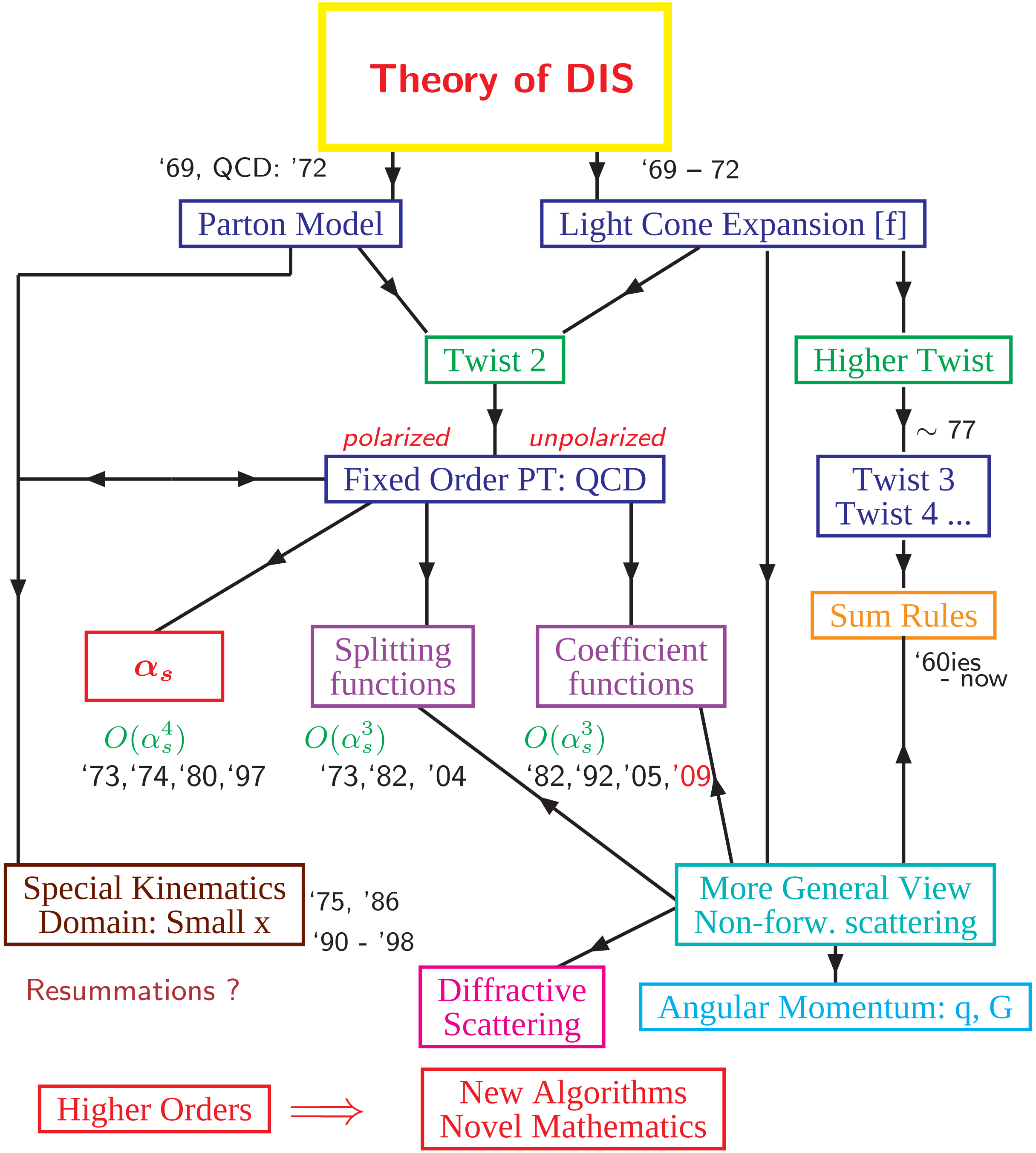
Theory Perspectives: Deep-Inelastic Scattering

Johannes Blümlein
DESY

- The Major Goals
- DIS Theory Status
- Unpolarized Parton Distribution Functions
- Polarized Parton Distribution Functions
- $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$
- Advanced Technologies to Evaluate Feynman Diagrams @ 3 Loops
- Outlook

1. The Major Goals

- Precision Measurement of the Strong Coupling Constant $\alpha_s(M_Z^2)$
- Precision Measurement of the Unpolarized Parton Densities
- Precision Measurement of the Polarized Parton Densities
- Who Carries the Spin of the Proton?
- Higher Twist Effects
- Is there Saturation in DIS at small x ? $\Longrightarrow$ answered by experiment.



Theory of DIS
'69, QCD: '72
'69 – 72
Parton Model
Light Cone Expansion [f]
Twist 2
Higher Twist
∼ 77
polarized
unpolarized
Fixed Order PT: QCD
Twist 3
Twist 4 ...
Sum Rules
'60ies - now
α_s
Splitting functions
Coefficient functions
$O(\alpha_s^4)$
'73,'74,'80,'97
$O(\alpha_s^3)$
'73,'82, '04
$O(\alpha_s^3)$
'82,'92,'05,'09
Special Kinematics Domain: Small x
'75, '86
'90 - '98
More General View Non-forw. scattering
Resummations ?
Diffractive Scattering
Angular Momentum: q, G
Higher Orders ⟹
New Algorithms Novel Mathematics

Status of Highest Order Calculations

- Running α_s: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$ Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995; $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike; Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Klein, B. Tödtli 2008
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993 Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$ Watson 1982
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L: $O(\alpha_s^3)$ Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996, Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_2: $O(\alpha_s^2 \varepsilon)$: all operators (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: Moments 2–10(12,14) of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008

= done at DESY (or in DESY collab.).

DIS Structure Functions @ Twist 2

$$F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma_j^i \left(\alpha_s, \frac{Q^2}{\mu^2}, x \right)$$

$$\uparrow \text{bare pdf} \qquad \uparrow \text{sub} - \text{system cross} - \text{sect.}$$

$$= \underbrace{\hat{f}_i(x, \mu^2) \otimes \Gamma_k^i \left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k \left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right)}_{\text{finite Wilson coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Evolution Equations

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_\psi(g)\right] F_i(N) = 0$$

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} + \gamma_\kappa^N(g) - 2\gamma_\psi(g)\right] f_k(N) = 0$$

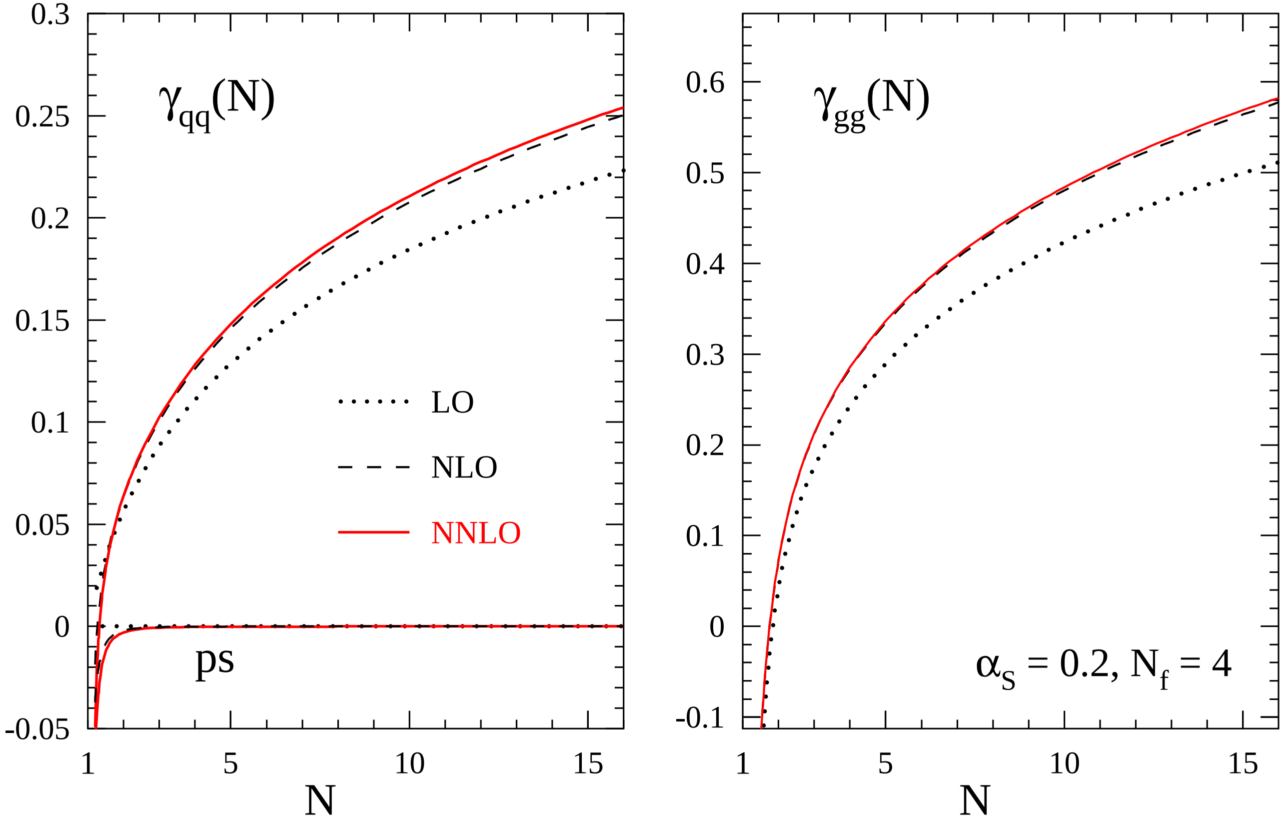
$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g} - \gamma_\kappa^N(g)\right] C_j^k(N) = 0$$

CALLAN–SYMANZIK equations for mass factorization $\equiv$
ALTARELLI–PARISI evolution equations
x-space :

$$\frac{d}{d\log(\mu^2)}\begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s) \otimes \begin{pmatrix} q^+(x,Q^2) \\ G(x,Q^2) \end{pmatrix}$$

$$\boldsymbol{P}(x,\alpha_s) = \boldsymbol{P}^{(0)}(x) + \frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 \boldsymbol{P}^{(2)}(x) + \ldots$$

Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004

The Basic Functions of massless QCD to w=5:$\equiv$ 3 Loops

Representative : $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight w=3 :

$$F_1(N) = \mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N)$$

$$F_2(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N), \qquad F_3(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_2(x)}{1-x}\right)_+\right](N)$$

Yndurain et al., 1981: $F_2(N)$

Weight w=4 :

$$F_4(N) = \mathbf{M}\left[\frac{S_{1,2}(x)}{1+x}\right](N), \qquad F_5(N) := \mathbf{M}\left[\left(\frac{S_{1,2}(x)}{1-x}\right)_+\right](N)$$

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_4(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_8(N) = \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N),$$

$$F_{9,10}(N) = \mathbf{M}\left[\left(\frac{S_{2,2}(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_{11}(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N),$$

$$F_{12,13}(N) := \mathbf{M}\left[\left(\frac{\ln(x)S_{1,2}(-x) - \mathrm{Li}_2^2(-x)/2}{1 \pm x}\right)_{(+)}\right](N)$$

$F_6(N) - F_{13}(N)$: J.B., S. Moch, 2004.

