Parton Distributions for the LHC

Johannes Blümlein
DESY

- The Major Goals

- DIS Theory Status

- Unpolarized Parton Distribution Functions

- Λ_{QCD} and $\alpha_s(M_Z^2)$

- PDFs and Inclusive Cross Sections at LHC

- Advanced Technologies

- Outlook

1. The Major Goals

- Precision Measurement of the Strong Coupling Constant $\alpha_s(M_Z^2)$

- Precision Measurement of the Unpolarized Parton Densities

- Precision Measurement of the Polarized Parton Densities

- Who Carries the Spin of the Proton?

- Higher Twist Effects

- Is there Saturation in DIS at small x ? $\implies$ answered by experiment.

Theory of DIS

'69, QCD: '72 → **Parton Model**

'69 − 72 → **Light Cone Expansion [f]**

Twist 2

polarized   *unpolarized*

Fixed Order PT: QCD

Higher Twist

~ 77

**Twist 3
Twist 4 ...**

Sum Rules

'60ies
− now

α_s

Splitting functions

Coefficient functions

$O(\alpha_s^4)$ $O(\alpha_s^3)$ $O(\alpha_s^3)$

'73,'74,'80,'97   '73,'82, '04   '82,'92,'05,'09

**Special Kinematics
Domain: Small x**

'75, '86
'90 - '98

Resummations ?

**More General View
Non-forw. scattering**

**Diffractive
Scattering**

Angular Momentum: q, G

Higher Orders $\Longrightarrow$ **New Algorithms
Novel Mathematics**

Status of Highest Order Calculations

- Running α_s: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$

  Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;

  $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;

  Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Klein, B. Tödtli 2008
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993

  Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$ Watson 1982
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L: $O(\alpha_s^3)$

  Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,

  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_2: $O(\alpha_s^2 \varepsilon)$: all operators
  (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: Moments 2–10(12,14)
  of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008

= done at DESY (or in DESY collab.).

$$F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma_j^i \left(\alpha_s, \frac{Q^2}{\mu^2}, x \right)$$

$$\uparrow \text{ bare pdf} \quad \uparrow \text{ sub} - \text{system cross} - \text{sect.}$$

$$= \underbrace{\hat{f}_i(x, \mu^2) \otimes \Gamma_k^i \left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k \left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right)}_{\text{finite Wilson coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx \, x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Evolution Equations

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_\psi(g) \right] F_i(N) = 0$$

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \gamma_\kappa^N(g) - 2\gamma_\psi(g) \right] f_k(N) = 0$$

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - \gamma_\kappa^N(g) \right] C_j^k(N) = 0$$

CALLAN–SYMANZIK equations for mass factorization $\equiv$
ALTARELLI–PARISI evolution equations
x-space :

$$\frac{d}{d\log(\mu^2)} \left(\begin{array}{c} q^+(x,Q^2) \\ G(x,Q^2) \end{array} \right) = \frac{\alpha_s}{2\pi} \boldsymbol{P}(x,\alpha_s) \otimes \left(\begin{array}{c} q^+(x,Q^2) \\ G(x,Q^2) \end{array} \right)$$

$$\boldsymbol{P}(x,\alpha_s) = \boldsymbol{P}^{(0)}(x) + \frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{P}^{(2)}(x) + \dots$$
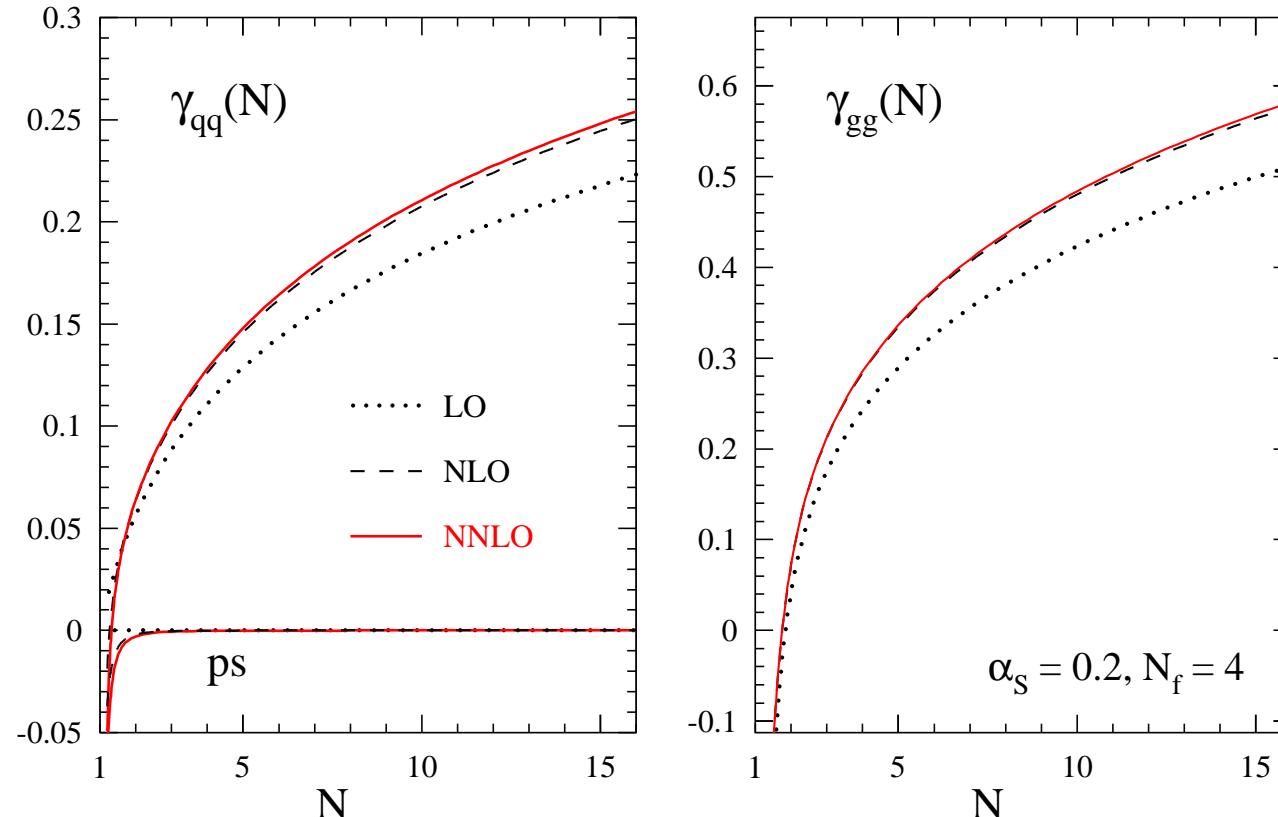
Vermaseren, Moch, Vogt 2004

Representative : $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight w=3 :

$$F_1(N) = \mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N)$$

$$F_2(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N), \quad F_3(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_2(x)}{1-x}\right)_+\right](N)$$

Yndurain et al., 1981: $F_2(N)$

Weight w=4 :

$$F_4(N) = \mathbf{M}\left[\frac{S_{1,2}(x)}{1+x}\right](N), \quad F_5(N) := \mathbf{M}\left[\left(\frac{S_{1,2}(x)}{1-x}\right)_+\right](N)$$

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_4(x)}{1 \pm x}\right)_{(+)}\right](N), \quad F_8(N) = \mathbf{M}\left[\frac{S_{1,3}(x)}{1 + x}\right](N),$$

$$F_{9,10}(N) = \mathbf{M}\left[\left(\frac{S_{2,2}(x)}{1 \pm x}\right)_{(+)}\right](N), \quad F_{11}(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1 + x}\right](N),$$

$$F_{12,13}(N) := \mathbf{M}\left[\left(\frac{\ln(x)S_{1,2}(-x) - \mathrm{Li}_2^2(-x)/2}{1 \pm x}\right)_{(+)}\right](N)$$

$F_6(N) - F_{13}(N)$: J.B., S. Moch, 2004.

Massless QCD to 3 Loops depends on 14 Functions.

Weight w=6 :

$\Rightarrow$ Representation for 3 Loop Wilson Coeff.: 35 Functions, J.B., 2009.

