STRUCTURE FUNCTIONS AT SMALL x

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1. INTRODUCTION
2. TWIST-2 TERMS AT SMALL x
3. FAN-DIAGRAMS & SCREENING
4. SEMI-CLASSICAL RESULTS
5. CONCLUSIONS

1. INTRODUCTION

DEEP INELASTIC SCATTERING : $ep \to eX$; (γ^*)

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[2xF_1(x,Q^2)\,y^2 + 2(1-y)F_2(x,Q^2)\right]$$

BEHAVIOR OF $F_1(x,Q^2)$, $F_2(x,Q^2)$?

- $x \sim O(1)$, $\ln(Q^2/\Lambda^2) \gg 1$ $\longrightarrow$ AP- equations

$$\begin{pmatrix} q_i(Q^2,x) \\ G(Q^2,x) \end{pmatrix} = A_{kj}(Q^2,Q_0^2,x) \otimes \begin{pmatrix} q_i(Q_0^2,x) \\ G(Q_0^2,x) \end{pmatrix}$$

- HIGH Q^2, $x \ll 1$ (STILL SUFFICIENTLY LARGE)
  'SEMI-HARD' RANGE

Fig.
Fig.

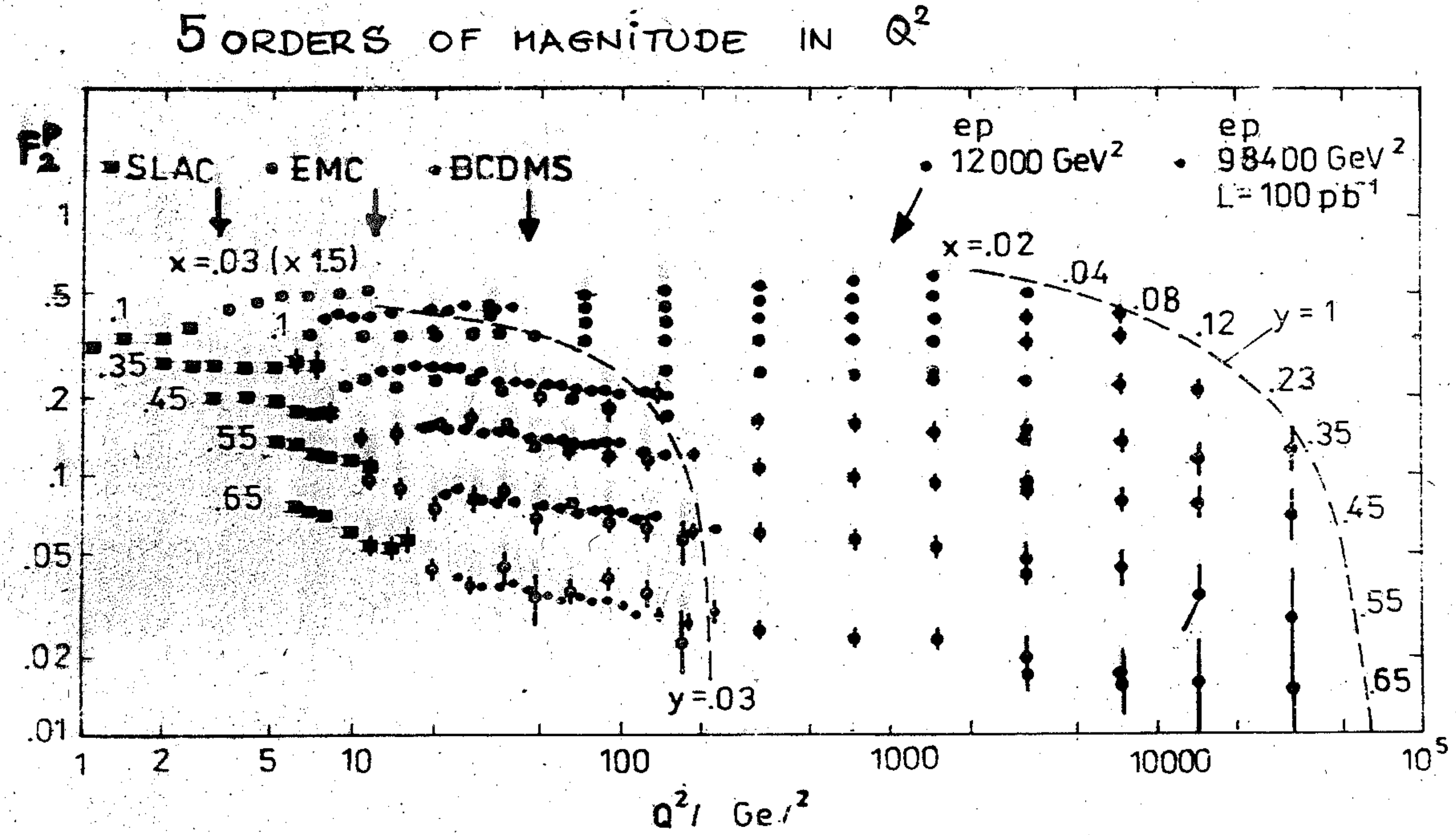
$\longrightarrow$ $F_2(x,Q^2)$, $F_L(x,Q^2)$, $xG(x,Q^2)$

BEHAVIOUR AS $x \to 0$ (OR x GETTING SMALL)
$Q^2 =$ CONST.

SATURATION ?