Massless QCD to 3 Loops depends on 14 Functions.

Weight w=6 :

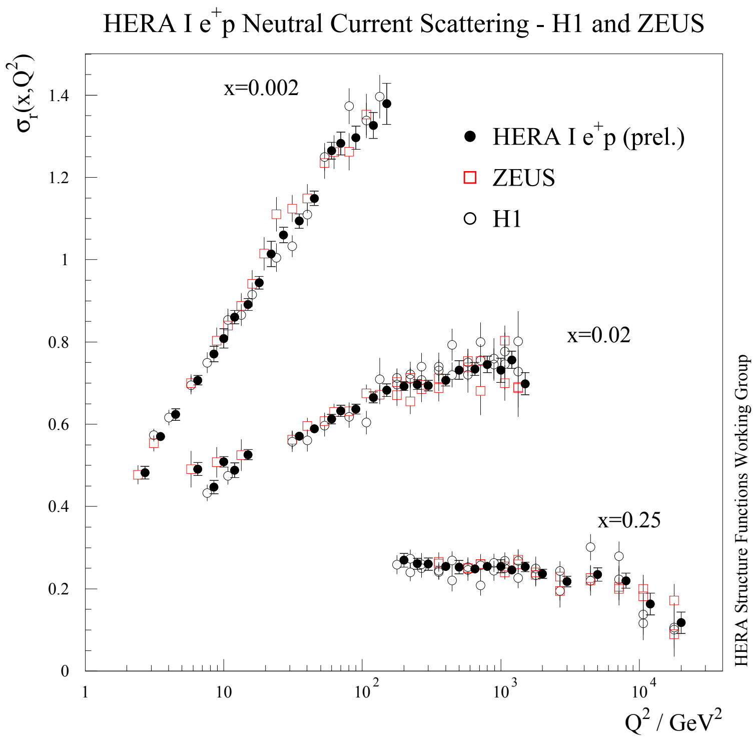
$\Rightarrow$ Representation for 3 Loop Wilson Coeff.: 35 Functions, J.B., 2009.

Complex Analysis of these Functions

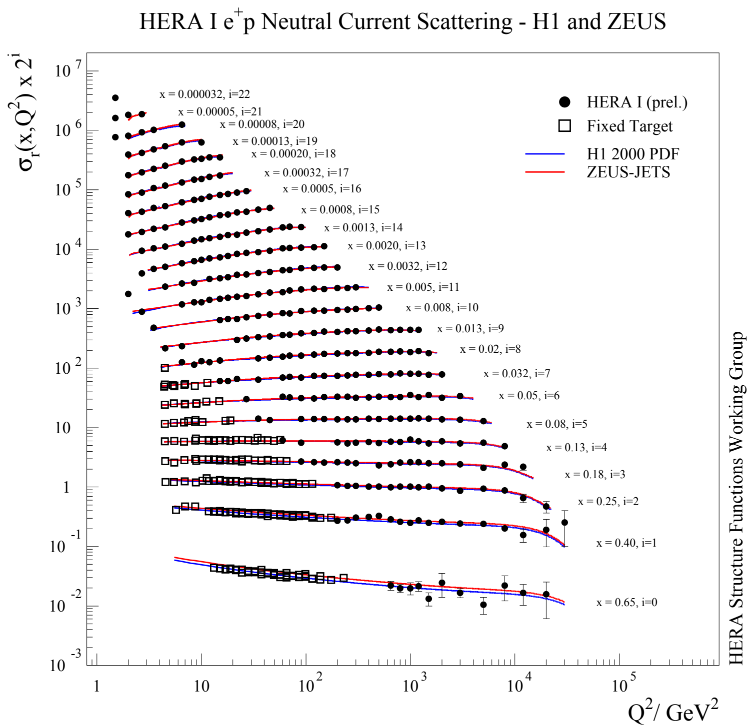
- Construct exact analytic continuations to complex N
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \to z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all Q^2
- Include the heavy flavor Wilson coefficients in Mellin Space
$\Longrightarrow$ nearly accomplished to $O(a_s^3)$ I. Bierenbaum, JB, S. Klein (2009)
- Perform a single fast, numerical Mellin inversion
(at high precision)

