XXV Int Symposium on
Multiparticle Dynamics
Tatry, September 1995

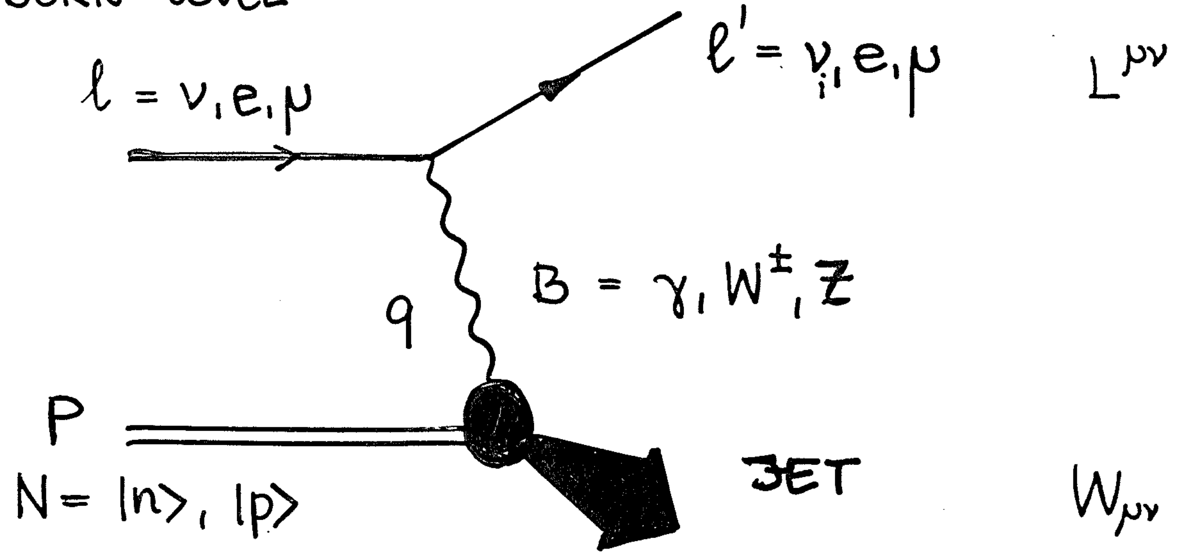
On the theoretical status of deep inelastic scattering

Johannes Blümlein

DESY

1. Introduction

BORN LEVEL:



$$\frac{d^2\sigma}{dx\,dy} \sim L_{\mu\nu} W^{\mu\nu}$$

$W_{\mu\nu} \longrightarrow F_i(x, Q^2)$ STRUCTURE FUNCTIONS

$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2Pq} = \frac{Q^2}{sy}, \quad y = \frac{Pq}{P\ell}$$

STRUCTURE FUNCTIONS AT BORN LEVEL:

UNPOLARIZED N, $N = p, n$.

$\ell^{\pm} N$:

$$F_2 = x \sum_q e_q^2 (q + \bar{q}) \qquad \gamma^2$$

$$G_2 = 2x \sum_q e_q v_q (q + \bar{q}) \qquad \gamma Z$$

$$H_2 = x \sum_q (v_q^2 + a_q^2)(q + \bar{q}) \qquad Z^2$$

$$x G_3 = 2x \sum_q e_q a_q (q - \bar{q}) \qquad \gamma Z$$

$$x H_3 = 2x \sum_q v_q a_q (q - \bar{q}) \qquad Z^2$$

$$W_2^{\pm}, \ x W_3^{\pm} \qquad W^{+2}, W^{-2}$$

$\overset{(-)}{\nu} N$:

$$W_2^{\pm}, \ x W_3^{\pm} \qquad W^{+2}, W^{-2}$$

$$F_{2Z}, \ x F_{3Z} \qquad Z^2$$

11 SF's

$O(\alpha_s)$: $F_L, G_L, H_L, W_L^{\pm}, F_{LZ}$ — 6 SF's

SIMILARLY: POL. ℓ – POL. N:

(NUMBER OF PARTON DENSITIES IS SMALLER (TWIST 2).)

g_1 $|\gamma|^2$

g_2

$\vdots$

- NOT ALL THESE STRUCTURE FUNCTIONS CAN BE MEASURED EASILY (NEITHER NOW, NOR IN FUTURE).

- GOOD ACCESS:

$$\left.\begin{array}{l} F_2^{\ell p}, \; F_2^{\ell d} \\ W_2^{\nu N} = \frac{1}{2}(W_2^+ + W_2^-) \\ xW_3^{\nu N} = \frac{1}{2}x\,(W_3^+ + W_3^-) \end{array}\right\} \text{QCD Analysis}$$

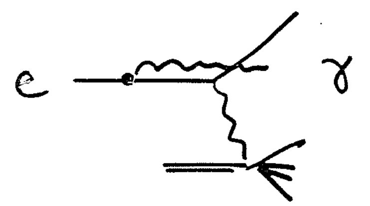
$F_L^{\ell p, \ell d}$ (x shape & may be more!)

$x G_3^{\ell p, d}$ (x shape) ↘ later: $G(x, Q^2)$.

F_{2Z} – x shape

QED RC's:

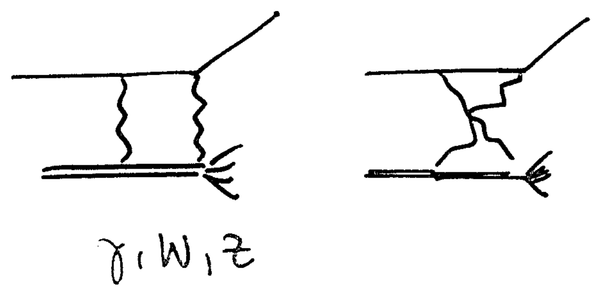
TO BE HANDELED BEFORE!



ISR

$|q^2| \to |z q^2| \quad \infty \quad \log\left(\frac{Q^2}{m_e^2}\right)$!

CAN BECOME VERY SMALL!



Q^2 ??

2. QED radiative corrections to DIS

$O(\alpha)$ corr.

EARLY PAPERS: HO, TSAI 1969

$e(\mu)$ N. BARDIN, FEDORENKO, SHUMEIKO 1981
CONSOLI, GRECO 1981

$\bar{\nu}$' N DE RUJULA, PETRONZIO, SAVOY-NAVARRO 197?
MARCIANO, SIRLIN 1980, 1982
LLEWELLYN SMITH, WHEATER 1981, 1982
PASCHOS, WIRBEL 1982
LIEDE 1983
BARDIN, DOKUCHAEVA 1984

MORE RECENT CALCULATIONS:

eN: BARDIN, BURDIK, CHRISTOVA, RIEMANN 1987 ew-loops
1989 a,b.
KURAEV, MERENKOV, FADIN 1988
JB 1989
SPIESBERGER (87), 1990, 91 ew. loops
KRIPFGANZ, MÖHRING, SPIESBERGER 1991 HO (lept.)
JB 1991
BARDIN et al. '92, 94, 93
JB 1994 HO other variables + expon.

PROGRAMS: HERACLES (MC)
KWIATKOWSKI, MÖHRING, SPIESBERGER 1992

$\underbrace{\text{HELIOS \& TERAD}}_{\text{HECTOR}}$ 1991
1995
ARBUZOV, BARDIN, BLÜMLEIN, KALINOWSKAYA, RIEMANN

BORN

$$\frac{d^2\sigma}{dxdy} = \frac{d^2\sigma^{(0)}}{dxdy} + \frac{d^2\sigma^{(1)}}{dxdy} + \frac{d^2\sigma^{(2)}}{dxdy}$$

O(α) $$\frac{d^2\sigma^{(1)}}{dxdy} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz P_{ee}^{(1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$

O(α²) $$\frac{d^2\sigma^{(2)}}{dxdy} = \left[\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\right]^2\int_0^1 dz P_{ee}^{(2,1)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dxdy}\right\}$$

(LLA) $$+\left(\frac{\alpha}{2\pi}\right)^2\int_{z_0}^1 dz\left\{\ln^2\left(\frac{Q^2}{m_e^2}\right)P_{ee}^{(2,2)}(z) + \sum_{f=l,q}\ln^2\left(\frac{Q^2}{m_f^2}\right)P_{ee,f}^{(2,3)}(z)\right\}\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dxdy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}}$$

$$\mathcal{J}(x,y,z) = \begin{vmatrix} \partial\hat{x}/\partial x & \partial\hat{y}/\partial x \\ \partial\hat{x}/\partial y & \partial\hat{y}/\partial y \end{vmatrix}$$

$$P_{ee}^{(1)}(z) = \frac{1+z^2}{1-z}$$ O(α)

$$P_{ee}^{(2,1)}(z) = \frac{1}{2}\left[P_{ee}^{(1)}\otimes P_{ee}^{(1)}\right](z)$$ O(α²)

$$= \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z - (1-z)$$

$$P_{ee}^{(2,2)}(z) = \frac{1}{2}\left[P_{e\gamma}^{(1)}\otimes P_{\gamma e}^{(1)}\right](z)$$

$$\equiv (1+z)\ln z + \frac{1}{2}(1-z) + \frac{2}{3}\frac{1}{z}(1-z^3)$$

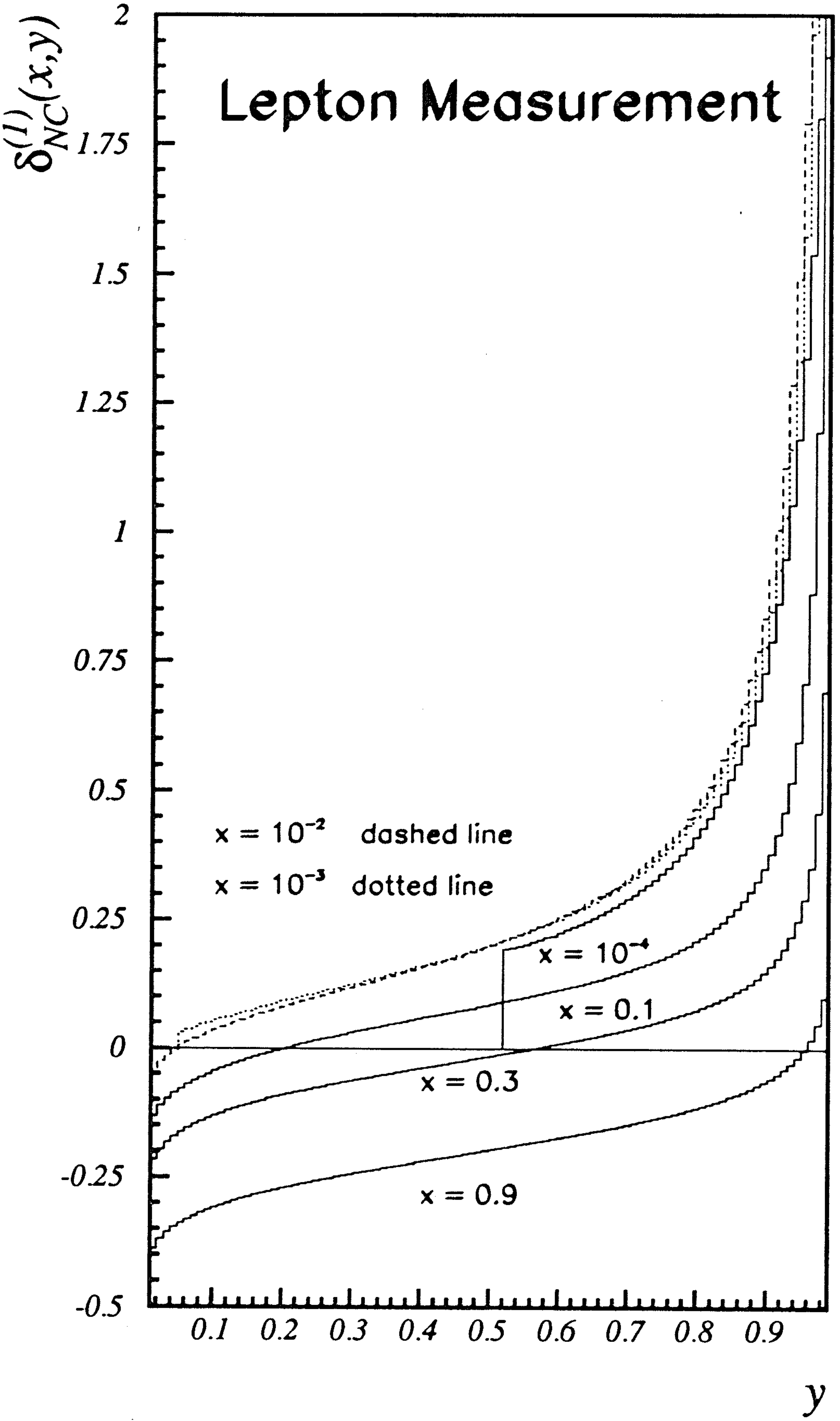
$$P_{ee,f}^{(2,3)}(z) = N_c(f)e_f^2\frac{1}{3}P_{ee}^{(1)}(z)\theta\left(1 - z - \frac{2m_f}{E_e}\right)$$

$$D_{NS}(z,Q^2) = \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2} - 2\gamma_E\right)\right]}{\Gamma(1+\zeta)}$$ SOFT EXPONENTIAT.

$$\zeta = -3\ln\left[1 - (\alpha/3\pi)\ln(Q^2/m_e^2)\right]$$

$$P_{ee}^{>2,\,soft}(z,Q^2) = D_{NS}(z,Q^2) - \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\frac{2}{1-z}\left\{1 + \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\left[\frac{11}{6} + 2\ln(1-z)\right]\right\}$$

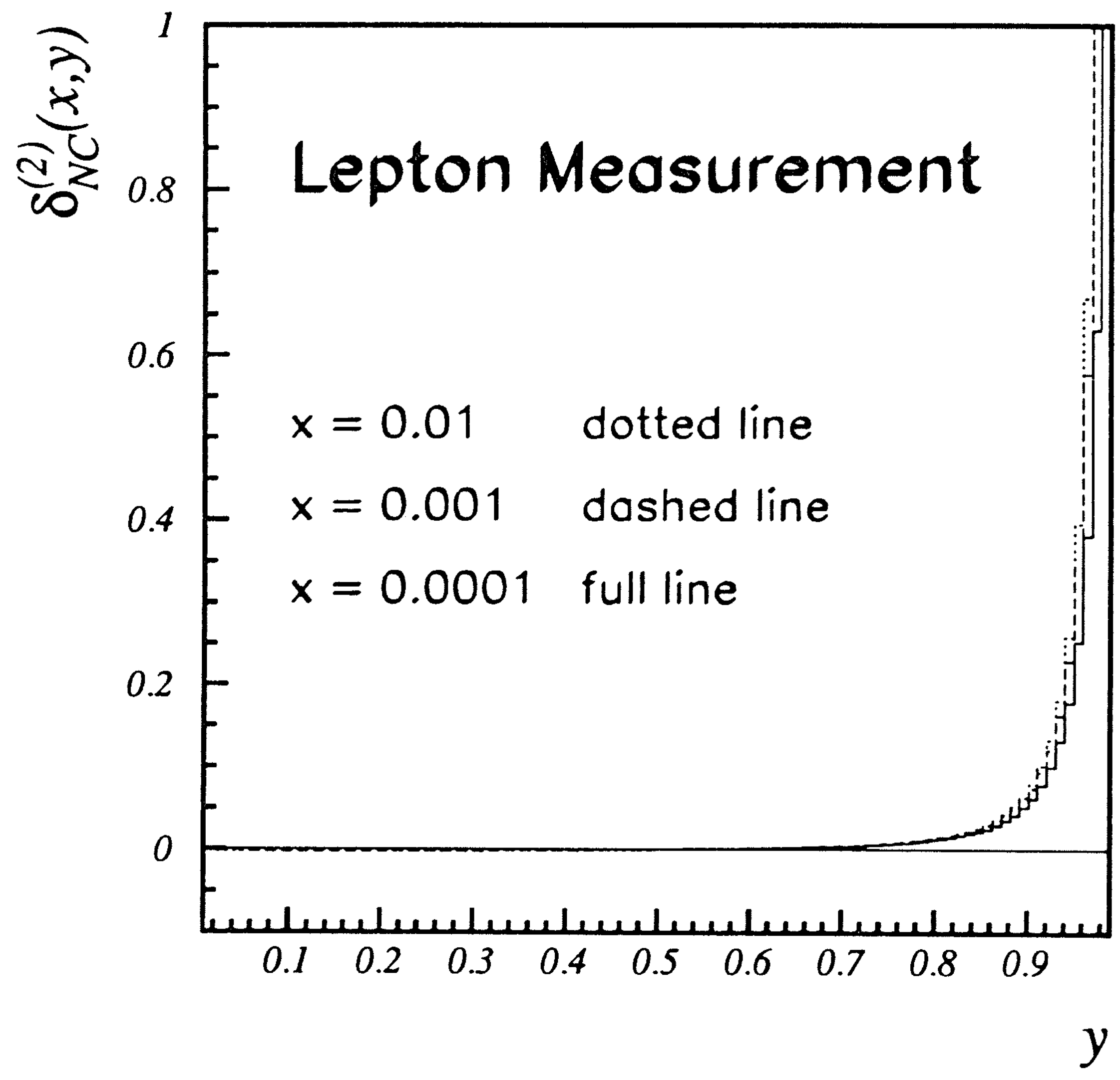
NS: SOFT. + VIRT. O(α³) AND HIGHER.



