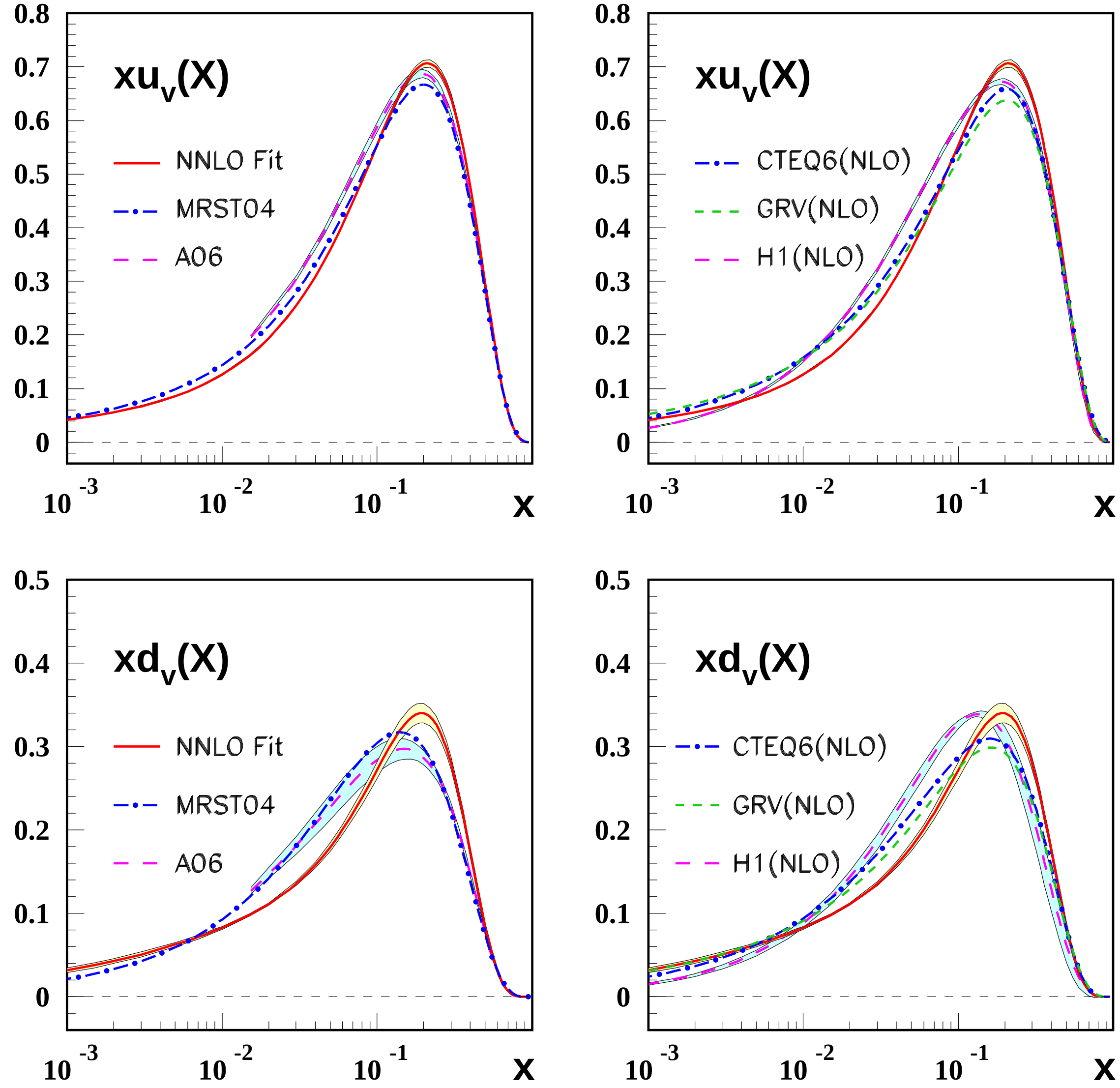

ZEUS
ZEUS NLO QCD fit
tot. error
ZEUS 96/97
BCDMS
E665
NMC
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
F2^em-log10(x)
Q^2(GeV^2)

# Parton Distributions: Overview




H1

# World Data Analysis: Valence Distributions




World data:
NS-analysis

$W^2 > 12.5$ GeV$^2$, $Q^2 > 4$ GeV$^2$

N$^3$LO :

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

# Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ $\pm 2$ MeV in $\Lambda_{QCD}$

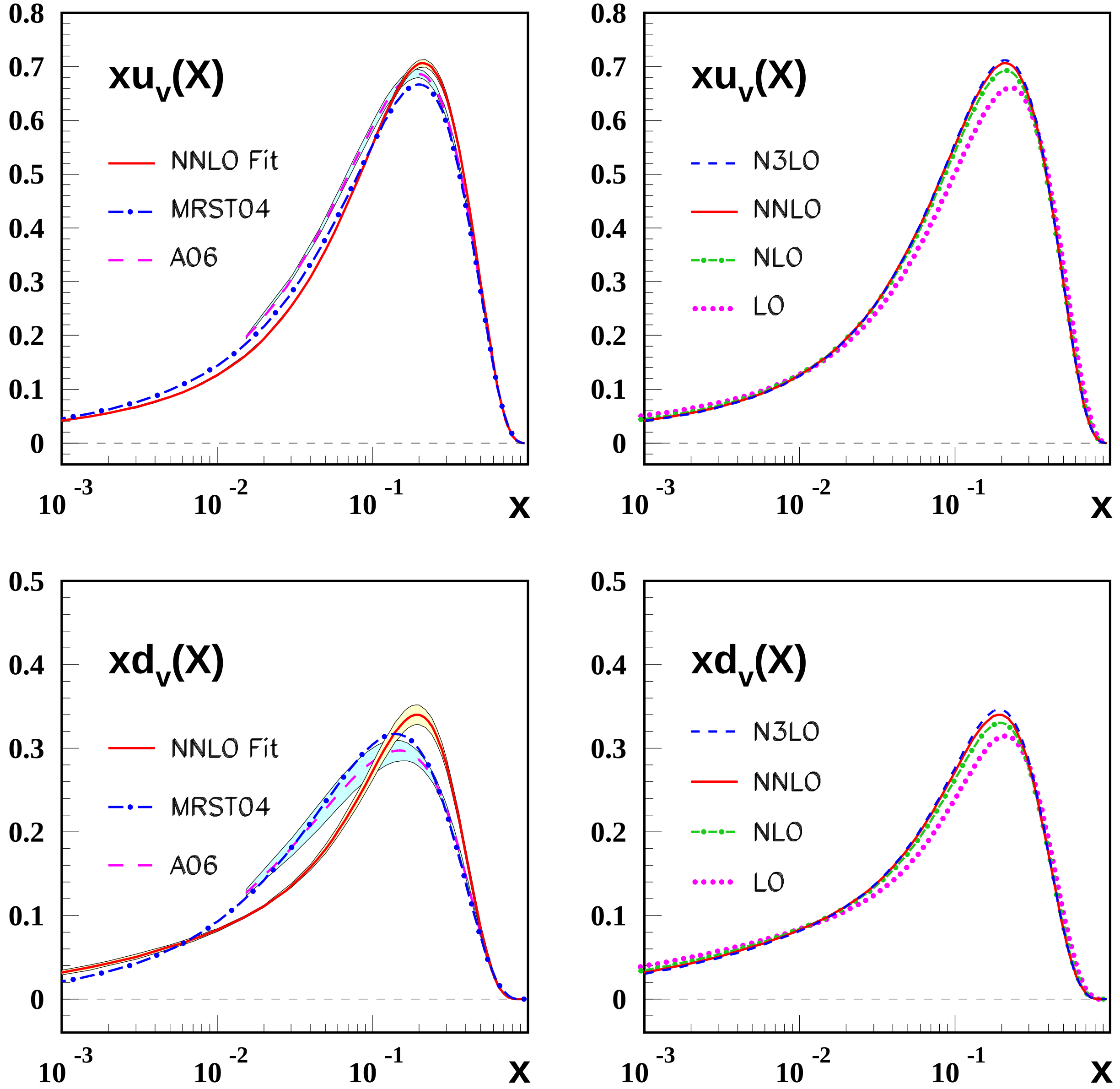
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