Complex Analysis of these Functions

- Construct exact analytic continuations to complex N

- The functions are meromorphic
(up to soft corrections, which have a simple structure)

- Asymptotic Representation

- Recursion $z + 1 \rightarrow z$

- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all Q^2

- Include the heavy flavor Wilson coefficients in Mellin Space
$\Longrightarrow$ nearly accomplished to $O(a_s^3)$ I. Bierenbaum, JB, S. Klein (2009)

- Perform a single fast, numerical Mellin inversion
(at high precision)

$\Rightarrow$ **Fastest and most Precise Way of Analysis**

# 3. Unpolarized Parton Distribution Functions

HERA I e$^+$p Neutral Current Scattering - H1 and ZEUS

HERA I e$^+$p Neutral Current Scattering - H1 and ZEUS

DIS08   Joël Feltesse
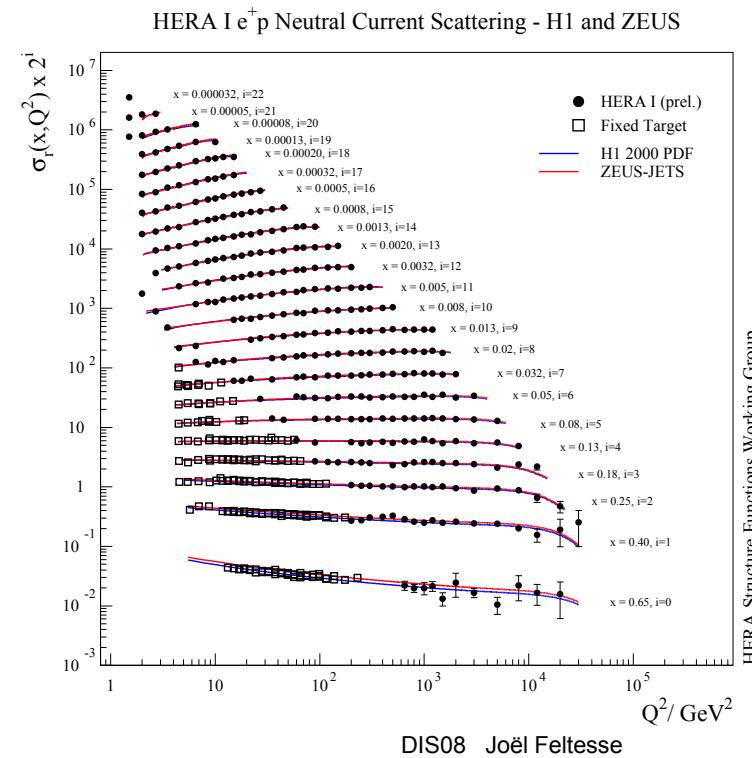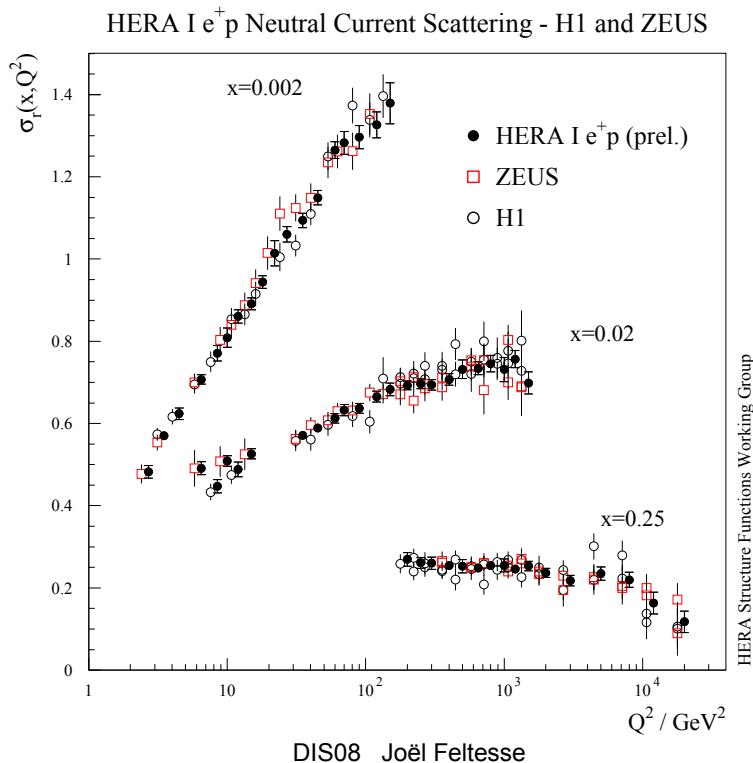
New ZEUS + H1 averaged $F_2(x, Q^2)$

Direct $F_L(x, Q^2)$ Measurement at HERA

H1 Preliminary F_L — medium & high Q^2
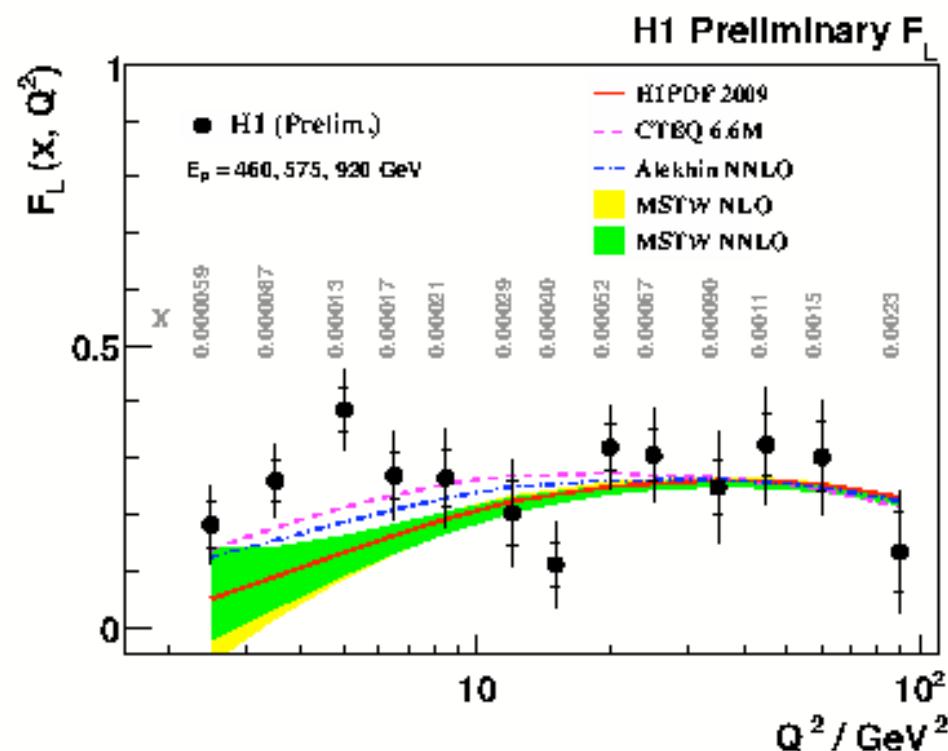
Plots of $F_L(x, Q^2)$ versus x for Q^2 = 12, 15, 20, 25, 35, 45, 60, 90, 120, 150, 200, 250, 300, 400, 500, 650, 800 GeV²

