SPIN '96
Amsterdam
September 1996

Small x Resummation for Polarized Structure Functions

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Phys. Lett. **B370** (1996) 149, Acta Phys. Pol. **B27** (1996) 1309
DESY 96–050, Phys. Lett. **B** (1996) in press.

1. Introduction

- FOR $x \to 0$ THE ANOMALOUS DIMENSIONS FOR NS & S BEHAVE AS

$$\propto \alpha_s \left(\alpha_s \ln^2 x\right)^\ell$$

- HOW STRONGLY $g_{1,5}^{e,p,en}\big|_{x\to 0}$ ARE AFFECTED ?

$\longrightarrow$ NONPERTURBATIVE INPUT & MELLIN CONVOLUTION

$$x \sim 0 \quad \longleftrightarrow \quad x \sim 1$$

$\longrightarrow$ LESS SINGULAR TERMS (STRUCTURE OF ANOM. DIMS., ... CONSERVATION LAWS)

$\Longrightarrow$ WHICH NUMERICAL CONCLUSION ?

2. The Non-Singlet Case

$$\frac{\partial f_{NS}^{\pm}}{\partial \log Q^2} = P_{NS}^{\pm}(x, a_s) \otimes f_{NS}^{\pm}(x, Q^2)$$

$$P_{NS}^{\pm}(x, Q^2) = \sum_{l=0}^{\infty} a_s^{l+1} P_l^{\pm}(x) \quad , \quad a_s \equiv \frac{\alpha_s}{4\pi}$$

$$\frac{\partial a_s}{\partial \log Q^2} = - \sum_{l=0}^{\infty} \beta_l a_s^{l+2}$$

$\longrightarrow$ TRANSFORM TO EVOLUTION eqs. OF OBSERVABLES

$$P_l^{\pm} \longrightarrow K_l^{\pm}$$

$$f_{NS}^{\pm} \longrightarrow F_{NS,i}^{\pm} = C_i^{\pm} \otimes f_i^{NS,\pm}$$

$\longrightarrow$ THE SMALL x RESUMMATION OF $K^{\pm}$ CAN BE DETERMINED

KIRSCHNER, LIPATOV 1983

$\longrightarrow$ ADD THIS RESUMMATION BEYOND NLO.

$$\mathcal{M}[K^{\pm}_{x\to 0}](N) = -\frac{1}{2}\Gamma^{\pm}(N, a_s)$$

$$\Gamma^{+}(N, a_s) = -N\left\{1-\left(1-\frac{8a_s C_F}{N^2}\right)^{1/2}\right\}$$

$$\Gamma^{-}(N, a_s) = -N\left\{1-\left[1-\frac{8a_s C_F}{N^2}\left(1-\frac{8N_c a_s}{N}\frac{d}{dN}\mathbb{F}\right)\right]^{1/2}\right\}$$

$$\mathbb{F} = \ln\left[\exp\left(\tfrac{1}{4}z^2\right) D_{-\frac{1}{2N_c^2}}(z)\right]$$

$$z = N/\sqrt{2N_c a_s}$$

	TAYLOR SERIES	ASYMPT. SERIES
l	K_l^+	K_l^-
0	2.667E0	2.667E0
1	3.556E0	5.333E0
2	1.580E0	1.432E0
3	3.512E-1	9.964E-1
4	4.682E-2	-2.078E-1
5	4.162E-3	1.448E-1
6	2.643E-4	-5.777E-2
7	1.258E-5	2.168E-2
8	4.661E-7	-7.173E-3
9	1.381E-8	2.143E-3
10	3.348E-10	-5.827E-4

$\longleftrightarrow$ THE COMPLETE EXPRESSIONS MAY BE USED AS WELL.

Table 1: The coefficients $K_l^{\pm}$ of the expansion of $K^{\pm}_{x\to 0}(x, a_s)$ in terms of $a_s(a_s \ln^2 x)^l$ as obtained from the resummations in eqs. (17) and (18).