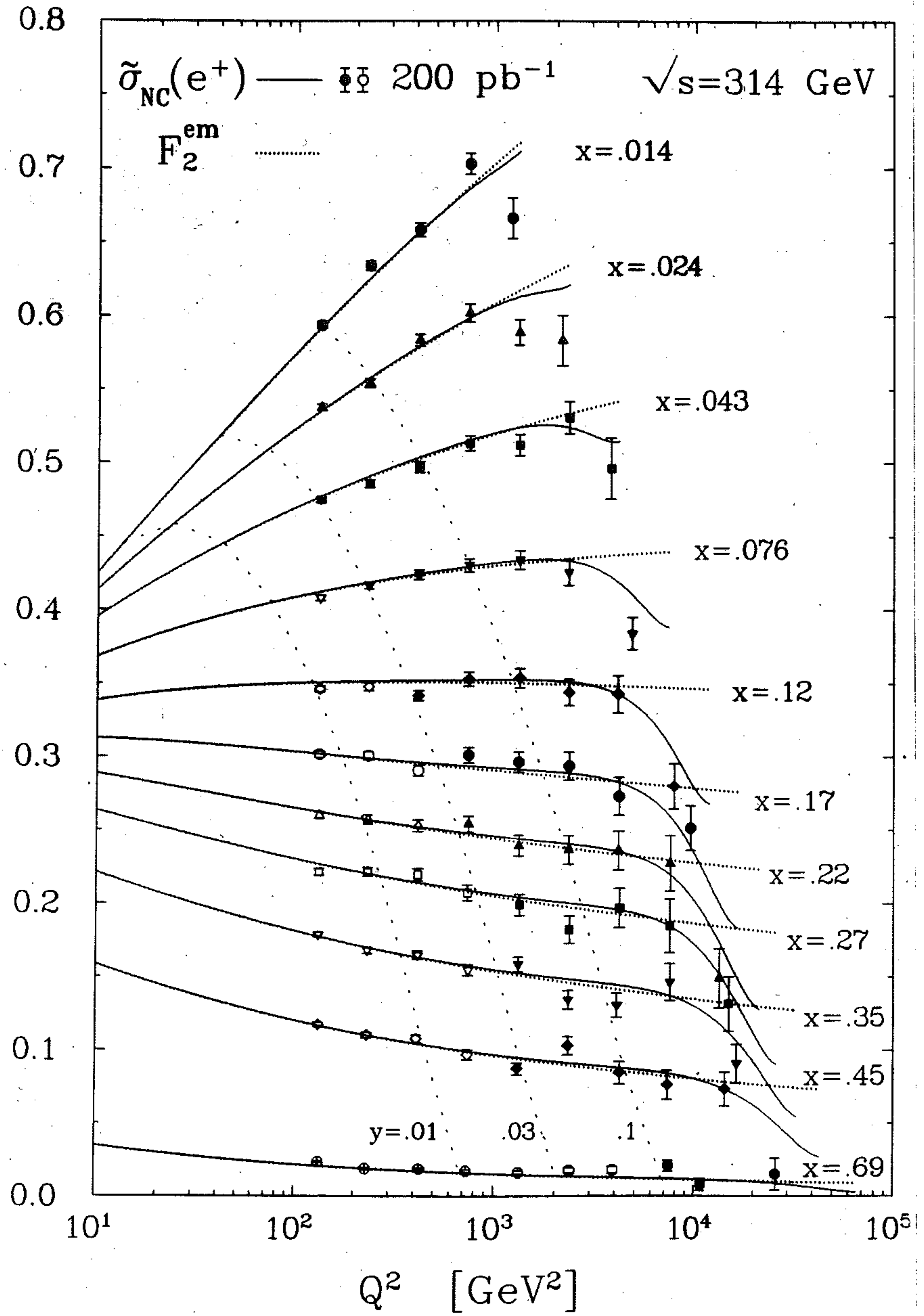
QUANTITATIVE THEORY REQUIRED.
MEASUREMENT : $\pm 1\%$! (HERA)
FIND A WAY FROM :

$$\infty \longrightarrow \approx \longrightarrow = .$$



5 ORDERS OF MAGNITUDE IN Q^2
F_2^p
SLAC
EMC
BCDMS
ep
12000 GeV^2
ep
98400 GeV^2
L = 100 pb^{-1}
x = .03 (x 1.5)
x = .02
.04
.08
.12
.23
.35
.45
.55
.65
y = 1
y = .03
.1
.35
.45
.55
.65
1
.5
.2
.1
.05
.02
.01
1
2
5
10
100
1000
10000
10^5
Q^2/ Ge$/^2$

Fig. 6



$\tilde{\sigma}_{NC}(e^+)$ — 200 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
x=.014
x=.024
x=.043
x=.076
x=.12
x=.17
x=.22
x=.27
x=.35
x=.45
x=.69
y=.01
.03
.1
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^1
10^2
10^3
10^4
10^5
Q^2 [GeV^2]