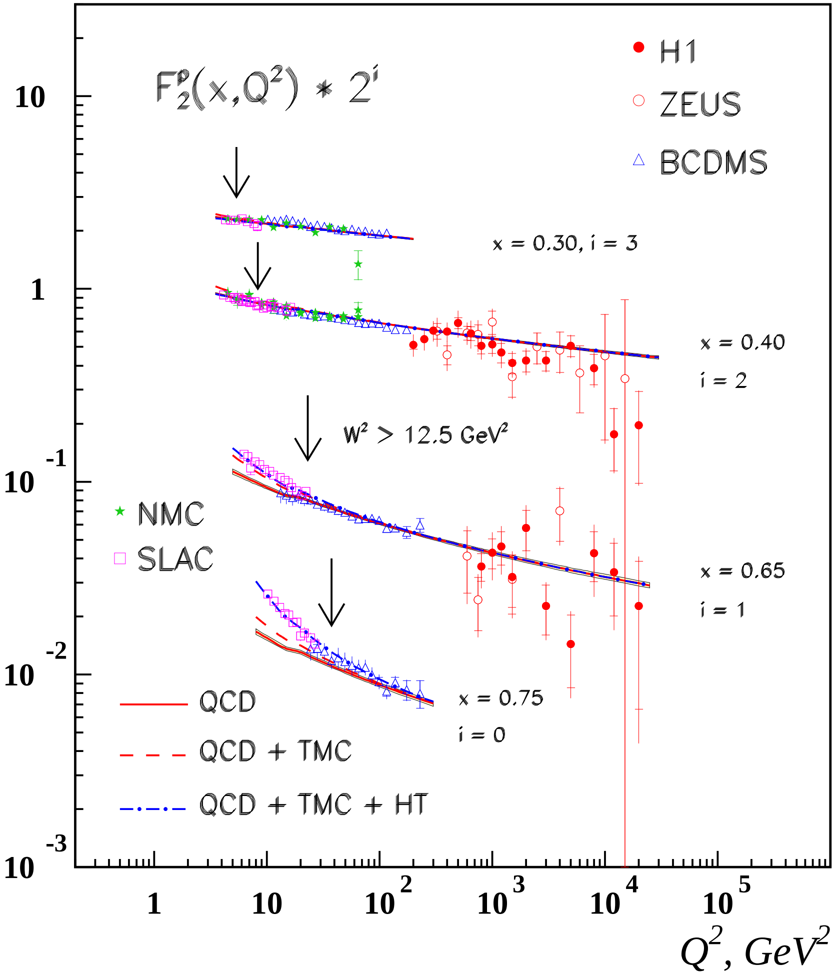
$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4$$

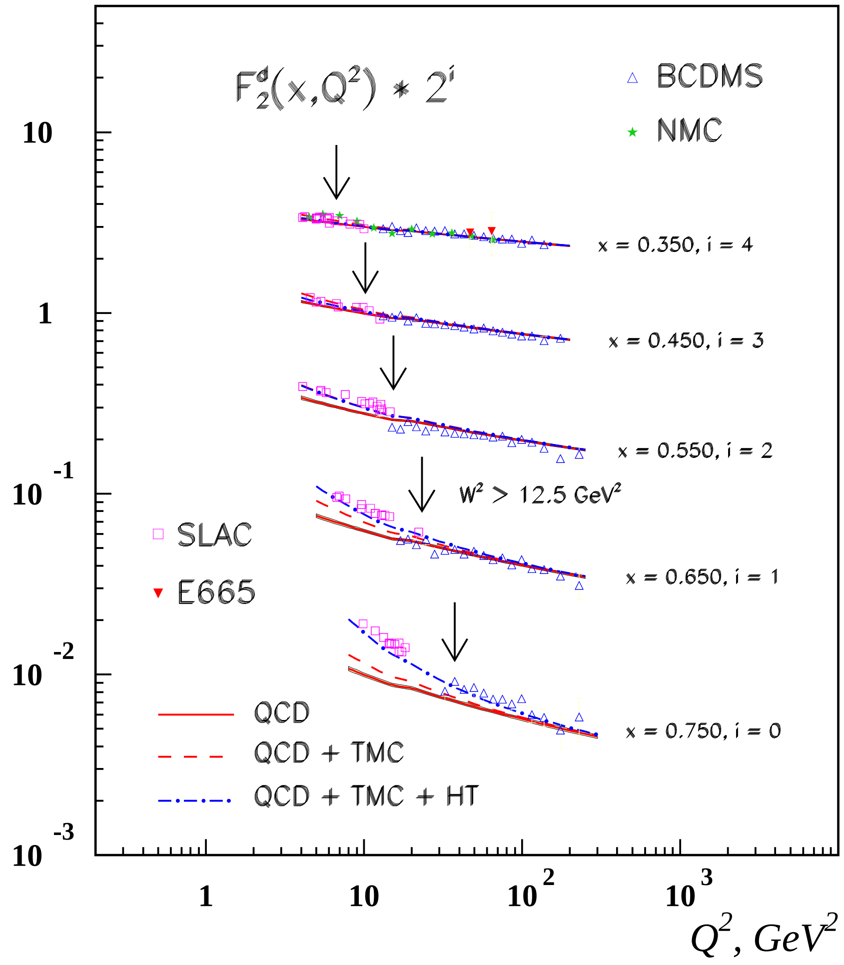
The results agree better than 20%.