3. The Singlet Case

SMALL x CONTRIBUTIONS TO THE EVOLUTION MATRIX:

$$\mathbb{P}(x, a_s)_{x\to 0} \equiv \sum_{\ell=0}^{\infty} \mathbb{P}^{(\ell)}_{x\to 0}\, a_s^{\ell+1} \ln^{2\ell} x = \frac{1}{8\pi^2} \mathcal{M}^{-1}[\mathbb{F}_0(N, a_s)]$$

BARTELS, ERMOLAEV, RYSKIN '96

$$\mathbb{F}_0(N, a_s) = 16\pi^2 \frac{a_s}{N} \mathbb{M}_0 - \frac{8 a_s}{N^2} \mathbb{F}_8(N, a_s)\, \mathbb{G}_0 + \frac{1}{8\pi^2} \frac{1}{N} \mathbb{F}_0^2$$

$$\mathbb{F}_8(N, a_s) = 16\pi^2 \frac{a_s}{N} \mathbb{M}_8 + \frac{2 a_s}{N} C_G \frac{d}{dN} \mathbb{F}_8(N, a_s) + \frac{1}{8\pi^2} \frac{1}{N} \mathbb{F}_8^2$$

$$\mathbb{M}_0 = \begin{pmatrix} C_F & -2T_f N_f \\ 2C_F & 4C_A \end{pmatrix} \qquad \mathbb{G}_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}$$

$$\mathbb{M}_8 = \begin{pmatrix} C_F - C_A/2 & -T_f N_f \\ C_A & 2C_A \end{pmatrix}$$

$\longrightarrow$ SOLVE FOR $\mathbb{P}^{(\ell)}_{x\to 0}\Big|_{\ell=0}^{\infty}$

$$P^{(0)}_{x\to0} = 2\begin{pmatrix} C_F & -2\,T_F N_f \\ 2\,C_F & 4\,C_A \end{pmatrix}, \tag{12}$$

$$P^{(1)}_{x\to0} = 2\begin{pmatrix} C_F(2C_A - 3C_F - 4T_F N_f) & -2T_F N_f(2C_A + C_F) \\ 2C_F(2C_A + C_F) & 8\,C_A^2 - 4\,T_F N_f\,C_F \end{pmatrix}. \tag{13}$$

They agree with the respective leading $\ln^{2l} x$ terms of the complete LO and NLO splitting function matrices [3, 4] [2]. We also list the entries of $P^{(2)}_{x\to0}$ and $P^{(3)}_{x\to0}$ explicitely in the colour factors:

$$\begin{aligned}
P^{(2)}_{qq} &= \frac{2}{3}C_F\left[-5\,C_F^2 - \frac{3}{2}\,C_A^2 + 6\,C_A\,C_F - 8\,T_F N_f\,C_F - 6\,T_F N_f\,C_A\right] \\
P^{(2)}_{qg} &= \frac{2}{3}T_F N_f\left[-15\,C_A^2 + 2\,C_F^2 - 6\,C_F\,C_A + 8\,T_F N_f C_F\right] \\
P^{(2)}_{gq} &= \frac{2}{3}C_F\left[15\,C_A^2 - 2\,C_F^2 + 6\,C_F\,C_A - 8\,T_F N_f C_F\right] \\
P^{(2)}_{gg} &= \frac{2}{3}\left[28\,C_A^3 + 2\,T_F N_f\,C_A^2 - 4\,T_F N_f\,C_F^2 - 24\,C_F\,T_F N_f\,C_A\right],
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
P^{(3)}_{qq} &= \frac{2}{45}C_F\Big[6\,C_A^3 - 20\,C_F C_A^2 + 22\,C_A C_F^2 - \frac{19}{2}\,C_F^3 - 74\,T_F N_f\,C_A^2 - 44\,C_F T_F N_f\,C_A \\
&\qquad + 2\,T_F N_f\,C_F^2 + 40\,(T_F N_f)^2 C_F\Big] \\
P^{(3)}_{qg} &= \frac{2}{45}T_F N_f\Big[-54\,C_F\,C_A^2 - 2\,C_F^2 C_A + 40\,T_F N_f C_F^2 - 128\,C_A^3 - 8\,T_F N_f C_A^2 + 15\,C_F^3 \\
&\qquad + 108\,T_F N_f C_F C_A\Big] \\
P^{(3)}_{gq} &= \frac{2}{45}C_F\Big[54\,C_F\,C_A^2 + 2\,C_F^2 C_A - 40\,T_F N_f C_F^2 + 128\,C_A^3 + 8\,T_F N_f C_A^2 - 15\,C_F^3 \\
&\qquad - 108\,T_F N_f C_F C_A\Big] \\
P^{(3)}_{gg} &= \frac{2}{45}\Big[-288\,T_F N_f\,C_F\,C_A^2 - 64\,C_F^2 T_F N_f\,C_A + 6\,T_F N_f\,C_F^3 + 40\,(T_F N_f)^2 C_F^2 \\
&\qquad + 20\,T_F N_f\,C_A^3 + 252\,C_A^4\Big].
\end{aligned} \tag{15}$$