$\Rightarrow$ **Fastest and most Precise Way of Analysis**

3. Unpolarized Parton Distribution Functions



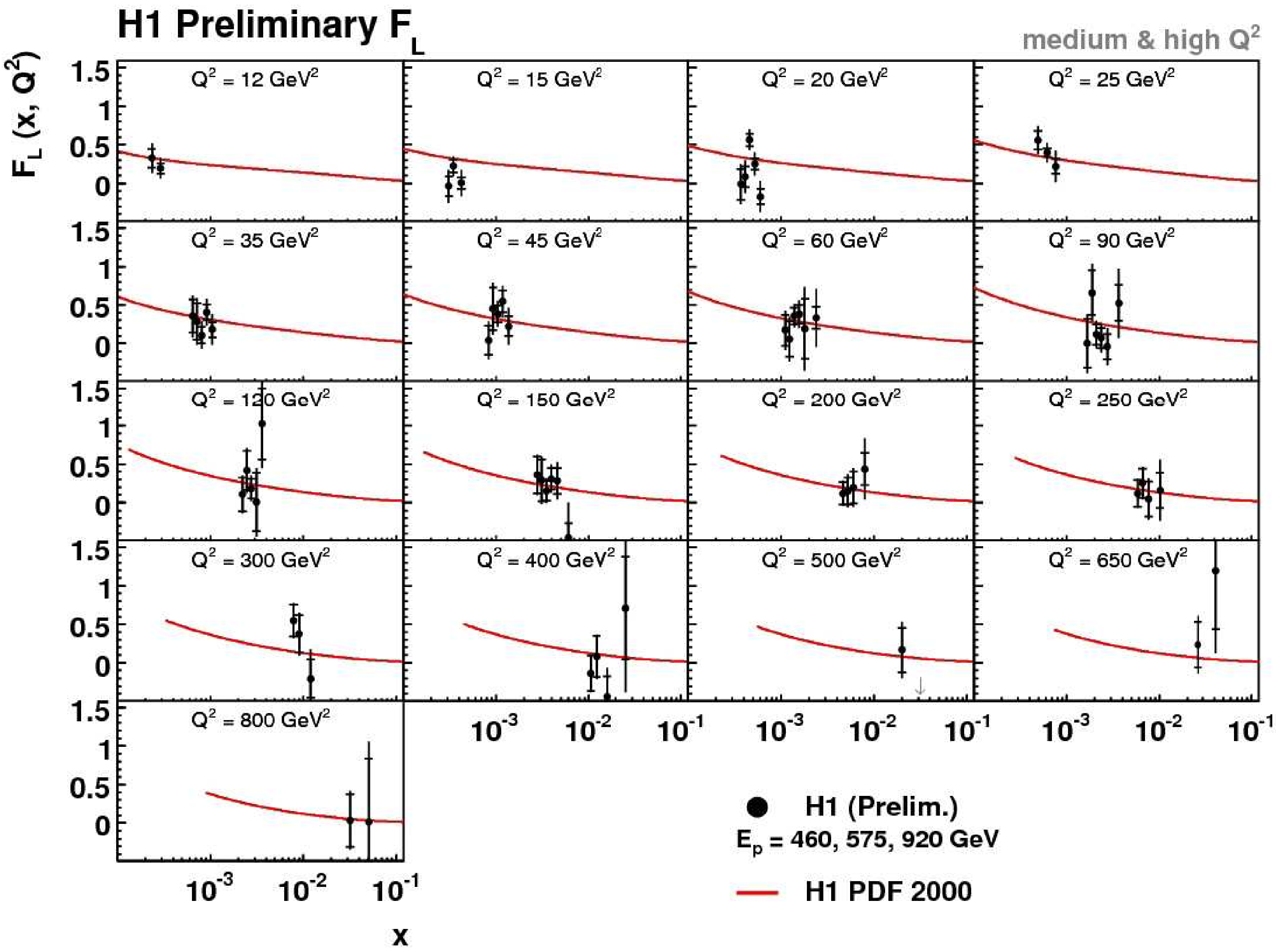
DIS08 Joël Feltesse



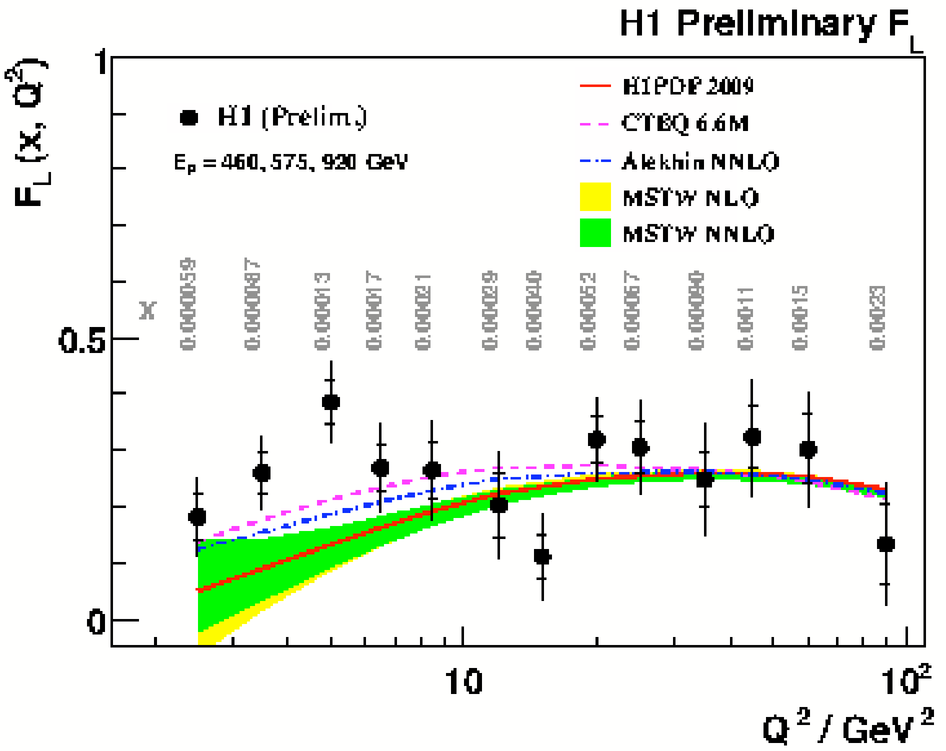
DIS08 Joël Feltesse

New ZEUS + H1 averaged $F_2(x, Q^2)$

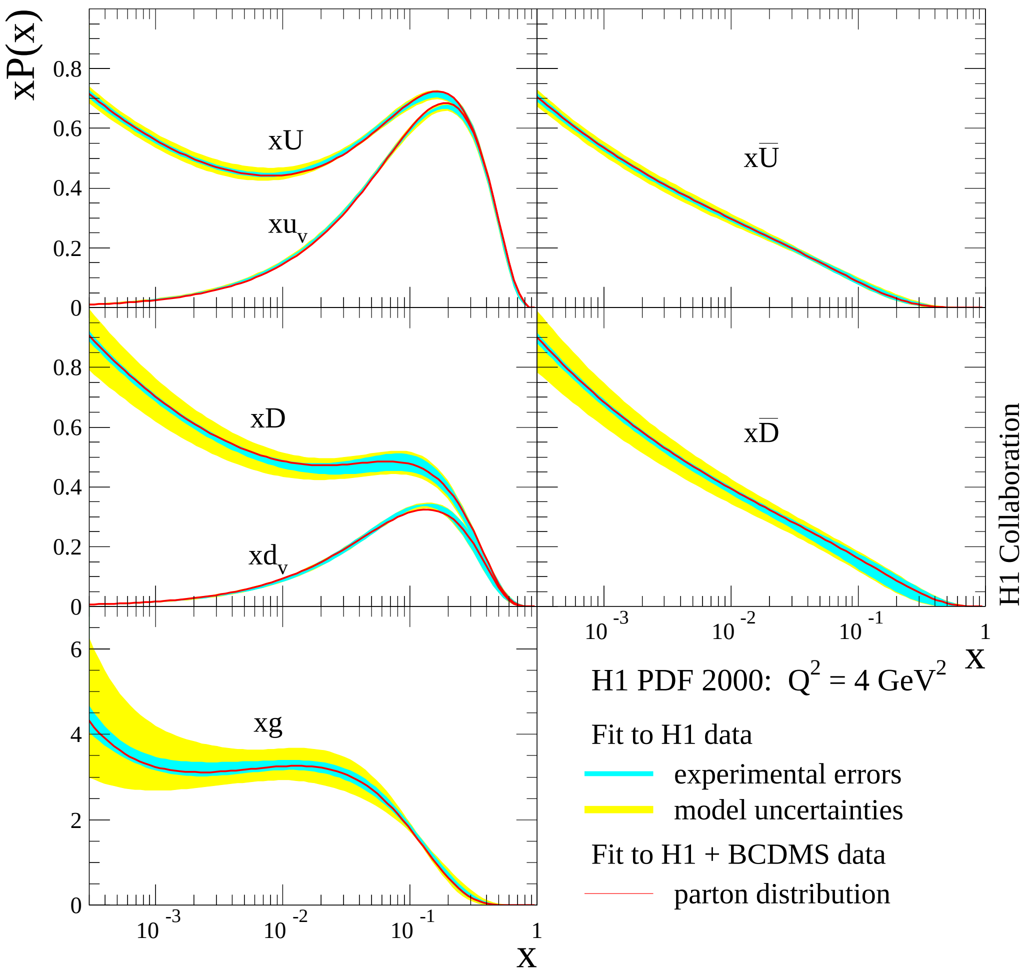
Direct $F_L(x, Q^2)$ Measurement at HERA



Direct $F_L(x, Q^2)$ Measurement at HERA (H1-prel.)

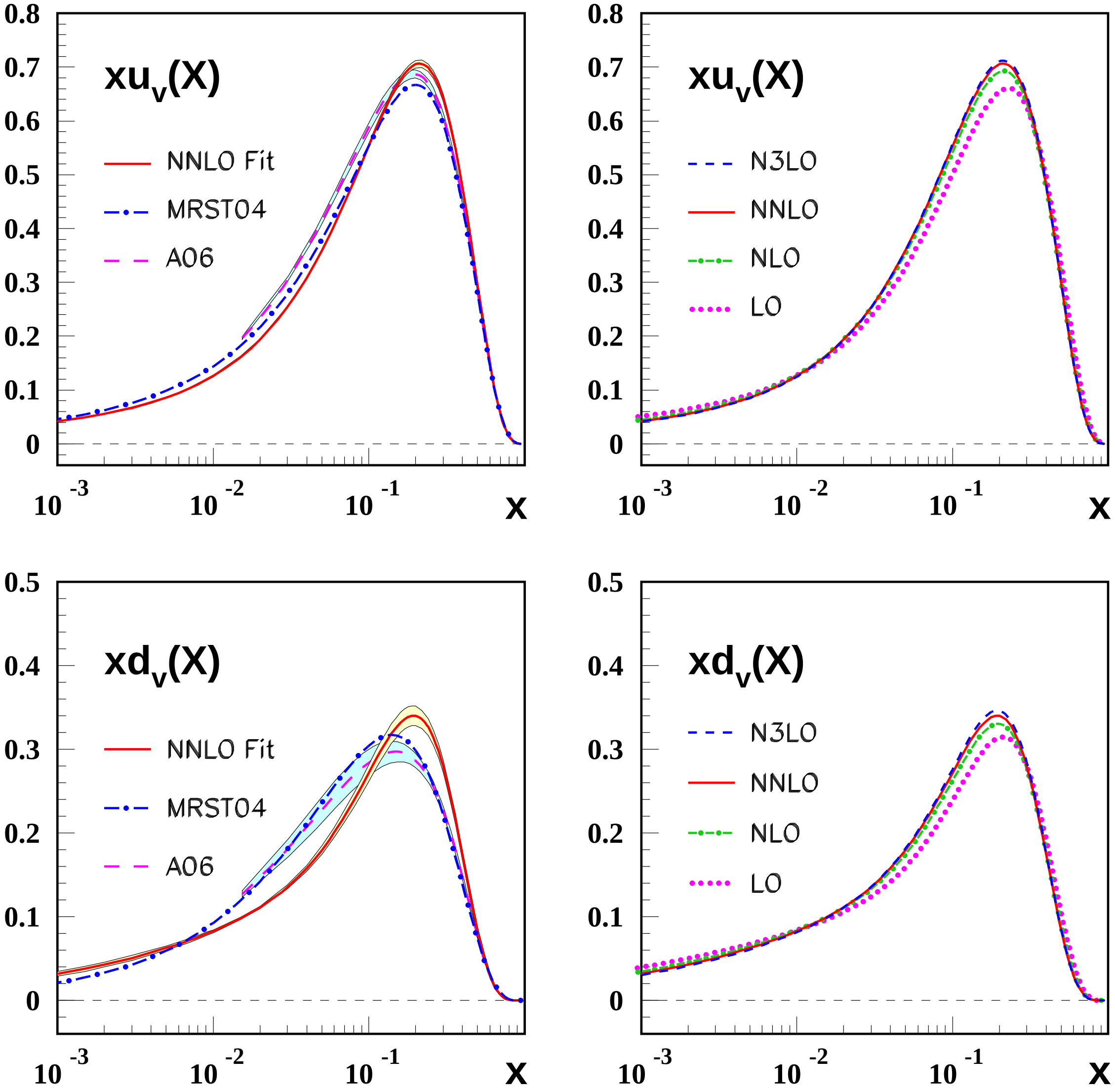


Parton Distributions: Overview



H1

World Data Analysis: Valence Distributions



World data:
NS-analysis

$W^2 > 12.5$ GeV^2, $Q^2 > 4$ GeV^2

N^{3}LO :

$$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ ± 2 MeV in Λ_{QCD}

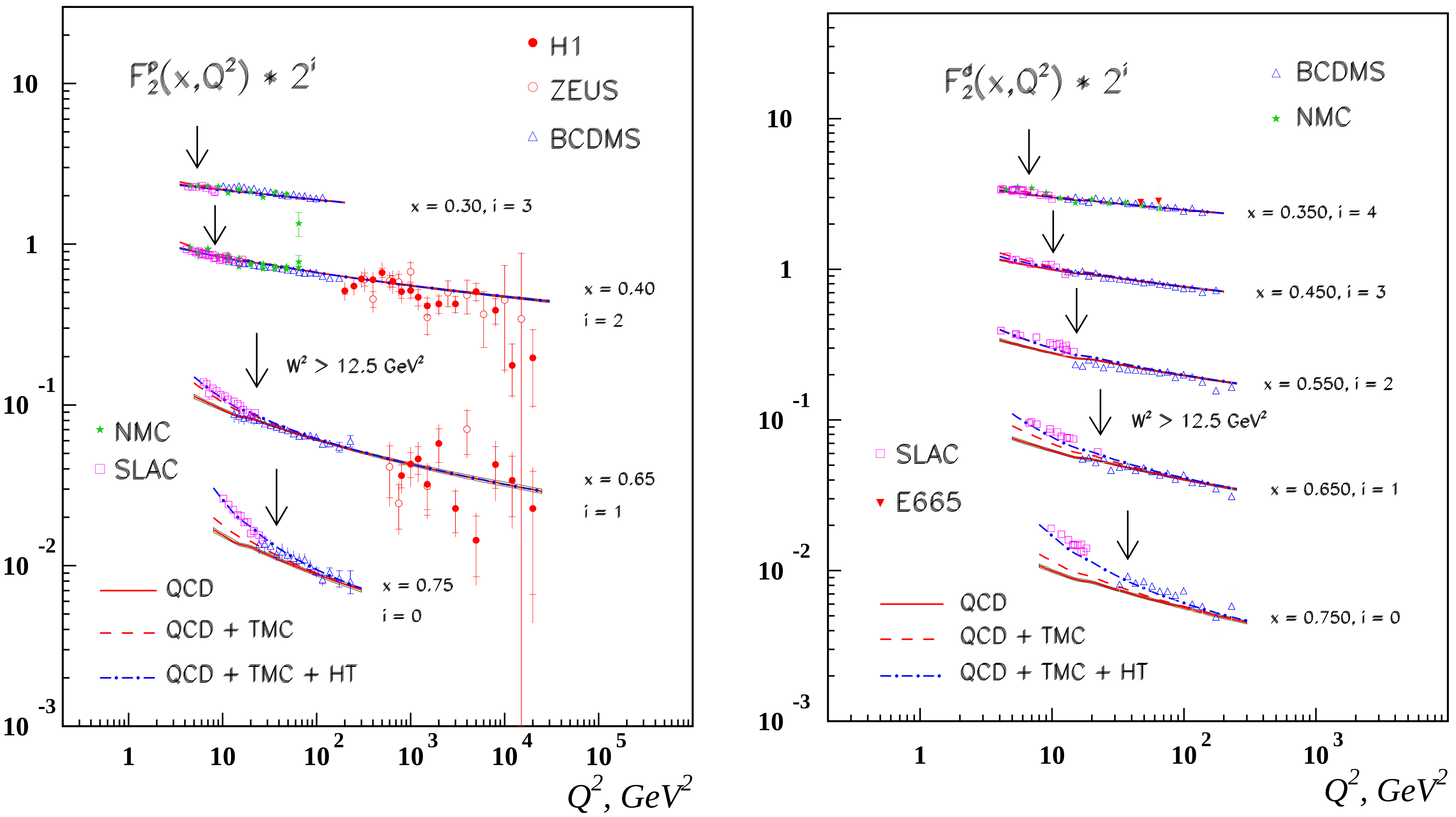
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