● H1 (Prelim.)
E_p = 460, 575, 920 GeV
— H1 PDF 2000

H1 PDF 2000: $Q^2 = 4$ GeV^2

Fit to H1 data

— experimental errors

— model uncertainties

Fit to H1 + BCDMS data

— parton distribution

H1
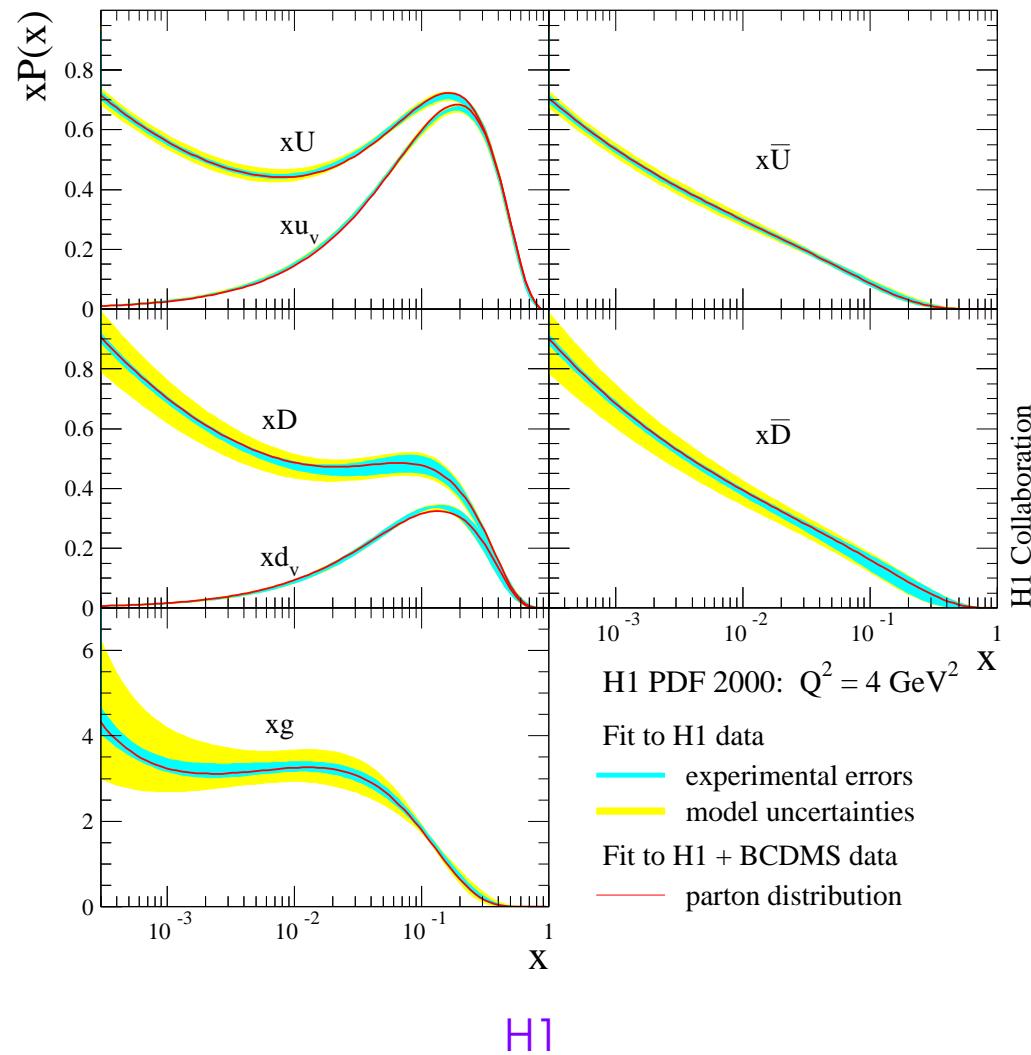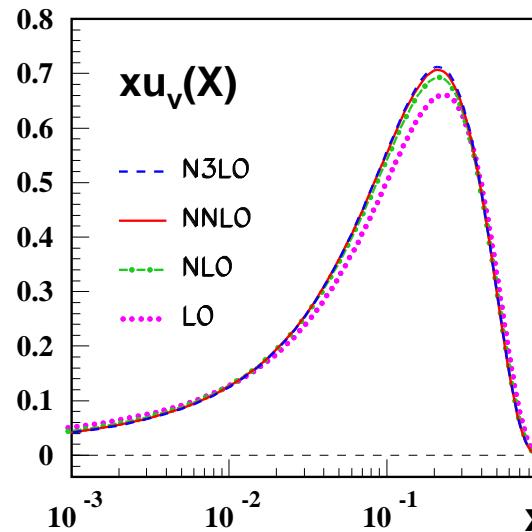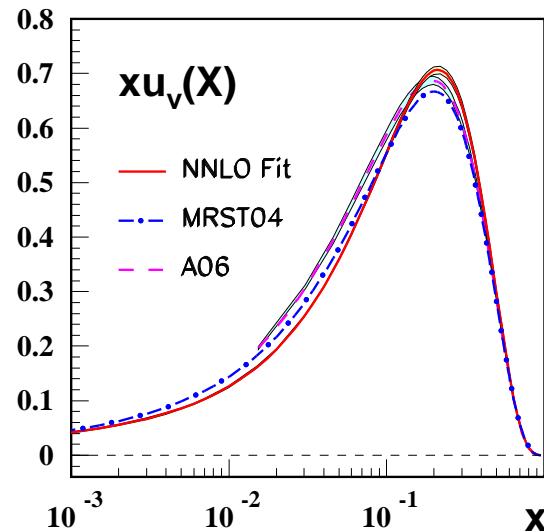
World data:
NS-analysis
$$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$$
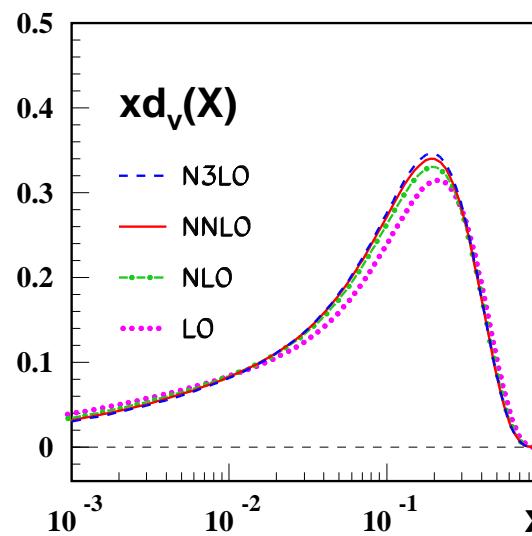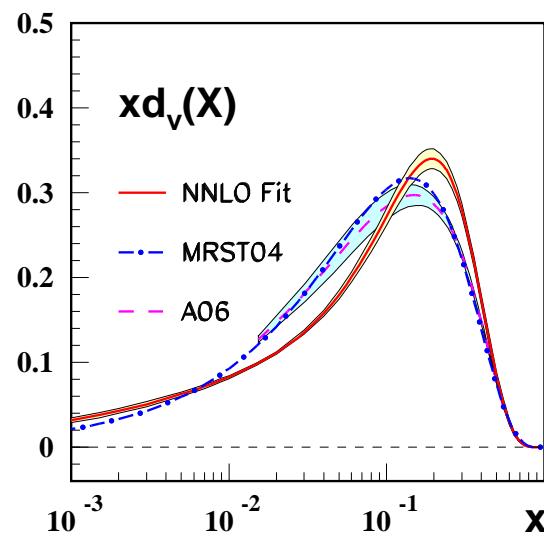
N^3LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ ± 2 MeV in Λ_{QCD}

$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)\,2}}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3$$
$$+ \left[\frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4$$

The results agree better than 20%.

Valence Distributions

Left panel: $F_2^p(x, Q^2) * 2^i$

- H1
- ZEUS
- BCDMS
- NMC
- SLAC

x = 0.30, i = 3
x = 0.40, i = 2
x = 0.65, i = 1
x = 0.75, i = 0

$W^2 > 12.5 \ GeV^2$

- QCD
- QCD + TMC
- QCD + TMC + HT

$Q^2, \ GeV^2$

Right panel: $F_2^d(x, Q^2) * 2^i$

- BCDMS
- NMC
- SLAC
- E665

x = 0.350, i = 4
x = 0.450, i = 3
x = 0.550, i = 2
x = 0.650, i = 1
x = 0.750, i = 0