	$N_f = 3$			
l	$P^{(l)}_{qq}$	$P^{(l)}_{qg}$	$P^{(l)}_{gq}$	$P^{(l)}_{gg}$
0	0.2666666667D1	-6.D0	0.5333333333D1	24.D0
1	-0.1066666667D2	-44.D0	0.3911111111D2	128.D0
2	-0.3679012346D2	-0.1394444444D3	0.1239506173D3	0.4188888889D3
3	-0.6642085048D2	-0.2288296296D3	0.2034041152D3	0.6981037037D3
4	-0.6110486315D2	-0.2154469209D3	0.1915083742D3	0.6685486038D3
5	-0.3858083350D2	-0.1347153415D3	0.1197469702D3	0.4201415122D3
6	-0.1679581048D2	-0.5955393524D2	0.5293683133D2	0.1868497637D3
7	-0.5631967112D1	-0.1979044200D2	0.1759150400D2	0.6213345892D2
8	-0.1424435573D1	-0.5081773820D1	0.4517132284D1	0.1601700122D2
9	-0.2991318204D0	-0.1049686114D1	0.9330543234D0	0.3302769905D1
10	-0.4868787168D-1	-0.1756718578D0	0.1561527625D0	0.5557315074D0
	$N_f = 4$			
l	$P^{(l)}_{qq}$	$P^{(l)}_{qg}$	$P^{(l)}_{gq}$	$P^{(l)}_{gg}$
0	0.2666666667D1	-8.D0	0.5333333333D1	24.D0
1	-16.D0	-0.5866666667D2	0.3911111111D2	0.1226666667D3
2	-0.4953086420D2	-0.1788148148D3	0.1192098765D3	0.3905185185D3
3	-0.8573278464D2	-0.2859456790D3	0.1906304527D3	0.6315654321D3
4	-0.7633439480D2	-0.2595199393D3	0.1730132928D3	0.5831456986D3
5	-0.4648843309D2	-0.1568583694D3	0.1045722463D3	0.3540380256D3
6	-0.1955536907D2	-0.6687789759D2	0.4458526506D2	0.1519476618D3
7	-0.6326464875D1	-0.2148071223D2	0.1432047482D2	0.4881870016D2
8	-0.1545655016D1	-0.5319199339D1	0.3546132893D1	0.1214527167D2
9	-0.3133281583D0	-0.1062889464D1	0.7085929761D0	0.2420644158D1
10	-0.4922813282D-1	-0.1712782960D0	0.1141855307D0	0.3928177051D0

Table 1: The elements of the coefficient matrices $P^{(l)}_{x\to0}$ in eq. (9) for $N_f = 3$ and $N_f = 4$.

PROPERTIES OF THE SOLUTION:

$$\boxed{P^{(\ell)}_{qg_{x\to 0}} \cdot \frac{1}{T_f N_f} \equiv - P^{(\ell)}_{gq} \cdot \frac{1}{C_F}}$$

- SUSY LIMIT: $C_A = C_F = N_F = 1; \quad T_F = \frac{1}{2}.$

$$P^{(\ell)}_{qq}(x) + P^{(\ell)}_{gq}(x) - P^{(\ell)}_{qg}(x) - P^{(\ell)}_{gg}(x) = 0$$

$$\mathbb{P}^{SUSY}_{x\to 0} = 2 a_s \mathbb{M}_1 + \sum_{\ell=1}^{\infty} a_s^{\ell+1} \ln^{2\ell} x \; \mathbb{M}_2$$