Fig. 3

2. KINEMATICAL RANGES

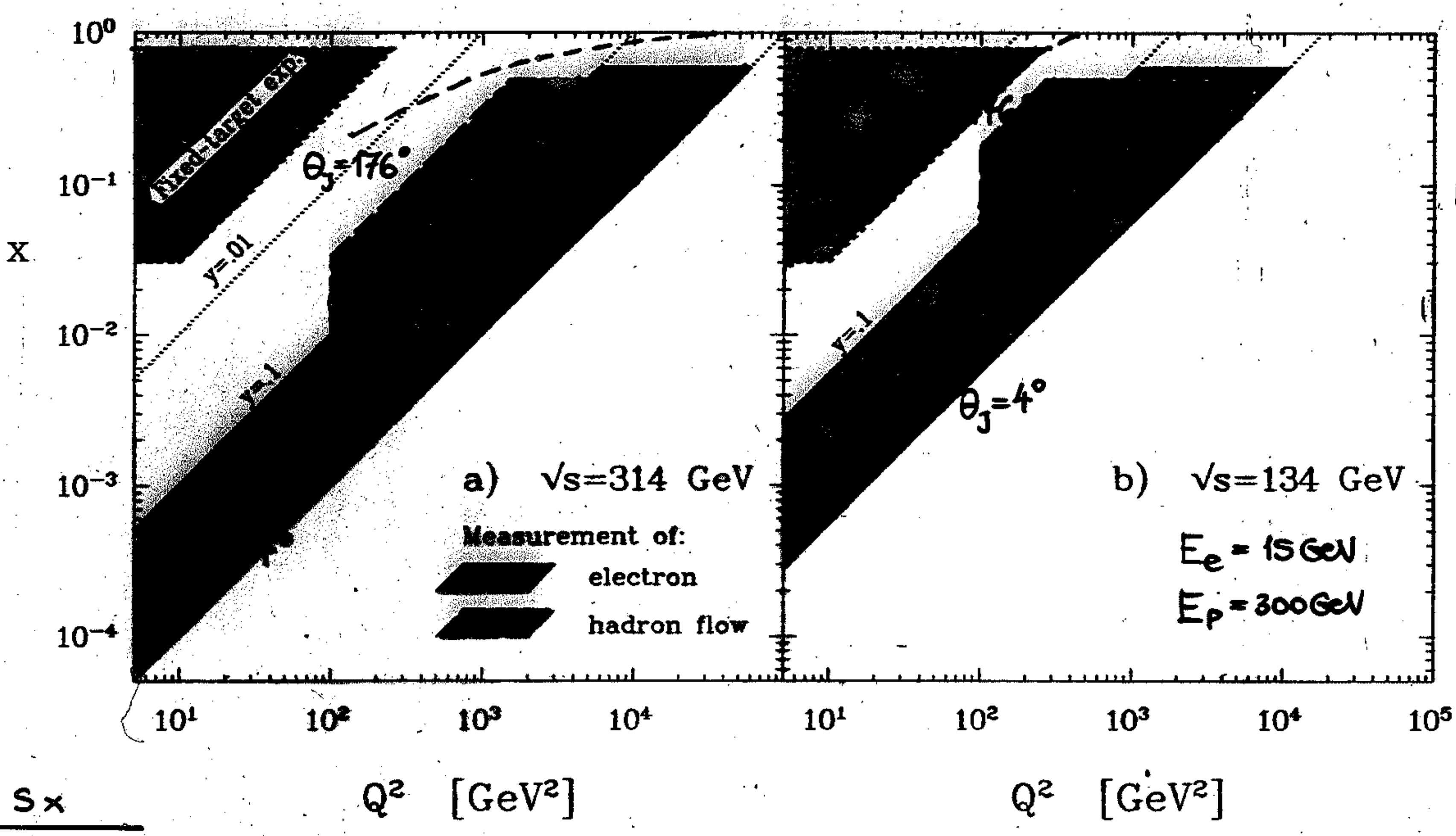
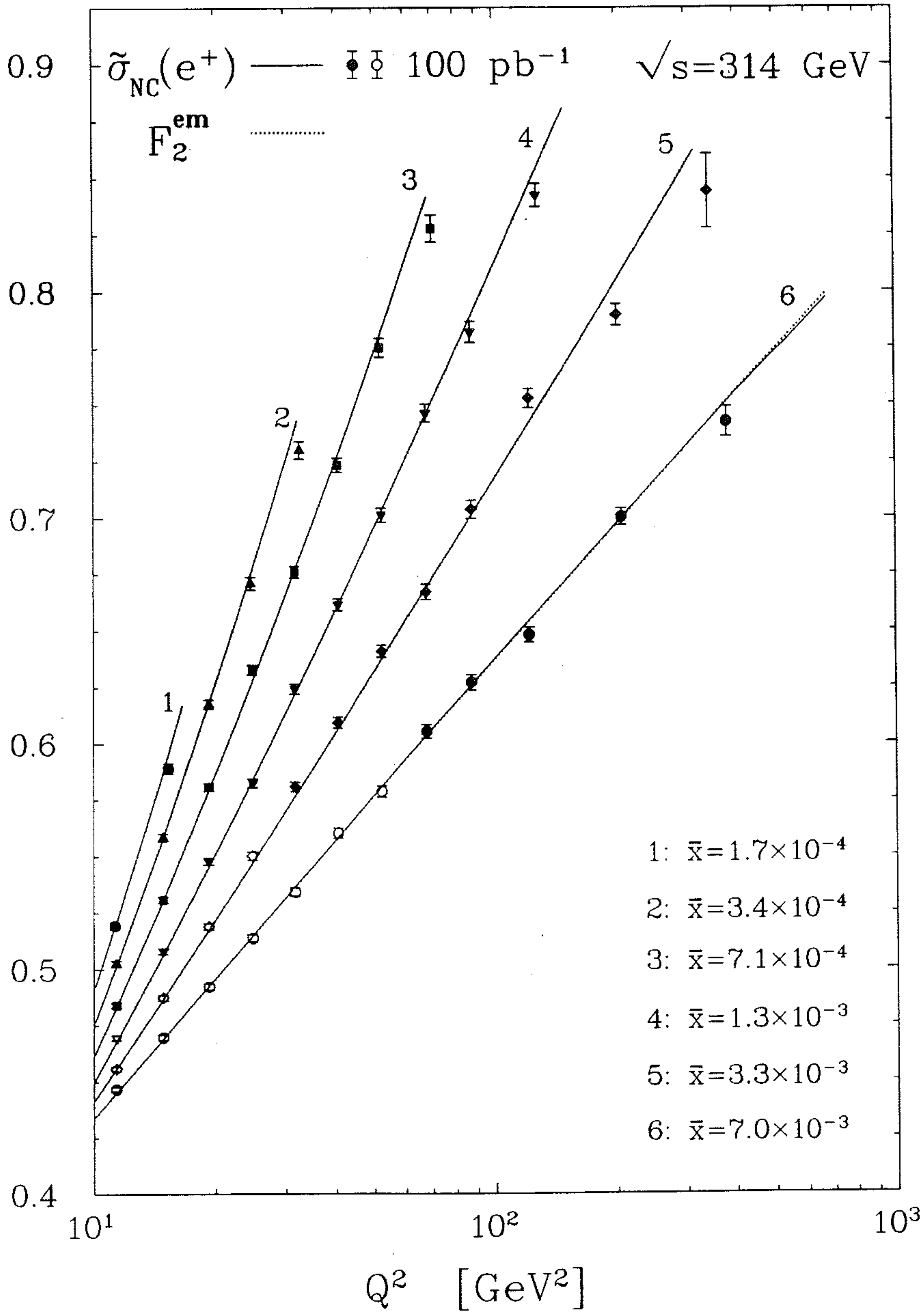


Fig. 1

$$Q_e^2 = \frac{sx}{1+\frac{xE_p}{E_e}\operatorname{ctg}^2\frac{\vartheta_e}{2}}$$

$$Q_J^2 = \frac{sx}{1+E_e/xE_p \operatorname{ctg}^2 \vartheta_J/2}$$



$\tilde{\sigma}_{NC}(e^+)$ — 100 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
1: $\bar{x}=1.7\times10^{-4}$
2: $\bar{x}=3.4\times10^{-4}$
3: $\bar{x}=7.1\times10^{-4}$
4: $\bar{x}=1.3\times10^{-3}$
5: $\bar{x}=3.3\times10^{-3}$
6: $\bar{x}=7.0\times10^{-3}$
Q^2 [GeV^2]

Fig. 6

2. TWIST-2 TERMS AT SMALL X

STRUCTURE FUNCTIONS: $W_1, W_2, W_3, \ldots$

$\longrightarrow$ OPE $\Longleftrightarrow$ AP-EQUATIONS : $\ln \frac{Q^2}{\Lambda^2} \gg 1$