$W^2 > 12.5 \ GeV^2$

- QCD
- QCD + TMC
- QCD + TMC + HT

$Q^2, \ GeV^2$
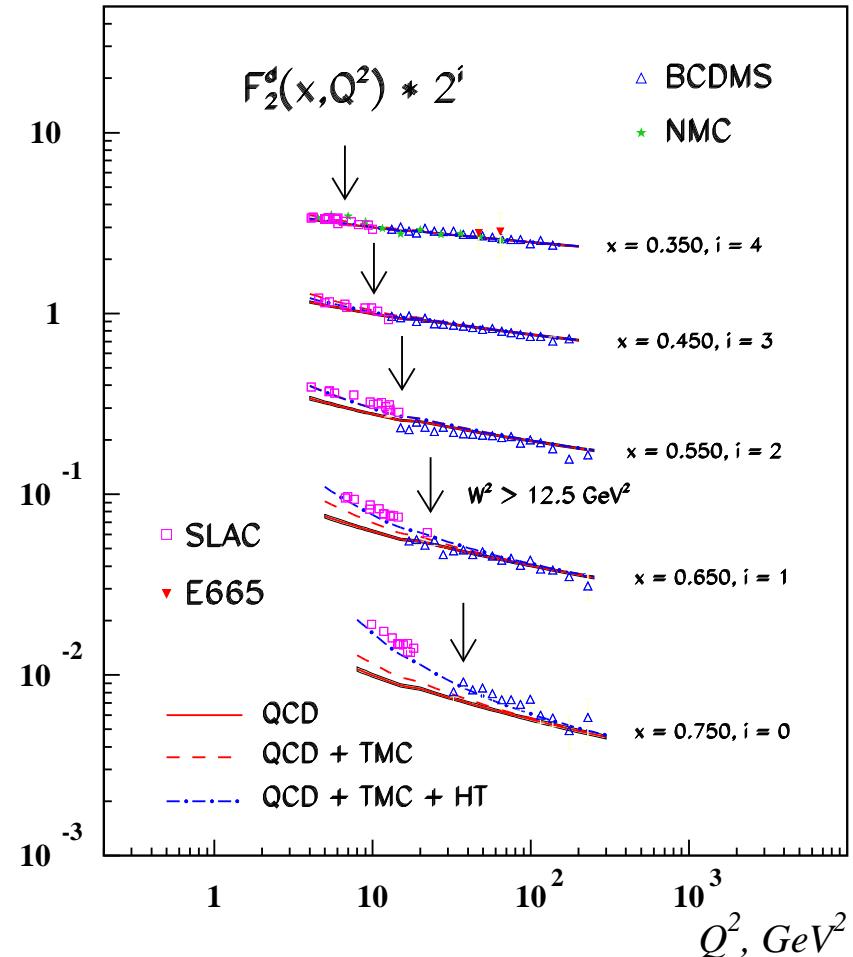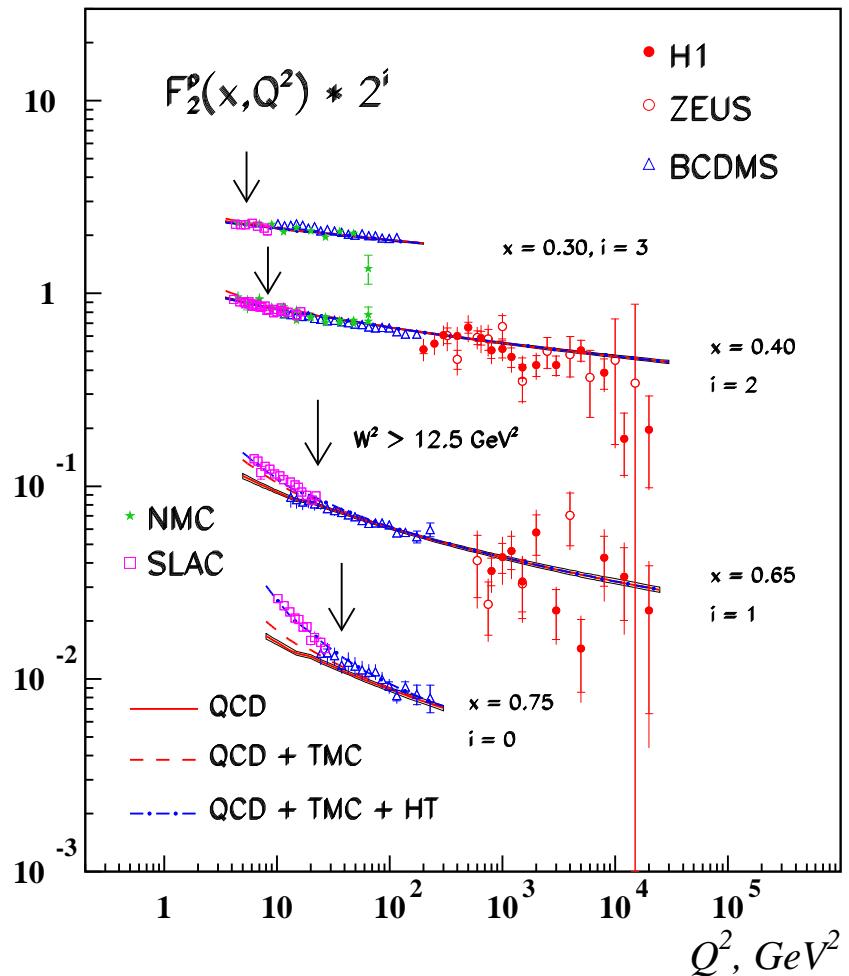
Valence Distributions: higher twist
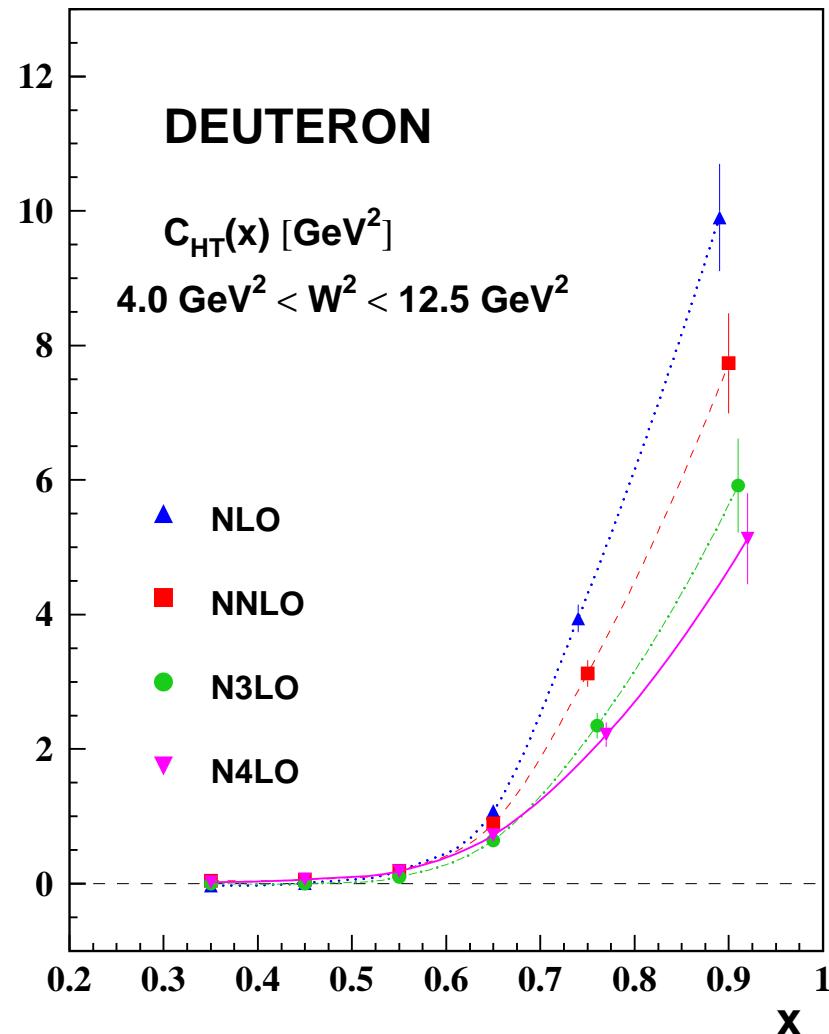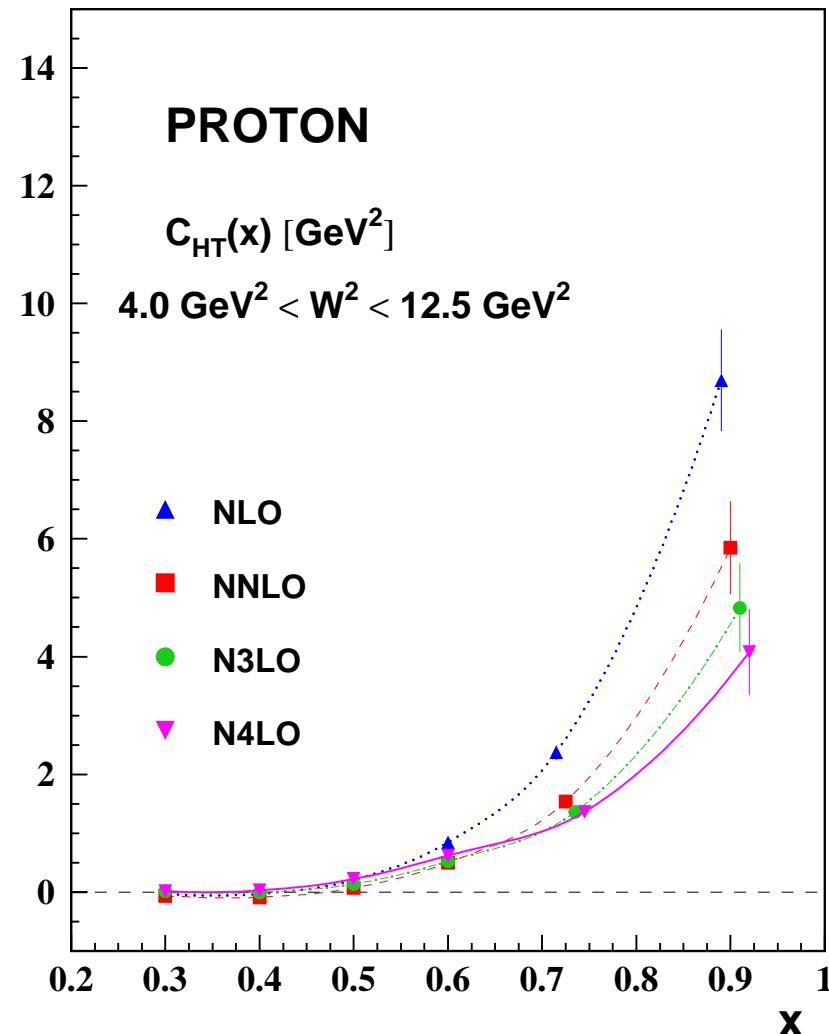

- agreement between p and d analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