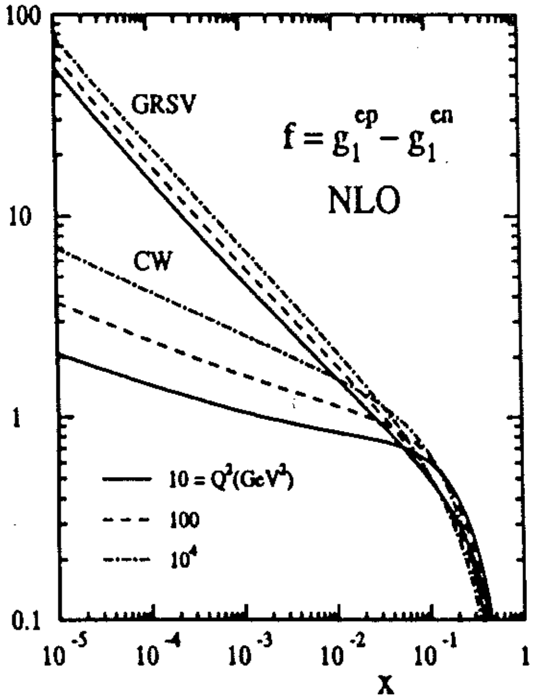
$$\mathbb{M}_1 = \mathbb{M}_0^{SUSY} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \quad ; \quad \mathbb{M}_2 = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix}.$$

$$\mathbb{M}_1 \mathbb{M}_2 = 3\,\mathbb{M}_2 , \quad \mathbb{M}_2^2 = \mathbb{M}_2 , \quad [\mathbb{M}_1, \mathbb{M}_2] = 0 .$$

4. Numerical Results

NS

i) NON SINGLET



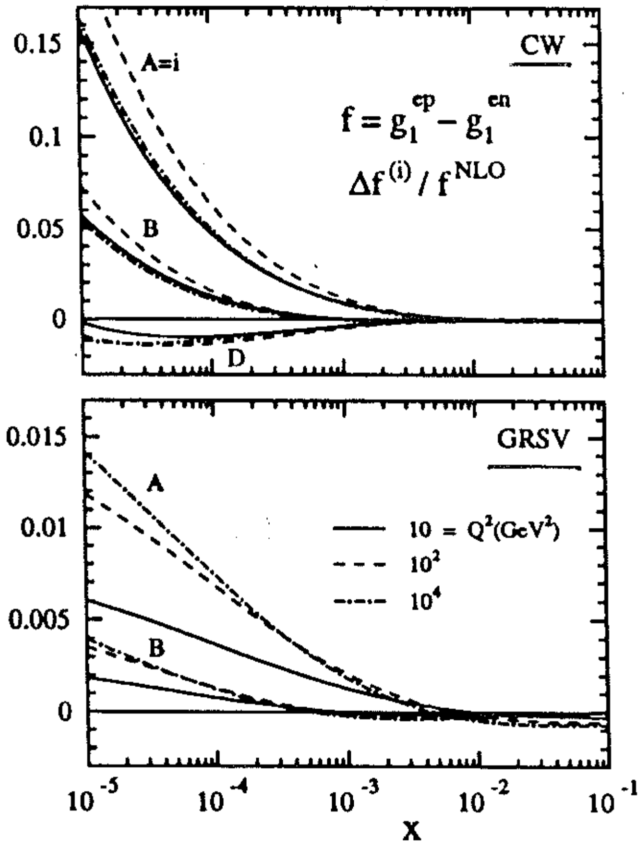
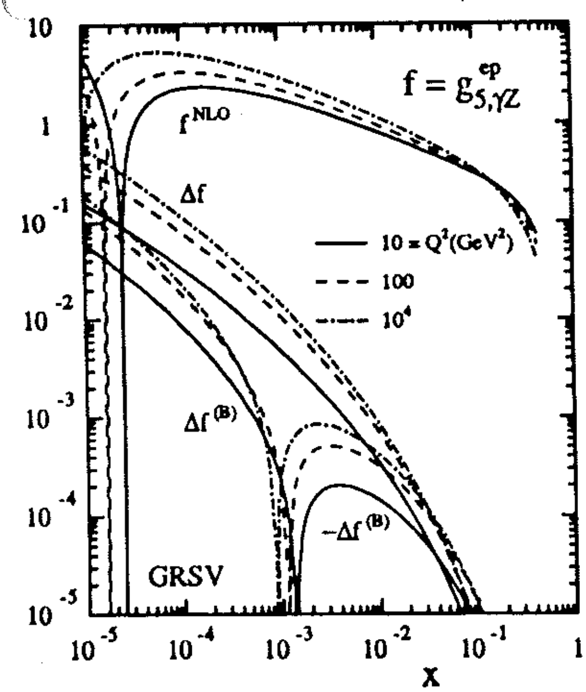