$e' \ (E'_e, \Theta'_e)$

$Q^2_e, x_e \ (y_e)$

$\log \frac{Q^2}{m_e^2}$ large.



$\delta_{NC}^{(2)}(x,y)$
Lepton Measurement
x = 0.01 dotted line
x = 0.001 dashed line
x = 0.0001 full line
0
0.2
0.4
0.6
0.8
1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

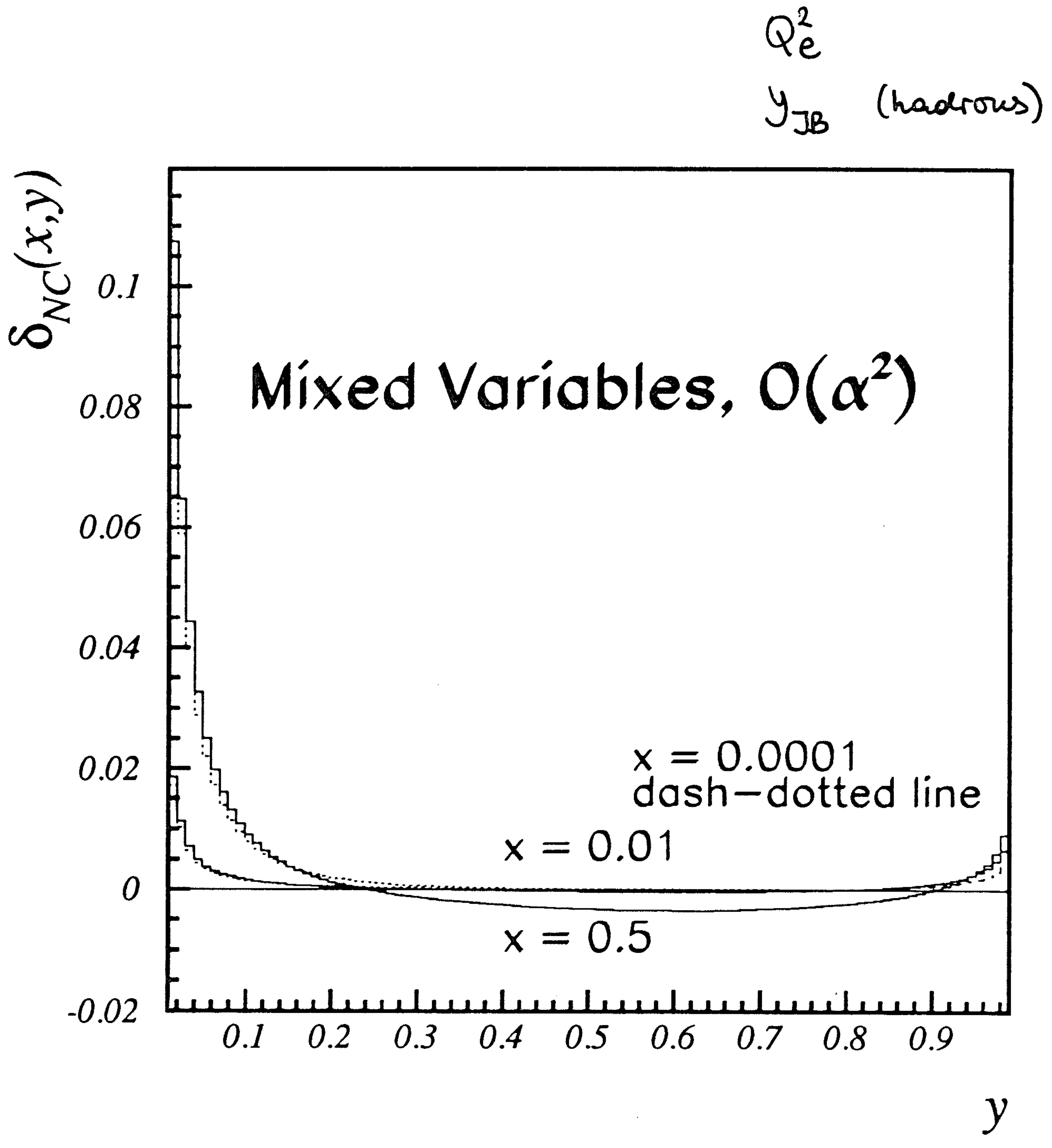


Figure 5: $\delta_{NC}(x,y)$ for the case of mixed variables. Dotted lines: $\delta_{NC}^{e^- \to e^+}(x,y)$; upper line: $x = 0.5$, lower line $x = 0.01$. The other parameters are the same as in figure 3.

Q^2, x from $\theta_{e'}, \theta_{Jet}$.

(ZEUS)

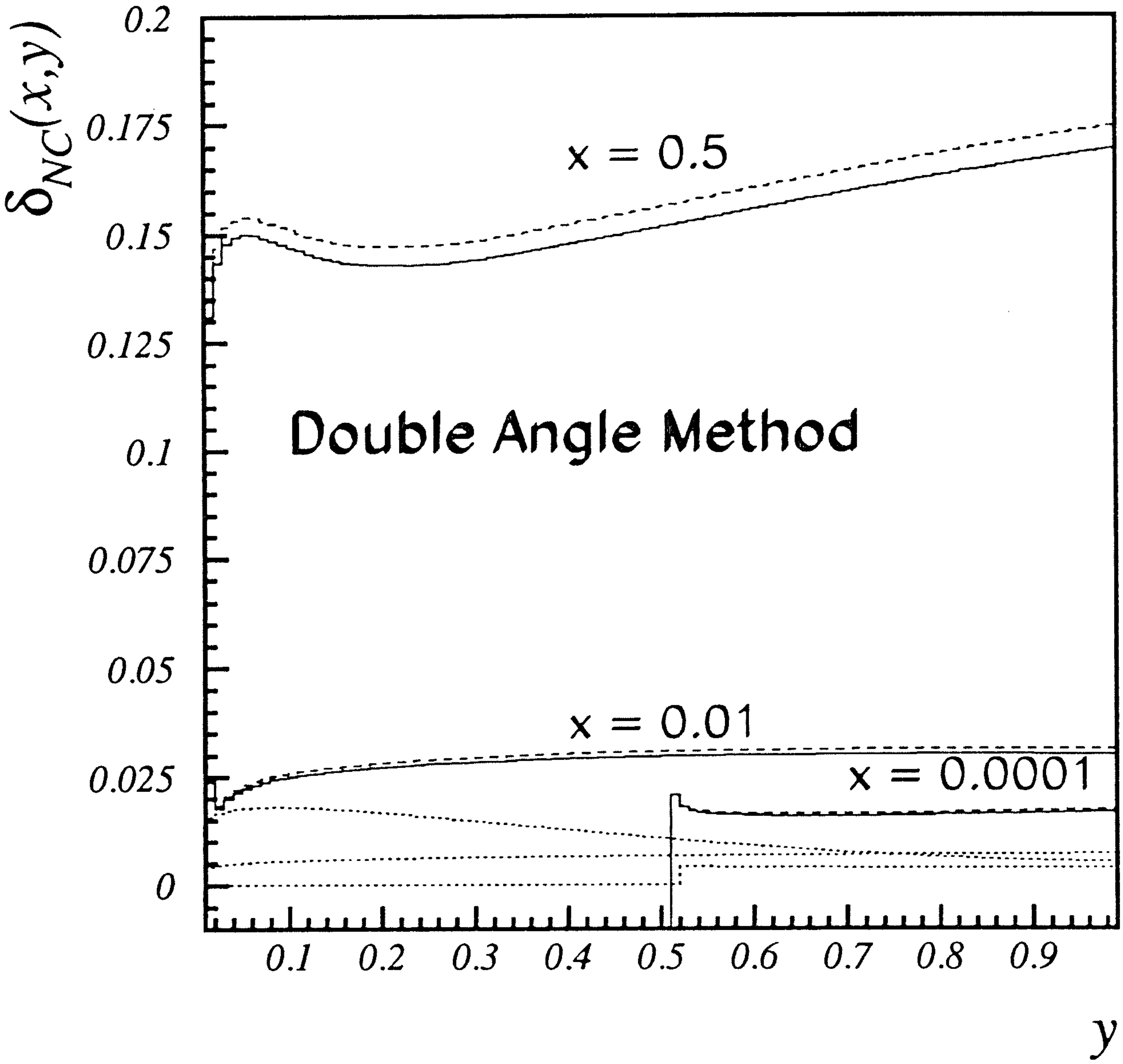
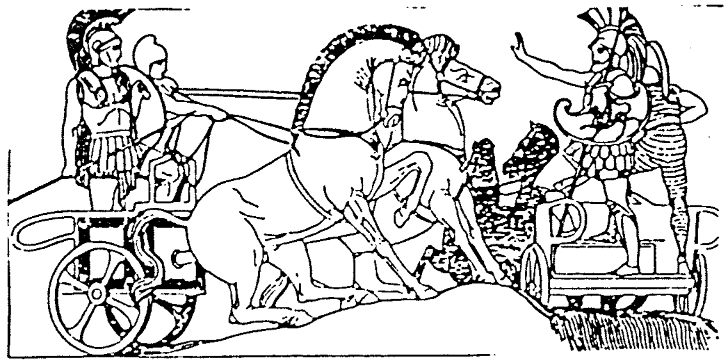


Figure 6: $\delta_{NC}(x,y)$ for the case of the double angle method for $\mathcal{A} = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dashed lines: $\delta_{NC}^{(1)}(x,y)$. Dotted lines: $\delta_{NC}^{e^- \to e^+}(x,y)$ scaled by $\times 100$; upper line: $x = 0.5$, middle line: $x = 0.01$, lower line: $x = 0.0001$. The other parameters are the same as in figure 3.



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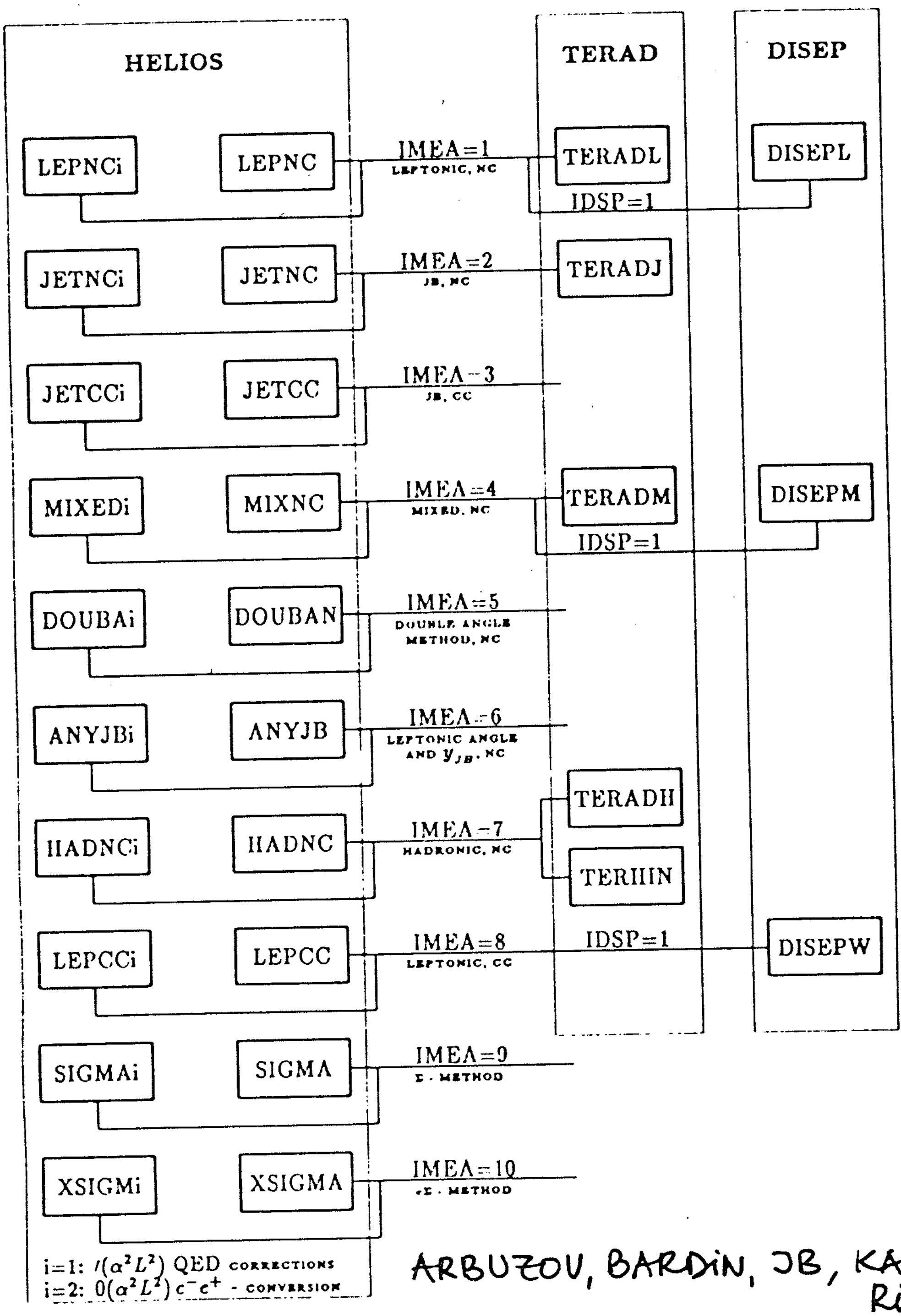


ARBUZOV, BARDIN, JB, KALINOVSKAYA, RIEMANN

10 [illegible].
Options so far.

3. The running coupling constant

- CENTRAL PARAMETER, NOT AN OBSERVABLE!
- CHARGE RENORMALIZATION IN QCD YIELDS: ($\overline{MS}$)

$$\frac{\partial \alpha_s(\mu^2)}{\partial \log \mu^2} = -\frac{\beta_0}{4\pi}\alpha_s^2 - \frac{\beta_1}{(4\pi)^2}\alpha_s^3 - \frac{\beta_2}{(4\pi)^3}\alpha_s^4 + \ldots$$

$\beta_0 = 11 - \frac{2}{3}N_f$	GROSS, WILCZEK POLITZER T'HOOFT	1973
$\beta_1 = 102 - \frac{38}{3}N_f$	CASWELL JONES	1974

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2$$

TARASOV, VLADIMIROV, ZHARKOV 1980

LARIN, VERMASEREN 1993

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(Q_0^2)} + \frac{\beta_0}{4\pi}\log\left(\frac{Q^2}{Q_0^2}\right)$$

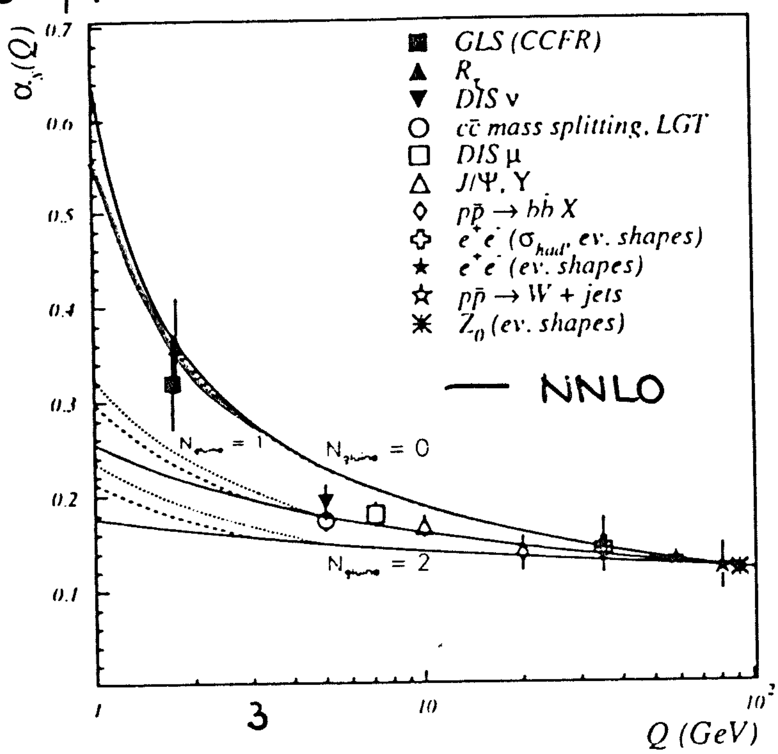
$$+ \phi^{(n)}(\alpha_s(Q^2);\beta_i) - \phi^{(n)}(\alpha_s(Q_0^2);\beta_i)$$

$$\phi_{(n)}(x;\beta_i) = -\frac{\beta_1}{8\pi\beta_0}\ln\left|\frac{16\pi^2 x^2}{16\pi^2\beta_0 + 4\beta_1\pi x + \beta_2 x^2}\right|$$

$$+ \frac{\beta_1^2 - 2\beta_0\beta_2}{8\pi\beta_0\sqrt{4\beta_2\beta_0 - \beta_1^2}}\arctan\left(\frac{2\pi\beta_1 + \beta_2 x}{2\pi\sqrt{4\beta_0\beta_2 - \beta_1^2}}\right)$$

$N_f \leq 5$: $4\beta_0\beta_2 - \beta_1^2 > 0$

$N_f = 6$: $4\beta_0\beta_2 - \beta_1^2 < 0$!