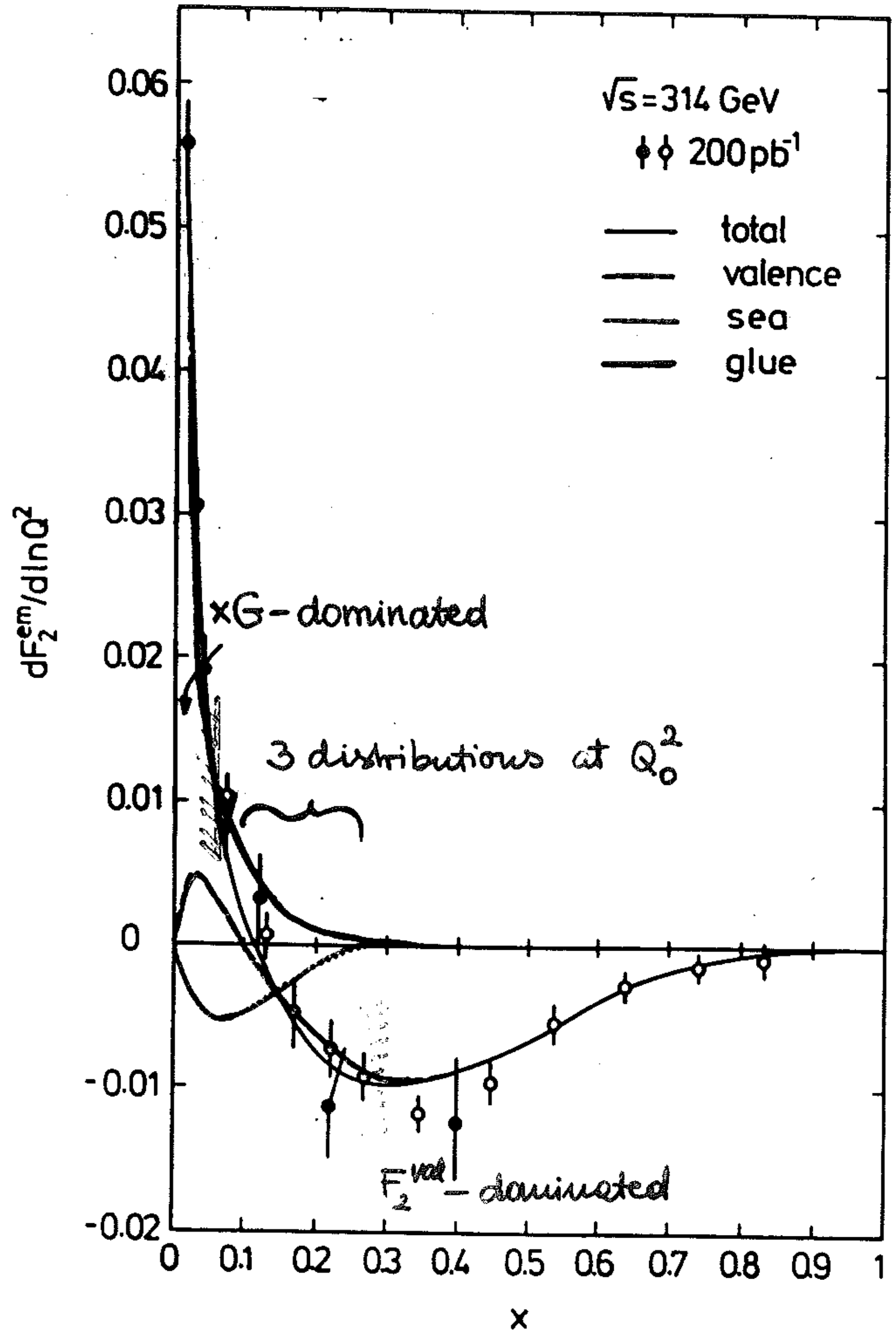
$$\frac{d\Sigma(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{qq}(x)\otimes\Sigma(x,Q^2) + P_{qG}(x)\otimes G(x,Q^2)\right]$$

$$\frac{dG(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\left[P_{Gq}(x)\,\Sigma(x,Q^2) + P_{GG}(x)\otimes G(x,Q^2)\right]$$

$\Longrightarrow$ $x \ll 1$

$$P_{GG}(z) = 6\left\{\frac{z}{(1-z)_+} - 1 + \frac{1}{z} + z(1-z) + \frac{\beta_0}{6}\delta(1-z)\right\}$$

- $P_{GG}(z) \longrightarrow \frac{6}{z}$
- VANISHING FERMION CONTRIBUTIONS



√s=314 GeV
200pb⁻¹
total
valence
sea
glue
dF₂ᵉᵐ/dlnQ²
xG-dominated
3 distributions at Q₀²
F₂ᵛᵃˡ-dominated
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
x

$$\frac{d\,xG(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dx'}{x'}\; 6\,\frac{x^2}{x'}\left[G(x',Q^2)\,x'\right]\frac{1}{x'}$$

REWRITE :

$$\alpha_s(Q^2)\,d\ln Q^2 = \frac{12\pi}{3\beta_0}\,d\ln\ln(Q^2/\Lambda^2) = \frac{4\pi}{\beta_0}\,d\xi$$

$$\frac{d\,xG(x,Q^2)}{d\xi} = \frac{2}{\beta_0}\cdot 6\,x^2\int_x^1\frac{dx'}{x'^3}\left[x'G(x',Q^2)\right]$$

$$\boxed{\frac{d^2 xG(x,Q^2)}{dy\,d\xi} = \frac{1}{2}\,xG(x,Q^2)}\;;\quad x\to 0$$

DLA - FORM
OF THE AP - EQU.

$y = \frac{8\cdot N_c}{\beta_0}\ln\frac{1}{x}$; $N_c = 3$.

- DE GRAND
- RALSTON, MC KAY.

GLR - VARIABLES :

$$\left.\begin{aligned}\xi &= \ln\ln\left(\frac{Q^2}{\Lambda^2}\right)\\ y &= \frac{8N_c}{\beta_0}\ln\frac{1}{x}\end{aligned}\right\}\left\{\begin{aligned}\xi &= \frac{8N_c}{\beta_0}\ln\ln\left(\frac{Q^2}{\Lambda^2}\right)\\ y &= \ln\frac{1}{x}\end{aligned}\right.$$

KWIECINSKI

SOLUTION OF THE DLA-AP-EQUATION:

- LINEAR HYPERBOLIC PARTIAL DEQU.
- DETERMINE GREENS FUNCTION
- SOLUTION:

$$xG(x,Q^2) = \mathbb{G}(y,\xi) = \sum_{\nu=0}^{\infty} \left\{ A_\nu \left(\frac{2\xi}{y}\right)^{\nu/2} + B_\nu \left(\frac{y}{2\xi}\right)^{\nu/2} \right] I_\nu\left(\sqrt{2\xi y}\right)$$

- DETERMINE A_ν, B_ν : STARTING VALUES.