# Valence Distributions

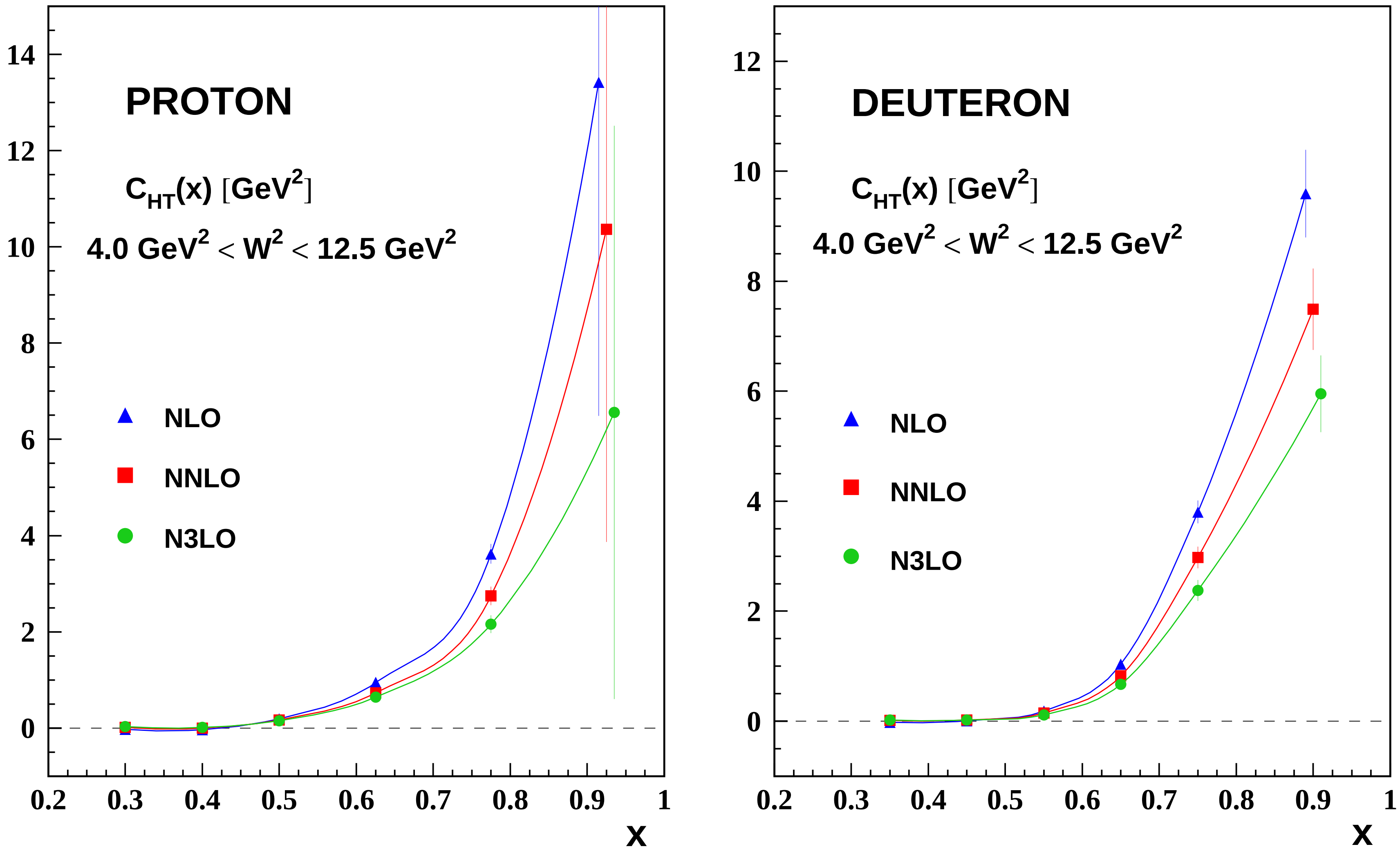


# Valence Distributions



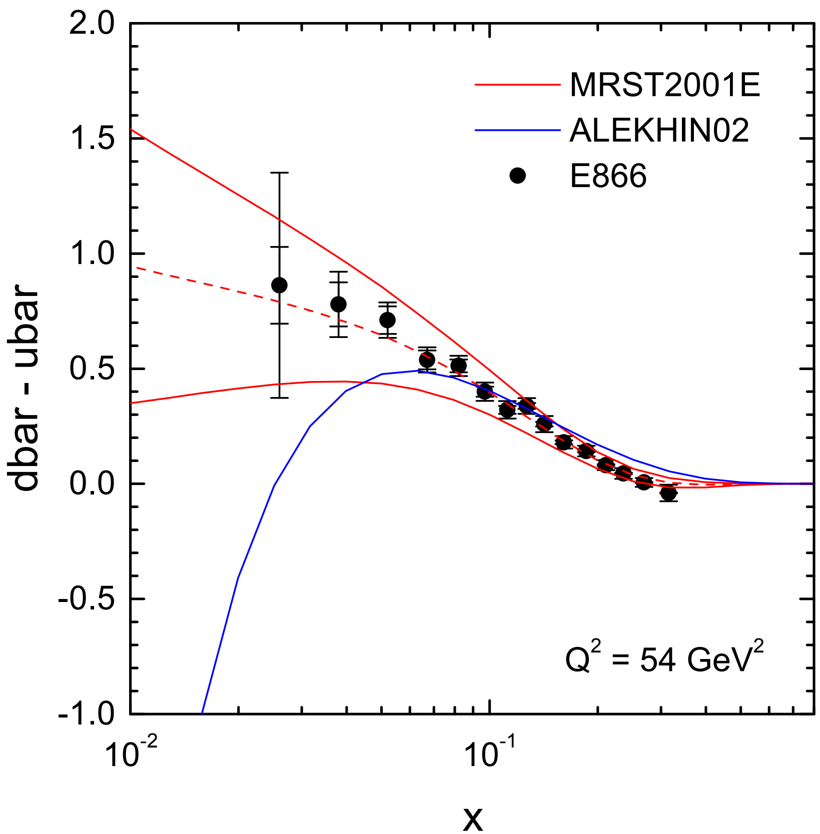



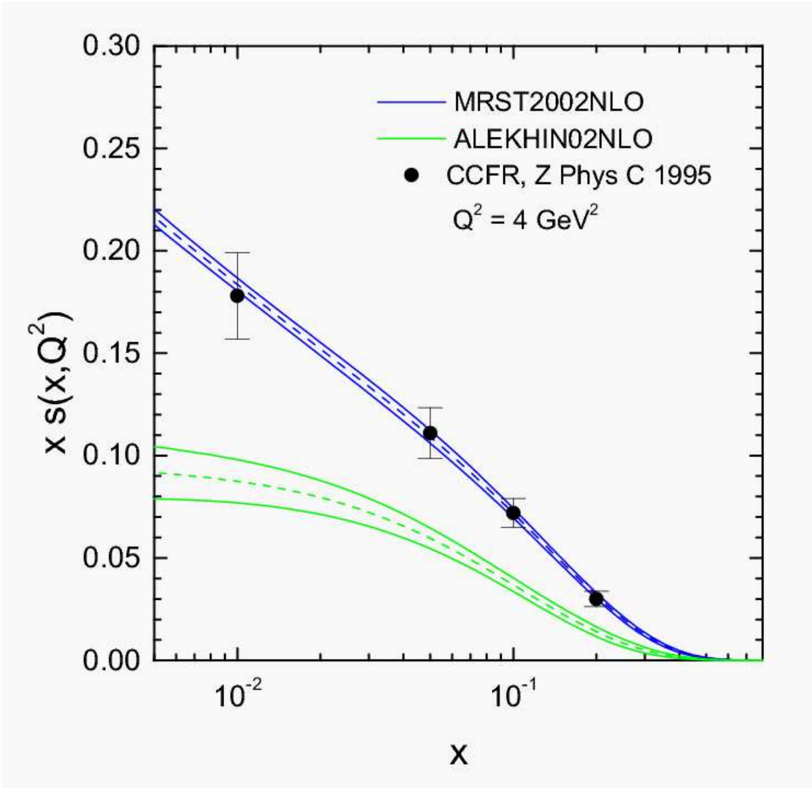
# Valence Distributions: higher twist




- agreement between $p$ and $d$ analysis
- LGT determination of interest