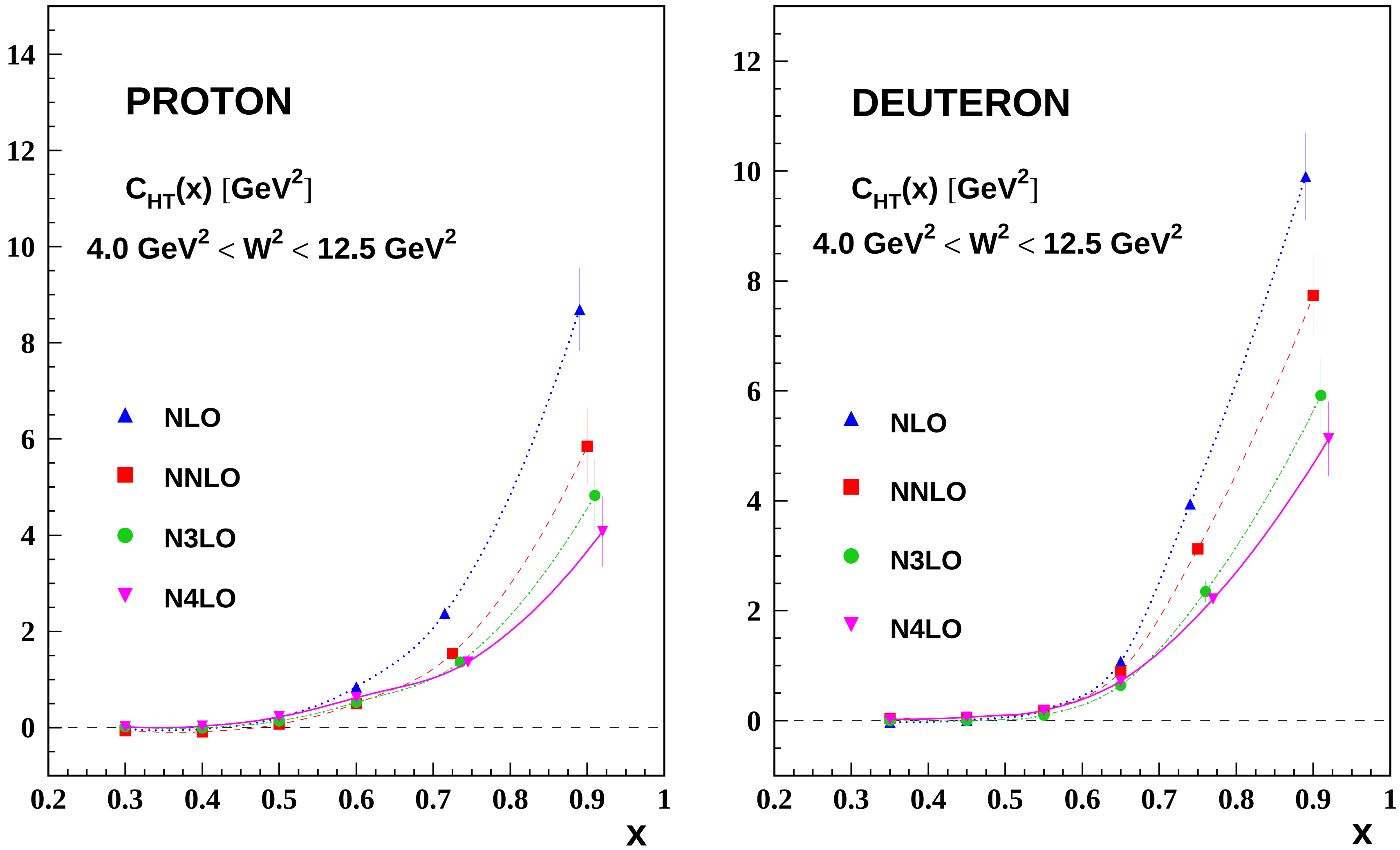
$$\begin{aligned}\gamma_2^{3;NS} &= \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 \\ &+ \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4\end{aligned}$$

The results agree better than 20%.

Valence Distributions

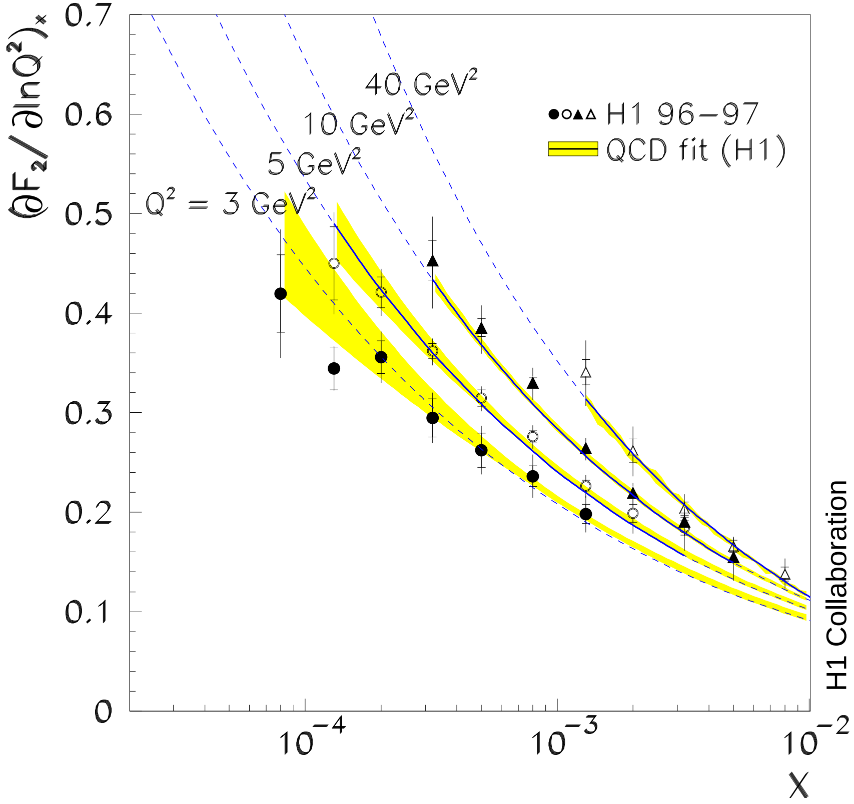


Valence Distributions: higher twist

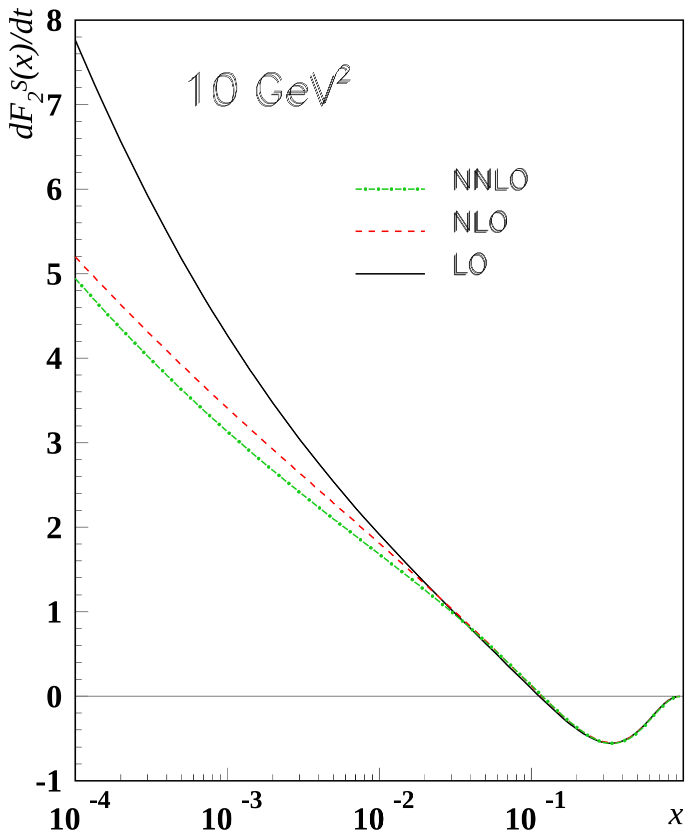


- agreement between p and d analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

Slope of F_2 at low x



H1



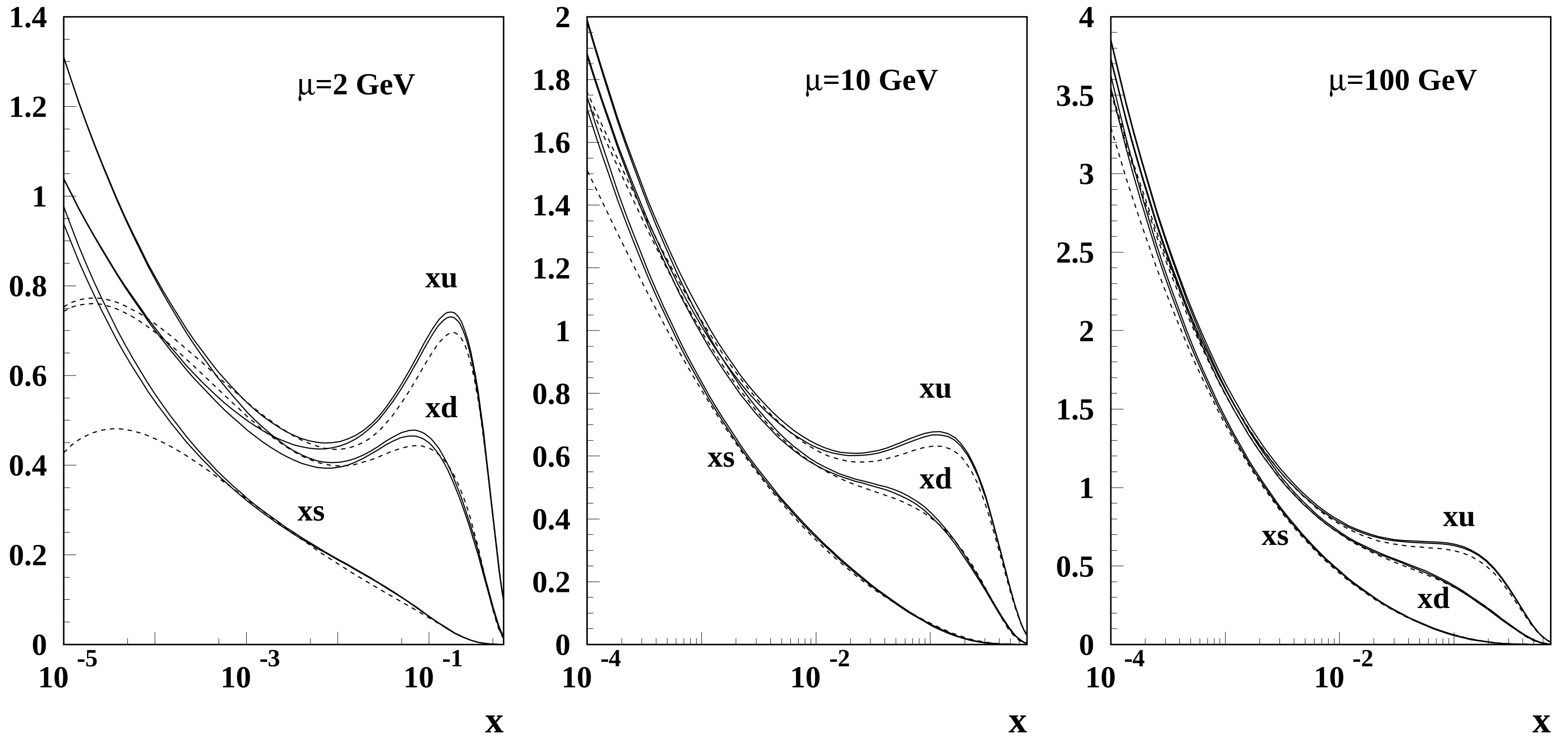
J.B., A. Guffanti 2005

Very likely, that the $\overline{\mathrm{MS}}$–gluon is remains positive!

Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO):

+ Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...



S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

Correct treatment of HQ very essential: FFNS, BSMN-schemes.

full lines: ABKM error band; dashed lines: MSTW08

Flavor distributions: strangeness

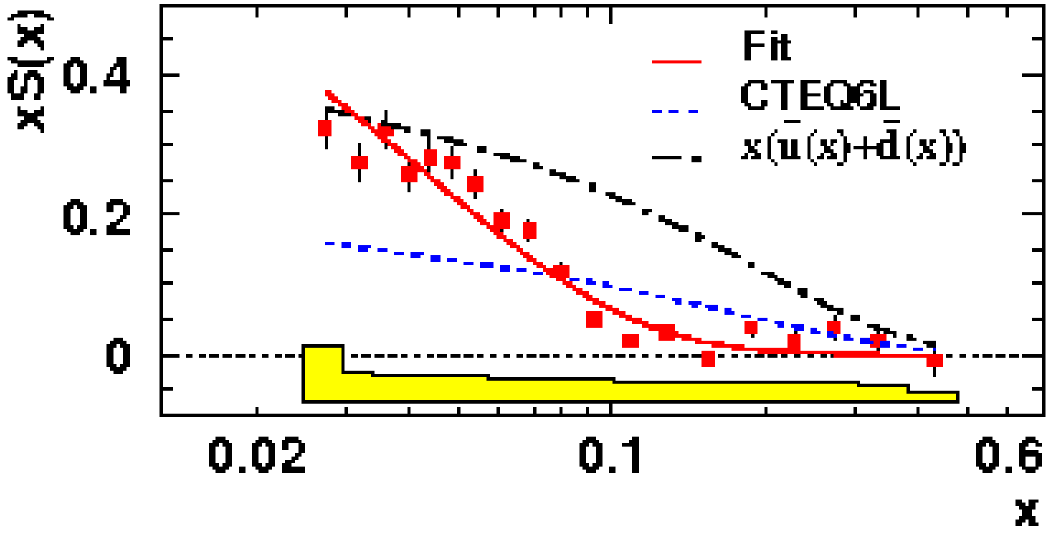
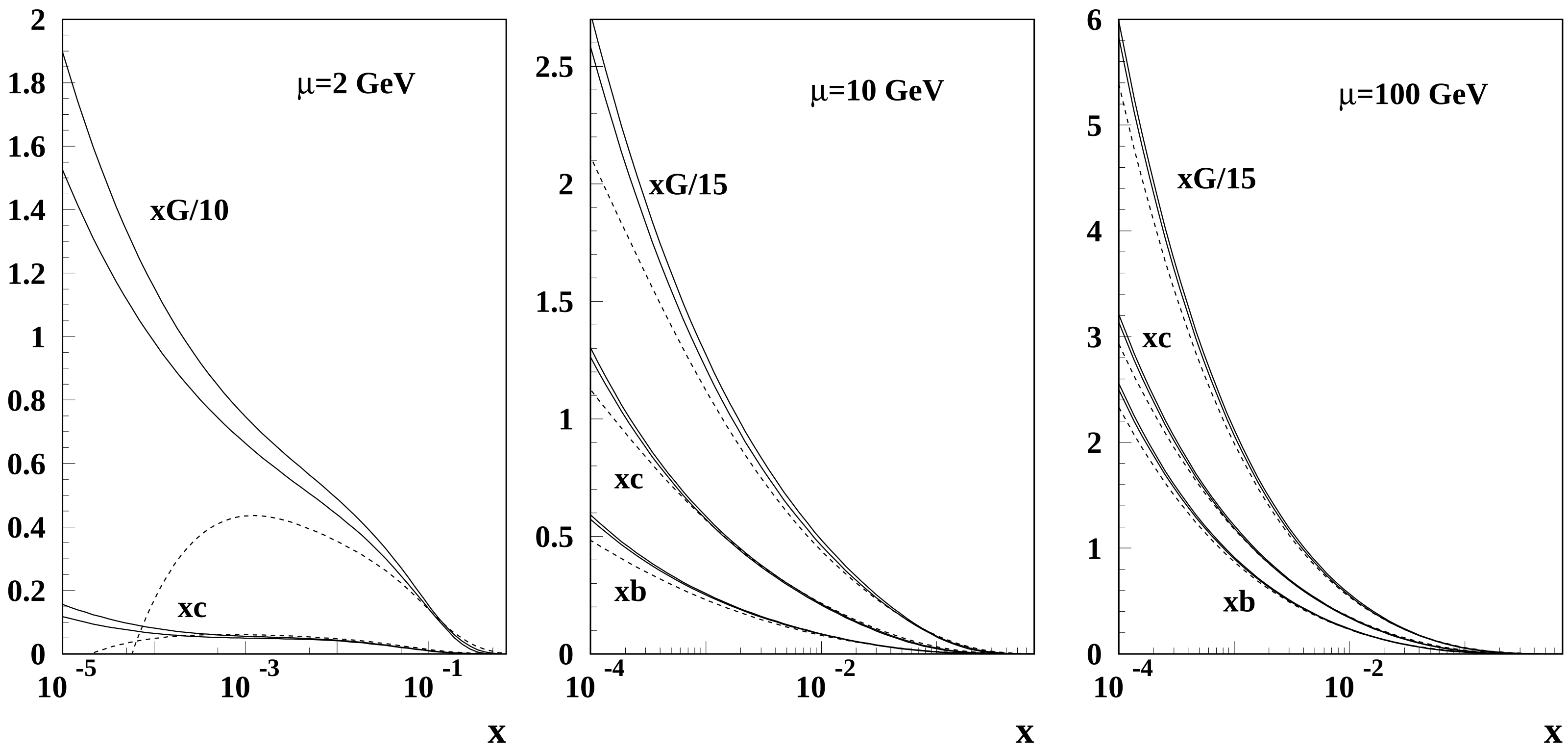


FIG. 3: The strange parton distribution $xS(x)$ from the measured HERMES multiplicity for charged kaons evolved to $Q_0^2 = 2.5$ GeV^2 assuming $\int \mathcal{D}_S^K(z)dz = 1.27 \pm 0.13$. The solid curve is a 3-parameter fit for $S(x) = x^{-0.924} e^{-x/0.0404}(1-x)$, the dashed curve gives $xS(x)$ from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

Nice HERMES measurement (hep-ex/0803.2993); still to be understood.

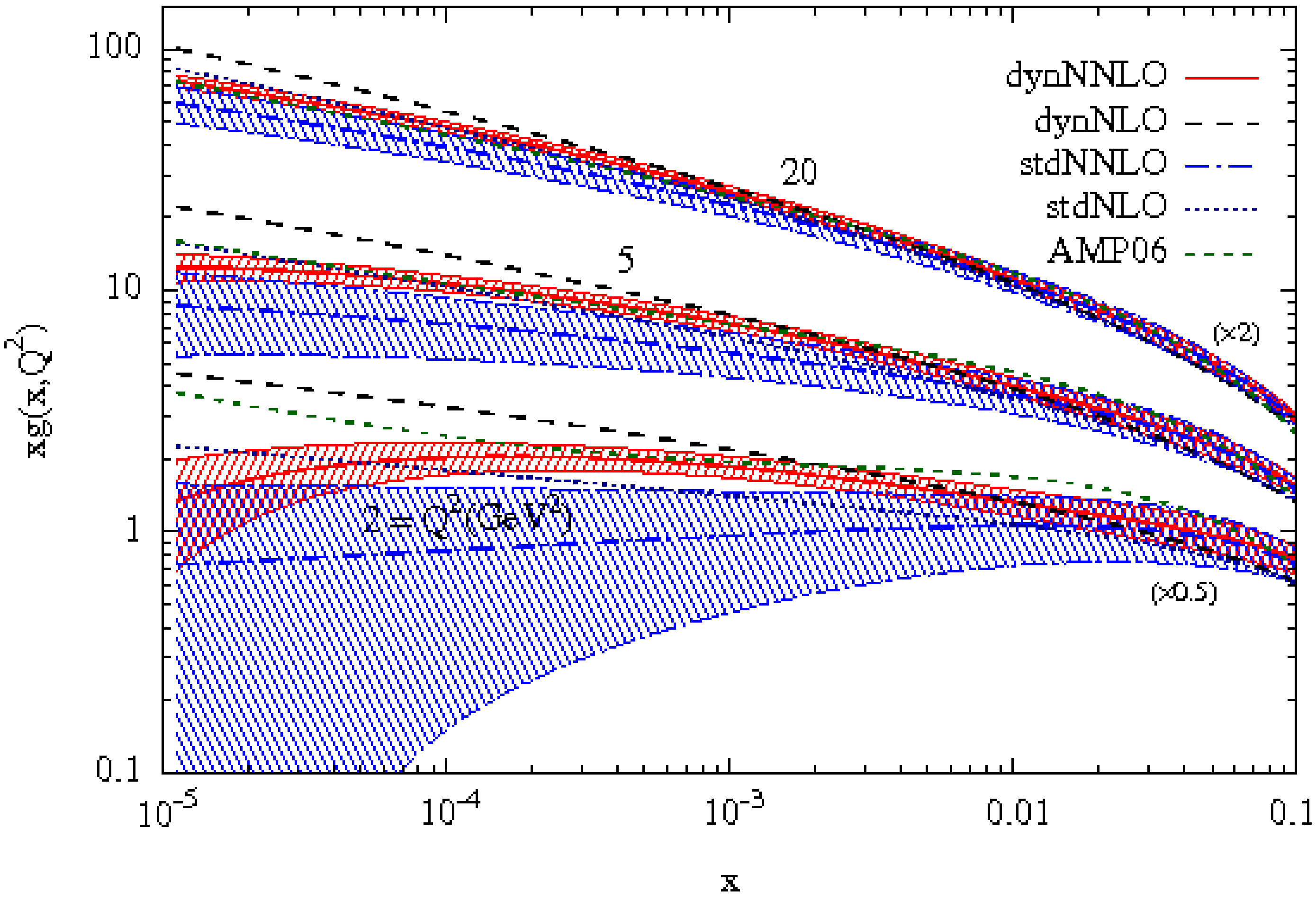
Heavy quarks and gluon (NNLO)



