STRUCTURE FUNCTIONS

AT SMALL X:

NEW DEVELOPMENTS

6/90
#1

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- WORKSHOP ON LOW-X STRUCTUREFUNCTIONS, HELD AT DESY, MAY 1990
- SUMMARIZE ASPECTS
  - LOW X BEHAVIOUR
    xG (F_2)
  - DYNAMICAL CONTRIBUTIONS
    TWIST 2 + LIPATOV TERMS
    + FAN-TERMS
  - LOW Q^2-BEHAVIOUR AT
    LOW X → RC & MC !

1) A REMINDER OF LL-QCD EVOLUTION

2) AP-EQUS. AT LOW X

3) LIPATOV LADDERS AT LOW X

4) FAN DIAGRAMS

5) LOW Q^2 AT LOW X

6) SUMMARY

1) A REMINDER OF LL-QCD EVOLUTION

EXP. ACCESIBLE: STRUCTUREFUNCTIONS

$F_2(x,Q^2)$, $xW_3(x,Q^2)$, $W_2(x,Q^2)$, ...

LEADING LOG QCD: MAPPING OF:

$$\begin{pmatrix} F_2 \,|_{Q_0^2} \\ G \,|_{Q_0^2} \end{pmatrix} \longrightarrow \begin{pmatrix} F_2 \,|_{Q^2} \\ G \,|_{Q^2} \end{pmatrix}$$

THIS MAPPING IS DETERMINED BY

$$\alpha_s(Q^2) = \frac{4\pi}{\frac{11}{3} - 2N_f\frac{1}{3}} \cdot \frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)}$$

↑ QCD SCALE

STRUCTURE OF EVOLUTION EQUATIONS:

CONSIDER:

$$F(x,Q^2) = \sum_i \left[\alpha_i \, q_i(x,Q^2) + \beta_i \bar{q}_i(x,Q^2)\right]$$

$$= a \, F_{NS}(x,Q^2) + b \, F_S(x,Q^2)$$

WITH:

$$F_{NS} = \sum_{ij} \alpha_{ij} \left[q_i(x,Q^2) - \bar{q}_j(x,Q^2)\right]$$

$$F_S = \sum_i \left[q_i(x,Q^2) + \bar{q}_i(x,Q^2)\right]$$

AP-equations: $t = 2/\beta_0 \ln\left[\ln(Q^2/\Lambda^2) / \ln(Q_0^2/\Lambda^2)\right]$