H1
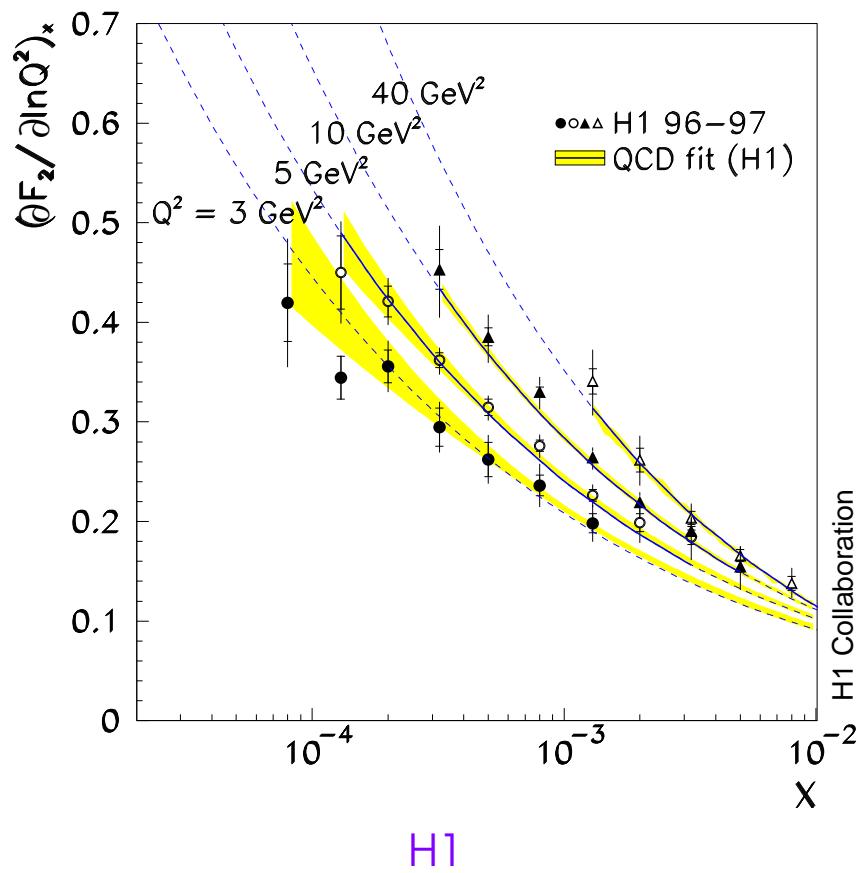
J.B., A. Guffanti 2005
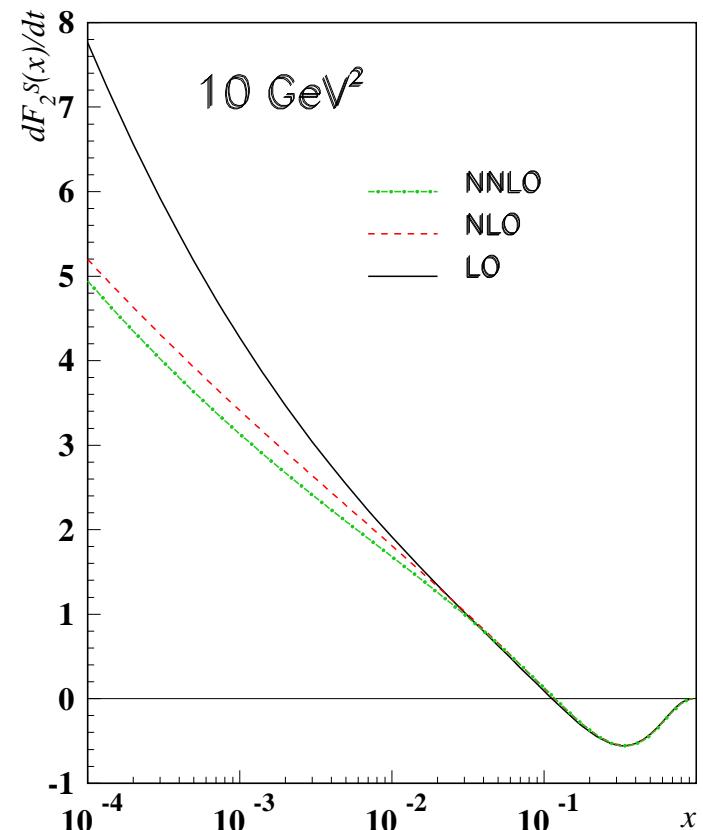
Very likely, that the $\overline{MS}$–gluon is remains positive!

Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO):
+ Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...
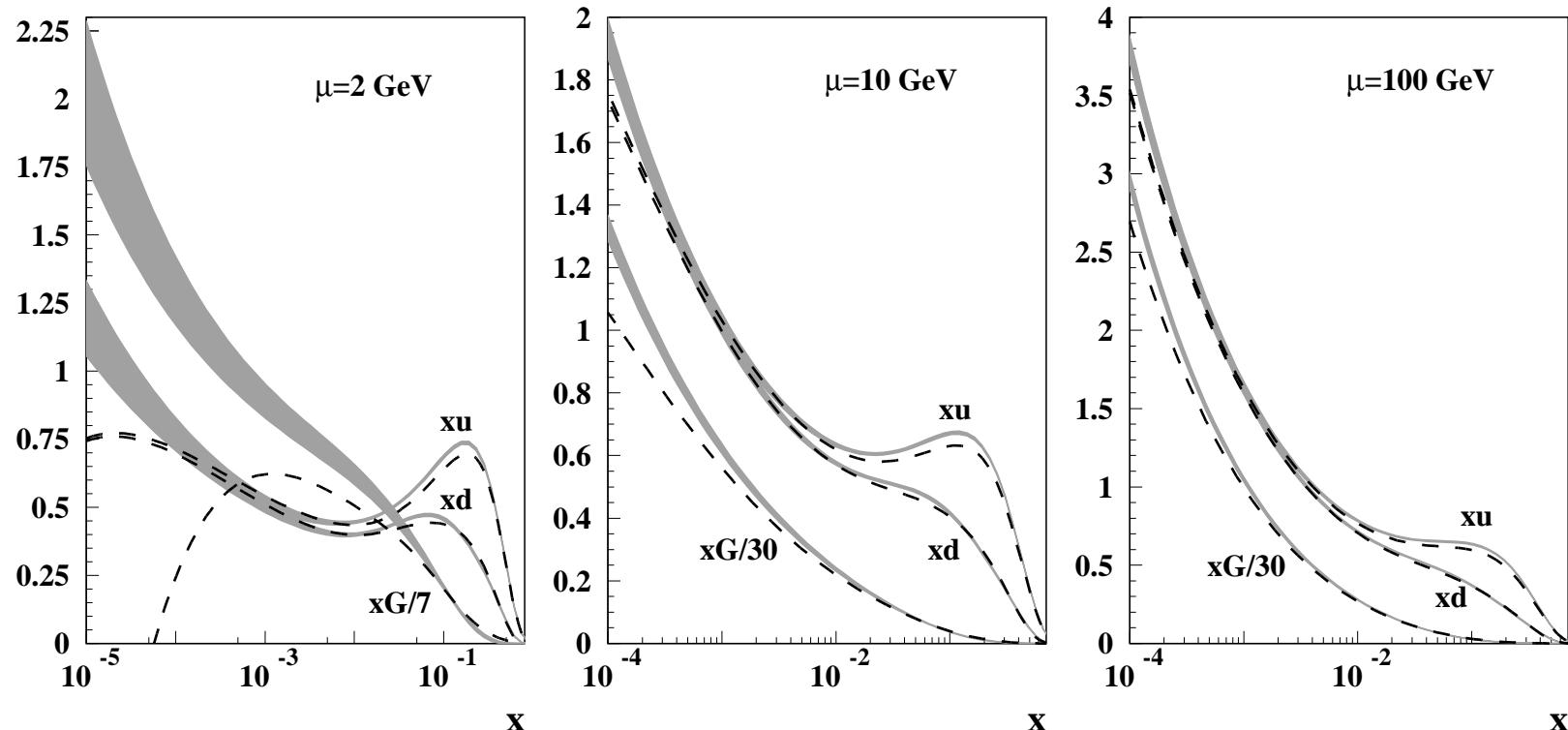
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
Correct treatment of HQ very essential: FFNS, BSMN-schemes.
full lines: ABKM error band; dashed lines: MSTW08

Heavy quarks and gluon (NNLO)

μ=2 GeV

μ=10 GeV

μ=100 GeV

1.5xs

xc

xb
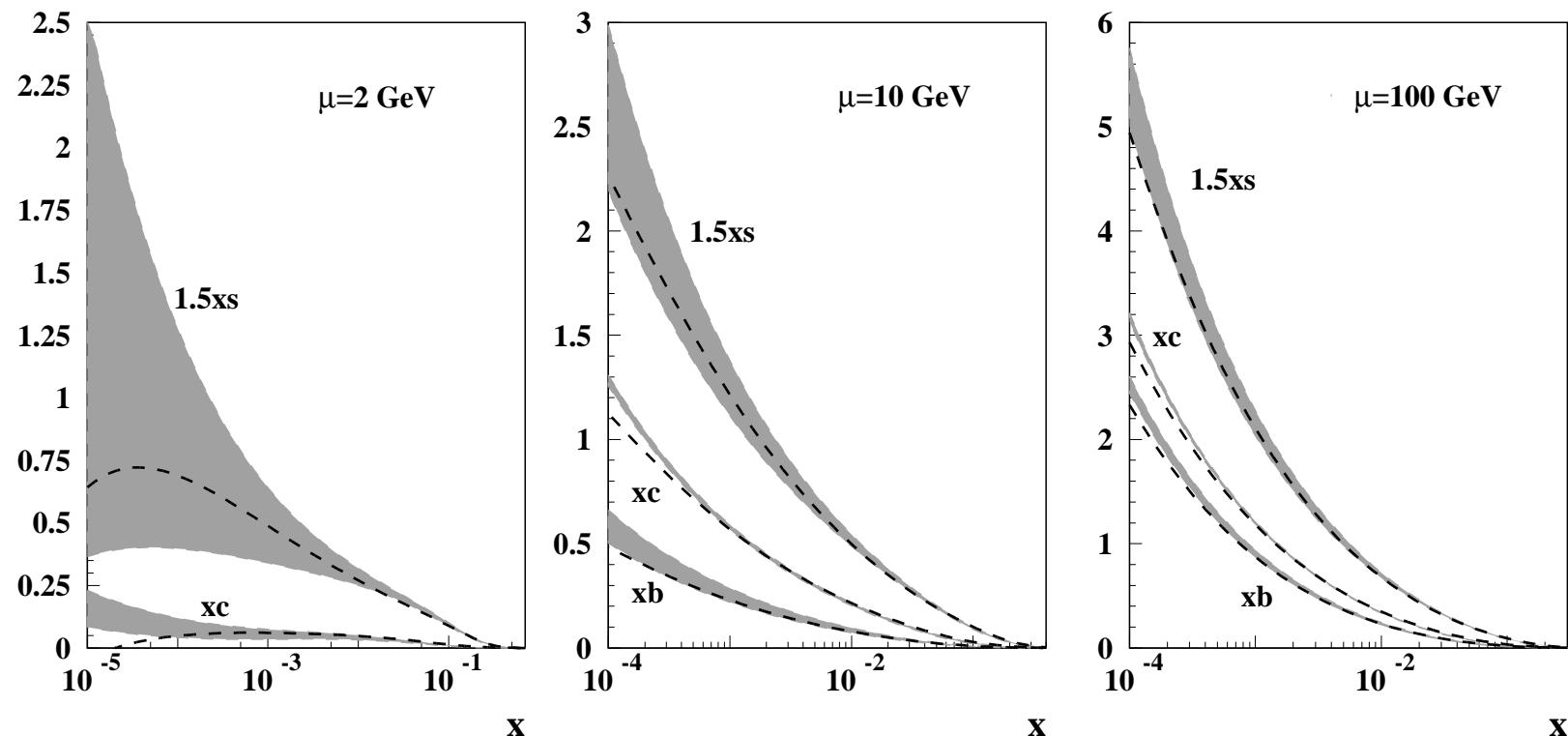
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
full lines: ABKM error band; dashed lines: MSTW08