S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

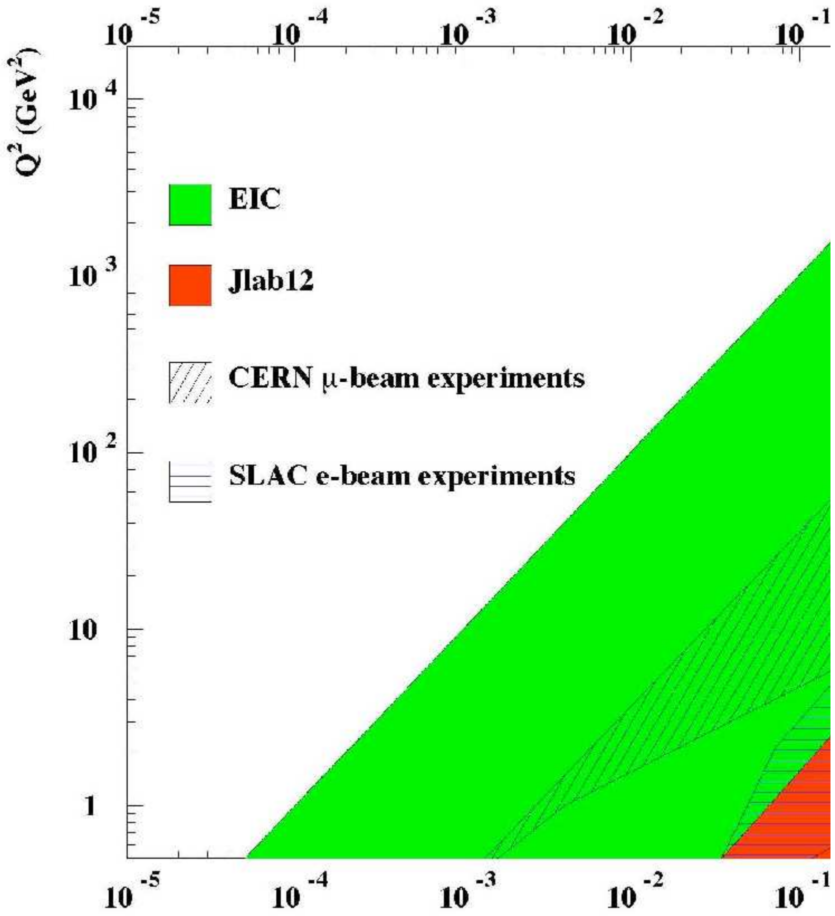
full lines: ABKM error band; dashed lines: MSTW08

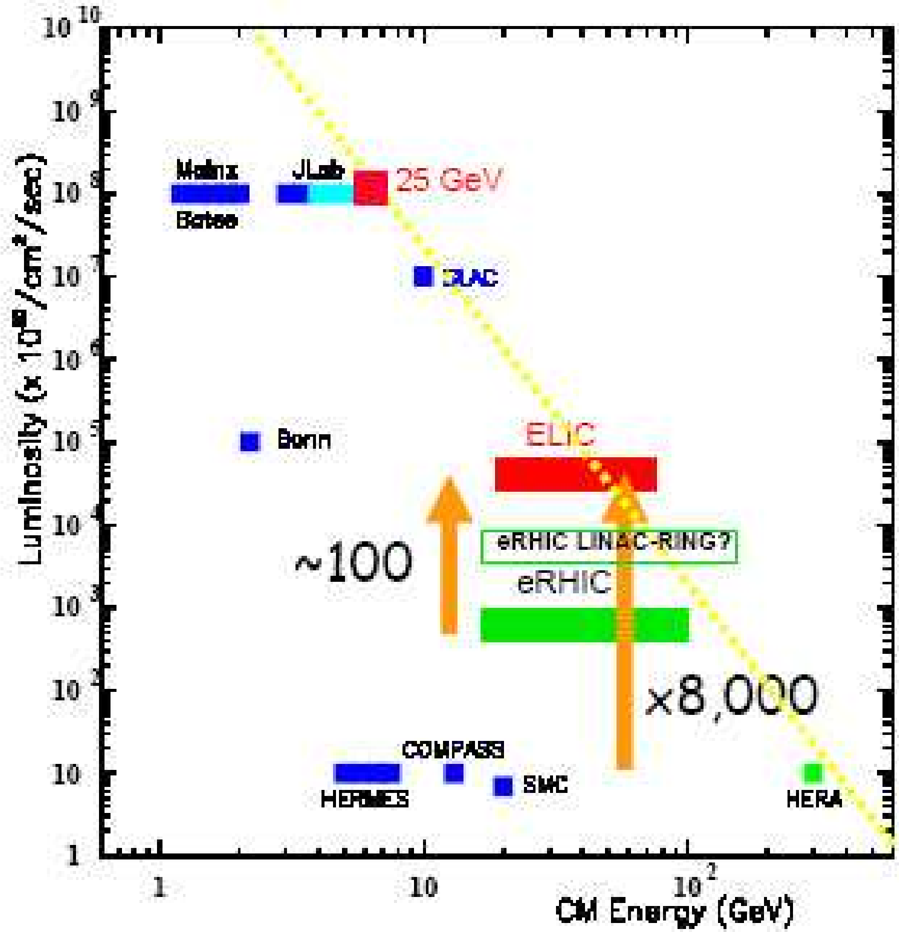
Gluon (NNLO)



Jimenez-Delgado/ Reya (2008)

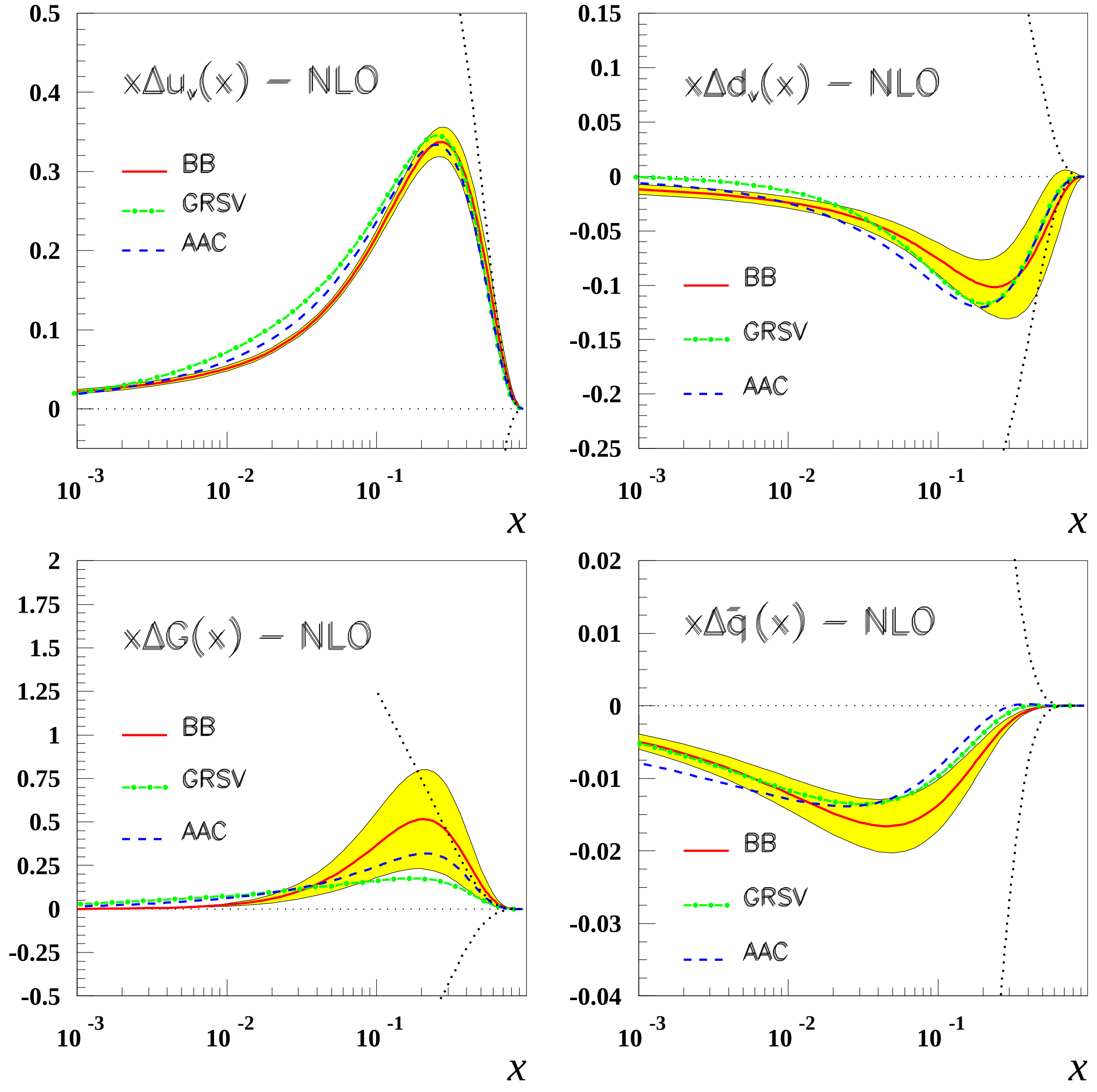
4. Polarized Structure Functions





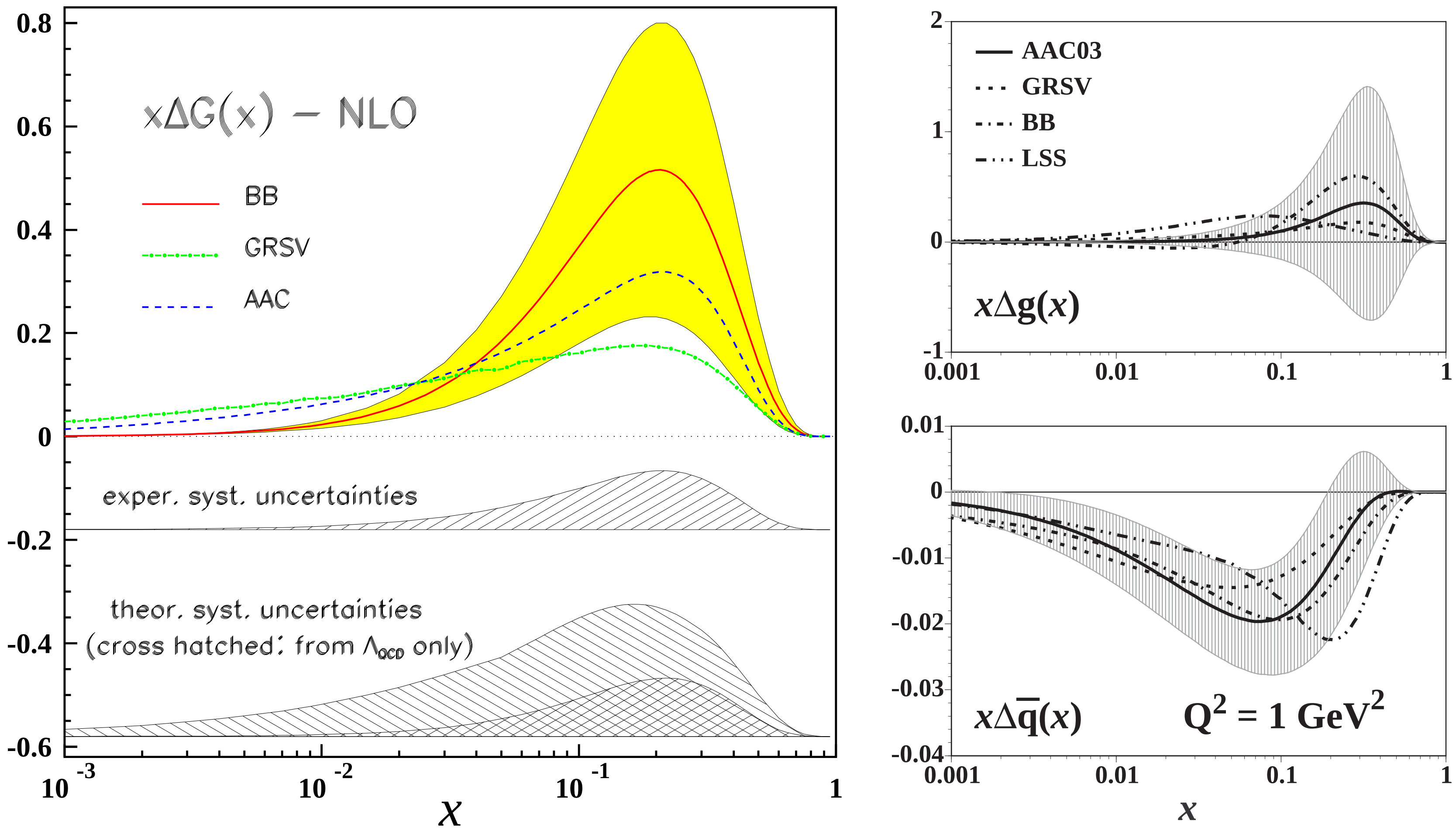
High Luminosity is most important: Various precision measurements.