JB, J. BOTTS
1994

Fig. 1. Comparison of different theoretical predictions for $\alpha_s(Q^2)$ with experimental results of α_s [1]. The full curves denote the NLO solution of Eq. (2) for $N_{\tilde{g}} = 0, 1, 2$ with $m_{\tilde{g}} = 0$ taking $\alpha_s(Q_0^2) = \alpha_s(M_Z^2) = 0.122$. The dash-dotted line denotes the NNLO solution in the case of QCD. The dashed and dotted lines describe the cases $m_{\tilde{g}} = 3$ and 5 GeV, respectively.

S.BETHKE, 1995

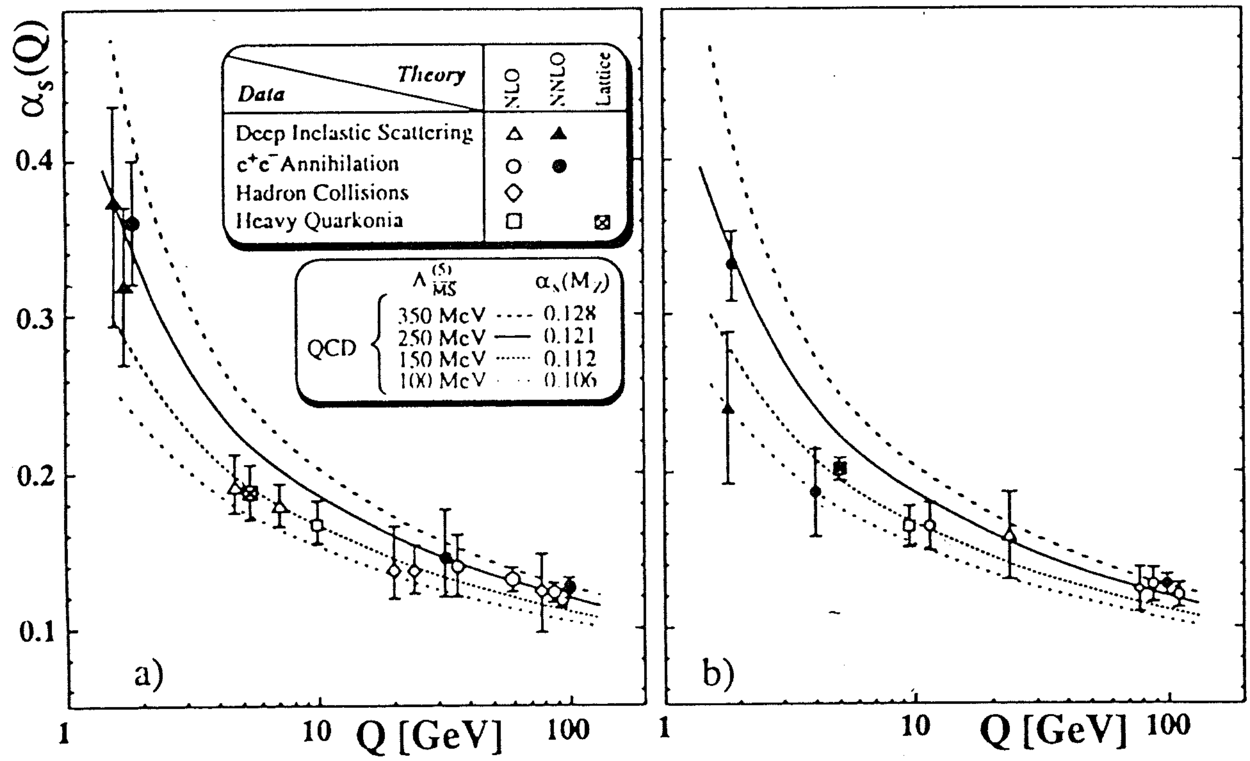


Figure 1. A Summary of measurements of α_s, compared with QCD expectations for four different values of $\Lambda_{\overline{MS}}$ which are given for $N_f = 5$ quark flavours. (a): Status before this conference. (b): Newest and mostly preliminary results, from Table 1. Curves and symbols are the same as in a).

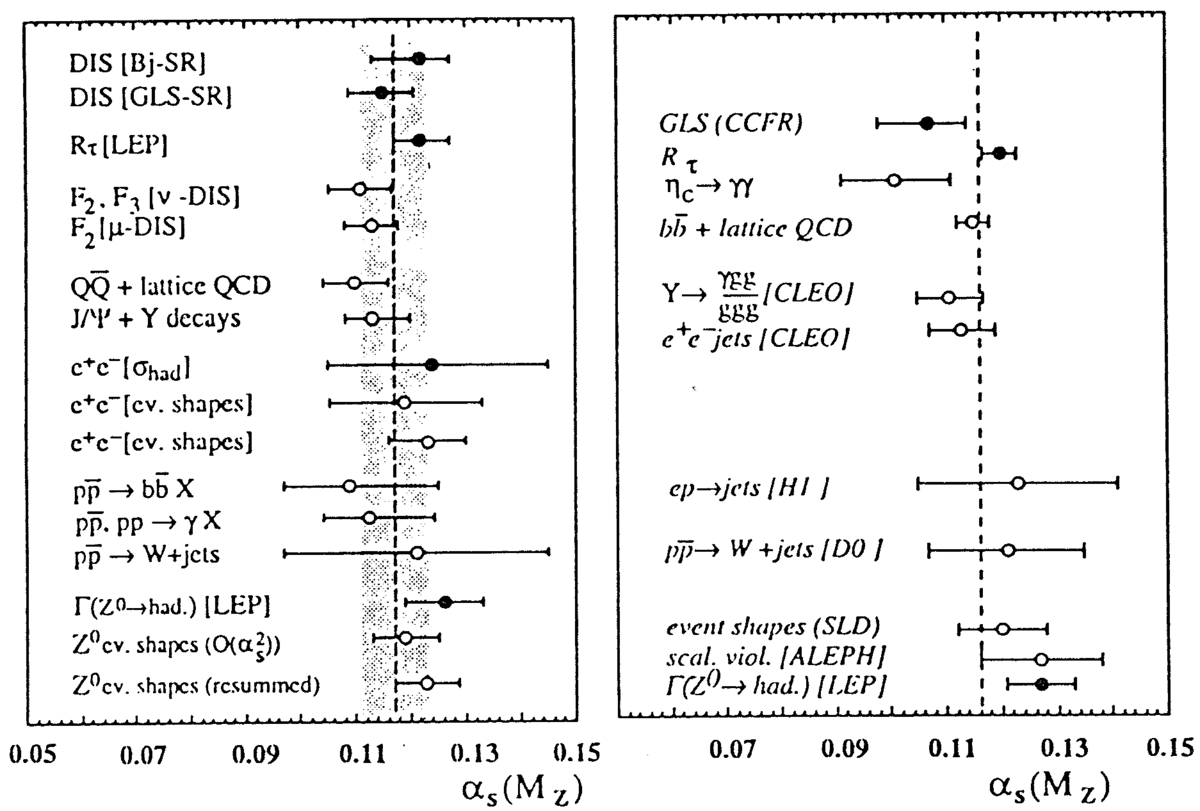


Figure 2. A Summary of measurements of $\alpha_s(M_{Z^0})$. Filled symbols are derived using $\mathcal{O}(\alpha_s^3)$ QCD; open symbols are in $\mathcal{O}(\alpha_s^2)$ or based on lattice calculations. (a): Status before this conference; vertical line and shaded area represent the world average of $\alpha_s(M_{Z^0}) = 0.117 \pm 0.006$. (b): Newest and mostly preliminary results, from Table 1; vertical line represents $\alpha_s(M_{Z^0}) = 0.116$.

- DIS $\nu F_2, F_3$ 5 $.193^{+0.019}_{-0.018}$ $.111 \pm .006$ .004 .004 NLO
- DIS μF_2 7.1 $.180 \pm .014$ $.113 \pm .005$ .003 .004 NLO

Process	Ref.	⟨Q⟩ [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
GLS (CCFR)	[15]	1.73	0.24 ± 0.047	$0.107^{+0.007}_{-0.009}$	$^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	NNLO
R_τ (CLEO)	[16]	1.78	0.302 ± 0.024	0.116 ± 0.003	0.002	0.002	NNLO
R_τ (ALEPH)	[17]	1.78	0.355 ± 0.021	0.122 ± 0.003	0.002	0.002	NNLO
R_τ (OPAL)	[17]	1.78	$0.375^{+0.032}_{-0.025}$	0.123 ± 0.003	0.002	0.002	NNLO
R_τ (Raczka)	[18]	1.78	0.333 ± 0.021	0.120 ± 0.003	0.002	0.002	NNLO
$\eta_c \to \gamma\gamma$ (CLEO)	[16]	2.98	0.187 ± 0.029	0.101 ± 0.010	0.008	0.006	NLO
$Q\overline{Q}$ states	[19]	5.0	0.188 ± 0.018	0.110 ± 0.006	0.000	0.006	q LGT
$b\overline{b}$ states	[19]	5.0	0.203 ± 0.007	0.115 ± 0.002	0.000	0.002	LGT
$\Upsilon(1S)$ (CLEO)	[16]	9.46	0.164 ± 0.013	0.111 ± 0.006	0.001	0.006	NLO
• $e^+e^- \to$ jets (CLEO)	[16]	10.53	0.164 ± 0.015	0.113 ± 0.006	0.002	0.006	NLO
$ep \to jets$ (H1)	[20]	5 - 60		0.123 ± 0.018	0.014	0.010	NLO
$pp \to$ W jets (D0)	[21]	80.6	0.123 ± 0.015	0.121 ± 0.014	0.012	0.005	NLO
• $e^+e^- \to Z^0$:							
• scal. viol. (ALEPH)	[17]	91.2		0.127 ± 0.011	-	-	NLO
• ev. shapes (SLD)	[22]	91.2		0.120 ± 0.008	0.003	0.008	resum.
• $\Gamma(Z^0 \to$ had.) (LEP)	[23]	91.2		0.127 ± 0.006	0.005	$^{+0.003}_{-0.004}$	NNLO

QUARKS

Table 1. Summary of most recent measurements of α_s, presented at this conference. Abbreviations: GLS-SR = Gross-Llewellyn-Smith sum rules; (N)NLO = (next-)next-to-leading order perturbation theory; LGT = lattice gauge theory (q stands for quenched approximation); resum. = resummed next-to-leading order. Most results are still preliminary.

S. BETHKE 1995

DIS: $\overline{\alpha}_s(M_Z) = 0.112 \pm 0.004$

e^+e^-: $\overline{\alpha}_s(M_Z) = 0.121 \pm 0.004$

TWO CLUSTERS !

- LGT WITHIN BETWEEN

MORE CALCULATIONS NEEDED

→ PROPER TREATMENT OF QUARKS.

4. The Evolution Equation

(TWIST 2)

- THE QCD CORRECTIONS CONTAIN COLLINEAR SINGULARITIES. THEY HAVE TO BE FACTORIZED & ABSORBED INTO THE NONPERTURBATIVE INPUT DISTRIBUTIONS AT Q_0^2.
- INDEPENDENTLY OF THE x RANGE CONSIDERED A SCHEME DEPENDENCE IS INDUCED DUE TO THIS.
- THE $\boxed{\text{RGE}}$ DETERMINES THE BEHAVIOUR OF THE ANOMALOUS DIMENSIONS AND THE COEFFICIENT FUNCTIONS & INDUCES THE EVOLUTION EQUATION.

EXAMPLE: NS

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_\psi(g)\right]\langle NS|JJ|NS\rangle = 0$$

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_{NS}^n(g) - 2\gamma_\psi(g)\right]\langle NS|O_{NS}^n|NS\rangle = 0$$

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_{NS}^n(g)\right] C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = 0$$

$$\text{LO:}\quad \int_0^1 dx\, x^{n-1}\Delta(x,Q^2) = \delta_{NS} A_n^{NS}(Q_0^2)\left[\frac{\bar{g}^2(Q^2)}{\bar{g}^2(Q_0^2)}\right]^{+d_{NS}^n}$$

$$d_{NS}^n = \frac{\gamma_{NS}^{(0),n}}{2\beta_0}.$$

QCD CORRECTIONS FOR DIS STRUCTURE FUNCTIONS

1) NTLO EVOLUTION EQUATIONS:

DEFINE COMBINATIONS OF PARTON DENSITIES:

$$\boxed{\begin{array}{l} q_i^- = q_i - \bar{q}_i \\ q_i^+ = q_i + \bar{q}_i \quad , \quad q^+ = \sum_{i=1}^{N_f} q_i^+ \\ G \end{array}}$$

$$A(x) \otimes B(x) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) \, A(x_1) B(x_2)$$

$$\frac{d}{d\log Q^2} q_i^-(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} P^-(x, \alpha) \otimes q_i^-(x, Q^2)$$

$$\frac{d}{d\log Q^2} \left[q_i^+(x, Q^2) - \frac{1}{N_f} q^+(x, Q^2) \right] = \frac{\alpha_s(Q^2)}{2\pi} P^+(x, \alpha) \otimes \left[q_i^+(x, Q^2) - \frac{1}{N_f} q^+(x, Q^2) \right]$$

$$\frac{d}{d\log Q^2} \begin{bmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{bmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \mathbb{P}(x, \alpha) \otimes \begin{bmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{bmatrix}$$

$$P^{\pm}(x,\alpha) = P^{(0)}_{NS}(x) + \frac{\alpha_s}{2\pi} P^{\pm,1}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{\pm,2}(x) + \ldots$$

$$\mathbb{P}(x,\alpha) = \mathbb{P}^{(0)}(x) + \frac{\alpha_s}{2\pi} \mathbb{P}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathbb{P}^{(2)}(x) + \ldots$$

FACTORIZING THE PARTON DISTRIBUTIONS AT Q_0^2:

$$q_i^-(x,t) := E^-(x,t) \otimes q_i^-(x)$$

$$q_i^+(x,t) = E^+(x,t) \otimes q_i^+(x) + \frac{1}{N_f}\left[E_{11}(x,t) - E^+(x,t)\right] \otimes q^+(x) + \frac{1}{N_f} E_{12}(x,t) \otimes G(x).$$

$$\begin{bmatrix} q_i^+(x,t) \\ G(x,t) \end{bmatrix} = \mathbb{E}(x,t) \begin{bmatrix} q_i^+(x) \\ G(x) \end{bmatrix}$$

BOUNDARY CONDITIONS:

$$\lim_{t\to 0} E^{\pm}(x,t) = \delta(1-x)$$

$$\lim_{t\to 0} \mathbb{E}(x,t) = \mathbb{1}\cdot\delta(1-x).$$

$$t \underset{\mathrm{Df}}{=:} -\frac{2}{\beta_0} \ln \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}$$ EVOLUTION VARIABLE.