# Flavor distributions: light quarks



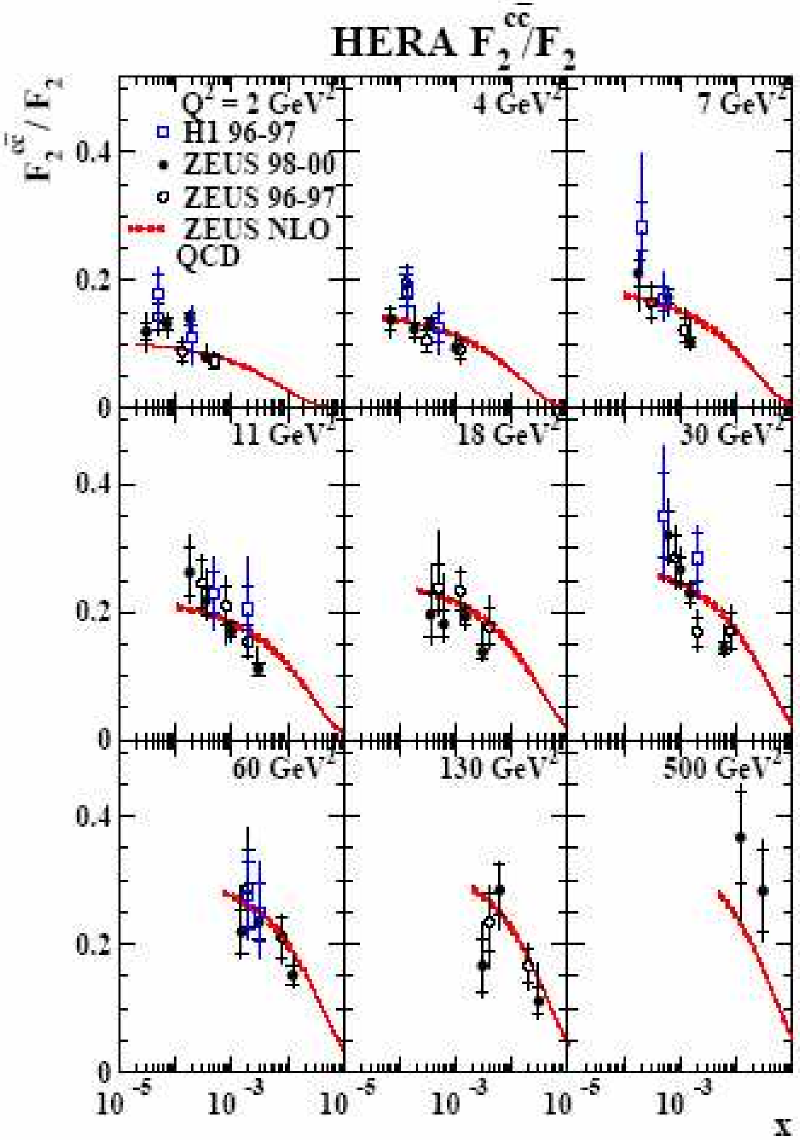




J. Stirling, 2004

More work needed.

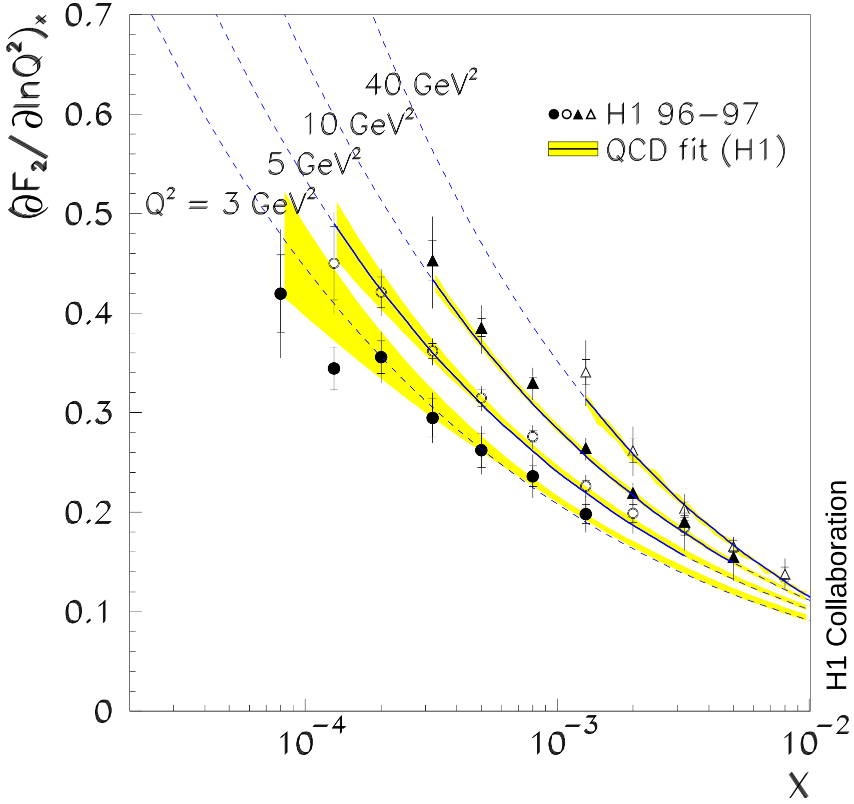
HERMES probably could measure $s(x, Q^2)$ in an independent way.

# Charm

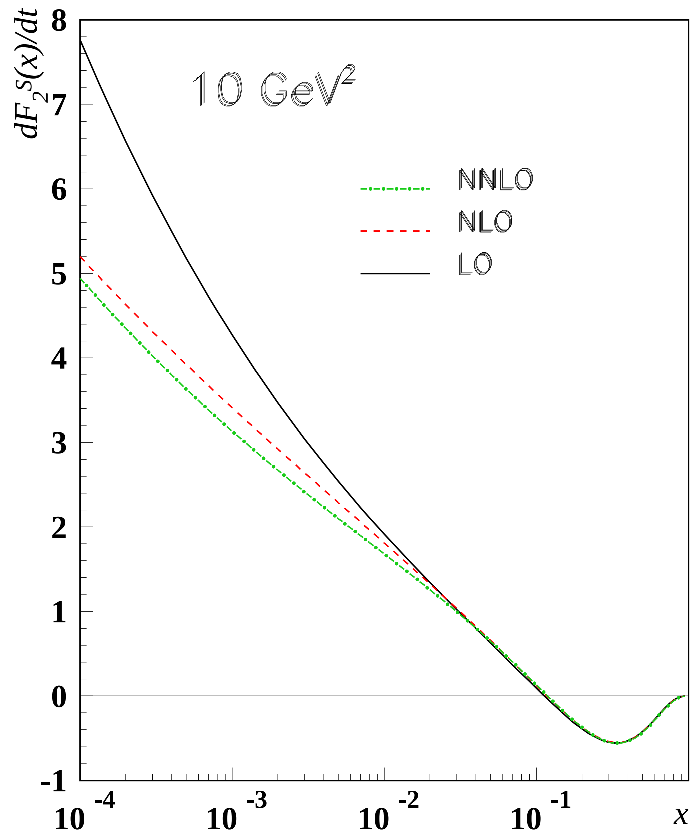



$F_2^{c\bar{c}}(x, Q^2)$ will be very well measured at HERA.