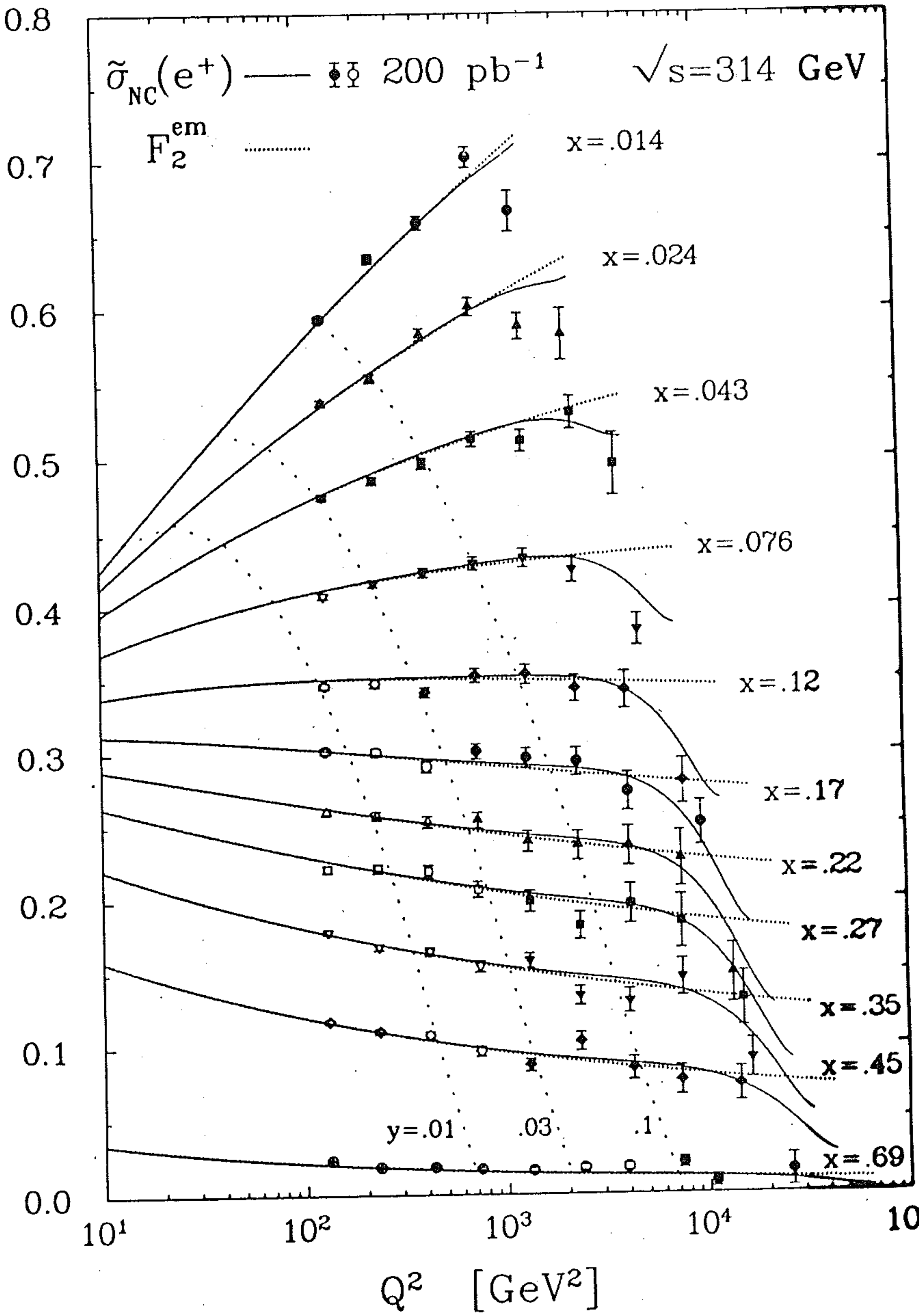
$$\boxed{\begin{aligned} &\frac{\partial}{\partial t} F_{NS}(x,t) = P_{qq}(x) \otimes F_{NS}(x,t) \\ &\frac{\partial}{\partial t} \begin{pmatrix} F_S(x,t) \\ G(x,t) \end{pmatrix} = \begin{pmatrix} P_{qq}(x) & 2N_f P_{qG}(x) \\ P_{Gq}(x) & P_{GG}(x) \end{pmatrix} \begin{pmatrix} F_S(x,t) \\ G(x,t) \end{pmatrix} \end{aligned}}$$

Solution: specify: • $F_{NS}(x,0)$

$F_S(x,0)$ i.e. $Q^2 = Q_0^2$.