CHANGE VARIABLES: $Q^2 \to t$

$$\boxed{\frac{\alpha_s(Q^2)}{2\pi}\, d\log Q^2 = \left(1 - \frac{\beta_1}{2\beta_0}\frac{\alpha_s(Q^2)}{2\pi} + \dots\right) dt}$$

EVOLUTION EQUS. FOR EVOLUTION OPERATORS:

$$\frac{d}{dt} E^{\pm}(x,t) = \left\{ P_{NS}(x) + \frac{\alpha_s(t)}{2\pi} R^{\pm}(x) + \dots \right\} \otimes E^{\pm}(x,t)$$

$$\frac{d}{dt} \mathbb{E}(x,t) = \left\{ \mathbb{P}^{0}(x) + \frac{\alpha(t)}{2\pi} \mathbb{R}(x) + \dots \right\} \otimes \mathbb{E}(x,t)$$

$$R^{\pm}(x) = P^{\pm,(1)}(x) - \frac{\beta_1}{2\beta_0} P_{NS}(x)$$

$$\mathbb{R}(x) = \mathbb{P}^{(1)}(x) - \frac{\beta_1}{2\beta_0} \mathbb{P}^{(0)}(x)$$

4.1. Splitting Functions

$\underline{O(\alpha_s): \quad (LO)}$

$$
\begin{aligned}
P_{NS}^{(0)}(z) \equiv P_{qq}(z) &= C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z) \right] \\
P_{qg}(z) &= T_f \left((1-z)^2 + z^2 \right) \\
P_{gq}(z) &= C_F \, \frac{1+(1-z)^2}{z} \\
P_{gg}(z) &= 2 C_G \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) \right] \\
&\quad + \frac{1}{2} \beta_0 \, \delta(1-z)
\end{aligned}
$$

GROSS, WILCZEK 1973
GEORGI, POLITZER 1973
GRIBOV, LIPATOV 1972
DOKSHITSER 1977
LIPATOV 1975

ALTARELLI, PARISI 1977
KIM, SCHILCHER 1977/78
et al.

$$\boxed{\int_0^1 dz \; z^{N-1} P_{ab}^{(0)}(z) = - \frac{\gamma_{ab}^{0,N}}{4}}$$

SPLITTING FUNCTION — ANOMALOUS DIMENSION

$\underline{O(\alpha_s^2)}$ CONTR. DUE TO:

FLORATOS, D ROSS, SACHRAJDA 1977-79, BARDEEN, BURAS, DUKE, MUTA 1978
CURCI, FURMANSKI, PETRONZIO 1980
FURMANSKI, PETRONZIO 1980
GONZALEZ-ARROYO, LOPEZ, YNDURAIN 1979/80
FLORATOS, KOUNNAS, LACAZE 1981 abc

NON-SINGLET :

$$\boxed{P_{\pm}(x,\alpha) = \hat{P}_{qq}(x,\alpha) \pm \hat{P}_{q\bar{q}}(x,\alpha)}$$

$\overline{MS}$

$$\hat{P}_{qq}(x,\alpha) = \left(\frac{\alpha}{2\pi}\right) C_F \left(\frac{1+x^2}{1-x}\right) + \left(\frac{\alpha}{2\pi}\right)^2 \left[C_F^2 P_F(x) + \tfrac{1}{2} C_F C_G P_G(x) + C_F N_F T_F P_{N_F}(x)\right], \tag{4.50}$$

$$\hat{P}_{q\bar{q}}(x,\alpha) = \left(\frac{\alpha}{2\pi}\right)^2 \left(C_F^2 - \tfrac{1}{2} C_F C_G\right) P_A(x), \tag{4.51}$$

$$P_F(x) = -2\frac{1+x^2}{1-x}\ln x \ln(1-x) - \left(\frac{3}{1-x} + 2x\right)\ln x - \tfrac{1}{2}(1+x)\ln^2 x - 5(1-x), \tag{4.52}$$

$$P_G(x) = \frac{1+x^2}{1-x}\left[\ln^2 x + \tfrac{11}{3}\ln x + \tfrac{67}{9} - \tfrac{1}{3}\pi^2\right] + 2(1+x)\ln x + \tfrac{40}{3}(1-x), \tag{4.53}$$

$$P_{N_F}(x) = \tfrac{2}{3}\left[\frac{1+x^2}{1-x}\left(-\ln x - \tfrac{5}{3}\right) - 2(1-x)\right], \tag{4.54}$$

$$P_A(x) = 2\frac{1+x^2}{1+x}\int_{x/(1+x)}^{1/(1+x)} \frac{dz}{z} \ln\frac{1-z}{z} + 2(1+x)\ln x + 4(1-x). \tag{4.55}$$

SINGLET:

$P_{ij}^{(1)}(x)$:

$$\hat{P}_{FF}^{(1,S)} = C_F^2[-1 + x + (\tfrac{1}{2} - \tfrac{3}{2}x)\ln x - \tfrac{1}{2}(1+x)\ln^2 x - (\tfrac{3}{2}\ln x + 2\ln x\ln(1-x))p_{FF}(x) + 2p_{FF}(-x)S_2(x)]$$
$$+ C_F C_G[\tfrac{14}{3}(1-x) + (\tfrac{11}{6}\ln x + \tfrac{1}{2}\ln^2 x + \tfrac{67}{18} - \tfrac{1}{6}\pi^2)p_{FF}(x) - p_{FF}(-x)S_2(x)]$$
$$+ C_F T_R N_F[-\tfrac{16}{3} + \tfrac{40}{3}x + (10x + \tfrac{16}{3}x^2 + 2)\ln x - \tfrac{112}{9}x^2 + \tfrac{40}{9}x^{-1} - 2(1+x)\ln^2 x - (\tfrac{10}{9} + \tfrac{2}{3}\ln x)p_{FF}(x)]\,,$$

$$\hat{P}_{FG}^{(1,S)} = C_F^2[-\tfrac{5}{2} - \tfrac{7}{2}x + (2 + \tfrac{7}{2}x)\ln x + (-1 + \tfrac{1}{2}x)\ln^2 x - 2x\ln(1-x) + (-3\ln(1-x) - \ln^2(1-x))p_{FG}(x)]$$
$$+ C_F C_G[\tfrac{28}{9} + \tfrac{65}{18}x + \tfrac{44}{9}x^2 + (-12 - 5x - \tfrac{8}{3}x^2)\ln x + (4+x)\ln^2 x + 2x\ln(1-x) + (-2\ln x\ln(1-x)$$
$$+ \tfrac{1}{2}\ln^2 x + \tfrac{11}{3}\ln(1-x) + \ln^2(1-x) - \tfrac{1}{6}\pi^2 + \tfrac{1}{2})p_{FG}(x) + p_{FG}(-x)S_2(x)]$$
$$+ C_F T_R N_F[-\tfrac{4}{3}x - (\tfrac{20}{9} + \tfrac{4}{3}\ln(1-x))p_{FG}(x)]\,,$$

$$\hat{P}_{GF}^{(1,S)} = C_F T_R N_F[4 - 9x + (-1 + 4x)\ln x + (-1 + 2x)\ln^2 x + 4\ln(1-x)$$
$$+ (-4\ln x\ln(1-x) + 4\ln x + 2\ln^2 x - 4\ln(1-x) + 2\ln^2(1-x) - \tfrac{2}{3}\pi^2 + 10)p_{GF}(x)]$$
$$+ C_G T_R N_F[\tfrac{182}{9} + \tfrac{14}{9}x + \tfrac{40}{9}x^{-1} + (\tfrac{136}{3}x - \tfrac{38}{3})\ln x - 4\ln(1-x) - (2+8x)\ln^2 x + (-\ln^2 x$$
$$+ \tfrac{44}{3}\ln x - 2\ln^2(1-x) + 4\ln(1-x) + \tfrac{1}{3}\pi^2 - \tfrac{218}{9})p_{GF}(x) + 2p_{GF}(-x)S_2(x)]\,,$$

$$\hat{P}_{GG}^{(1,S)} = C_F T_R N_F[-16 + 8x + \tfrac{20}{3}x^2 + \tfrac{4}{3}x^{-1} + (-6 - 10x)\ln x + (-2 - 2x)\ln^2 x]$$
$$+ C_G T_R N_F[2 - 2x + \tfrac{26}{9}x^2 - \tfrac{26}{9}x^{-1} - \tfrac{4}{3}(1+x)\ln x - \tfrac{20}{9}p_{GG}(x)]$$
$$+ C_G^2[\tfrac{27}{2}(1-x) + \tfrac{67}{9}(x^2 - x^{-1}) + (-\tfrac{25}{3} + \tfrac{11}{3}x - \tfrac{44}{3}x^2)\ln x + 4(1+x)\ln^2 x + (\tfrac{67}{9} - 4\ln x\ln(1-x)$$
$$+ \ln^2 x - \tfrac{1}{3}\pi^2)p_{GG}(x) + 2p_{GG}(-x)S_2(x)]\,.$$

$$S_2(x) \equiv \int_{(1+x)/x}^{1/(1+x)} \frac{dz}{z}\ln\left(\frac{1-z}{z}\right); \quad S_1(x) \equiv \int_0^{1-x} \frac{dz}{z}\ln(1-z)\,.$$

4.2. Coefficient Functions

$O(\alpha_s)$:

$$C_{F2}^{(1)} = C_F \left[\frac{1+x^2}{1-x} \left(\ln \frac{1-x}{x} - \frac{3}{4} \right) + \frac{1}{4}(9+5x) \right]_+$$

$$C_{F1}^{(1)} = C_{F2}^{(1)} - 2x\, C_F$$

$$C_{F3}^{(1)} = C_{F2}^{(1)} - C_F\,(1+x)$$

$$C_{G2}^{(1)} = 2N_f T_R \left[(x^2+(1-x)^2) \ln\left(\frac{1-x}{x}\right) - 1 + 8x(1-x) \right]$$

$$C_{G1}^{(1)} = C_{G2}^{(1)} - 2N_f T_R\, 4x(1-x)$$

cf. FURMANSKI, PETRONZIO 1982 and refs. therein.

$O(\alpha_s^2)$:

F_2, F_L, xF_3 :

ZIJLSTRA, VAN NEERVEN, 1991 abc, 1992
LARIN, VERMASEREN (moments) 1991 (93).

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GRV '91

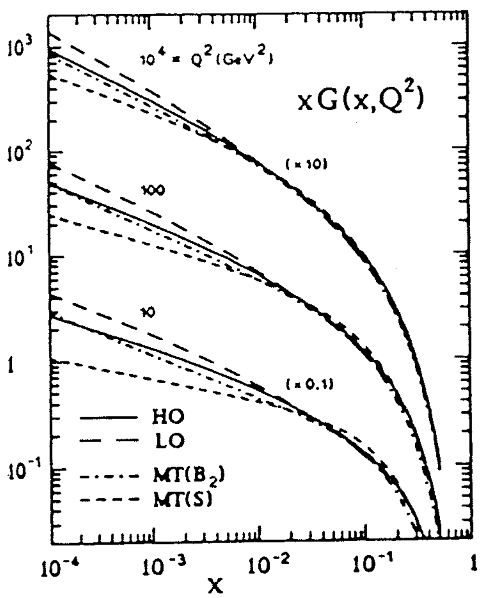

Fig. 9. The detailed small-x behavior of our radiatively generated gluon distributions in LO and HO at fixed values of Q^2, compared with the MT(S) and MT(B_2) fits [16]. The KMRS(B_0) [3] and MT(B_1) parametrizations are similar to MT(S), although slightly flatter at $Q^2 = 10\,\mathrm{GeV}^2$. The 'steep' gluon distributions [our HO, MT(B_2), KMRS(B_-) unshadowed] differ very little in the kinematic region shown

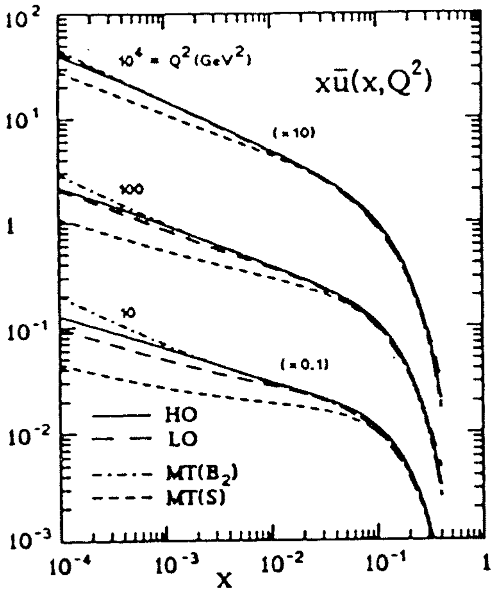

Fig. 10. The detailed small-x behavior of our radiatively generated sea distributions $\bar{u} = \bar{d}$ in LO and HO, compared with the MT(S) and MT(B_2) fits [16]. For $x < 10^{-2}$ the MT(B_1) and KMRS(B_0) [3] parametrizations are significantly ($\lesssim 30\%$) below the MT(S) fit. KMRS(B_-) lies between MT(S) and our results

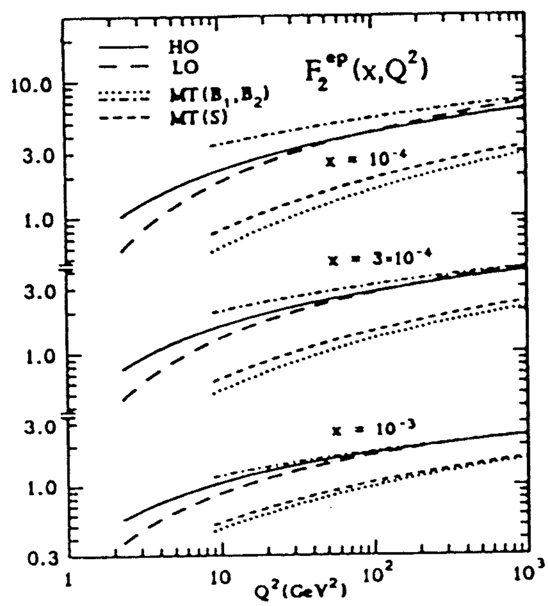

Fig. 12. Radiative LO and HO predictions for F_2^{ep} in the small-x region. For comparison we show expectations from conventional fit approaches MT [16], extrapolated to the experimentally not yet available $x < 10^{-2}$ region

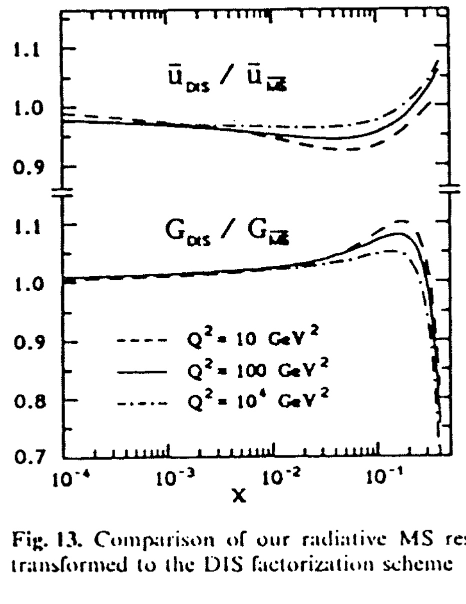

Fig. 13. Comparison of our radiative MS results with the ones transformed to the DIS factorization scheme

2) $\mathcal{O}(\alpha_s^2)$: $F_L(x,Q^2)$

$$F_L(x,Q^2) = \int_x^1 \frac{dy}{y} K^{NS}(y,Q^2)\mathcal{F}(x/y,Q^2)$$
$$+\int_x^1 \frac{dy}{y} K^{S}(y,Q^2)\mathcal{F}^{S}(x/y,Q^2) + \int_x^1 \frac{dy}{y} K^{G}(y,Q^2)\mathcal{G}(x/y,Q^2),$$

where

$$\mathcal{F}(x,Q^2) = \sum_{i=1}^{n_f} e_i^2 x(q_i(x,Q^2) + \bar{q}_i(x,Q^2)),$$
$$\mathcal{F}^{S}(x,Q^2) = \delta_\psi^2 \sum_{i=1}^{n_f} x(q_i(x,Q^2) + \bar{q}_i(x,Q^2)),$$
$$\mathcal{G}(x,Q^2) = xG(x,Q^2),$$

$$\delta_\psi^2 = \left(\frac{\sum_{i=1}^{n_f} e_i^2}{n_f}\right)$$

$$K^{NS}(x,Q^2) = \frac{\alpha_s}{4\pi} f_{L,q}^{(1)}(x) \quad + \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{NS(2)}(x),$$
$$K^{S}(x,Q^2) = \qquad\qquad \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{S(2)}(x),$$
$$K^{G}(x,Q^2) = \frac{\alpha_s}{4\pi} \delta_\psi^2 f_{L,G}^{(1)}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 \delta_\psi^2 f_{L,G}^{(2)}(x),$$

1st Ord.

$$f_{L,q}^{(1)}(x) = 4C_F x^2,$$
$$f_{L,G}^{(1)}(x) = 8n_f x^2(1-x),$$

SANCHEZ-G.

2nd Ord.