Figure 1: The NLO small-x evolution of the polarized non–singlet structure function combination $g_1^{ep} - g_1^{en}$, and the relative corrections due to the resummed kernels, for the initial distributions of refs. [7] and [8]. The dependence on possible less singular terms is illustrated by the prescriptions 'A', 'B', and 'D' of eq. (1). The figure has been adapted from ref. [6].



NS A: $P(N) \longrightarrow P(N) - P(1)$

B: $P(N) \longrightarrow P(N)(1-N)$

(F-Number cons.)

Figure 6: The small-x evolution of the non–singlet interference structure function $g_{5,\gamma Z}^{ep}$ in NLO and the resummation corrections to these results. The possible importance of less singular terms in the higher–order splitting functions is illustrated by the prescription '(B)' for this '+'-case.

NLO and SUBLEADING TERMS:

$$
\begin{aligned}
\gamma^{\mathrm{pol}}_{qq,1}(N)_{x\to 0} &= \frac{-64+64N_f}{3N^3}+\frac{464-128N_f}{9N^2} \\
&\overset{N_f=3}{=} +\frac{42.67}{N^3}+\frac{8.889}{N^2}\,, \\
\gamma^{\mathrm{pol}}_{qg,1}(N)_{x\to 0} &= +\frac{176N_f}{3N^3}-\frac{24N_f}{N^2} \\
&\overset{N_f=3}{=} +\frac{176.0}{N^3}-\frac{72.00}{N^2}\,, \qquad (42) \\
\gamma^{\mathrm{pol}}_{gq,1}(N)_{x\to 0} &= -\frac{1408}{9N^3}+\frac{896}{9N^2} \\
&\overset{N_f=3}{=} -\frac{156.4}{N^3}+\frac{99.56}{N^2}\,, \\
\gamma^{\mathrm{pol}}_{gg,1}(N)_{x\to 0} &= \frac{-1728+64N_f}{3N^3}+\frac{2088-208N_f}{3N^2} \\
&\overset{N_f=3}{=} -\frac{512.0}{N^3}+\frac{488.0}{N^2}\,.
\end{aligned}
$$

ii) SINGLET :

GRSV (STAND.)