$\longrightarrow$ GLR: $\mathbb{G}(y,0) = 1 \quad \curvearrowright \quad \mathbb{G}(y,\xi) = I_0\left(\sqrt{2\xi y}\right)$

$\Lambda^2 = Q_0^2$

$\longrightarrow$

$$I_0\left(\sqrt{2y(\xi-\xi_0)}\right) \underset{\xi y \gg 1}{\simeq} \frac{\exp\left(\sqrt{2(\xi-\xi_0)y}\right)}{\sqrt{2\pi}\left[2(\xi-\xi_0)\right]^{1/4}}$$

WELL KNOWN FORMULA.

> FOR $\xi = \text{const.}$ & $y \to 0$ GROWS $I_\nu\left(\sqrt{2(\xi-\xi_0)y}\right)$ FASTER THAN ANY POWER OF $\ln\frac{1}{x}$.
>
> UNITARITY WILL BE VIOLATED.

3. FAN DIAGRAMS & SCREENING

CAN UNITARITY BE RE-INSTALLED IN QCD ?

TWIST - 2 : NO

⟹ HIGHER TWIST TERMS SHOULD DO IT, IF QCD IS THE CORRECT THEORY.

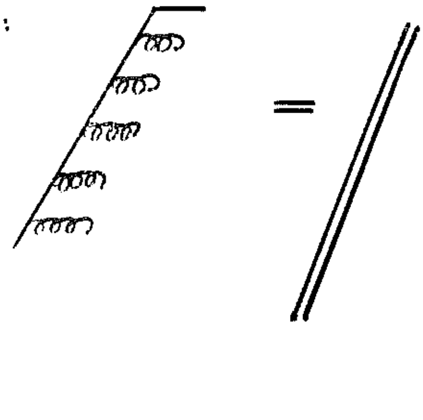
- THERE IS RATHER AN INTUITIVE PICTURE THAN A RIGOROUS PROOF OF THE SCREENING MECHANISM.
- STILL MUCH WORK TO BE DONE!

THE PICTURE

a) ASYMPTOTIC FREEDOM : INDIVIDUAL PARTONS

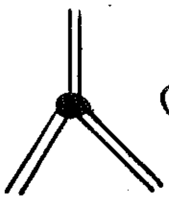
b) SEMIHARD RANGE : 'OVERLAPPING' PARTONS
'NON'-INDIVIDUAL

PARTON EVOLUTION:



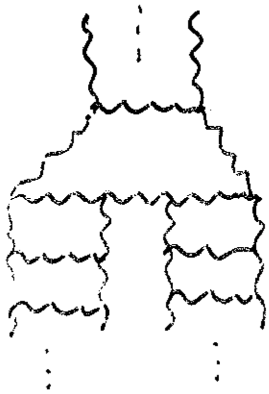
LADDER

OVERLAPPING PARTONS MAY RECOMBINE.

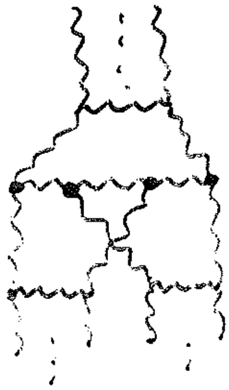


G_{3P}

CONSIDER ALL POSSIBLE CUTS OF 2 → 1 LADDER



PLANAR DIAGRAMS



NONPLANAR DIAGRAMS

ABRAMOVSKII - GRIBOV - KANCHELI (AGK) CUTTING RULE:

$$\sum_{i \in \text{PLANAR DIAGRAMS}} cut_i = 0$$

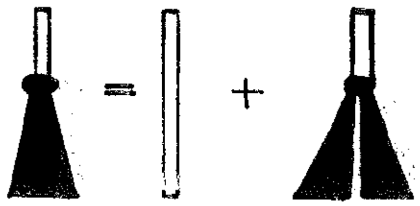
$\Rightarrow$ G_{3P} arises from $\sum_{i \in \text{non-planar}}$

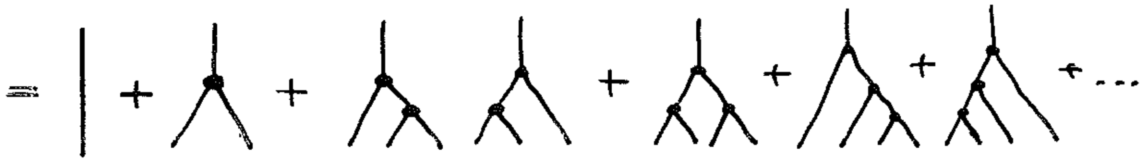
ONLY KNOWN RESULT: MÜLLER & QIU DLA

$$\boxed{G_{3P}(\xi) = \frac{3}{4}\pi^2 \frac{1}{\beta_0} e^{e^{\xi_0}} \exp\left[-(e^{\xi} + \xi)\right]}$$

IN LOWEST ORDER ONLY $2L \to 1L$ CONTRIBUTE (GLR).

THE FAN DIAGRAM EQU.





$$\frac{\partial^2 F(\xi, y)}{\partial \xi \partial y} = \frac{1}{2} F(\xi, y) - C \exp\left[-(e^{\xi}+\xi)\right] F^2(\xi, y) \qquad (1)$$

$$C = \frac{3}{4}\pi^2 \frac{1}{\beta_0} \frac{Q_0^2}{\Lambda^2} .$$

HIGHER TWIST NATURE: $\exp\left[-(e^{\xi}+\xi)\right] = \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \cdot \frac{\Lambda^2}{Q^2}$

ANY G_{3P} INSERTION YIELDS ONE FACTOR $\left(\frac{Q_0^2}{Q^2}\right)$.

WHAT IS $F(\xi, y)$?

IT STANDS FOR THE SINGLE GLUON DENSITY ON TOP OF THE FAN DIAGRAMS, IF ON BOTTOM OF THEM SINGLE GLUON DENSITIES ARE CONNECTED TO EACH LADDER.