↓

$$f_{L,q}^{S(2)}(x) = \tfrac{16}{9} C_F n_f \big[3(1-2x-2x^2)(1-x)\ln(1-x)$$
$$+9x^2(\mathrm{Li}_2(x) + \ln^2(x) - \zeta(2)) + 9x(1-x-2x^2)\ln x$$
$$-9x^2(1-x) - (1-x)^3\big],$$

$$f_{L,q}^{NS(2)}(x) = 4C_F(C_A - 2C_F)x^2$$

$$\times\Bigg[4\frac{6-3x+47x^2-9x^3}{15x^2}\ln x - 4\,\mathrm{Li}_2(-x)(\ln x - 2\ln(1+x)) - 8\zeta(3)$$

$$-2\ln^2 x\ln(1-x^2) + 4\ln x\ln^2(1+x) - 4\ln x\,\mathrm{Li}_2(x)$$

$$+\tfrac{2}{5}(5-3x^2)\ln^2 x - 4\frac{2+10x^2+5x^3-3x^5}{5x^3}$$

$$\times(\mathrm{Li}_2(-x) + \ln x\ln(1+x)) + 4\zeta(2)\left(\ln(1-x^2) - \frac{5-3x^2}{5}\right)$$

$$+8S_{1,2}(-x) + 4\,\mathrm{Li}_3(x) + 4\,\mathrm{Li}_3(-x) - \tfrac{23}{3}\ln(1-x)$$

$$-\frac{144+294x-1729x^2+216x^3}{90x^2}\Bigg]$$

$$+8C_F^2x^2\Bigg[\mathrm{Li}_2(x) + \ln^2\left(\frac{x}{1-x}\right) - 3\zeta(2) - \frac{3-22x}{3x}\ln x$$

$$+\frac{6-25x}{6x}\ln(1-x) - \frac{78-355x}{36x}\Bigg] - \tfrac{8}{3}C_F n_f x^2\left[\ln\left(\frac{x^2}{1-x}\right) - \frac{6-25x}{6x}\right],$$

Since we disagree with the result for the longitudinal gluonic Wilson coefficient $xC_L^{(2),G}$ given in eq. (10) of ref. [14], it is appropriate to give our result below. In the $\overline{\mathrm{MS}}$ scheme it reads

$$C_L^{(2),G}(x,1) = n_f C_F\Bigg[16x[\mathrm{Li}_2(1-x) + \ln x\ln(1-x)] + \left(-\tfrac{32}{3}x + \tfrac{64}{5}x^3 + \frac{32}{15x^2}\right)[\mathrm{Li}_2(-x) + \ln x\ln(1+x)]$$

$$+(8+24x-32x^2)\ln(1-x) - \left(\tfrac{32}{3}x + \tfrac{32}{5}x^3\right)\ln^2 x + \frac{1}{15}\left(-104-624x+288x^2-\frac{32}{x}\right)\ln x$$

$$+\left(-\tfrac{32}{3}x + \tfrac{64}{5}x^3\right)\zeta(2) - \tfrac{128}{15} - \tfrac{104}{5}x + \tfrac{336}{5}x^2 + \frac{32}{15x}\Bigg]$$

$$+n_f C_A\Bigg[-64x\,\mathrm{Li}_2(1-x) + (32x+32x^2)[\mathrm{Li}_2(-x) + \ln x\ln(1+x)] + (16x-16x^2)\ln^2(1-x)$$

$$+(-96x+32x^2)\ln x\ln(1-x) + \left(-16-144x+\tfrac{464}{3}x^2+\frac{16}{3x}\right)\ln(1-x) + 48x\ln^2 x$$

$$+(16+128x-208x^2)\ln x + 32x^2\zeta(2) + \tfrac{16}{3} + \tfrac{272}{3}x - \tfrac{848}{9}x^2 - \frac{16}{9x}\Bigg].$$

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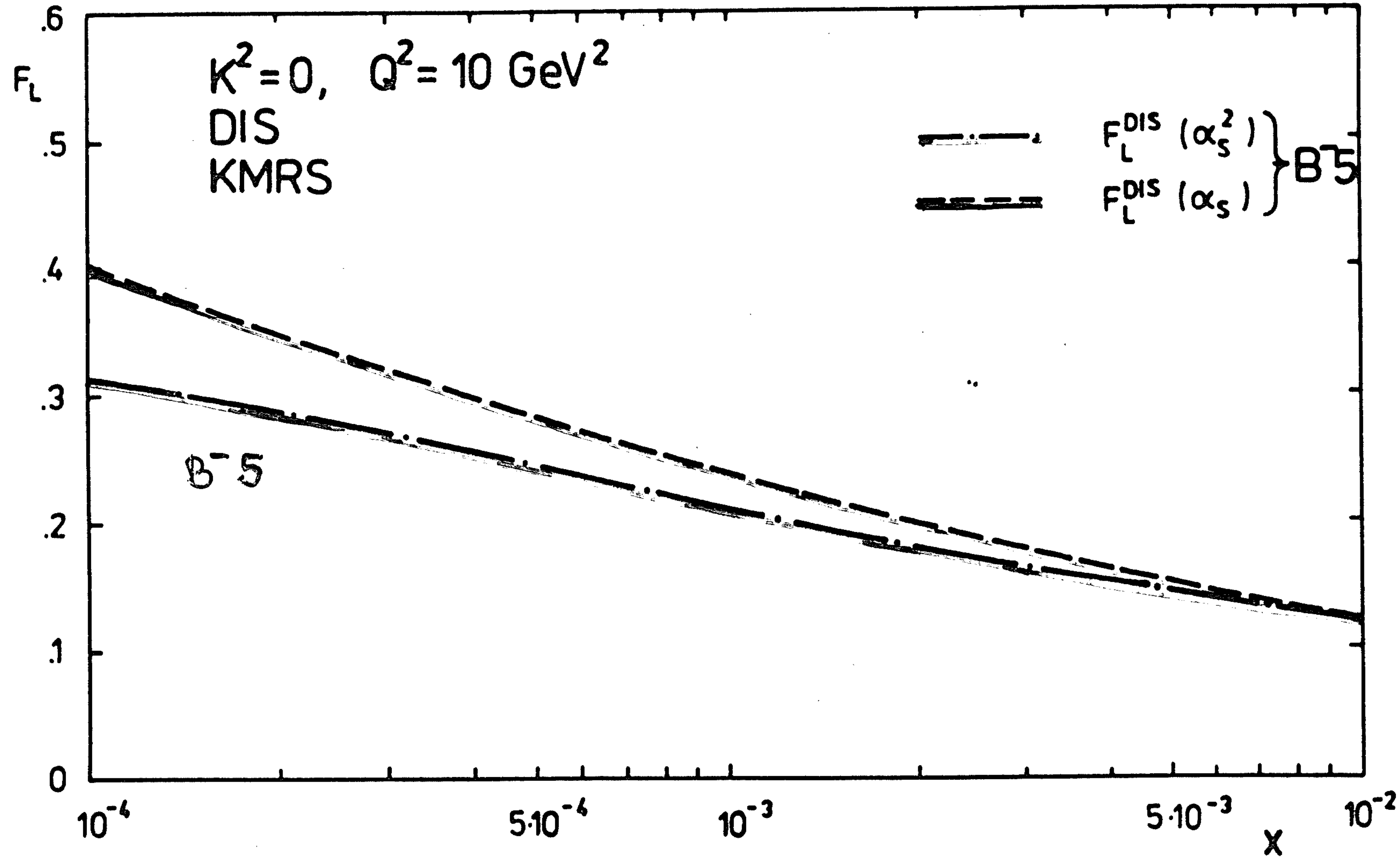
IMPORTANCE OF HIGHER ORDER CORRECTIONS

ZIJLSTRA, VAN NEERVEN



4.3. $O(\alpha_s^3)$ corrections

SUM RULES:

$$\int_0^1 dx\,(F_1^{\bar{\nu}p} - F_1^{\nu p}) = 1 - \frac{2}{3}\frac{\alpha_s}{\pi} - 2.3519\left(\frac{\alpha_s}{\pi}\right)^2 - 8.4852\left(\frac{\alpha_s}{\pi}\right)^3$$

LARIN, TKACHOV, VERMASEREN 1991

$$\int_0^1 dx\,(F_3^{\bar{\nu}p} + F_3^{\nu p}) = 6\Big[1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left(-\frac{55}{12} + \frac{1}{3}N_f\right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\Big[-\frac{13841}{216} - \frac{44}{9}\zeta_3 + \frac{55}{2}\zeta_5$$
$$+ N_f\left(\underline{\frac{10009}{1296}} + \underline{\frac{91}{54}\zeta_3} - \frac{5}{3}\zeta_5\right)$$
$$- \frac{115}{648}N_f^2\Big]\Big]$$

$$\int_0^1 dx\,(g_1^{ep} - g_1^{en}) = \frac{1}{3}\left|\frac{g_A}{g_V}\right|\Big\{1 - \frac{\alpha_s}{\pi} \;\ldots\ldots$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\Big[+ N_f\left(\underline{\frac{10339}{1296}} + \underline{\frac{61}{54}\zeta_3} - \ldots\right) \ldots\Big]\Big\}$$

LARIN, VERMASEREN, 1991

LARIN, RITBERGEN, VERMASEREN
1994.

NS:

8th moment of $C_L(x)$

$$
\begin{aligned}
C_{L,8}(1,a_s) &= a_s C_F \cdot \tfrac{4}{9} \\
&\quad + a_s^2\Big[C_F C_A\big(\tfrac{14741729}{1190700} - \tfrac{16}{3}\zeta_3\big) + C_F n_f\big(-\tfrac{14234}{8505}\big) \\
&\quad + C_F^2\big(-\tfrac{21694349}{3572100} + \tfrac{32}{3}\zeta_3\big)\Big] \\
&\quad + a_s^3 C_F C_A n_f\big(-\tfrac{23153083641529}{198037224000} + \tfrac{18459136}{363825}\zeta_3\big) \\
&\quad + a_s^3 C_F C_A^2\big(\tfrac{7653142193467}{18003384000} - \tfrac{9508318}{19845}\zeta_3 + \tfrac{2240}{9}\zeta_5\big) + a_s^3 C_F n_f^2 \cdot \tfrac{1435876}{229635} \\
&\quad + a_s^3 C_F^2 C_A\big(-\tfrac{766700762180089}{277252113600000} + \tfrac{2476582549}{2182950}\zeta_3 - \tfrac{2720}{3}\zeta_5\big) \\
&\quad + a_s^3 C_F^2 n_f\big(\tfrac{11844644404289}{198037224000} - \tfrac{914992}{10395}\zeta_3\big) \\
&\quad + a_s^3 C_F^3\big(-\tfrac{861671664694184 57}{499053804480000} - \tfrac{111668693}{218295}\zeta_3 + \tfrac{7360}{9}\zeta_5\big) \\
&\quad + a_s^3 \cdot 3\left(\sum_{f=1}^{n_f} q_f\right)\frac{d^{abc}d^{abc}}{n_c}\big(-\tfrac{35555777437}{3000564000} - \tfrac{85378}{14175}\zeta_3 + \tfrac{160}{9}\zeta_5\big) \\
&= a_s \cdot 0.5925925926 + a_s^2(35.87664404 - 2.231471683 n_f) \\
&\quad + a_s^3\left(2215.210878 - 305.4730331 n_f + 8.337149534 n_f^2\right. \\
&\qquad \left. - 8.741107731 \sum_{f=1}^{n_f} q_f\right).
\end{aligned}
$$

NS ANOM. DIM. (F_2)

$$
\begin{aligned}
\gamma_8(a_s) &= a_s C_F \cdot \tfrac{9883}{1260} + a_s^2\Big[C_F C_A \cdot \tfrac{25870049}{762048} + C_F n_f\big(-\tfrac{36241943}{4762800}\big) \\
&\quad + C_F^2\big(-\tfrac{27040578211}{4000752000}\big)\Big] + a_s^3 C_F C_A n_f\big(-\tfrac{1573915745223}{72013536000} - \tfrac{19766}{315}\zeta_3\big) \\
&\quad + a_s^3 C_F C_A^2\big(\tfrac{8101059985033}{41150592000} + \tfrac{2510407}{132300}\zeta_3\big) + a_s^3 C_F n_f^2\big(-\tfrac{38920977797}{18003384000}\big) \\
&\quad + a_s^3 C_F^2 C_A\big(-\tfrac{36625766990 59}{112021056000} - \tfrac{2510407}{44100}\zeta_3\big) \\
&\quad + a_s^3 C_F^2 n_f\big(-\tfrac{91675209372043}{1680315840000} + \tfrac{19766}{315}\zeta_3\big) \\
&\quad + a_s^3 C_F^3\big(-\tfrac{1093087100097437993}{635159387520000 0} + \tfrac{2510407}{66150}\zeta_3\big) \\
&= a_s \cdot 10.45820106 + a_s^2(123.7764525 - 10.14583662 n_f) \\
&\quad + a_s^3(2164.091836 - 352.3116596 n_f - 2.882493484 n_f^2).
\end{aligned}
$$

NS: MOMENTS F_2.

$M_{2,N}$ for $n_f = 5$:

$$M_{2,2}(n_f=5) = a_s^{32/69}(1 + 2.348059464 a_s - 6.052509330 a_s^2) A_2(\mu^2),$$

$$M_{2,4}(n_f=5) = a_s^{314/345}(1 + 8.457076895 a_s + 73.59702078 a_s^2) A_4(\mu^2),$$

$$M_{2,6}(n_f=5) = a_s^{2836/2415}(1 + 13.71561575 a_s + 192.6174600 a_s^2) A_6(\mu^2),$$

$$M_{2,8}(n_f=5) = a_s^{9883/7245}(1 + 18.17792372 a_s + 324.5935524 a_s^2) A_8(\mu^2).$$

The calculation of the 8th non-singlet moment took the equivalent of more than 600 CPU hours on an SGI Challenge workstation with a 100 MHZ MIPS 4400 chip.

5. Resummation of small x contributions

- AT SMALL x : DOMINANT TERMS IN P_{ab}, C_{ab}

$\longrightarrow$ LARGE CONTRIBUTIONS, INTEND TO RESUM THESE TERMS.

STRUCTURE FUNCTIONS :

$\longrightarrow$ BFKL CONTRIBUTIONS (BALITSKII, FADIN KURAEV, LIPATOV 1976-78.

CHARACTERISTIC EQU.

$$\boxed{\begin{aligned} \ell-1 &= \bar{\alpha}_s\, \chi(\gamma_L(\ell,\alpha_s)) \quad , \quad \bar{\alpha}_s = \frac{C_A\alpha_s}{\pi} \\ \chi(z) &= 2\psi(1) - \psi(z) - \psi(1-z) \end{aligned}}$$

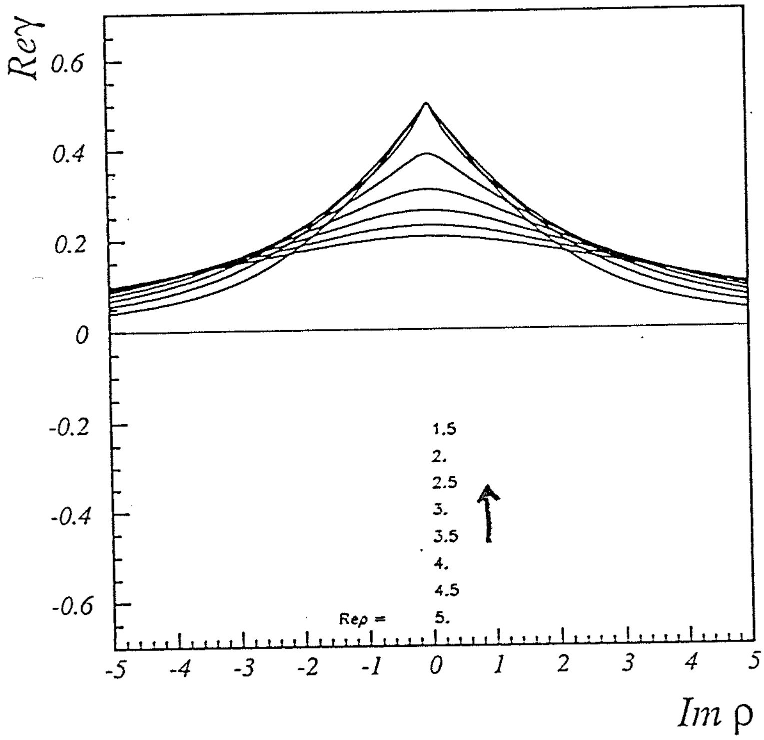
$$\gamma_L(\ell,\bar{\alpha}_s) = \frac{\bar{\alpha}_s}{\ell-1}\left\{1 + 2\sum_{k=1}^{\infty} b_{2k+1}\,\gamma_L^{2k+1}(\ell,\bar{\alpha}_s)\right\}$$