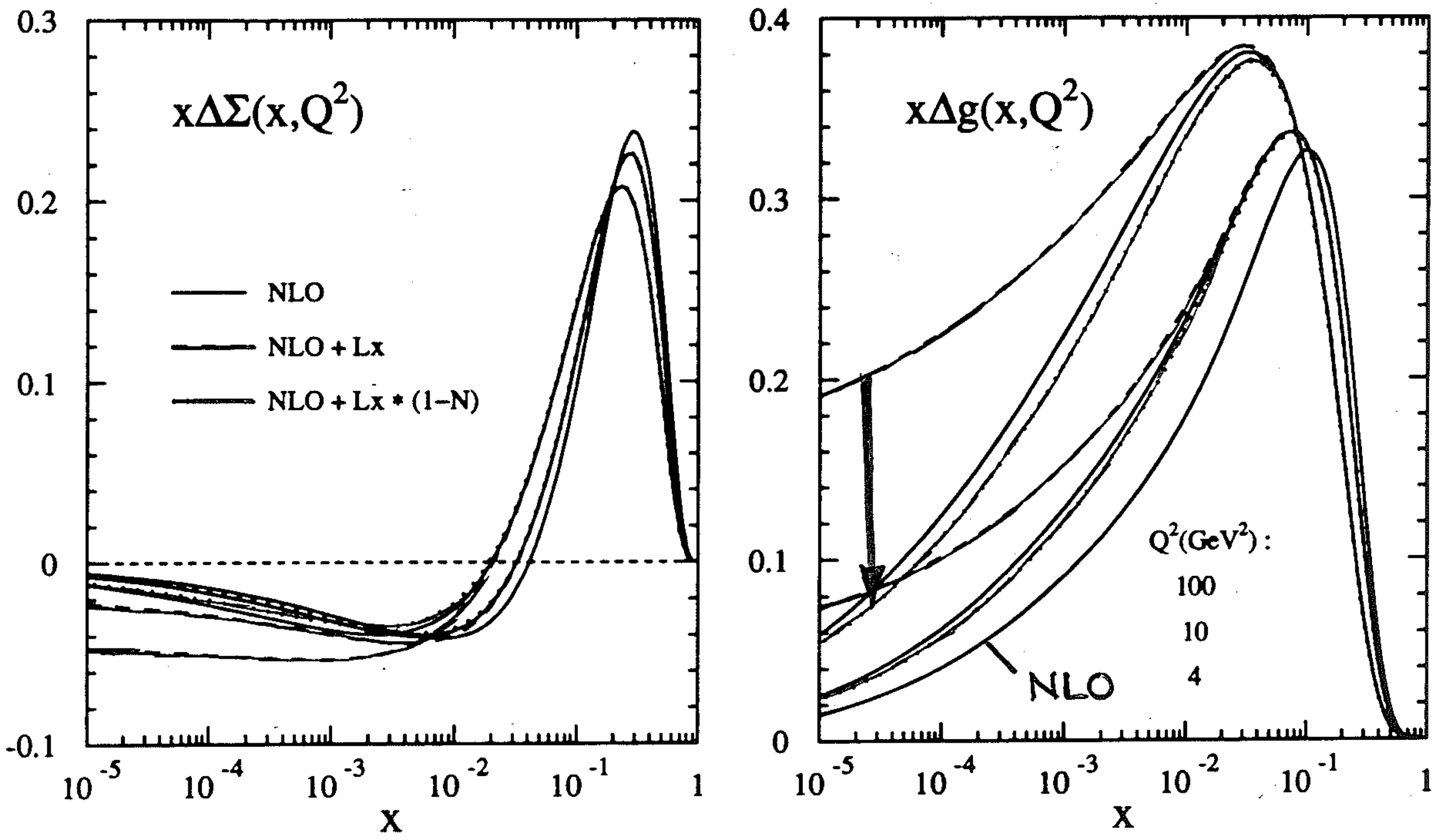
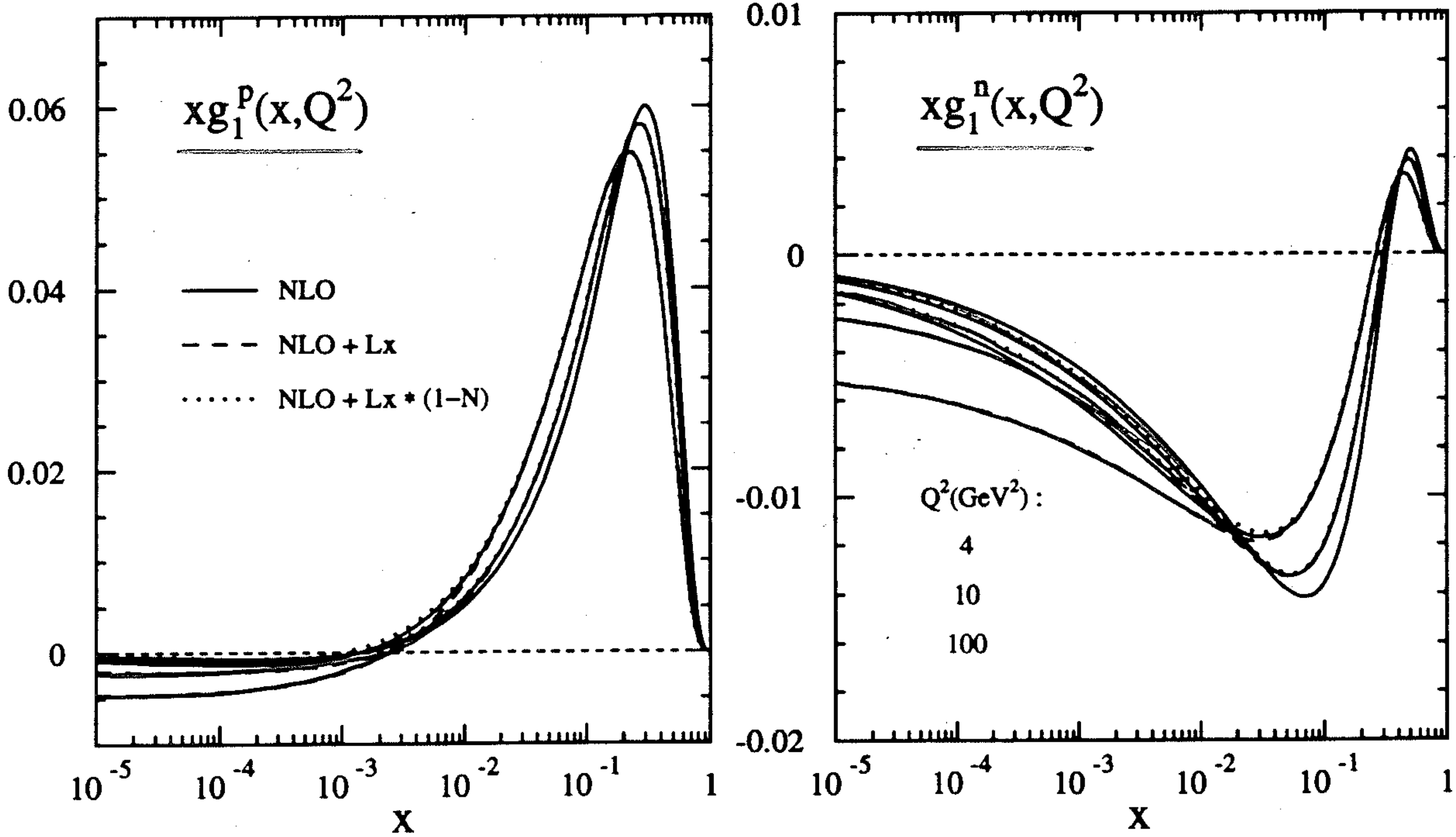