# Slope of $F_2$ at low $x$



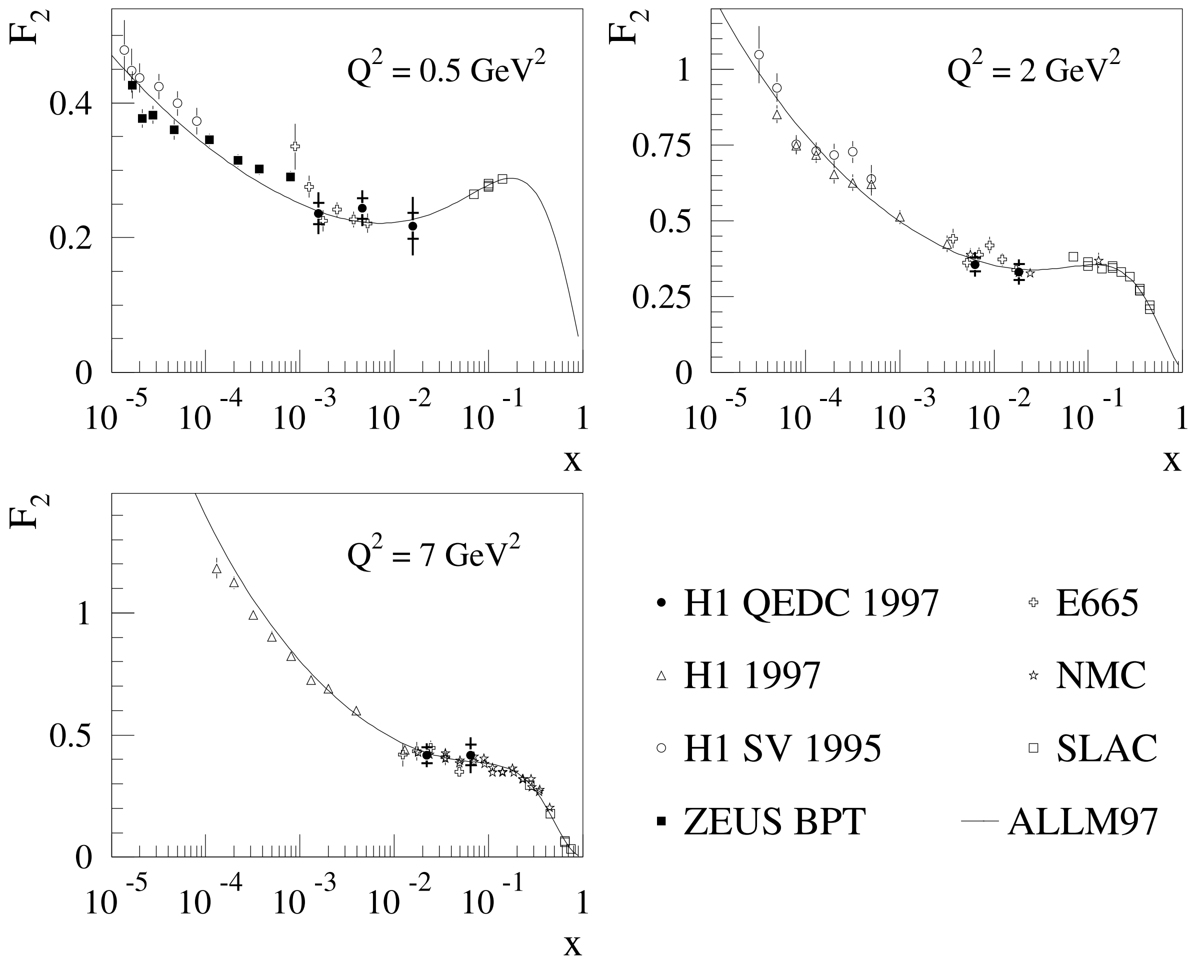

H1



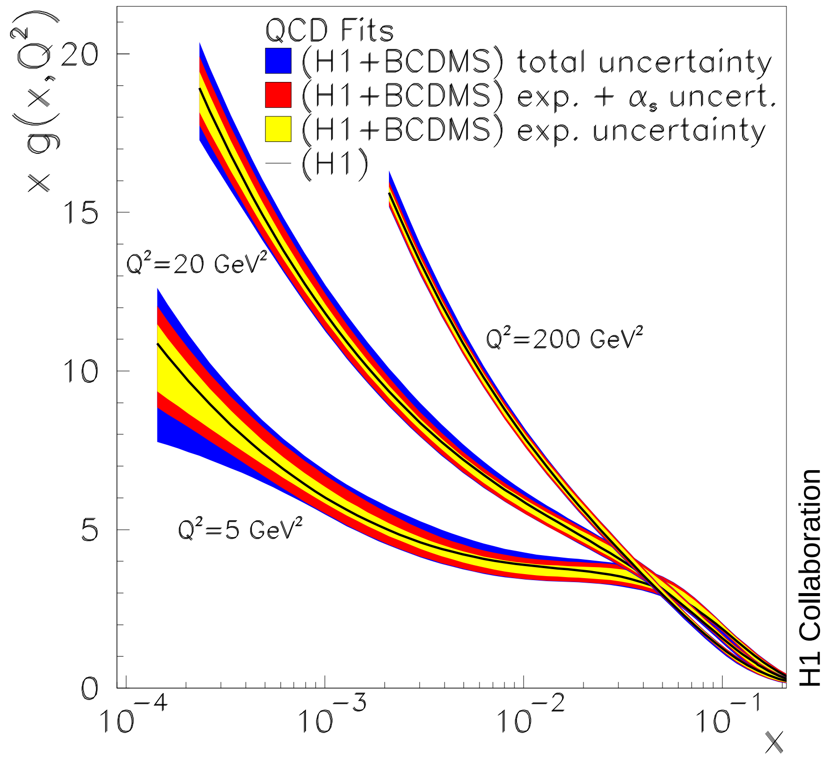

J.B., A. Guffanti 2005

Very likely, that the $\overline{\rm MS}$–gluon is remains positive!

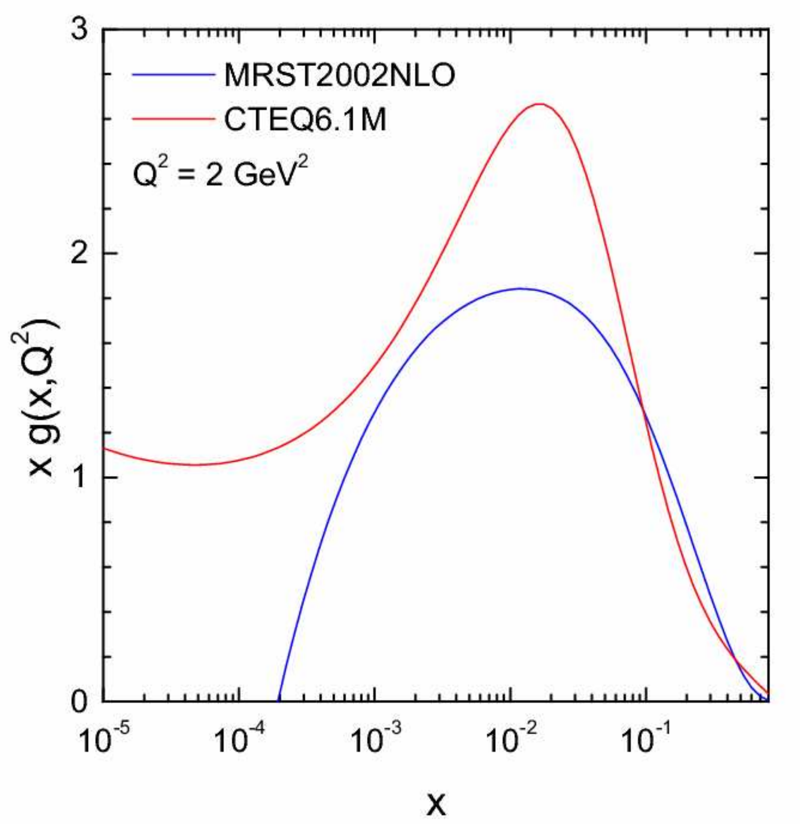
# Perturbative or non-perturbative growth?