Polarized Parton Densities at Present



J.B., H. Böttcher (2002)

The Polarized Gluon Distribution at Present

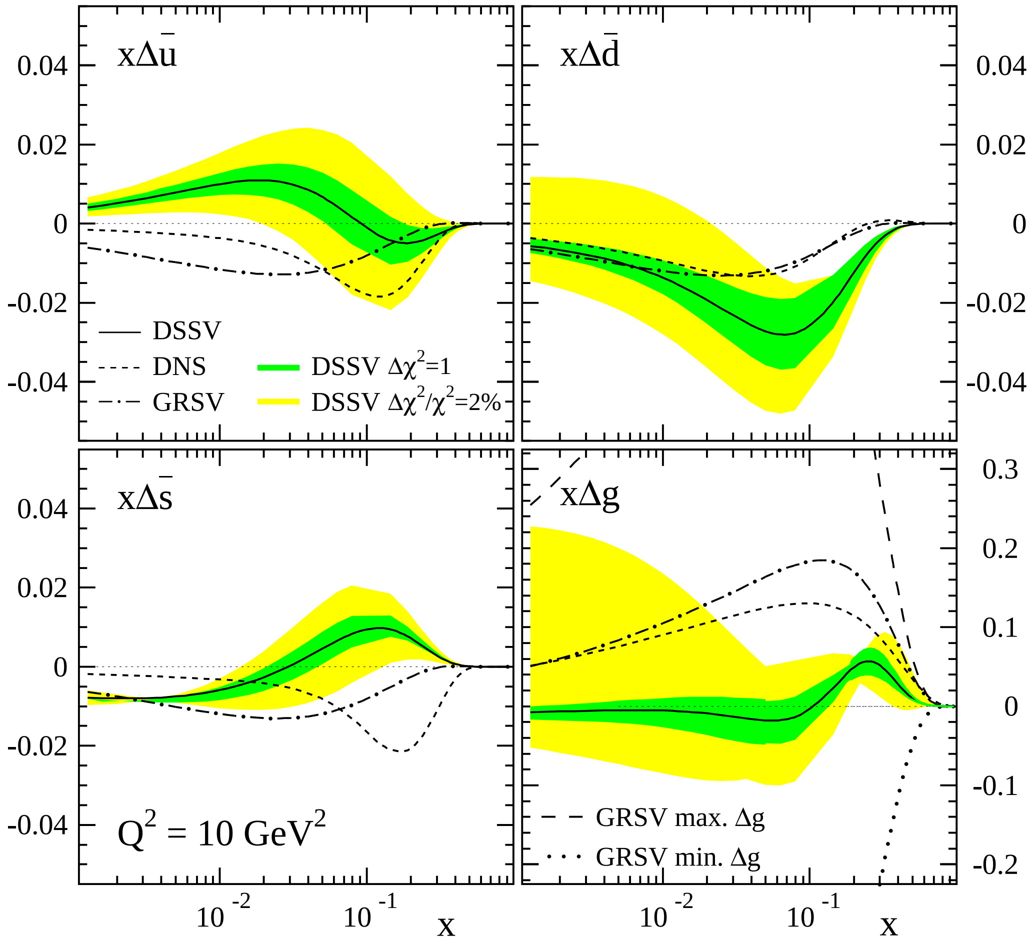


J.B., H. Böttcher (2002) AAC

$\Longrightarrow$ Currently slight move of ΔG towards lower values

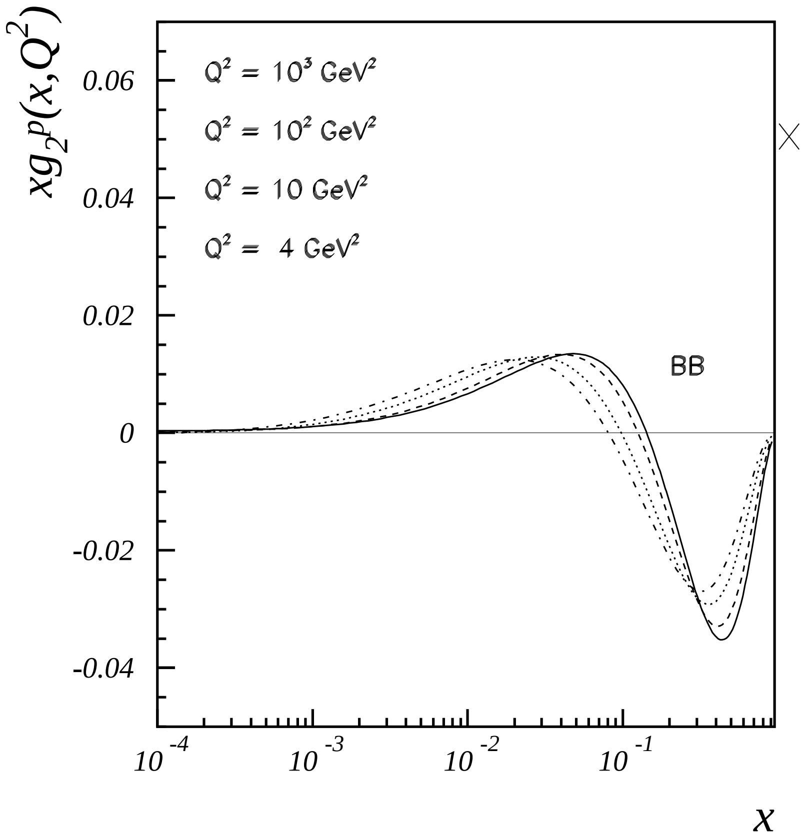
$\Longrightarrow$ 3-loop analysis would settle theory error.

Unfolding the Sea Quarks

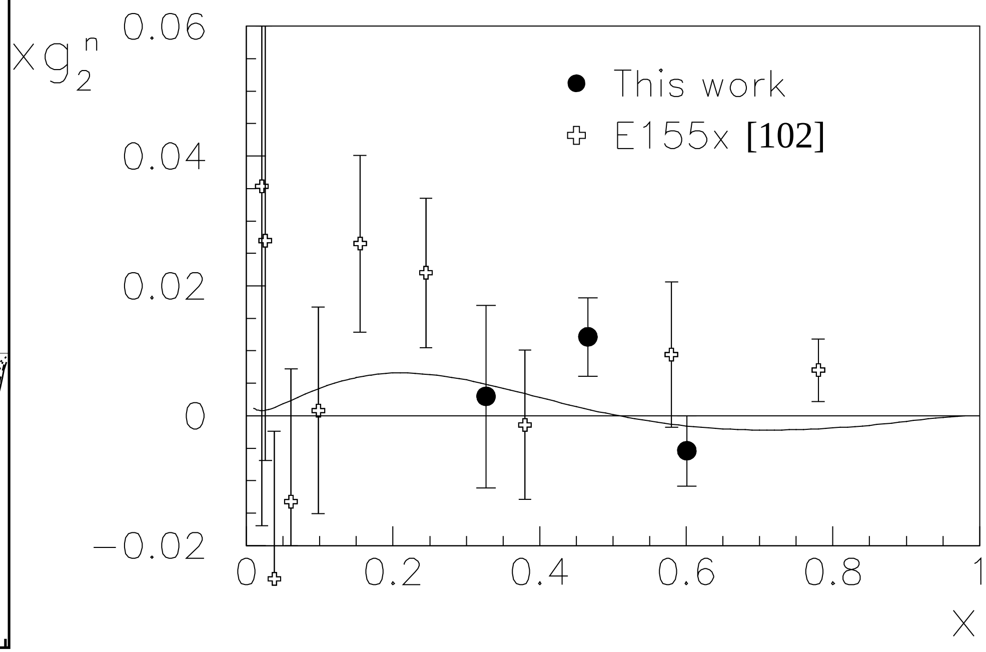


De Florian, Sassot, Stratmann, Vogelsang, 2008

$g_2(x, Q^2)$ - a Window to Higher Twist



$g_2^{\tau=2}(x, Q^2)$ (light partons)



JLAB Hall A, 2004

Accurate measurement highly desired.
How big is the $\tau = 3$ contribution ?

Moments of PDF's: PT + data

f	n	This Fit N^{3}LO	MRST04 NNLO	A02 NNLO
u_v	2	0.3006 ± 0.0031	0.285	0.304
	3	0.0877 ± 0.0012	0.082	0.087
	4	0.0335 ± 0.0006	0.032	0.033
d_v	2	0.1252 ± 0.0027	0.115	0.120
	3	0.0318 ± 0.0009	0.028	0.028
	4	0.0106 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1754 ± 0.0041	0.171	0.184
	3	0.0559 ± 0.0015	0.055	0.059
	4	0.0229 ± 0.0007	0.022	0.024

J.B., H. Böttcher, A. Guffanti, 2006

	Moment	BB, NLO
Δu_v	0	0.926
	1	0.163 ± 0.014
	2	0.055 ± 0.006
Δd_v	0	-0.341
	1	-0.047 ± 0.021
	2	-0.015 ± 0.009
$\Delta u_v - \Delta d_v$	0	1.267
	1	0.210 ± 0.025
	2	0.070 ± 0.011

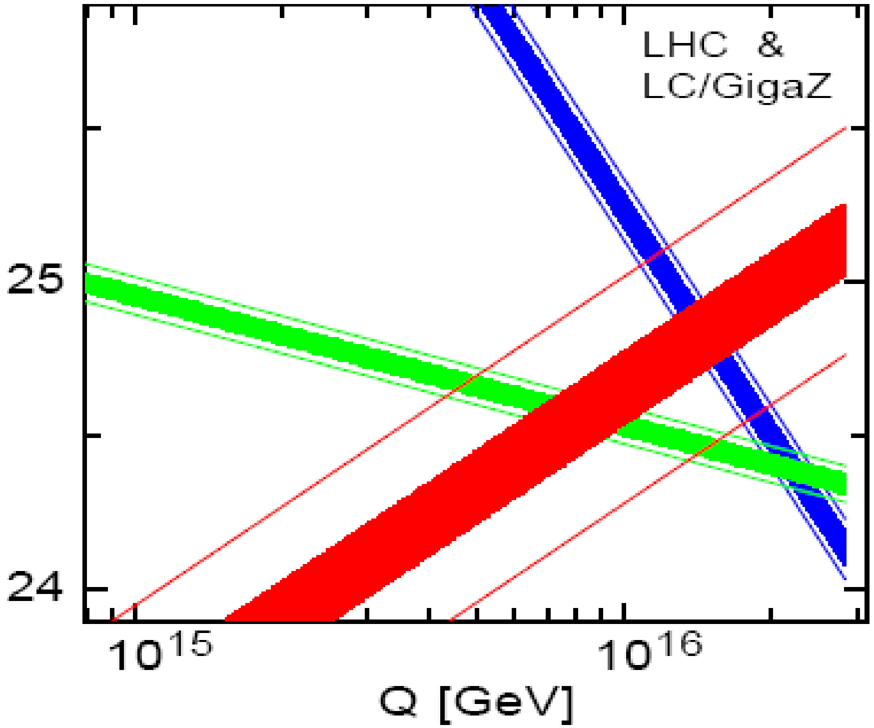
J.B., H. Böttcher, 2002

Lattice Results : developing; different fermion-types studied. Low values of m_π crucial; values approach 270 MeV now.