Fig. 1

Q^2	10 GeV^2		100 GeV^2	
x	10^{-4}	10^{-3}	10^{-4}	10^{-3}
$x\Delta\Sigma$	-0.0100 -0.0285 -0.0473	-0.0169 -0.0396 -0.0560	-0.0171 -0.0505 -0.0855	-0.0218 -0.0523 -0.0772
$x\Delta g$	0.019 0.101 0.201	0.034 0.152 0.294	0.053 0.226 0.432	0.071 0.281 0.528

Table 2: A comparison of the resummed evolution of the polarized parton distributions for different assumptions on the gluon distribution Δg. Upper lines: minimal gluon, middle lines: standard set, lower lines: maximal gluon (and corresponding quark distributions) of ref. [9] at $Q_0^2 = 4$ GeV^2.



$xg_1^p(x,Q^2)$
NLO
NLO + Lx
NLO + Lx * (1−N)
0.06
0.04
0.02
0
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1
x
$xg_1^n(x,Q^2)$
Q^2(GeV^2) :
4
10
100
0.01
0
-0.01
-0.02
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1
x

Fig. 2

5. Conclusions

- THE SMALL x LEADING TERMS WERE RESUMMED USING THE CORRESPONDING RGE'S

- NS : CURRENT PARAMETRIZATIONS
  ↪ $\lesssim 1\%$ EFFECT DUE TO THE RESUMMED TERMS (F-NUMBER C. DOWN BY $\times 1/3$

- S : STRONG EFFECT
  $P \to P \cdot (1-N)$: COMPLETE DISAPPEARANCE !
  ↑
  NOT UNPROBABLE ↔ NLO

- FIND RESUMMATION FOR THE NEXT TO SINGULAR TERMS;
  3-LOOP RESULTS ↔ SMALL x.