FFNS, $N_f = 3$
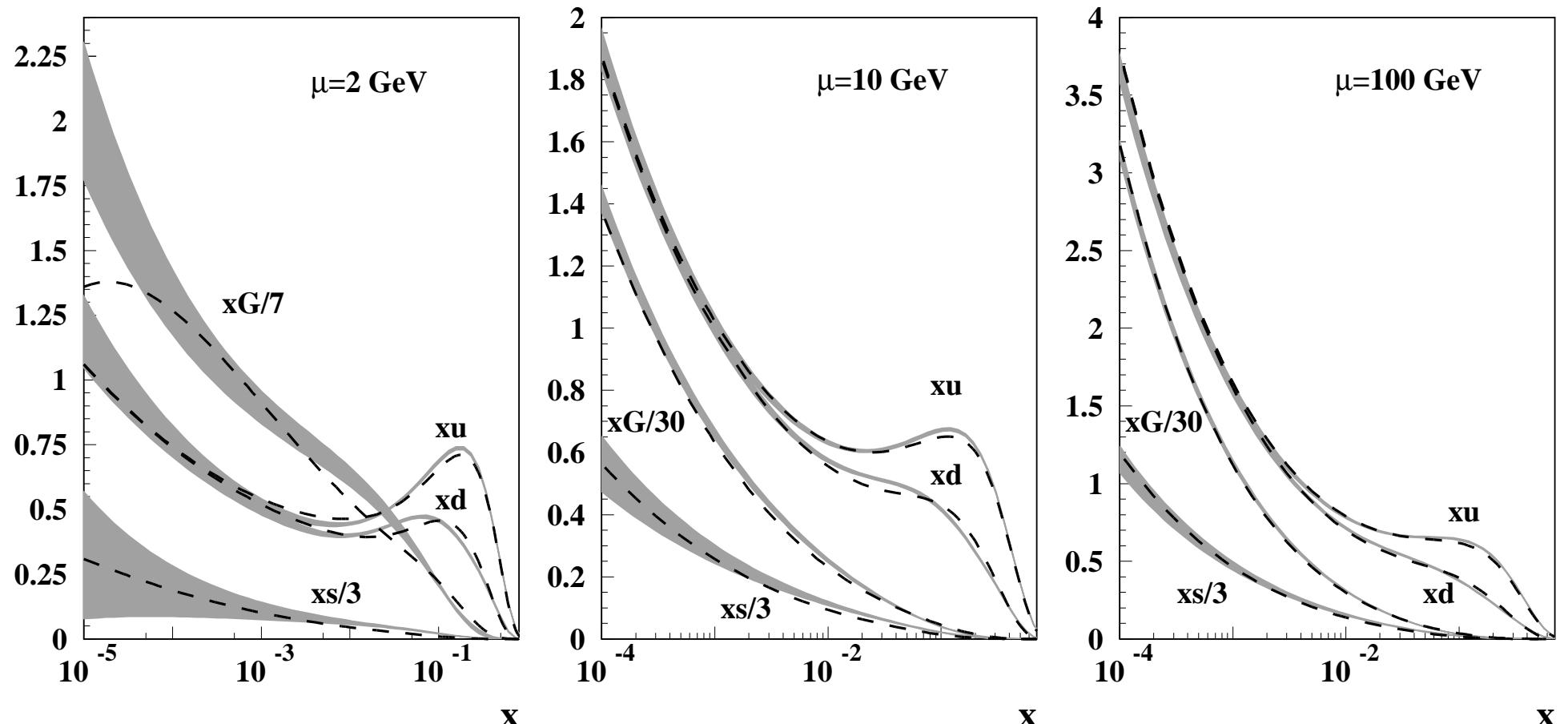
comparison: ABKM (2009) vs. Jimenez-Delgado/ Reya (2008)

Flavor distributions: strangeness
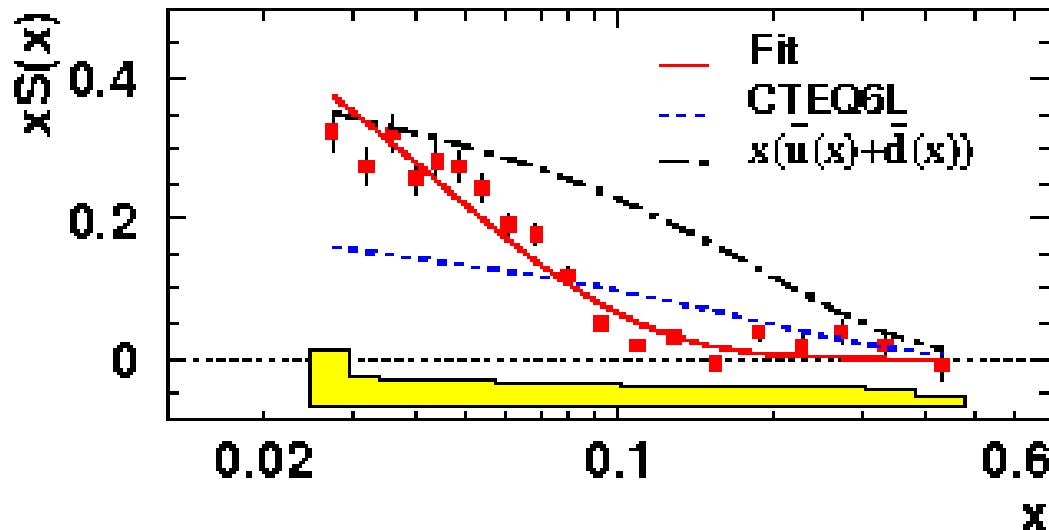
FIG. 3: The strange parton distribution $xS(x)$ from the measured HERMES multiplicity for charged kaons evolved to $Q_0^2 = 2.5\,\mathrm{GeV}^2$ assuming $\int \mathcal{D}_g^K(z)dz = 1.27 \pm 0.13$. The solid curve is a 3-parameter fit for $S(x) = x^{-0.924}e^{-x/0.0404}(1-x)$, the dashed curve gives $xS(x)$ from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

Nice HERMES measurement (hep-ex/0803.2993); still to be understood.

$$\frac{\delta\alpha_{\text{em}}(0)}{\alpha_{\text{em}}(0)} \sim 3 \cdot 10^{-11} \qquad \frac{\delta\alpha_{\text{weak}}}{\alpha_{\text{weak}}} \sim 7 \cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}$$

(until recently)
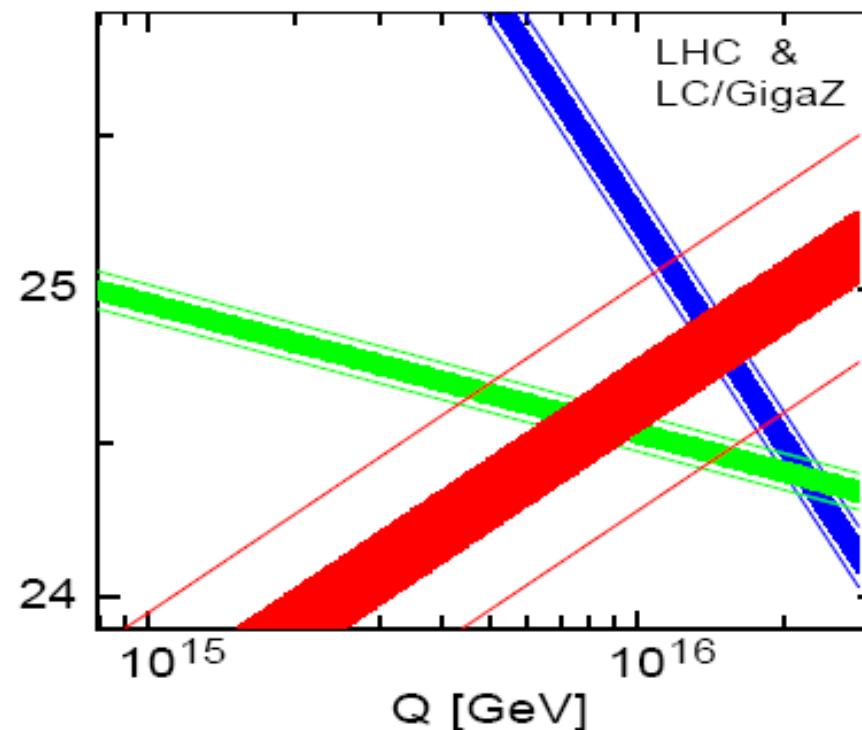
P. Zerwas, 2004

$$\alpha_s(M_Z^2)$$

S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \approx 1.2\%$$

	$\alpha_s(\mathrm{M_Z^2})$	
ABKM	0.1135 ± 0.0014	HQ: FFS $N_f = 3$
A.Hoang et al.	$0.1135 \pm 0.0011 \pm 0.0006$	e^+e^- thrust
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach
BBG (2006)	$0.1134 \, ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
JR (2008)	0.1124 ± 0.0020	dynamical approach
MSTW (2008)	0.1171 ± 0.0014	
BBG (2006)	$0.1141 \, ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO

$\alpha_s(M_Z^2)$
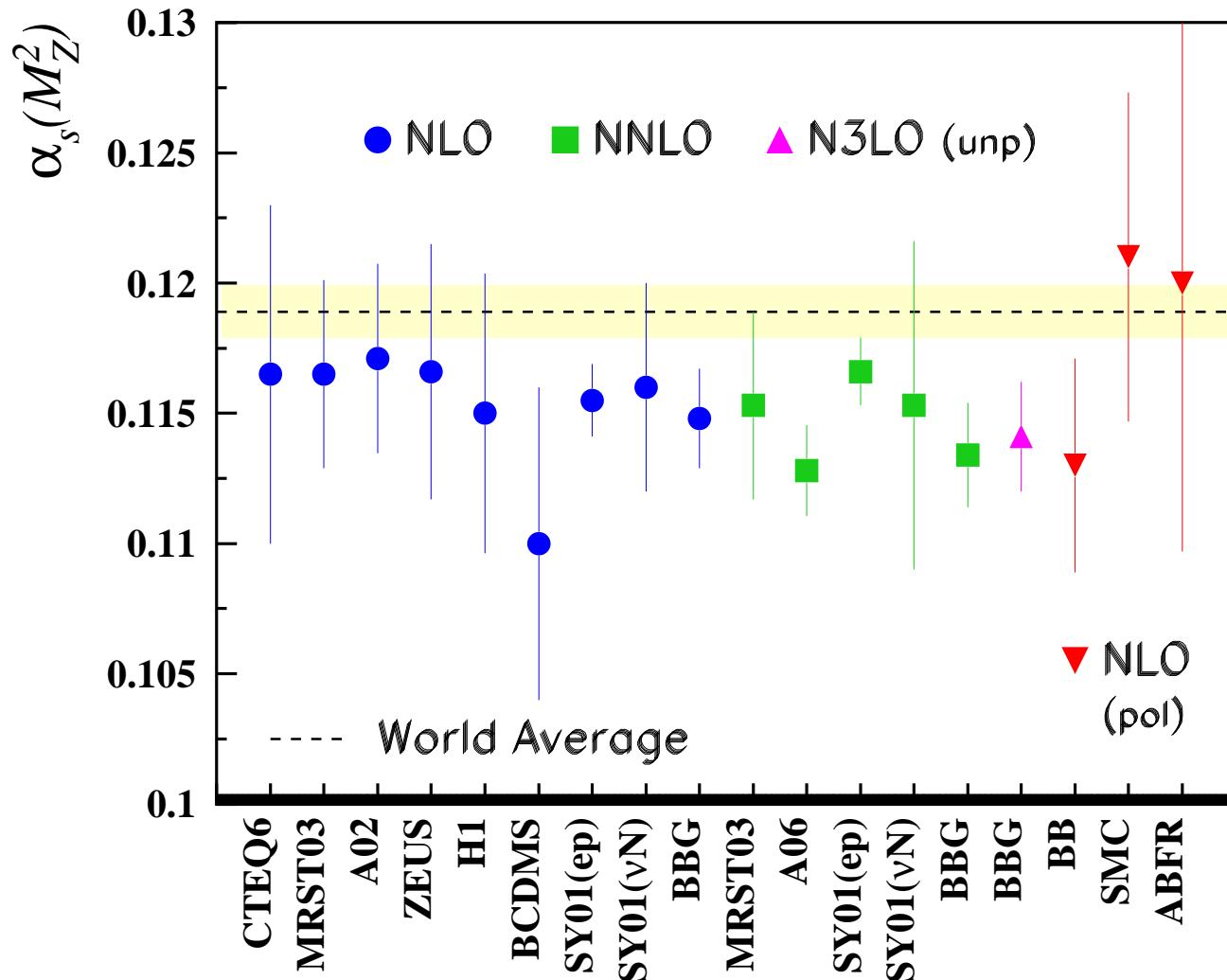
J.B., H. Böttcher, A. Guffanti, 2006

- $\alpha_s(M_Z^2)$ for different data sets included are too different !
  $\Rightarrow$ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet
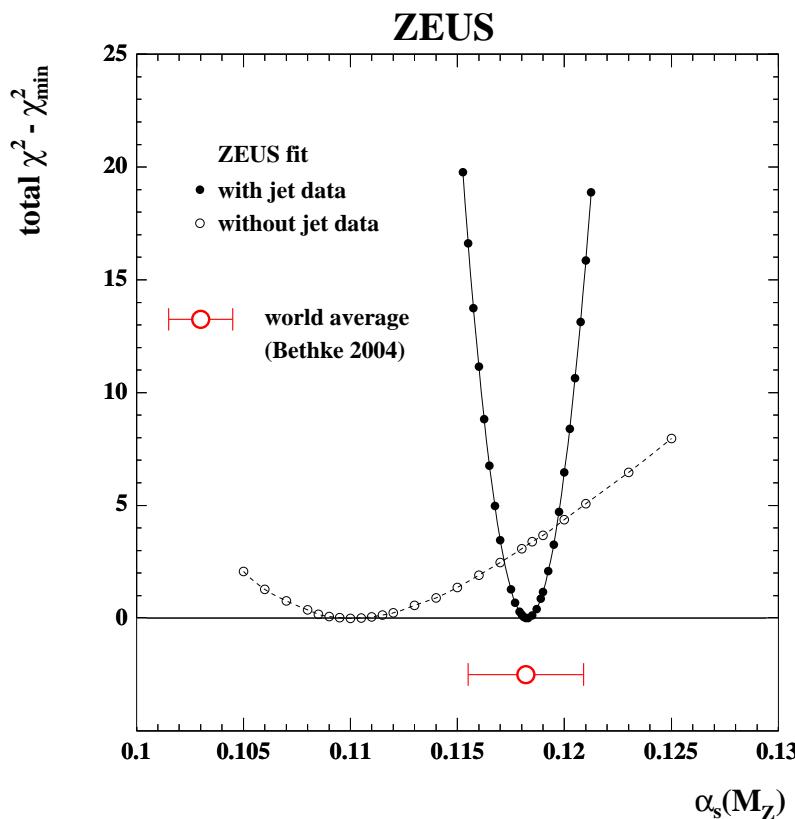
M. Cooper-Sarkar, 2005

5. Some Predictions for Tevatron and the LHC

Drell-Yan Process (NNLO)