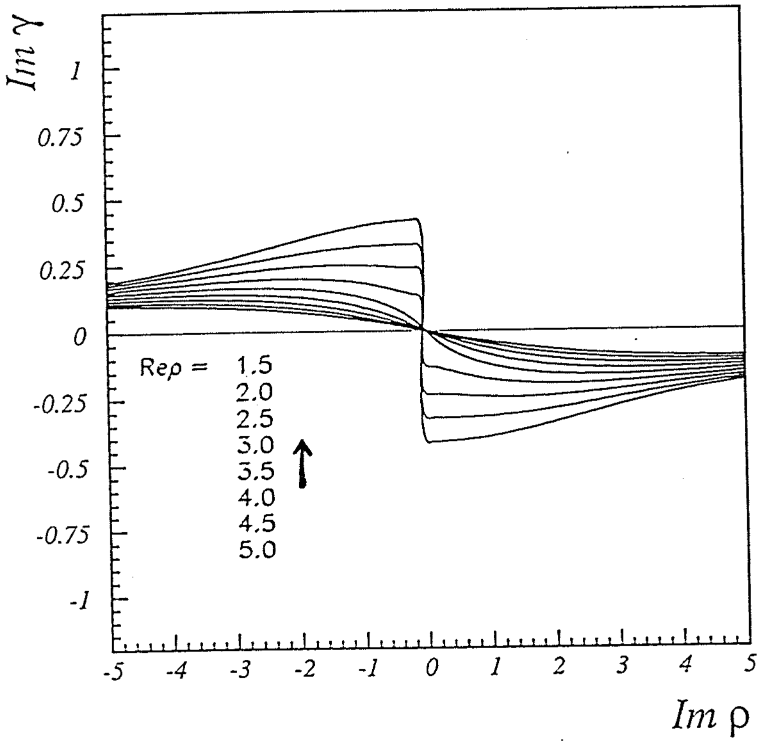
$$= A + \underbrace{2b_3A^4}_{\propto\, \bar{\alpha}_s^4} + 2b_5A^6 + 12b_3^2A^7 + \ldots$$

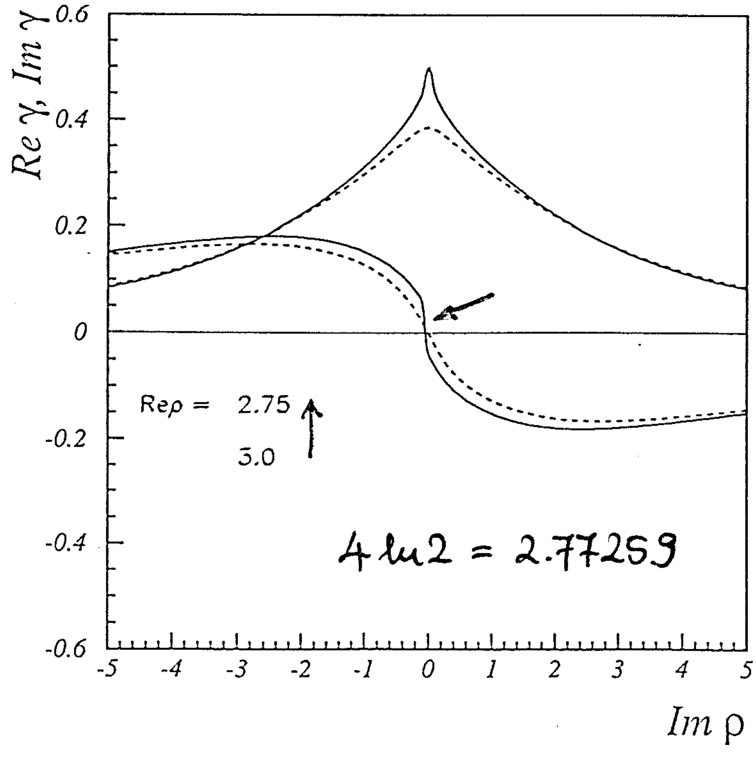
$$A = \frac{\bar{\alpha}_s}{\ell-1}.$$

The behaviour of $\gamma_c(\rho)$ for $\rho \in C$

$Re\rho \geq 1.5$

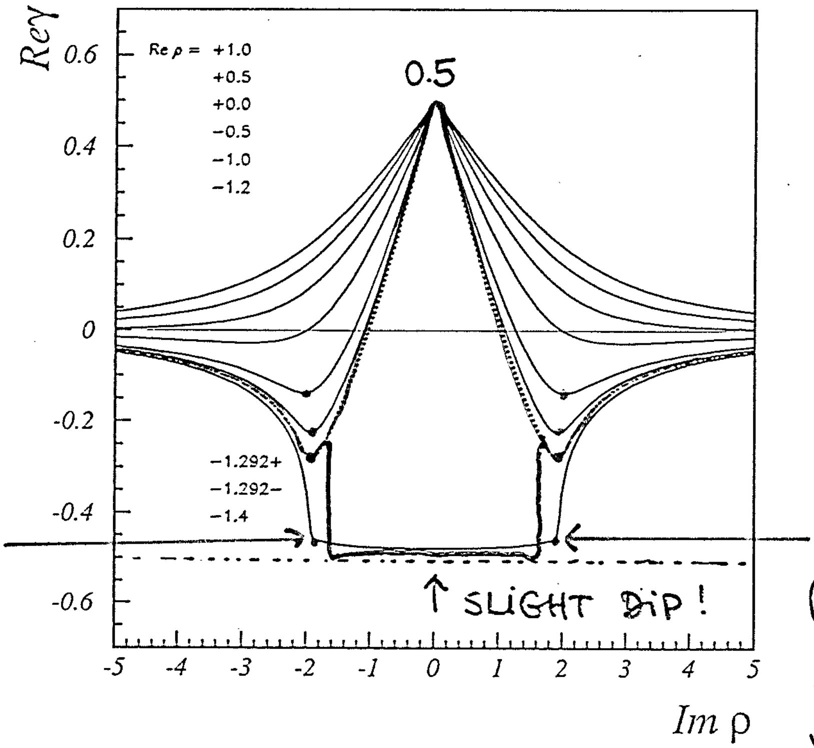




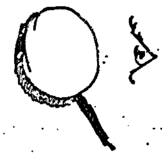


(USE: ADAPTIVE NEWTON ALGORITHM).

$1.5 > Re\rho > -1.5$



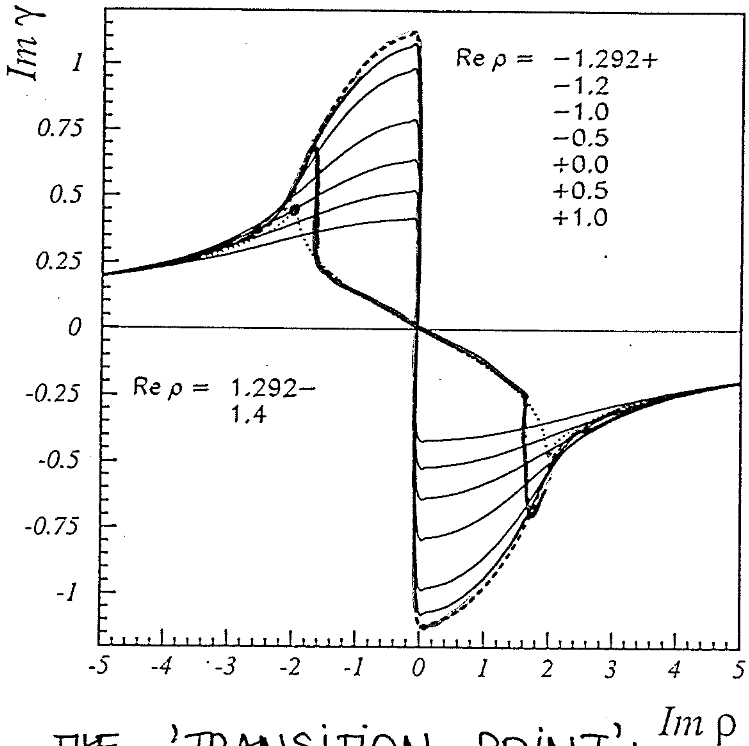
RIDGE
↓
BATH TUB



$(-1.41, \pm 1.97)$

BRANCH POINTS

√ ELLIS/HAUTMANN/WEBBER.



POSITION OF THE 'TRANSITION POINT': EXPAND AROUND $\gamma_c \sim -\frac{1}{2}$:

$$\rho = \underbrace{4(\log 2 - 1)}_{-1.22741} - \frac{8\alpha}{1-2\alpha} + \sum_{k=0}^{\infty} b_{2k+1} \left(2^{2(k+1)} - 2\right) \alpha^{2k}$$

$\mathrm{Im}\,\alpha = 0$, $\mathrm{Re}\,\alpha = 0.0082$ $\curvearrowright$ $\mathrm{Re}\,\rho \approx -1.292$.

LOCATION OF THE BRANCH POINTS

$$\rho = \frac{\ell - 1}{\bar{\alpha}_s} = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma).$$

$$1 = \left[-\psi'(\gamma) + \psi'(1-\gamma)\right] \frac{\partial\gamma}{\partial\rho}$$

$$\frac{1}{\partial\gamma/\partial\rho} = \psi'(1-\gamma) - \psi'(\gamma) = 0$$

$$\boxed{\psi'(z) - \frac{\pi^2}{2}\frac{1}{\sin^2 \pi z} = 0}$$

$$\gamma_1 = \frac{1}{2} + 0i \qquad \rho_1 = 4\ln 2$$

$$\gamma_{2,3} = -0.425214 \pm i\,0.473898$$

$$\rho_{2,3} = -1.4105 \pm i\,1.9721.$$

CATANI, HAUTMANN 1994
ELLIS, HAUTMANN, WEBBER
ROBERTS et al. 1995

EVOLUTION EQU.

$$\frac{d f_a(\omega,\mu)}{d \ln \mu^2} = \sum_b \gamma_{ab}(\omega, \alpha_s(\mu^2))\, f_b(\omega,\mu^2)$$

$$f_a(\omega) = \int_0^1 dx\, x^\omega f_a(x)$$

$$\gamma_{ab}(\omega,\alpha_s) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{\omega}\right)^k A_{ab}^{(k)} + \sum_{k=0}^{\infty} \alpha_s \left(\frac{\alpha_s}{\omega}\right)^k B_{ab}^{(k)} + O\left(\alpha_s^2 \left(\frac{\alpha_s}{\omega}\right)^k\right).$$

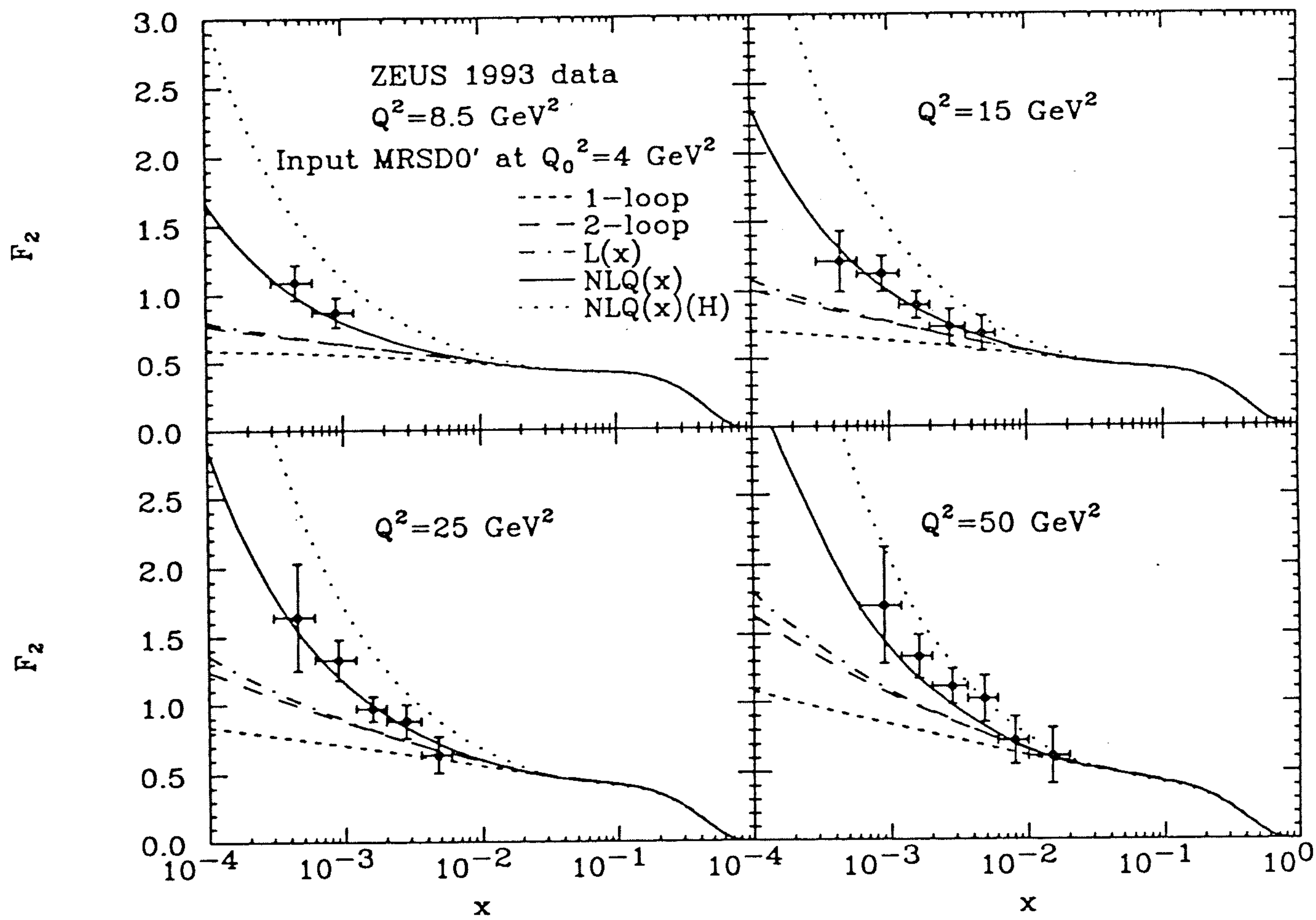
$$\frac{d}{d\mu^2}\begin{pmatrix} f_S \\ f_g \end{pmatrix} = \begin{pmatrix} \gamma_{SS} & \gamma_{SG} \\ \gamma_{GS} & \gamma_{GG} \end{pmatrix} \begin{pmatrix} f_S \\ f_G \end{pmatrix}$$

$$\gamma_L = \begin{pmatrix} 0 & 0 \\ \frac{C_F}{C_A}\gamma_L(\omega) & \gamma_L(\omega) \end{pmatrix}$$

$$\gamma_{NL} = \begin{pmatrix} \frac{C_F}{C_A}\gamma_{NL}(\omega) - \frac{2\alpha_s}{3\pi}T_f & \gamma_{NL}(\omega) \\ \gamma_\delta & \gamma_\eta \end{pmatrix} + \dots$$

$$\gamma_{NL} \simeq \frac{2\alpha_s}{3\pi} T_f \left\{ 1 + 2.17\frac{\alpha_s}{\omega} + 2.30\left(\frac{\alpha_s}{\omega}\right)^2 + 8.27\left(\frac{\alpha_s}{\omega}\right)^3 + \dots \right\}$$

$\longrightarrow$ TAKE NTLO RESULTS COMPL INTO ACC.
(SUBTR. ACC. TERMS IN γ_L, γ_{NL} !)



ZEUS 1993 data
Q^2=8.5 GeV^2
Input MRSD0' at $Q_0{}^2$=4 GeV^2
1-loop
2-loop
L(x)
NLQ(x)
NLQ(x)(H)
Q^2=15 GeV^2
Q^2=25 GeV^2
Q^2=50 GeV^2
F_2
F_2
x
x

COMBINED AP & BFKL RESUMMATION:

G. MARCHESINI, ERICE 1990 (PUB: 1992)
CIAFALONI 1988
CATANI, FIORANI, MARCHESINI 1990 a,b.

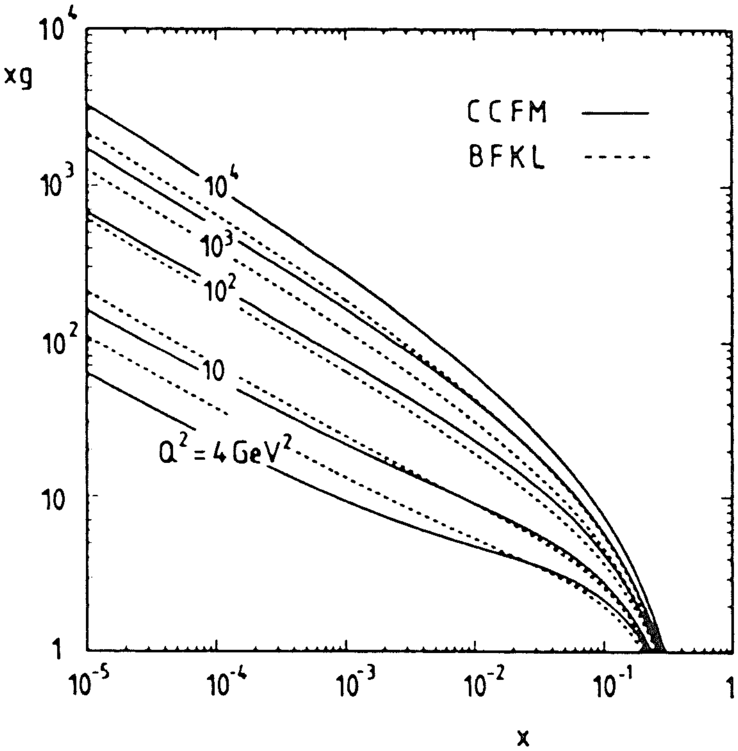
$$F(x, Q_T, Q) = F^0(x, Q_T, Q) + \int_x^1 dz \int \frac{dq^2}{\pi q^2}\, \theta(Q - zq)\, \Delta_S(Q, z, q)\, \tilde{P}(z, q, Q_T) \cdot F\left(\frac{x}{z}, Q_T', q\right)$$

$$Q_T' = |Q_T + (1-z)q|$$

$$\Delta_S = \exp\left(-\int_{(z'q')^2}^{q^2} \frac{dk^2}{k^2} \int_0^1 dz \frac{\bar{\alpha}_s}{1-z}\right)$$

$$\tilde{P}_S = \bar{\alpha}_s \left[\frac{1}{1-z} + \Delta_{NS} \frac{1}{z} + z(1-z) - 2\right]$$

$$\Delta_{NS} = \exp\left(-\bar{\alpha}_s \ln\frac{z_0}{z} \ln\frac{Q_T^2}{z_0 z q^2}\right)$$



... solution KWIECINSKI et al. 1995

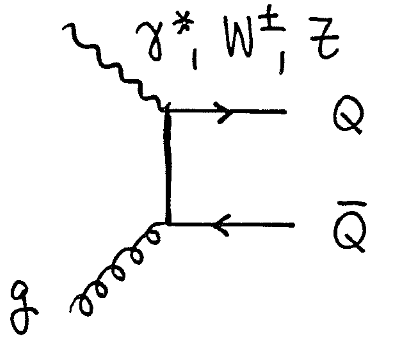
xG rises stronger than for BFKL!