- THIS IS NOT NECESSARILY THE CASE:

  ONE CAN LINK j LADDERS OUT OF n TO A

  G_m-DISTRIBUTION, $j \leq m \leq n$. (AT Q_0^2)

- THE RELATION OF $G_m|_{\geq 2}$ TO G_1 AT Q_0^2 IS UNKNOWN AND BEYOND PERTURBATION THEORY.

- STRICTLY SPEAKING, ONE HAS TO FIT ALL THIS DISTRIBUTION FUNCTIONS AT Q_0^2 ALONG WITH Λ FROM DATA.

- IT IS NOT KNOWN WHETHER OR NOT THERE IS SOME HIERARCHY (AS IN STATISTICAL PHYSICS), WHICH COULD BE OF RELEVANCE HERE FOR THE CORRELATIONS:

  e.g. $g_1 \gg g_2 \gg g_3 \gg \dots$ AT Q_0^2.

THE GLR (TOY) MODEL

INTEGRATE (1):

$$F(\xi,y) = \underset{\substack{\uparrow \\ G(y,\xi_0)=1}}{1} + \int_{\xi_0}^{\xi} d\xi' \int_{0}^{y} dy' \, F(\xi',y') \left[\frac{1}{2} - \underset{\substack{\uparrow \\ C=1}}{C} \exp\left[-(e^{\xi'} + \xi')\right] F(\xi',y') \right]$$

- TRIVIAL INPUT
- IF $C=0$ $\curvearrowright$ DLA-AP equ.: $F(\xi,y) = I_0\left(\sqrt{2(\xi-\xi_0)y}\right)$

(iteration of 1)

MORE COMPLETE EQU. (BBS)

$$F(\xi,y) = G(y,\xi_0) + \int_{\xi_0}^{\xi} d\xi' \int_{0}^{y} dy' \, F(\xi',y') \left[\frac{1}{2} - C_{3P} \exp\left(-(e^{\xi'} + \xi')\right) \right] F(\xi',y')$$

$C_{3P} = 0$ $\curvearrowright$ DLA-evolution of $G(y)$.

PROPERTIES OF THIS EQUATION:

- SINCE C_{3P} IS POSITIVE, THE NON-LINEAR TERM IS SREENING THE AP-RESULT.

$$\curvearrowright \qquad F(\xi,y) \leqslant F(\xi,y;\, C_{3P}=0)$$

- $F(\xi,y)$ MUST BE POSITIVE (NUMBER-DENSITY).

$\curvearrowright$ $F(\xi,y)$ IS BOUNDED FOR ANY $[\xi,\xi_0]\times[y,y_0]$.

- $\boxed{\text{LIMIT } y\to\infty \ (x\to 0)}$ $\xi = \text{const.}$

$$\lim_{y\to\infty} \int_0^y dy'\, F(\xi,y')\left[\tfrac{1}{2} - C\exp[-(e^{\xi}+\xi)]\,F(\xi,y')\right]$$

TO YIELD A FINITE RESULT: $\begin{cases} F(\xi,y) \xrightarrow[y\to\infty]{} 0 & \text{UN-REASON.} \\ F(\xi,y) = \frac{1}{2C}\exp[e^{\xi}+\xi] & \end{cases}$

"FROISSART BOUND".

THIS RESULT IS INDEPENDENT OF $G(y,\xi_0)$.

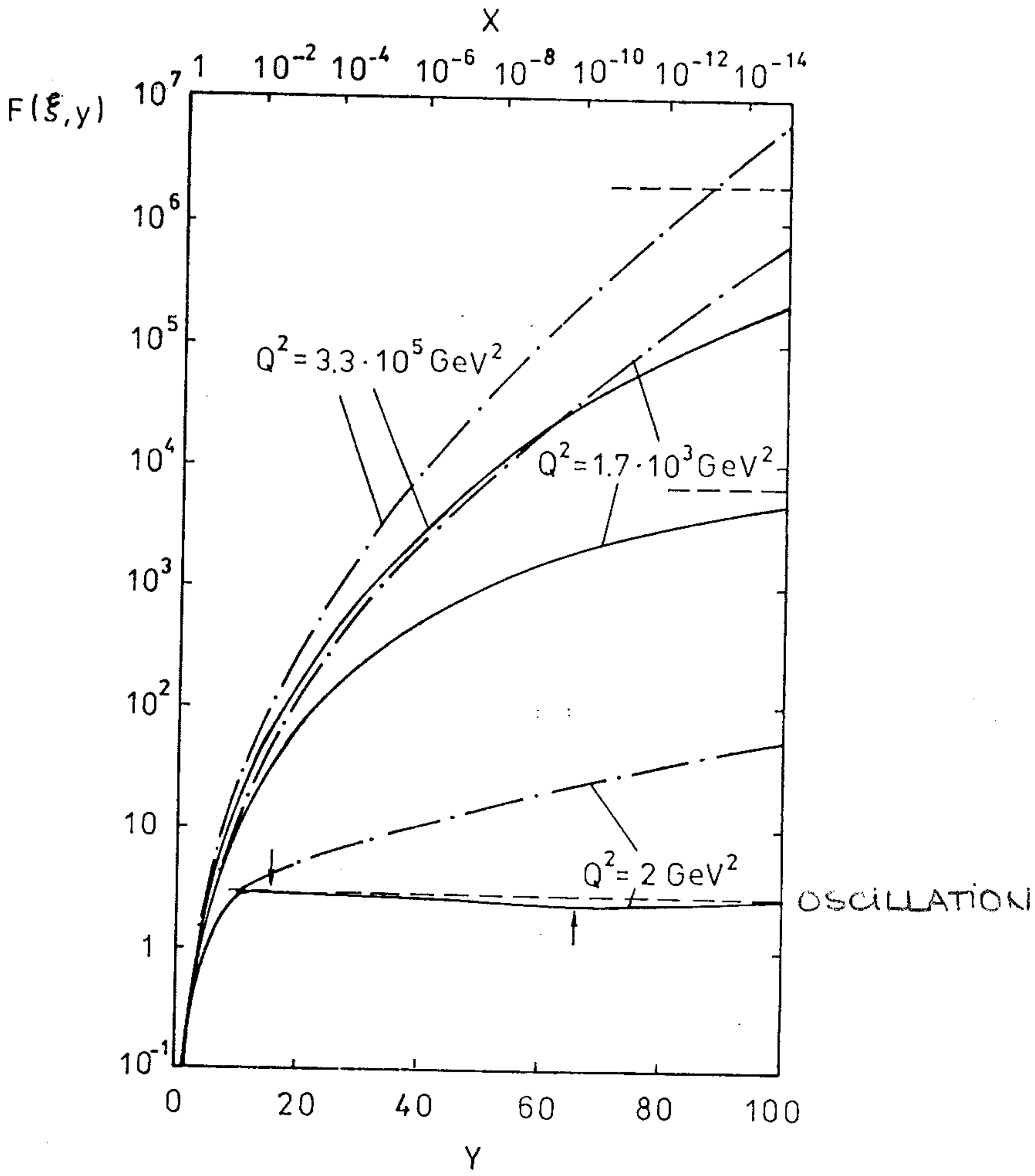
- ONE MAY SHOW THAT THE SOLUTION OF THE (NONLINEAR) HAMMERSTEIN INTEGRAL-EQU. OF VOLTERRA TYPE:

$$\exists!\ F(\xi,y)$$

ONE MAY THUS SOLVE THIS EQUATION DISCRETIZING IT ON A (ξ, y) LATTICE, THANKS TO THE VOLTERRA TYPE IN TERMS OF AN ITERATIVE PROCEDURE.

FIG

FIG



$G(y) \approx 3(1-x)^5$

$Q_0^2 = \Lambda^2$, $\Lambda = 200$ MeV

Fig. 2

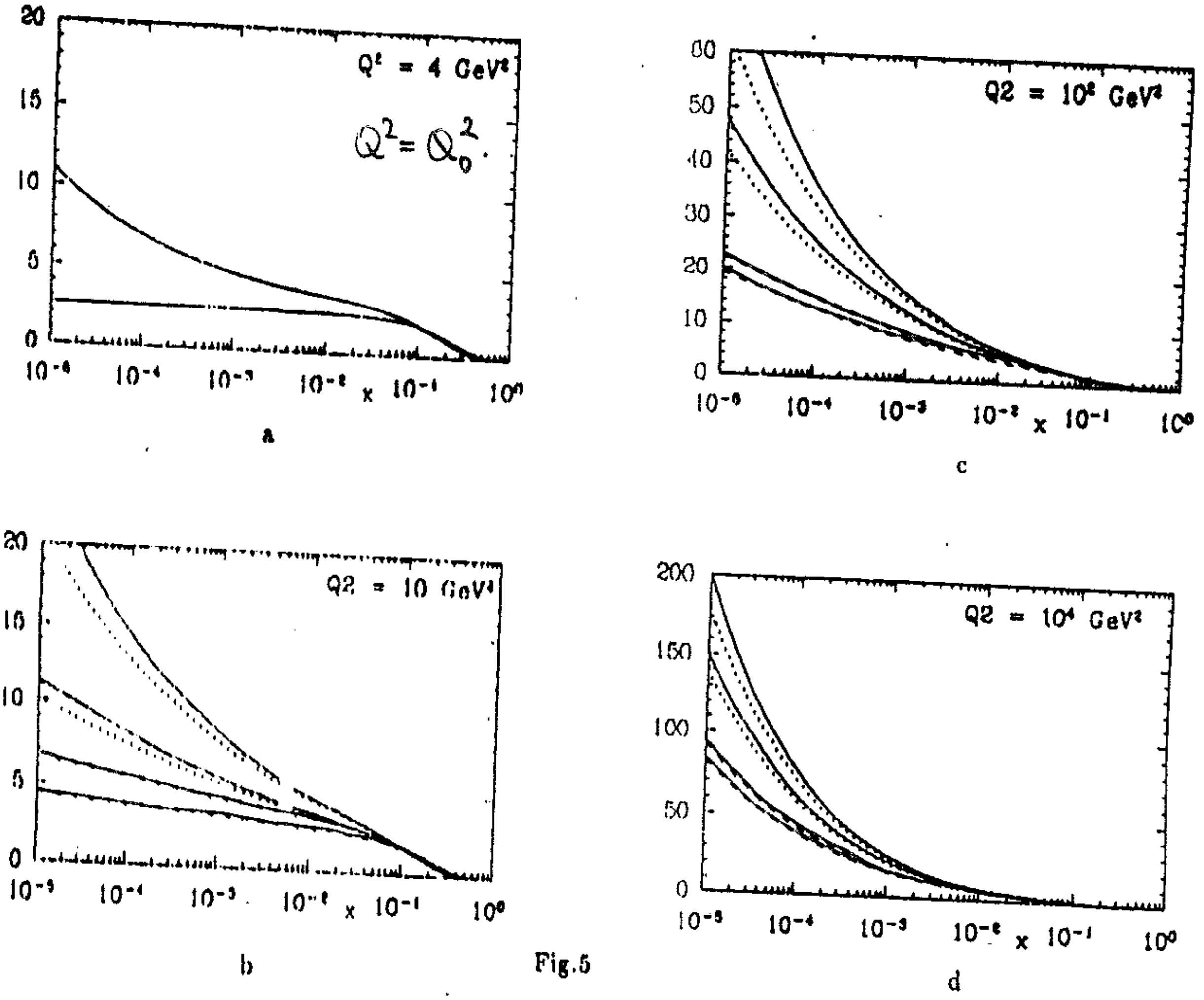


Fig.5

4. SEMI-CLASSICAL RESULTS

$$\partial_\xi \partial_y F(\xi,y) = \frac{1}{2} F(\xi,y) - C \exp[-(e^\xi+\xi)]\, F^2(\xi,y)$$

ANSATZ: $F = \exp(S)$ (TRAJECTORIES)

$$(S_{|y} F)_{|\xi} = S_{|y\xi} F + S_{|y} S_{|\xi} F = \frac{1}{2} F - C \exp[S - e^\xi - \xi] F$$

$$\boxed{S_{|y} S_{|\xi} = \frac{1}{2} - C \exp[S - e^\xi - \xi] \quad , \quad S_{|y} S_{|\xi} \gg S_{|y\xi}}$$

THIS LEADS TO A SYSTEM OF ORDINARY DEQS.

$$\begin{aligned}
\dot{y} &= S_{|\xi} \qquad\qquad (\text{BBS} \; ; \; \text{CK}) \\
\dot{\xi} &= S_{|y} \\
\dot{S} &= 2 S_{|y} S_{|\xi} \\
\dot{S}_{|y} &= - C \exp(S - e^\xi - \xi) S_{|y} \\
\dot{S}_{|\xi} &= - C \exp(S - e^\xi - \xi)(S_{|\xi} - 1 - e^\xi)
\end{aligned}$$

THE LINEAR CASE: $\ddot{y} = \ddot{\xi} = 0 \curvearrowright$ y & ξ are linear in T.

$$\left.\begin{aligned} y &= y_0 + T y_1 \\ \xi &= \xi_0 + T \xi_1 \end{aligned}\right\} \qquad \boxed{y = y_0 + \frac{y_1}{\xi_1}(\xi - \xi_0)}$$

THE DLA-AP equ. IS CHARACTERIZED BY <u>RAYS</u>.