# Gluon Density




H1

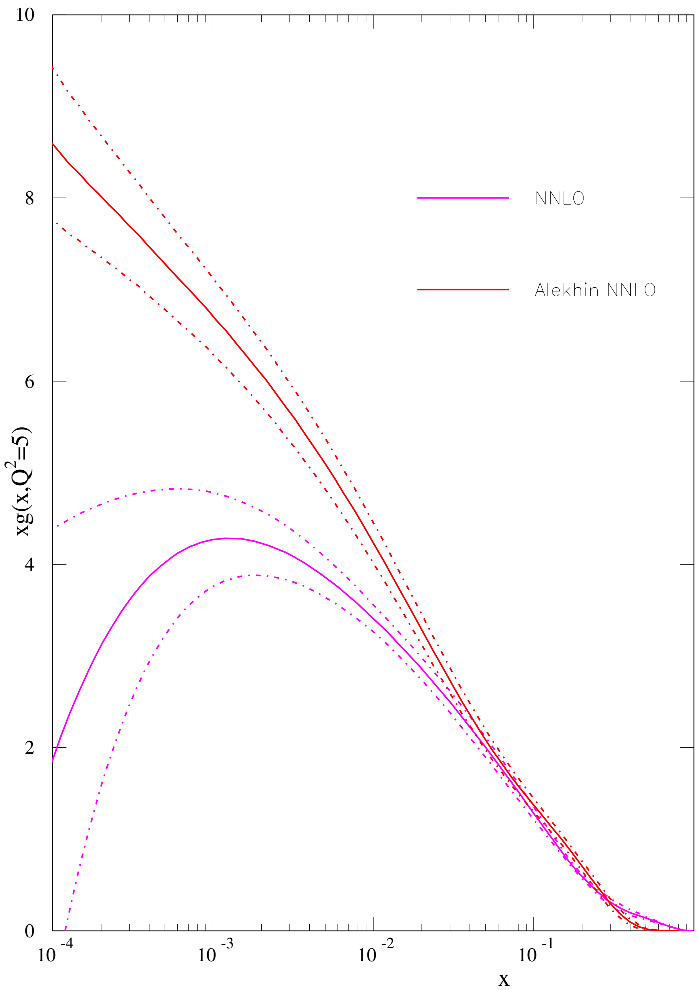



MRST 02 vs CTEQ 6

More work needed; MS– vs scheme-invariant evolution.

$F_L(x, Q^2)$ could be decisive.

# Gluon Density




MRST06 vs Alekhin: 2006

Not both distributions can be correct.

$F_L(x, Q^2)$ could be decisive.

More work needed ! BBG Analysis in progress.

# Moments of PDF's: PT + data

| $f$ | $n$ | This Fit N$^3$LO | MRST04 NNLO | A02 NNLO |
|---|---|---|---|---|
| $u_v$ | 2 | $0.3006 \pm 0.0031$ | 0.285 | 0.304 |
| | 3 | $0.0877 \pm 0.0012$ | $0.082$ | $0.087$ |
| | 4 | $0.0335 \pm 0.0006$ | 0.032 | 0.033 |
| $d_v$ | 2 | $0.1252 \pm 0.0027$ | 0.115 | 0.120 |
| | 3 | $0.0318 \pm 0.0009$ | $0.028$ | $0.028$ |
| | 4 | $0.0106 \pm 0.0004$ | 0.009 | 0.010 |
| $u_v - d_v$ | 2 | $0.1754 \pm 0.0041$ | 0.171 | 0.184 |
| | 3 | $0.0559 \pm 0.0015$ | $0.055$ | $0.059$ |
| | 4 | $0.0229 \pm 0.0007$ | 0.022 | 0.024 |

J.B., H. Böttcher, A. Guffanti, 2006

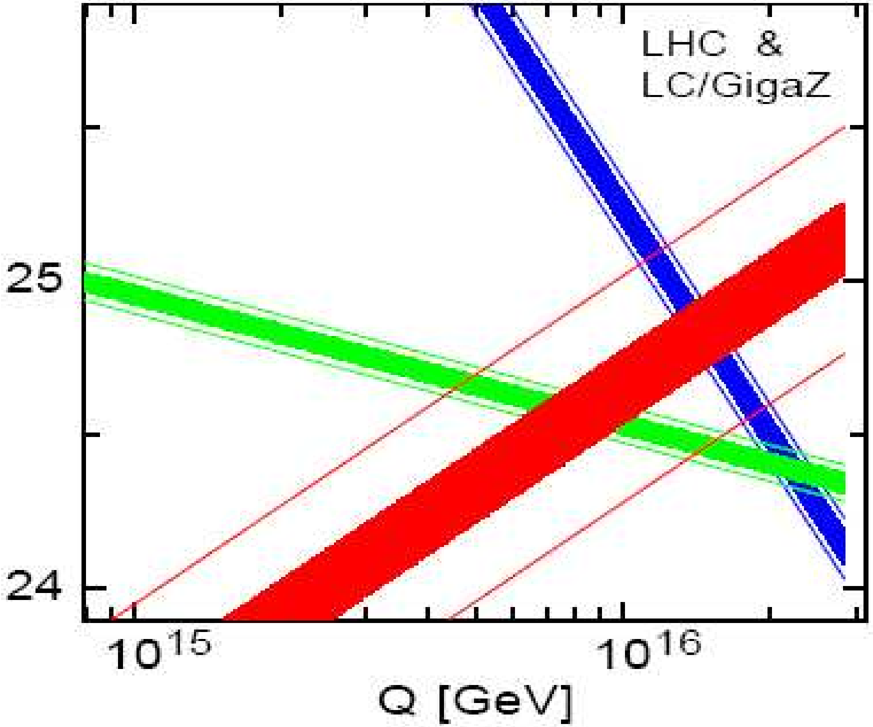
| | Moment | BB, NLO |
|---|---|---|
| $\Delta u_v$ | 0 | $0.926$ |
| | 1 | $0.163 \pm 0.014$ |
| | 2 | $0.055 \pm 0.006$ |
| $\Delta d_v$ | 0 | $-0.341$ |
| | 1 | $-0.047 \pm 0.021$ |
| | 2 | $-0.015 \pm 0.009$ |
| $\Delta u_v - \Delta d_v$ | 0 | $1.267$ |
| | 1 | $0.210 \pm 0.025$ |
| | 2 | $0.070 \pm 0.011$ |

J.B., H. Böttcher, 2002

Lattice Results : developing; different fermion-types studied. Low values of $m_\pi$ crucial; values approach 270 MeV now.