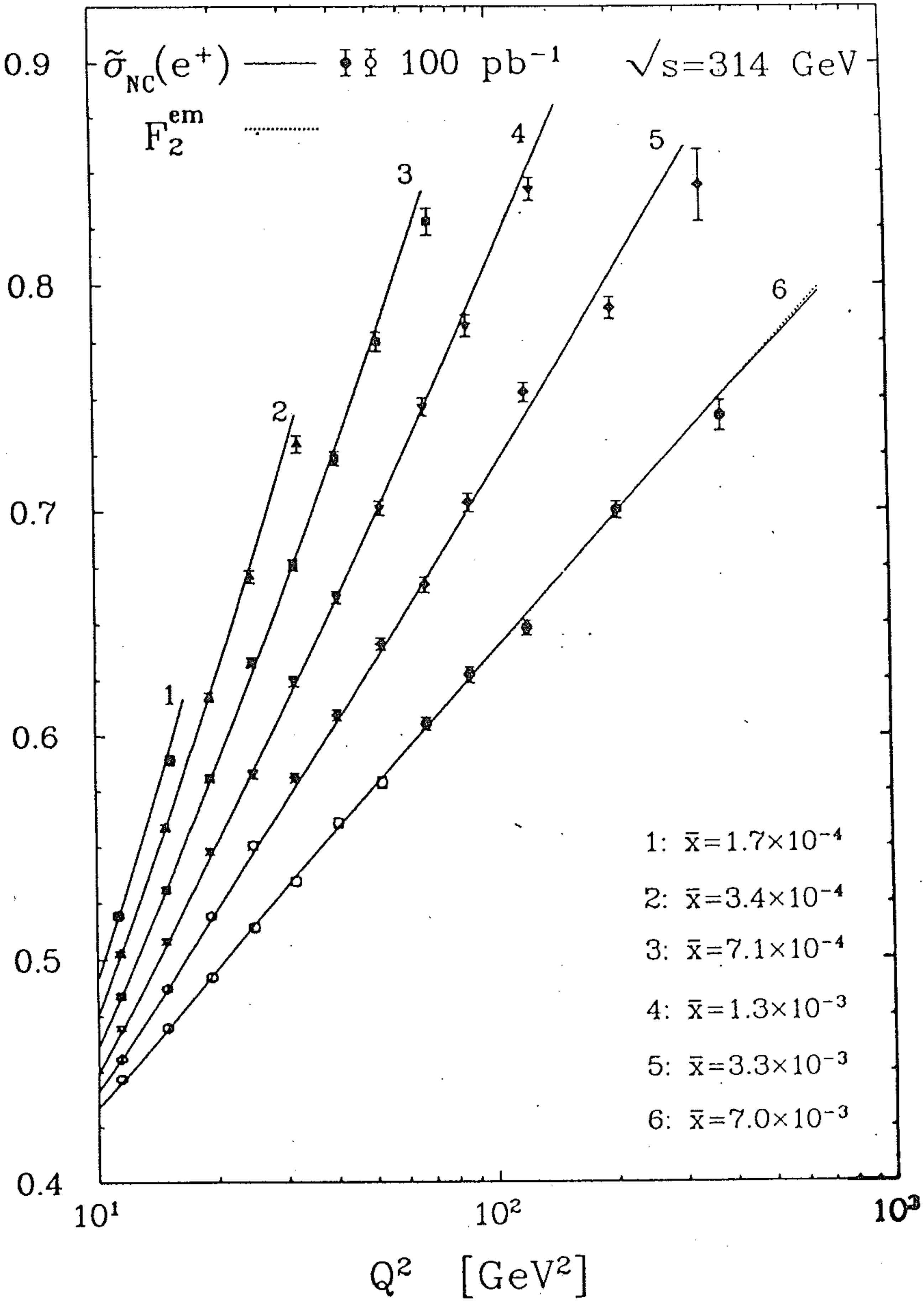
$G(x,0)$

• Integrate the IDE



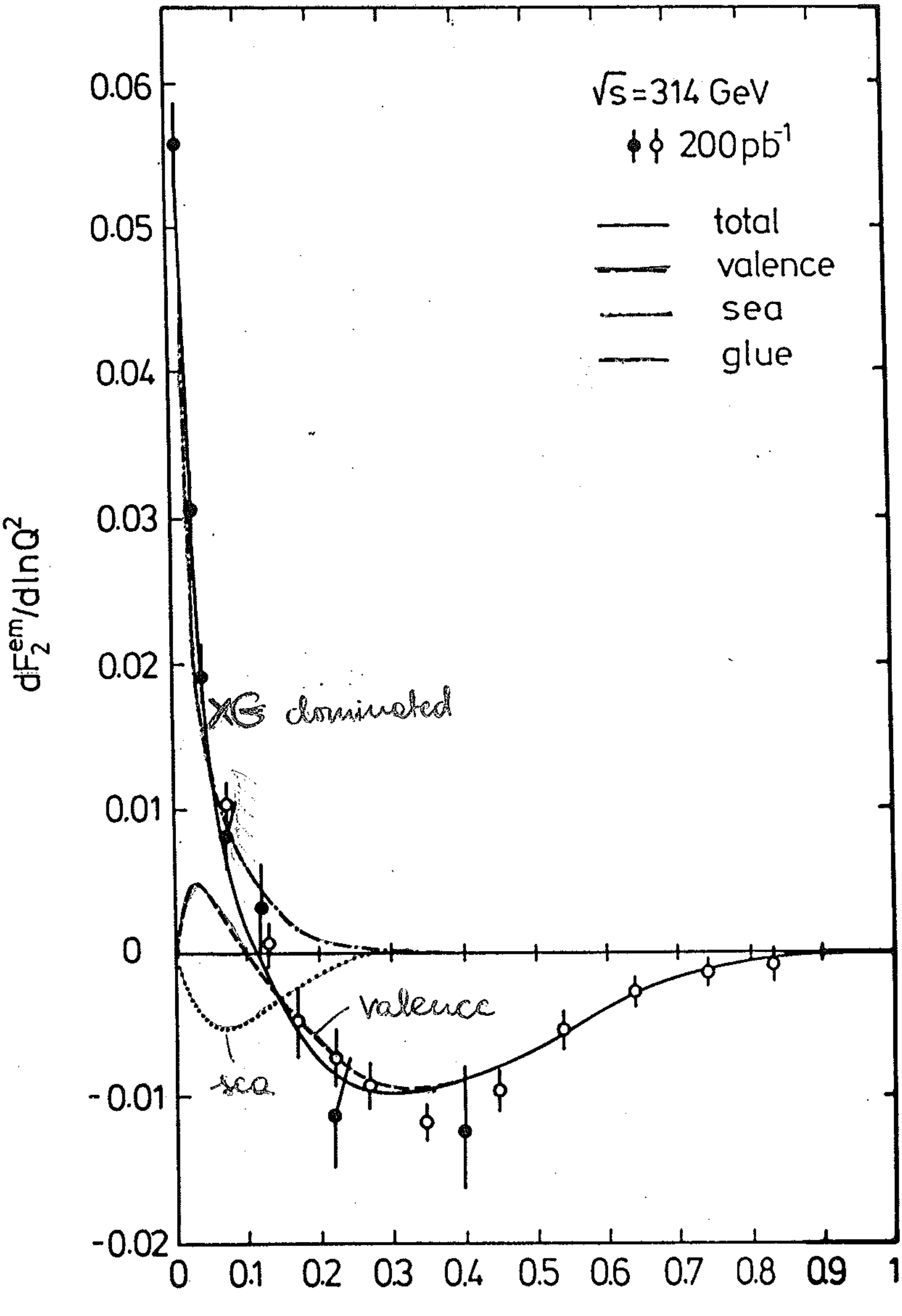
$\tilde{\sigma}_{NC}(e^+)$ — 200 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
x=.014
x=.024
x=.043
x=.076
x=.12
x=.17
x=.22
x=.27
x=.35
x=.45
x=.69
y=.01
.03
.1
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10^1
10^2
10^3
10^4
10^5
Q^2 [GeV^2]