$(y \xi \gg 1)$

- SOLVE THE COUPLED SYSTEM NUMERICALLY.

VERY IMPORTANT: EXACT STARTING POINT VALUE.
($\longmapsto$ FULL EQU. $\leftrightarrow$ COLLINS, KWIECINSKI)

EVOLVE BACKWARD IN ξ & y.

$\Longrightarrow$ TRAJECTORIES MARK A CRITICAL REGION

$\boxed{\text{FIG}}$

$$y_c(\xi) \simeq \frac{1}{4C} e^{2\xi}$$

$$S(\xi) = e^{\xi}$$

$\longrightarrow$ LIMIT OF VALIDITY OF THE GLR-EQ.
"BOUNDARY" OF THE SEMIHARD RANGE.
(FROM ABOVE & BELOW)

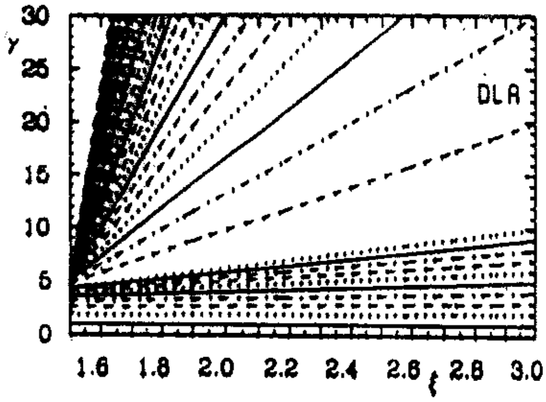
THE EQUATION PREDICTS ITS RANGE OF VALIDITY.

REASON, WHY THIS SEPARATION OCCURS:

$$\boxed{\frac{d^2y}{d\xi^2} = \frac{dy}{d\xi} - (1+e^{\xi})\frac{1}{\xi}}$$

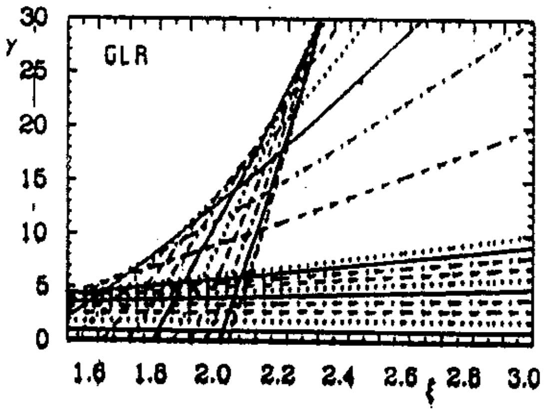
y_c INDUCES A SWITCH FROM $\frac{dy}{d\xi} > 0$ to $\frac{dy}{d\xi} < 0$.

TRAJECTORIES NEVER CROSS THE BOUDARY LINE.



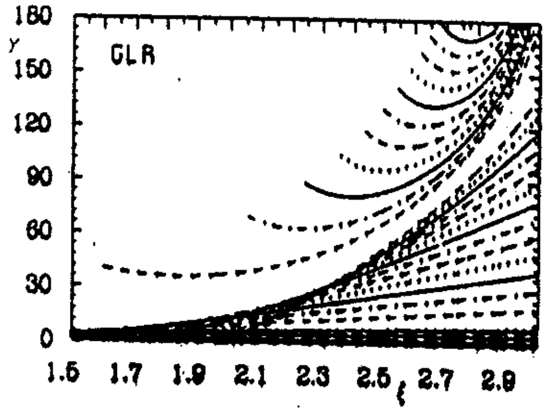
30
25
20
15
10
5
0
Y
DLR
1.6 1.8 2.0 2.2 2.4 2.6 ξ 2.8 3.0

a



30
25
20
15
10
5
0
Y
GLR
1.6 1.8 2.0 2.2 2.4 2.6 ξ 2.8 3.0

b



180
150
120
90
60
30
0
Y
GLR
1.5 1.7 1.9 2.1 2.3 2.5 ξ 2.7 2.9

c

Fig.3

5. CONCLUSIONS & OUTLOOK

i) THE APPARENT VIOLATION OF UNITARITY OF PARTON DENSITIES AT TWIST-2 CAN BE RESTORED IN QCD.

ii) THE FAN-DIAGRAMS (AMONG OTHERS ?) ARE RESPONSIBLE FOR THIS.

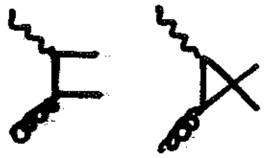
iii) WE HAVE, FOR THE FIRST TIME, GIVEN A COMPLETE NUMERICAL SOLUTION TO THE NON-LINEAR GRIBOV-LEVIN-RYSKIN EQUATION, TAKING INTO ACCOUNT APPROPRIATE NL. COUPLINGS AND INPUT DISTRIBUTIONS.

iv) THE SEMI-CLASSICAL ANALYSIS YIELDS A BOUNDARY REGION OF THE GLR-EQ., WHICH CAN BE CONSIDERED AS ITS LIMIT OF VALIDITY.

v) QUANTITATIVELY ONE EXPECTS NONLINEAR EFFECTS BELOW $x \simeq 10^{-2}$ & $Q^2 = 10\,GeV^2$

vi) THERE IS SOME DEPENDENCE ON C_{3P} DUE TO THE RELATIVE NORMALIZATION OF G_2/G_1^2.

vii) F_2 & F_L HAVE TO BE CALCULATED.



(LEVIN, RYSKIN)

TO BE RECALCULATED & IMPROVED

viii) G_{3P} HAS TO BE CALCULATED BETTER THAN IN DLA ONLY.

ix) A CONSISTENT ACCOUNT FOR FERMION-DGF IS NEEDED TO BRIDGE THE GAP BETWEEN DLA & AP-EQUS.

x) THE INTERRELATION BETWEEN FAN-DIAGRAMS & HIGHER TWIST OPERATORS HAVE TO BE CLARIFIED FURTHER.