# 3. $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{\rm em}(0)}{\alpha_{\rm em}(0)} \sim 3\cdot 10^{-11} \qquad \frac{\delta\alpha_{\rm weak}}{\alpha_{\rm weak}} \sim 7\cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2\cdot 10^{-2}$$




P. Zerwas, 2004

# Overview of the Analyses

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses $\nu N$ world data
- NS-N$^3$LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

# $\alpha_s(M_Z^2)$

| NLO | $\alpha_s(M_Z^2)$ | expt | theory | Ref. |
|---|---|---|---|---|
| CTEQ6 | 0.1165 | $\pm 0.0065$ | | [1] |
| MRST03 | 0.1165 | $\pm 0.0020$ | $\pm 0.0030$ | [2] |
| A02 | 0.1171 | $\pm 0.0015$ | $\pm 0.0033$ | [3] |
| ZEUS | 0.1166 | $\pm 0.0049$ | | [4] |
| H1 | 0.1150 | $\pm 0.0017$ | $\pm 0.0050$ | [5] |
| BCDMS | 0.110 | $\pm 0.006$ | | [6] |
| GRS | 0.112 | | | [10] |
| BBG | 0.1148 | $\pm 0.0019$ | | [9] |
| BB (pol) | 0.113 | $\pm 0.004$ | $^{+0.009}_{-0.006}$ | [7] |

NLO

| NNLO | $\alpha_s(M_Z^2)$ | expt | theory | Ref. |
|---|---|---|---|---|
| MRST03 | 0.1153 | $\pm 0.0020$ | $\pm 0.0030$ | [2] |
| A02 | 0.1143 | $\pm 0.0014$ | $\pm 0.0009$ | [3] |
| SY01(ep) | 0.1166 | $\pm 0.0013$ | | [8] |
| SY01($\nu$N) | 0.1153 | $\pm 0.0063$ | | [8] |
| GRS | 0.111 | | | [10] |
| A06 | 0.1128 | $\pm 0.0015$ | | [11] |
| BBG | 0.1134 | $+0.0019/-0.0021$ | | [9] |
| **$N^3$LO** | $\alpha_s(M_Z^2)$ | expt | theory | Ref. |
| BBG | 0.1141 | $+0.0020/-0.0022$ | | [9] |

NNLO and $N^3$LO

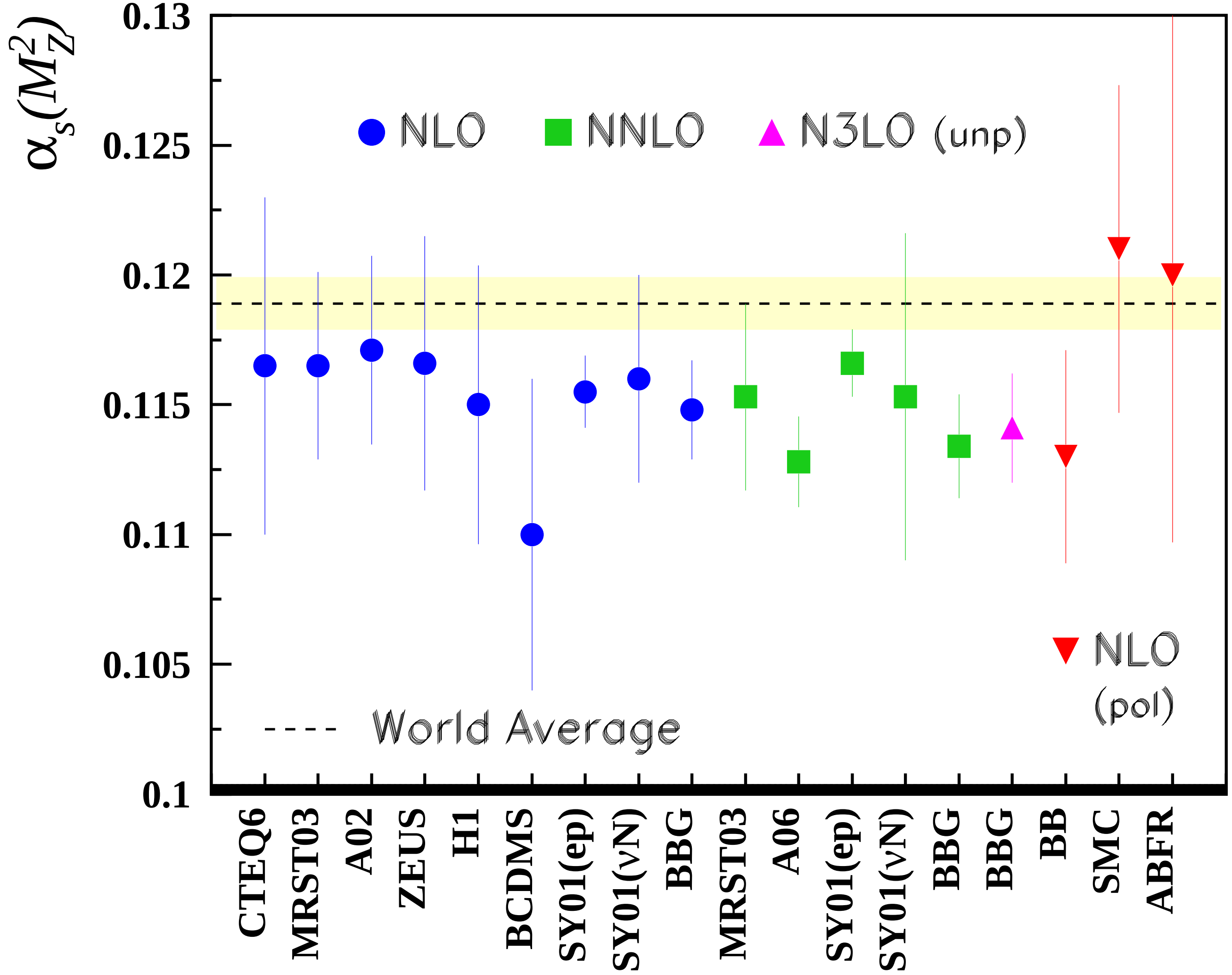
BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

# $\alpha_s(M_Z^2)$




J.B., H. Böttcher, A. Guffanti, 2006

# 4. The Needs : What would we like to know ?

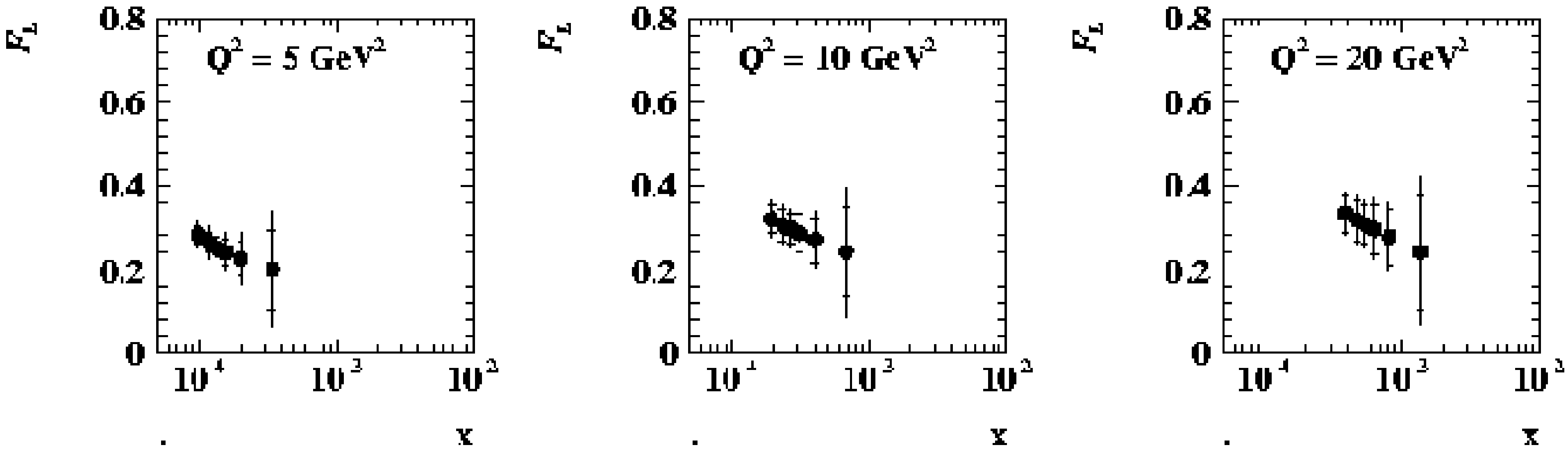
## HERA:

- Collect high luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.
- Measure : $F_L(x,Q^2)$. This is a key-question for HERA.