(flat input $G(x, Q_0^2)$).

FIG. 10. The integrated gluon distribution xg versus x, obtained from the CCFM (solid curves) and the BFKL (dashed curves) equations for $Q^2 = 4$, 10, 10^2, 10^3, and $10^4\,GeV^2$. Recall that our solutions are obtained from a "flat" gluon input.

6. Heavy flavour contributions to structure functions

LO :



+ crossed

WITTEN 1976
BABCOCK, SIVERS 1978
SHIFMAN, VAINSTEIN, ZAKHAROV 1978
GLÜCK, REYA 1979
LEVEILLE, WEILER 1979
BAUR, VAN DER BIJ, 1988
SCHULER 1987
GLÜCK, REYA, GODBOLE 1988

$$\frac{d^2\sigma}{dx\,dy}(ep \to eQ\bar{Q}X) = \frac{4\pi\alpha_s^2}{Q^4}\left[\left(1-y+\tfrac{1}{2}y^2\right)F_2^{Q\bar{Q}} - \tfrac{1}{2}y^2 F_L^{Q\bar{Q}}\right]$$

$$F_i = \int_{ax}^{1} dz\, G(z,\mu^2)\, f_i\left(\frac{x}{z}, Q^2\right)$$

$$a = 1 + 4m_Q^2/Q^2$$

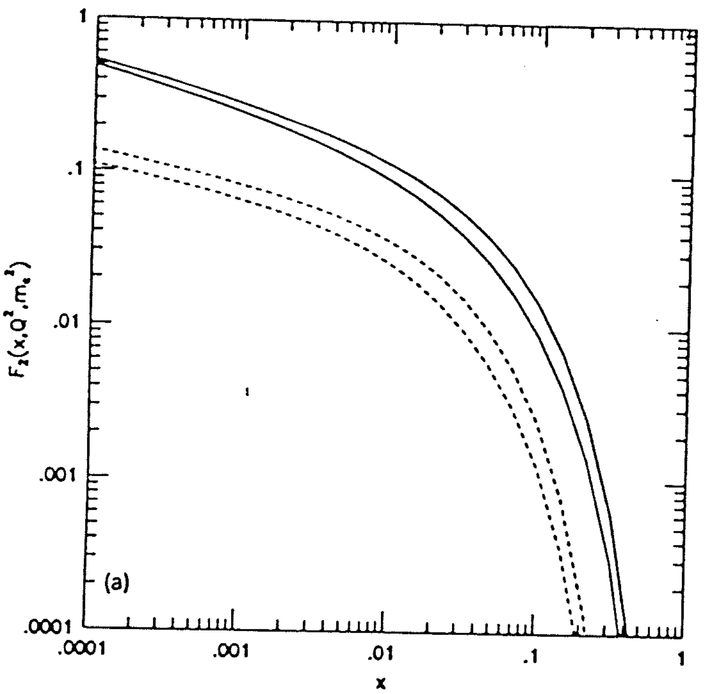
$$f_2 = e_q^2 \frac{\alpha_s}{\pi}\Big\{ v\left[4w^2(1-w) - \tfrac{1}{2}w - 2\frac{m_Q^2}{Q^2}w^2(1-w)\right] + \left[\tfrac{1}{2}w - w^2(1-w) + 2\frac{m_Q^2}{Q^2}w^2(1-3w) - 4\frac{m_Q^4}{Q^4}w^3\right] \cdot \ln\left|\frac{1+w}{1-w}\right|$$

$$f_L = e_q^2\frac{\alpha_s}{\pi}\left\{2w^2(1-w)v - 4\frac{m_Q^2}{Q^2}w^3 \ln\left|\frac{1+w}{1-w}\right|\right\}, \quad w = \frac{x}{z}$$

$$v^2 = 1 - 4m^2/Q^2\,(w/1-w)$$

NTLO: $O(\alpha_s^2)$

LAENEN, RIEMERSMA, SMITH, VAN NEERVEN 1993



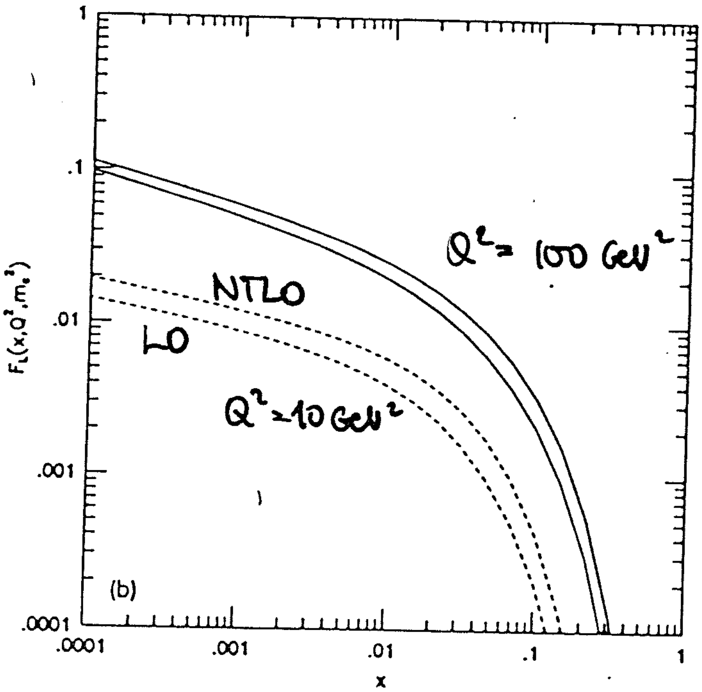
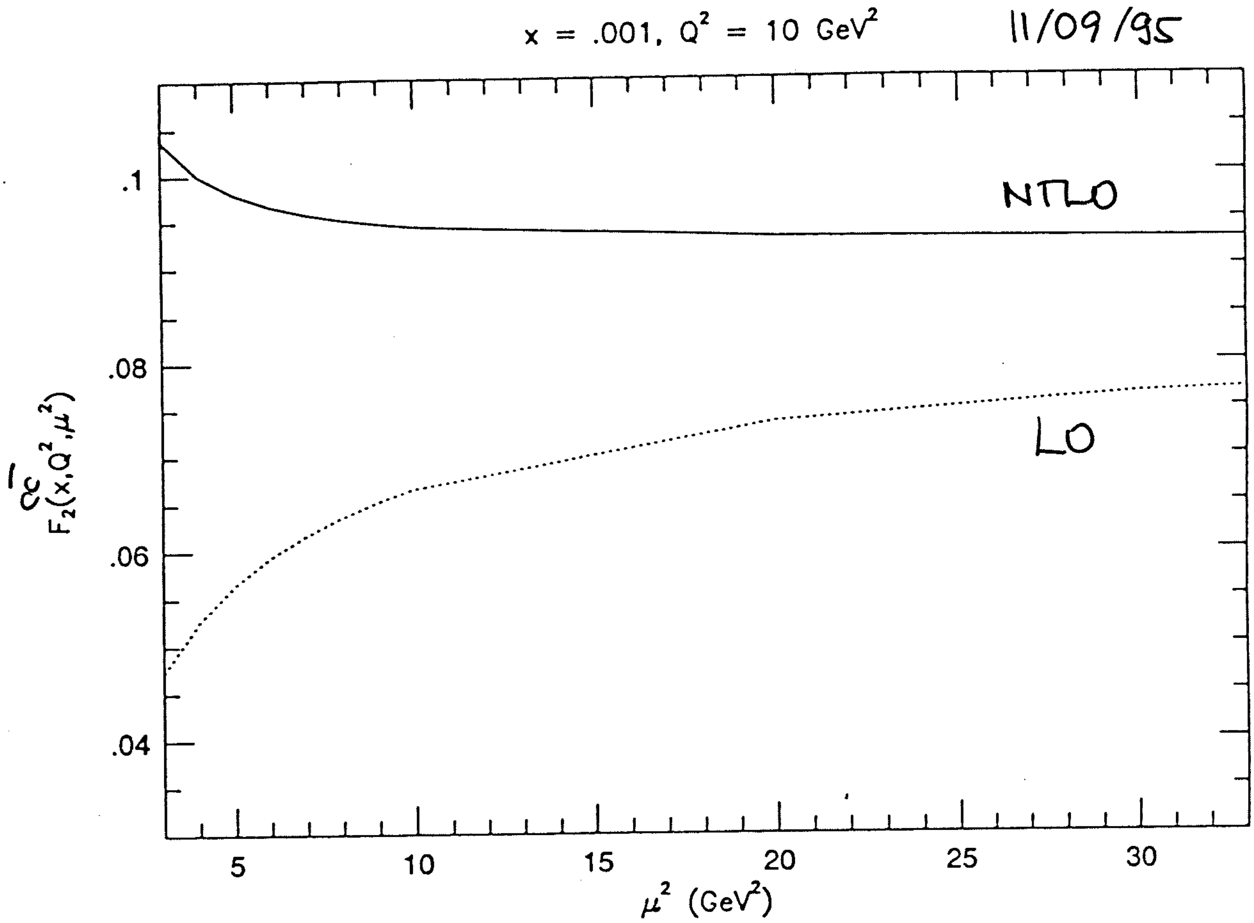


Fig. 14. (a) The x-dependence of $F_{2,c}^{(0)}(x, Q^2, m_c^2)$ (lower pair) and $F_{2,c}^{(0)}(x, Q^2, m_c^2) + F_2^{(1)}(x, Q^2, m_c^2)$ (upper pair) at fixed Q^2. The solid lines are for $Q^2 = 100$ (GeV/c)^2 and the dashed lines are for $Q^2 = 10$ (GeV/c)^2. (b) The x-dependence of $F_{L,c}^{(0)}(x, Q^2, m_c^2)$ (lower pair) and $F_{L,c}^{(0)}(x, Q^2, m_c^2) + F_L^{(1)}(x, Q^2, m_c^2)$. The notation is the same as in fig. 14a.

Scale dependence:

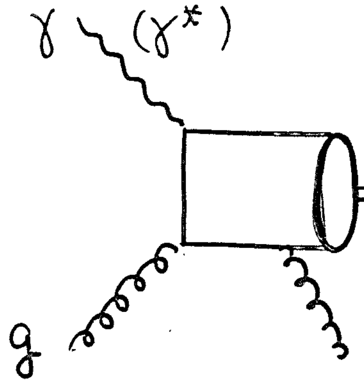
S. Riemersma
11/09/95



CTEQ distrib's.

7. J/ψ production

BORN:



J/ψ + 5 diagrams

COLOR-SINGLET MODEL

BERGER, JONES 1981

$$\frac{d\sigma^0}{dt_1} = \frac{128\pi^2}{3} \frac{\alpha \alpha_s^2 e_c^2}{s^2} M_{J/\psi}^2 \frac{|\varphi(0)|^2}{M_{J/\psi}} \cdot \frac{s^2 s_1^2 + t^2 t_1^2 + u^2 u_1^2}{s_1^2 t_1^2 u_1^2}$$

$$s_1 = s - M_{J/\psi}^2 \,, \quad t_1 = \ldots \,, \quad u_1 = \ldots$$

$$\sigma \sim G(x, \hat{Q}^2)$$

$Q^2 > 0$:

BAIER, RÜCKL 1982

KÖRNER, CLEYMANS, KURODA, GOUNARIS 1982

$O(\alpha \alpha_s^3)$:

$$\gamma g \longrightarrow J/\psi\, X \qquad (Q^2 = K^2 = 0).$$

KRÄMER, ZUNFT, STEEGBORN, ZERWAS 1994

KRÄMER 1995

KRÄMER

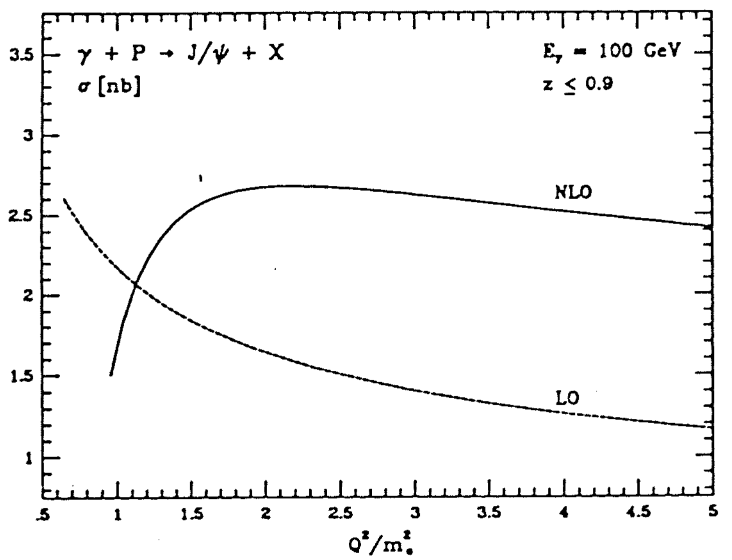


Figure 10: Dependence of the total cross section $\gamma + P \to J/\psi + X$ on the renormalization/factorization scale Q at an initial photon energy of $E_\gamma = 100$ GeV.

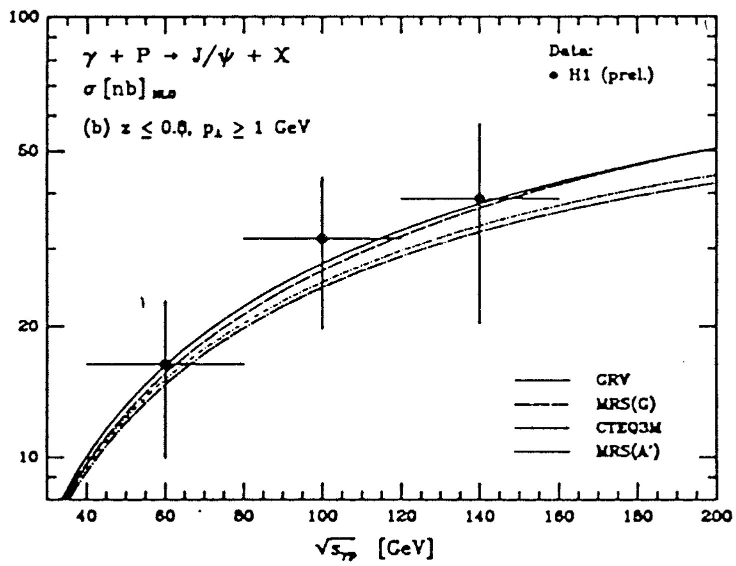


Figure 19: The total cross section as a function of the photon-proton centre of mass energy for different parametrizations of the gluon distribution of the proton compared with preliminary data from H1 [53] and ZEUS [54]. The results are shown in two kinematic domains: (a) $z \leq 0.9$; (b) $z \leq 0.8$ and $p_\perp \geq 1$ GeV.

Q^2 - dependence

BAIER, RÜCKL

1982.