5. Λ_{QCD} and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{\rm em}(0)}{\alpha_{\rm em}(0)} \sim 3 \cdot 10^{-11} \qquad \frac{\delta\alpha_{\rm weak}}{\alpha_{\rm weak}} \sim 7 \cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}$$

(until recently)



P. Zerwas, 2004

Overview of the Analyses

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N^{3}LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
GRS	0.112			[10]
BBG	0.1148	± 0.0019		[9]
BB (pol)	0.113	± 0.004	${}^{+0.009}_{-0.006}$	[7]

NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]
N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

NNLO and N^{3}LO

- BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

Lattice results :

- Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV
- QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger, but no quenched result.

$\alpha_s(M_Z^2)$

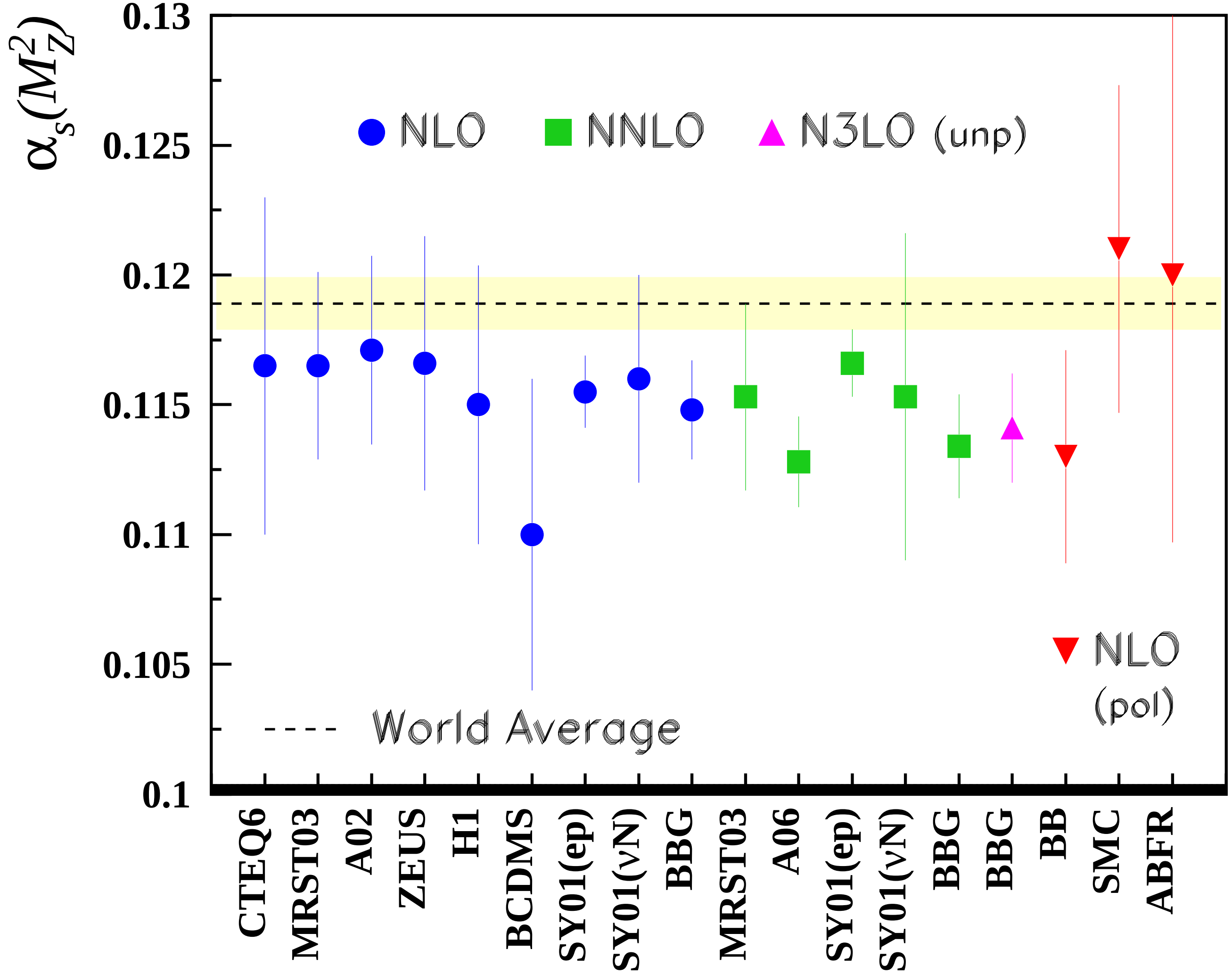
S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102

$$\frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \approx 1.2\%$$

(obtained by July 1st)

	$\alpha_s(\mathrm{M_Z^2})$	
ABKM	0.1135 ± 0.0014	HQ: FFS $N_f = 3$
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach
BBG (2006)	$0.1134 \begin{array}{l} +0.0019 \\ -0.0021 \end{array}$	valence analysis, NNLO
JR (2008)	0.1124 ± 0.0020	dynamical approach
MSTW (2008)	0.1171 ± 0.0014	
BBG (2006)	$0.1141 \begin{array}{l} +0.0020 \\ -0.0022 \end{array}$	valence analysis, $\mathrm{N^3LO}$

$\alpha_s(M_Z^2)$

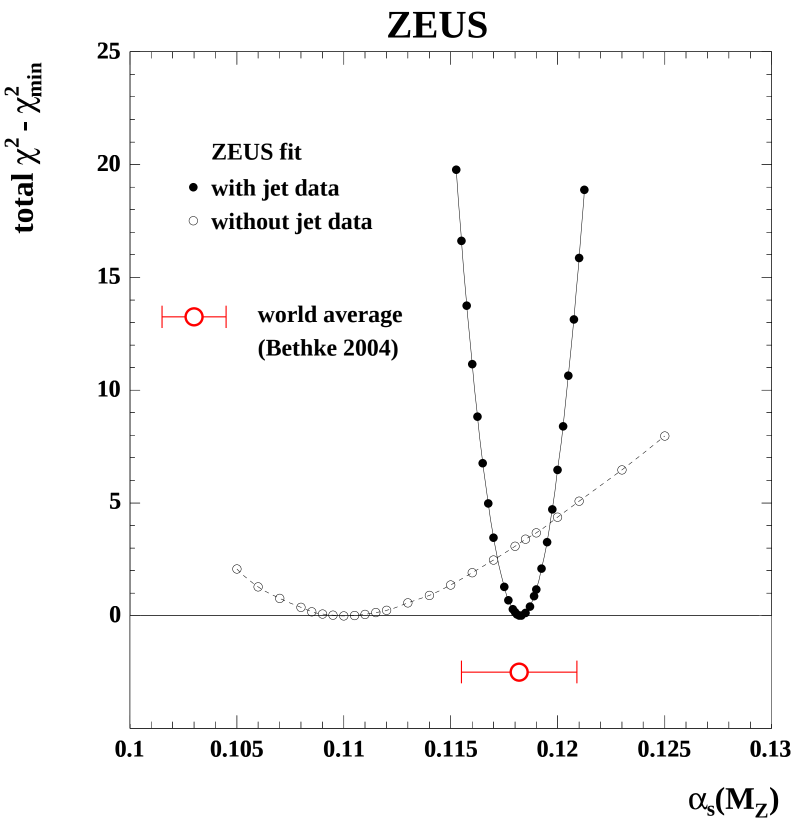


J.B., H. Böttcher, A. Guffanti, 2006

More Global Analyses

- $\alpha_s(M_Z^2)$ for different data sets included are too different !

$\Rightarrow$ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet



M. Cooper-Sarkar, 2005

6. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation mandatory: `QGRAF`
  # 2500 - 15000 diagrams

- The 'Only' problem: Calculation of Feynman Parameter Integrals; everything else automated: `FORM`-codes

- Renormalization still not always trivial: γ_5, mass(es), ...

- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier values, etc. - Avoids the problem of Babel  in analytic integration

- Generalized Hypergeometric Functions and their Generalizations are to the Heart of the Matter. M. Kalmykov et al., JB et al.

- Need: advanced Difference Equation Establishers & Solvers: `Sigma`

- **Do not proliferate !,** i.e. avoid IBP, MB, and other methods causing gigantic Zeroes.

- What remains is : **Integrating the hard way.**

Advanced Technologies to Evaluate Feynman Diagrams

Some Examples:

- Zero-scale Problems : Euler-Zagier and Multiple Zeta Values
  JB, D. Broadhurst, J. Vermaseren, DESY 09-03
  find all relations : $\Longrightarrow$ **Tera-Terms** to be processed
  alternating: all relations up to w = 12 (6-loop level);
  non-alternating: all relations up to w = 22; determined.
  Interesting relations: to w = 30;

- Reconstructing recurrent quantities from Mellin Moments
  JB, M. Kauers, S. Klein, C. Schneider DESY 09-02
  Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments ? Yes - recurrent quantities in Mellin space.
  $\leq$ 5114 Moments; difference equation fills 440 books
  Complete computation: 5 CPU Months

- Massive Wilson coefficients at 3 Loops
  I. Bierenbaum, JB, S. Klein, DESY 09-57
  first analytic massive 1-scale calculation @ 3-loops
  Moments 2–10 (12/14) have been calculated for all unpolarized channels
  Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers

7. Outlook

Theory:

- Polarized Anomalous Dimensions & massless Wilson coefficients @ 3 Loops
- Unpolarized Heavy Flavor Wilson coefficients @ 3 Loops : general N
- Polarized Heavy Flavor Wilson coefficients @ 3 Loops
- Along with this: Development of efficient analytic calculation methods being suited for 3-Loops and higher
- ep & pp jet cross sections at HO; progress in pdf Lattice calculations

Code:

- Creation of an Open Source Code for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

Experiment:

- Precision Data from LHC, JLAB and EIC.

Can we get $\delta\alpha_s$ even smaller ?