# $F_L(x, Q^2)$

M. Klein, 2004: Projection for a possible measurement at HERA

$\Longrightarrow$ of central importance to study the small $x$ behaviour of the gluon distribution



# 4. Future Avenues : What would we like to know ?

## HERA:

- Collect high luminosity for $F_2(x,Q^2)$, $F_2^{c\bar{c}}(x,Q^2)$, $g_2^{c\bar{c}}(x,Q^2)$, and measure $h_1(x,Q^2)$.
- Measure : $F_L(x,Q^2)$. This is a key-question for HERA.

## RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized. DIS PDF's $\Longleftrightarrow$ Collider PDF's

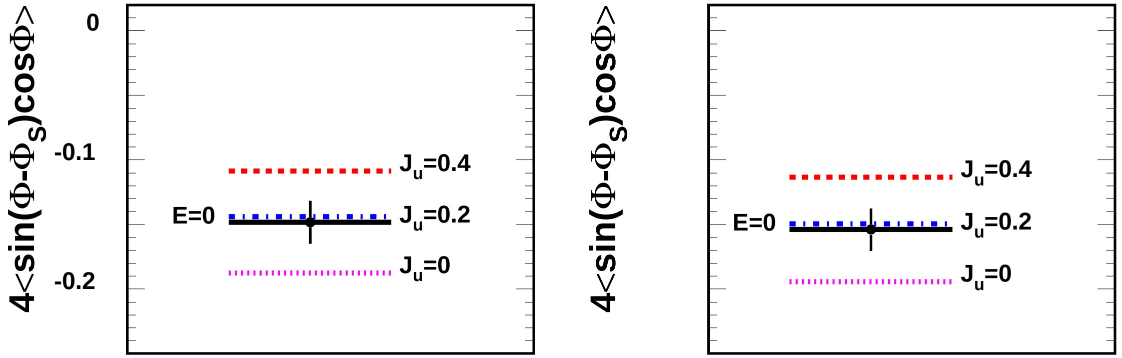
## JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small $x$.

# $L_q$ from DVCS

- HERA and JLAB : Improve DVCS data

Theory widely developed, cf. rev. Belitsky & Radyushkin, 2005




Expected DVCS asymmetry $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ with $b_v = 1$, $b_s = \infty$, $J_u = 0.4(0.2, 0.0)$, $J_d = 0.0$ in the Regge (left panel) and factorized (right panel) ansatz, at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPD E. The projected statistical error for 8M DIS events is shown. The systematic error is expected to not exceed the statistical one.

F. Ellinghaus et al. 2005

The measurement of $L_q$ off data is model–dependent at the moment.
Lattice calculations at low pion masses are needed to complete the picture

# Graph Resummation and Saturation

Further study of proposed mechanisms needed: RHIC, LHC for nucleus-nucleus collisions.

ep scattering: partly different mechanisms

more studies would be welcome; link to higher twist contributions in gluon–dynamics

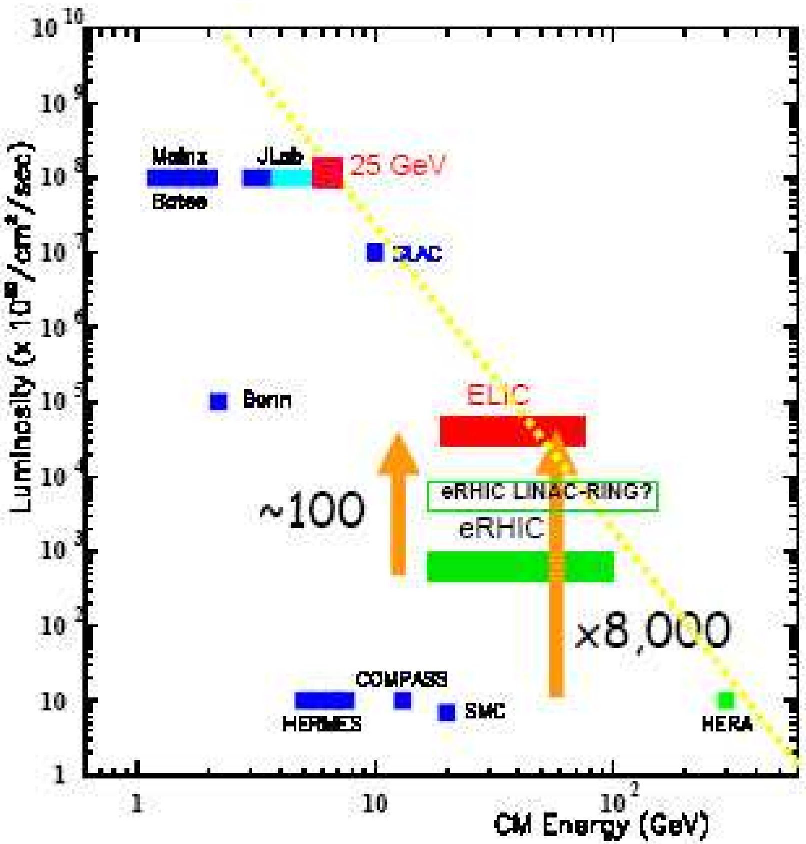
How do the non-perturbative and perturbative parts factorize ?

Conservation laws and interplay between the small $x$ and medium $x$ range behaviour

# New DIS Machines

## Where to go ?

- High energies : small $x$, large $Q^2$ desirable.
- High luminosities : ELIC: $\sqrt{s}$ between CERN and HERA energies




R. Ent,2004
high precision physics
polarized and unpolarized

Would be an important extension of the present programmes in many respects.

# Enhancing Precision Further...

- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\mathrm{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor. (Theory & Experiment)
- Flavor Structure of Sea-Quarks: More studies needed.(All Experiments)
- Revisit polarized data upon arrival of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed. (Theory)
- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions, Transversity, diffraction in polarized scattering (HERMES, High Precision polarized experiments, JLAB, ELIC)
- Comparison with Lattice Results: $\alpha_s$, Moments of Parton Distributions, Angular Momentum.

# Enhancing Precision Further...

- Calculation of more hard scattering reactions at the 3–loop level: LHC
- Further perfection of the mathematical tools:
  $\Longrightarrow$ Algorithmic simplification of Perturbation theory in higher orders.
- Even higher order corrections needed ?

# Jiro will be commemorated by a large community