TABLE I
Photoproduction cross section $\sigma(\gamma_v N \to J/\psi X)$ in nb expected from the process $\gamma_v g \to J/\psi g$ for $z \leqslant 0.7$ ($\alpha_s = 0.4$, $m_c = 1.55$ GeV)

Q^2 (GeV)^2	$E_\gamma = 90$ GeV	$E_\gamma = 150$ GeV
0.0	2.3	3.4
0.4	2.1	3.2
1.0	2.0	2.9
4.0	1.3	2.0
10.0	0.66	1.1
20.0	0.26	0.54

8. QCD corrections to polarized structure functions

$g_1(x, Q^2)$

$O(\alpha_s)$: ANOMALOUS DIMENSIONS / SPLITTING FCT.:

$$P_{NS,qq} \equiv P_{qq,S} = C_F\left[8\left(\frac{1}{1-x}\right)_+ - 4(1-x) + 6\,\delta(1-x)\right]$$

$$P_{qg,S} = T_f\left[16x - 8\right]$$

$$P_{gq,S} = C_F\left[8 - 4x\right]$$

$$P_{gg,S} = C_A\left[8\left(\frac{1}{1-x}\right)_+ + 8 - 16x + \frac{22}{3}\delta(1-x)\right] - T_f\left[\frac{8}{3}\delta(1-x)\right]$$

K. SASAKI 1975

M. AHMED, G. ROSS 1975/76

G. ALTARELLI, G. PARISI 1977

- NO TERMS $\propto \frac{1}{x}$.

PARAMETRIZATIONS FOR $g_1(x, Q^2)$:

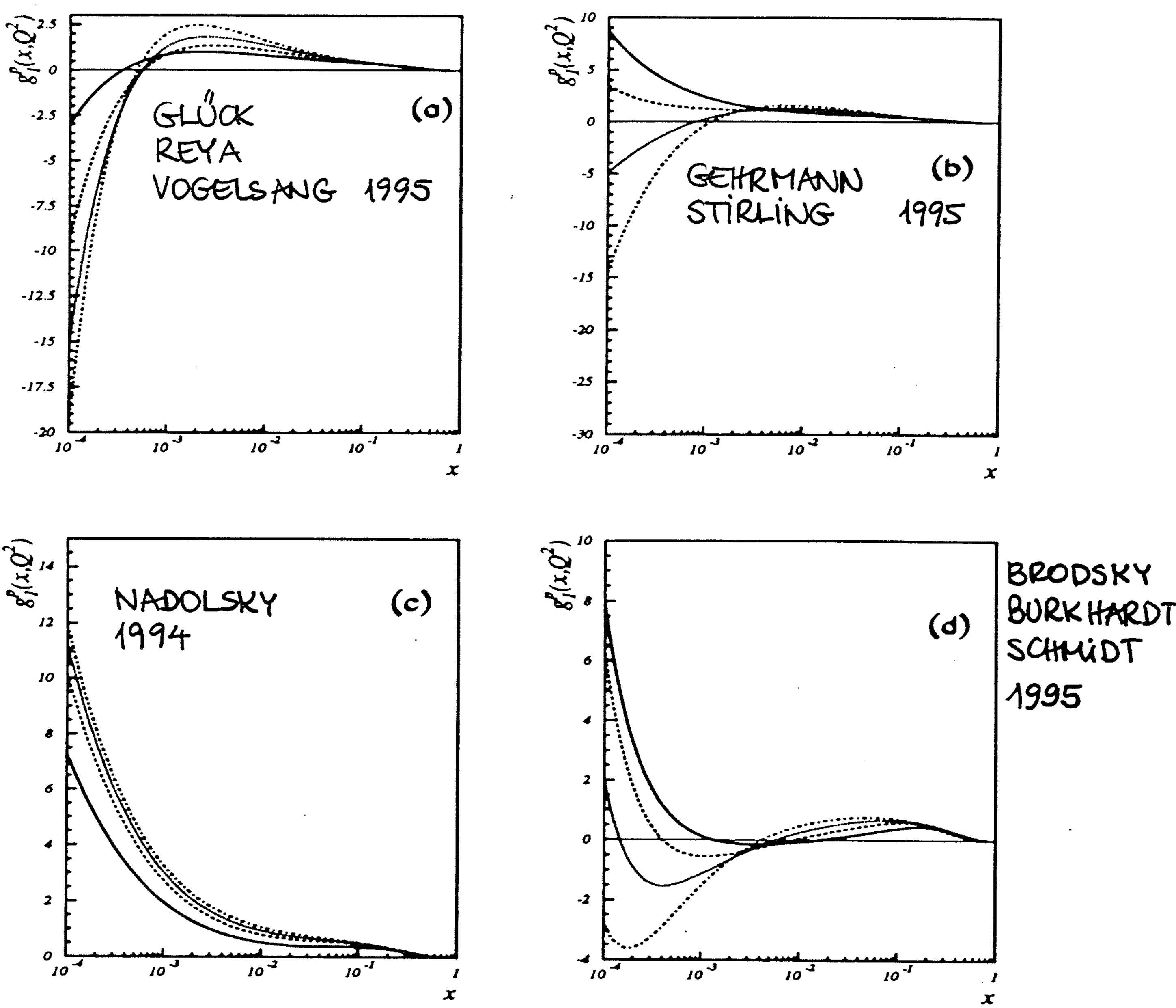


Figure 1: The structure function $g_1^p(x, Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].

COEFFICIENT FUNCTIONS:

$O(\alpha_s)$: $M^2 \equiv Q^2$

$$C_q^{NS} = \delta(1-z) + \frac{\alpha_s}{4\pi} C_F \Big\{ 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 3\left(\frac{1}{1-z}\right)_+ - 2(1+z)\ln(1-z) - 2\frac{1+z^2}{1-z}\ln z + 4 + 2z + \delta(1-z)\left(-4\zeta(2)-9\right)\Big\}$$

ALTARELLI, ELLIS, MARTINELLI 1979
HUMPERT, VAN NEERVEN 1981

$$C_g = \frac{\alpha_s}{4\pi} N_f T_f \Big\{ 4(2z-1)\left(\ln(1-z) - \ln z\right) + 4(3-4z)\Big\}$$

BODWIN, QIU 1990

$O(\alpha_s^2)$

ZIJLSTRA, VAN NEERVEN 1994, ALSO $M^2 \neq Q^2$.

- NTLO ANALYSES ARE POSSIBLE NOW
  $\longrightarrow$ NEED MORE PRECISE DATA STILL! IN A WIDER Q^2 RANGE.

RESUMMATION OF $\alpha_s \ln^2 x$ TERMS:

- BARTELS, ERMOLAEV, RYSKIN
- J.B.

$O(\alpha_s^2)$

$$P^{(1)}_{NS,qq} \equiv P^{(1,\text{unpol.})}_{NS,qq} \qquad (\text{NO } \tfrac{1}{x} \text{ TERMS!})$$

$$P^{(1)}_{PS,qq} = C_F T_f \left[-16(1+x)\underline{\ln^2 x} - 16(1-3x)\ln x + 16(1-x)\right].$$

ZIJLSTRA
VAN NEERVEN
MERTIG '94

$$\begin{aligned} P^{(1)}_{S,qg} = \; & 4 C_A T_f \, [-8(1+2x)\mathrm{Li}_2(-x) - 8\zeta(2) - 8(1+2x)\ln x \ln(1+x) \\ & \quad + 4(1-2x)\ln^2(1-x) - 4(1+2x)\underline{\ln^2 x} \\ & \quad -16(1-x)\ln(1-x) + 4(1+8x)\ln x - 44x + 48] \\ & +4 C_F T_f \left[8(1-2x)\zeta(2) - 4(1-2x)\ln^2(1-x)\right. \\ & \quad + 8(1-2x)\ln x \ln(1-x) - 2(1-2x)\underline{\ln^2 x} \\ & \quad \left. +16(1-x)\ln(1-x) - 2(1-16x)\ln x + 4 + 6x\right], \end{aligned} \tag{3.66}$$

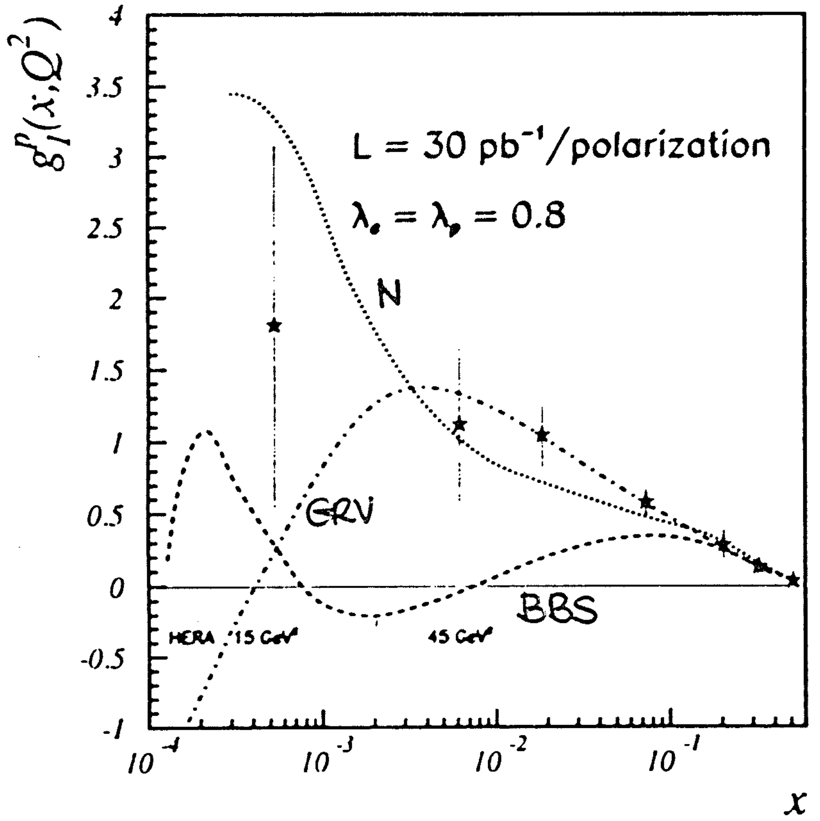
$$\begin{aligned} P^{(1)}_{S,gq} = \; & C_A C_F \left[16(2+x)\mathrm{Li}_2(-x) + 16x\zeta(2) + 8(2-x)\ln^2(1-x)\right. \\ & \quad + 16(2+x)\ln x \ln(1+x) + 8(2+x)\underline{\ln^2 x} \\ & \quad + 16(x-2)\ln x \ln(1-x) + \left(\frac{80}{3} + \frac{8}{3}x\right)\ln(1-x) \\ & \quad \left. +8(4-13x)\ln x + \frac{328}{9} + \frac{280}{9}x\right] \\ & +C_F^2 \left[8(x-2)\ln^2(1-x) - 4(x-2)\underline{\ln^2 x} - 164 + 128x\right. \\ & \quad \left. -8(x+2)\ln(1-x) - 4(20+7x)\ln x\right] \\ & +C_F T_f \left[-\frac{32}{9}(4+x) + \frac{32}{3}(x-2)\ln(1-x)\right], \end{aligned} \tag{3.67}$$

MERTIG
VAN NEERVEN
1995

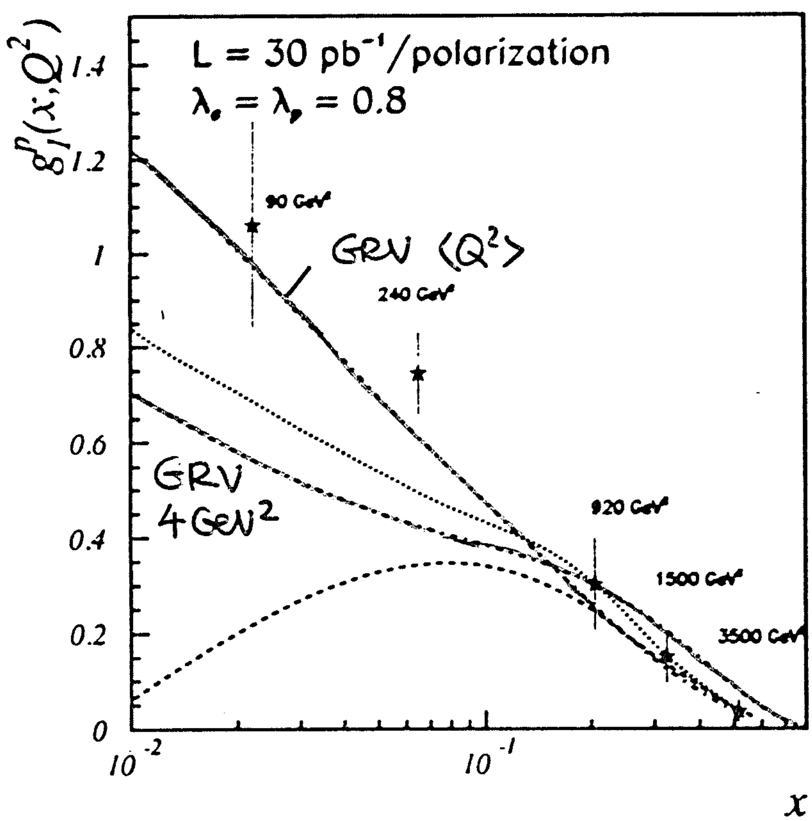
$$\begin{aligned} P^{(1)}_{S,gg} = \; & C_A^2 \left[\left(64x + \frac{32}{1+x} + 32\right)\mathrm{Li}_2(-x) + \left(64x - 16\left(\frac{1}{1-x}\right)_+ + \frac{16}{1+x}\right)\zeta(2)\right. \\ & \quad + \left(\frac{8}{1-x} - \frac{8}{1+x} + 32\right)\underline{\ln^2 x} + \left(64x + \frac{32}{1+x} + 32\right)\ln x \ln(1+x) \\ & \quad + \left(64x - \frac{32}{1-x} - 32\right)\ln x \ln(1-x) + \left(\frac{232}{3} - \frac{536}{3}x\right)\ln x \\ & \quad \left. + \frac{536}{9}\left(\frac{1}{1-x}\right)_+ - \frac{388}{9}x - \frac{148}{9} + \delta(1-x)\left(24\zeta(3) + \frac{64}{3}\right)\right] \\ & +C_A T_f \left[-\frac{160}{9}\left(\frac{1}{1-x}\right)_+ - \frac{32}{3}(1+x)\ln x - \frac{448}{9} + \frac{608}{9}x\right. \\ & \quad \left. -\frac{32}{3}\delta(1-x)\right] \\ & +C_F T_f \left[-16(1+x)\underline{\ln^2 x} + 16(x-5)\ln x - 80(1-x)\right. \\ & \quad \left. -8\delta(1-x)\right]. \end{aligned} \tag{3.68}$$

LARGEST SMALL x SINGULARITY $\propto \alpha_s \ln^2 x$.

POSSIBLE FUTURE MEASUREMENT AT HERA:
POL. e & POL. p (820 GeV).



☆ GS



SCALING VIOL. !

LOW Q^2 ⇔ HERA.

Figure 6: Statistical precision of a measurement of $g_1^p(x, Q^2)$ in the kinematical domain of HERA at larger values of x. The data points represent averages over the accessible Q^2 range and were calculated using the parametrization [6]. The dashed, dotted, and upper dash-dotted line correspond to the values of $g_1^p(x, \langle Q^2 \rangle)$ for the parametrizations [8], [7], and [5], respectively. The lower dash-dotted line shows $g_1^p(x, Q_0^2)$ for $Q_0^2 = 4\,\mathrm{GeV}^2$ for parametrization [5].

9. Open Problems

- QED : LARGE y NLO CORRECTION
  $\longrightarrow$ ALLOW TO MEASURE THERE DESPITE OF LARGE CORRECTIONS
  F_L, g_1, xG_3

- QCD : 3 LOOP ANOMALOUS DIMENSIONS F_2 : NS, S
  FIXED & SPLITTING FCT.
  ORDER NNLO ANALYSIS : F_2, W_2, xW_3 (TWIST 2)

- RESUMMATION : • NTLO BFKL TERMS
  • $(\alpha_s \ln^2 x)^n$ RESUMMABLE ?
  (& CORRECTIONS TO IT)

- DIFFRACTIVE PART OF F_2 AT HIGH Q^2:
  IS THERE A CONSISTENT DESCRIPTION WITHIN PERTURBATIVE QCD ?

- NTLO CORRECTIONS TO DIFFERENT PROCESSES IN POLARIZED LEPTON- POLARIZED NUCLEON SCATTERING

- NTLO CORRECTIONS TO $\gamma^* N \rightarrow J/\psi X$, $Q^2 > 0$.

- CONSISTENT APPROACH FOR HIGHER TWIST TERMS :
  - TWIST 3 OPERATORS : $g_2(x, Q^2)$
  - TWIST 4 OPERATORS : OPE , FWD COMPTON AMPLITUDE

    $\longleftrightarrow$ REGGE THY APPROACHES