$$L_{q\bar{q}}^{-}V_{q\bar{q}}^{2-}$$

pp --> W$^+$X $\sqrt{s}$=14 TeV ud̄ cs̄ us̄

p̄p --> W$^+$X $\sqrt{s}$=1.96 TeV ud̄ cs̄ us̄
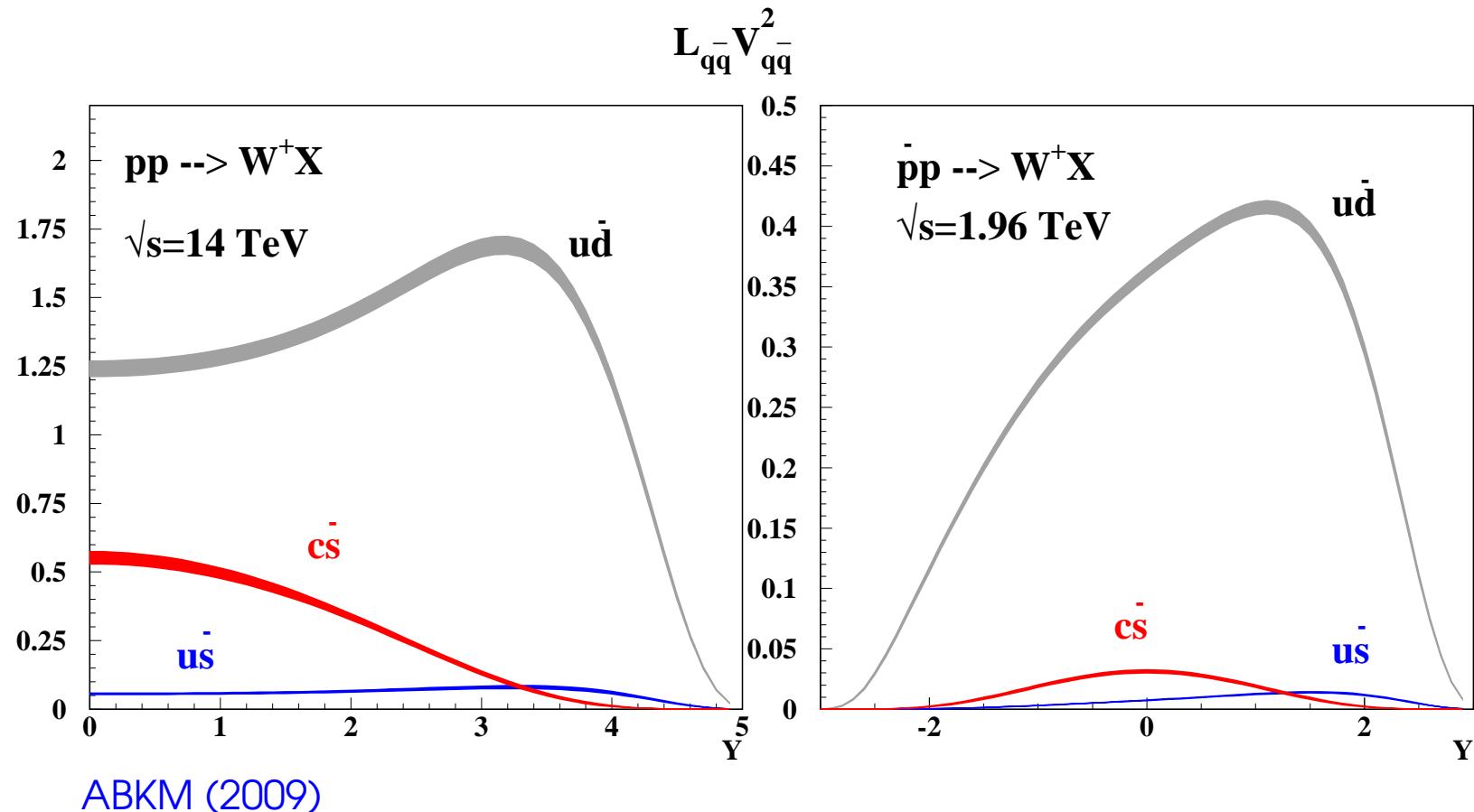
ABKM (2009)

$t\bar{t}$ Cross Section in $pp(\bar{p})$ scattering at (NNLO)

$\sqrt{s}$ (TeV)	this paper	MSTW2008
1.96 $(\bar{p}p)$	6.91 ± 0.17	7.04
7 (pp)	131.3 ± 7.5	160.5
10 (pp)	343 ± 15	403
14 (pp)	780 ± 28	887

ABKM (2009) vs MSTW08

Higgs Cross Section in $pp(\bar{p})$ scattering at (NNLO)

- σ_H (pb)
- pp(14 TeV)
- pp(10 TeV)
- pp(7 TeV)
- $\bar{p}p$(1.96 TeV)
- M_H (GeV)
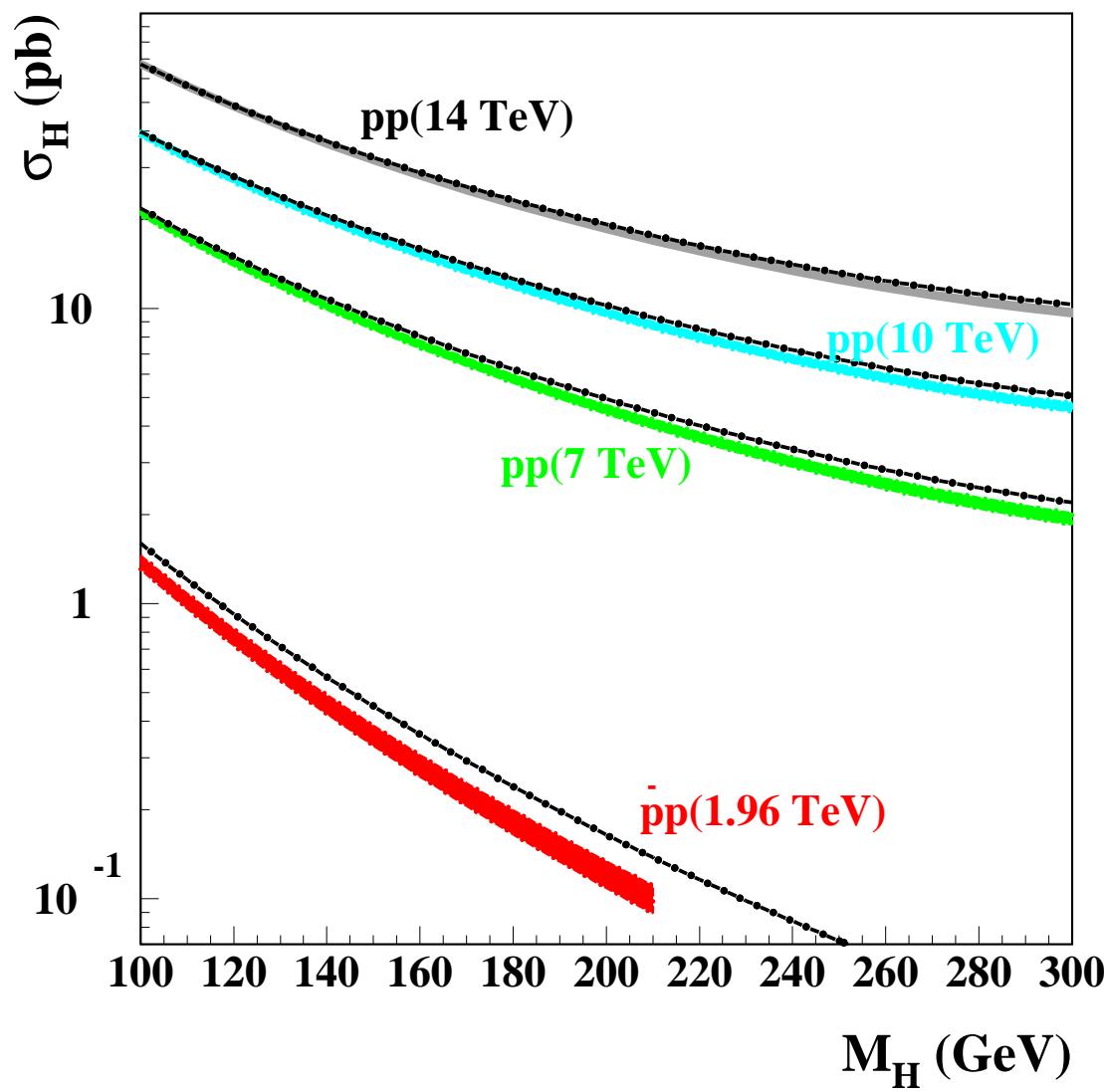
bands: ABKM (2009); lines: MSTW08

6. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation mandatory: QGRAF
  # 2500 - 15000 diagrams

- The 'Only' problem: Calculation of Feynman Parameter Integrals; everything else automated: FORM-codes

- Renormalization still not always trivial: γ_5, mass(es), ...

- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier values, etc. - Avoids the problem of Babel in analytic integration

- Generalized Hypergeometric Functions and their Generalizations are to the Heart of the Matter. M. Kalmykov et al., JB et al.

- Need: advanced Difference Equation Establishers & Solvers: Sigma

- **Do not proliferate !,** i.e. avoid IBP, MB, and other methods causing gigantic Zeroes.

- What remains is : **Integrating the hard way.**

Advanced Technologies to Evaluate Feynman Diagrams

Some Examples:

- **Zero-scale Problems :** Euler-Zagier and Multiple Zeta Values

  JB, D. Broadhurst, J. Vermaseren, DESY 09-03

  find all relations : $\Longrightarrow$ **Tera-Terms** to be processed

  alternating: all relations up to w = 12 (6-loop level);

  non-alternating: all relations up to w = 22; determined.

  Interesting relations: to w = 30;

- **Reconstructing recurrent quantities from Mellin Moments**

  JB, M. Kauers, S. Klein, C. Schneider DESY 09-02

  Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments ? Yes - recurrent quantities in Mellin space.

  $\leq$ 5114 Moments; difference equation fills 440 books

  Complete computation: 5 CPU Months

- **Massive Wilson coefficients at 3 Loops**

  I. Bierenbaum, JB, S. Klein, DESY 09-57

  first analytic massive 1-scale calculation @ 3-loops

  Moments 2–10 (12/14) have been calculated for all unpolarized channels
  Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers

7. Outlook

Theory:

- Polarized Anomalous Dimensions & massless Wilson coefficients @ 3 Loops

- Unpolarized Heavy Flavor Wilson coefficients @ 3 Loops : general N

- Polarized Heavy Flavor Wilson coefficients @ 3 Loops

- Along with this: Development of efficient analytic calculation methods being suited for 3-Loops and higher

- ep & pp jet cross sections at HO; progress in pdf Lattice calculations

Code:

- Creation of an Open Source Code for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

Experiment:

- Precision Data from LHC, JLAB and EIC.

Can we get $\delta\alpha_s$ even smaller ?