Fig. 3



$\tilde{\sigma}_{NC}(e^+)$ —— 100 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
1
2
3
4
5
6
1: $\bar{x}=1.7\times10^{-4}$
2: $\bar{x}=3.4\times10^{-4}$
3: $\bar{x}=7.1\times10^{-4}$
4: $\bar{x}=1.3\times10^{-3}$
5: $\bar{x}=3.3\times10^{-3}$
6: $\bar{x}=7.0\times10^{-3}$
0.9
0.8
0.7
0.6
0.5
0.4
10^1
10^2
10^3
Q^2 [GeV^2]

Fig. 6



$\sqrt{s}=314$ GeV
200 pb^{-1}
total
valence
sea
glue
$dF_2^{em}/d\ln Q^2$
x
xG dominated
valence
sea
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig.5

2.) AP AT LOW X

- GLUON DOMINATED EVOLUTION
- FERMIONIC DEGREES ARE SMALL

$$xG \equiv \overline{G}$$

$$P_{GG}(x) = 6\left\{\frac{z}{(1-z)_+} - 1 + \frac{1}{z} + z(1-z) + \frac{1}{6}\beta_0\delta(1-z)\right\}$$

$$\simeq \frac{6}{z} \quad \text{at low } x.$$

$$\frac{d\overline{G}}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dx'}{x'^2} P_{GG}\left(\frac{x'}{x}\right)\overline{G}(x')\frac{x}{x'}$$

$$\approx \frac{3\alpha_s(Q^2)}{2\pi}\cdot 2\int_x^1 \frac{dx'}{x'^3} x^2\,\overline{G}(x')$$

$\longrightarrow$ α_s to LHS & differentiate for $\left(\ln\frac{1}{x}\right)$

$$\boxed{\begin{gathered}\frac{d\overline{G}(x,Q^2)}{dy\,d\xi} = \frac{1}{2}\overline{G}(x,Q^2) \\[1em] y = \frac{8N_c}{\beta_0}\ln\frac{1}{x}, \quad \xi = \ln\ln\left(\frac{Q^2}{\Lambda^2}\right)\end{gathered}}$$

DLA-equ. (contained AP $\rightarrow$ low x)

3) LIPATOV LADDERS AT LOW X

(Kwiecinski)

Allow also for large logs $\alpha_s \ln 1/x$!

$$f_n(k^2) = \frac{dg_n(k^2)}{dk^2} \quad ; \quad g_n(k^2) = \int_0^1 dx\, x^{n-1} g(x,k^2)$$

$$f_n(k^2) = f_n^0(k^2) + \frac{3}{\pi}\alpha_s(k^2)\left\{ \frac{k^2}{n-1}\int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2}\left(\left[\frac{f_n(k'^2)-f_n(k^2)}{|k'^2-k^2|}\right] + \frac{f_n(k^2)}{\sqrt{4k'^4+k^4}}\right)\right.$$

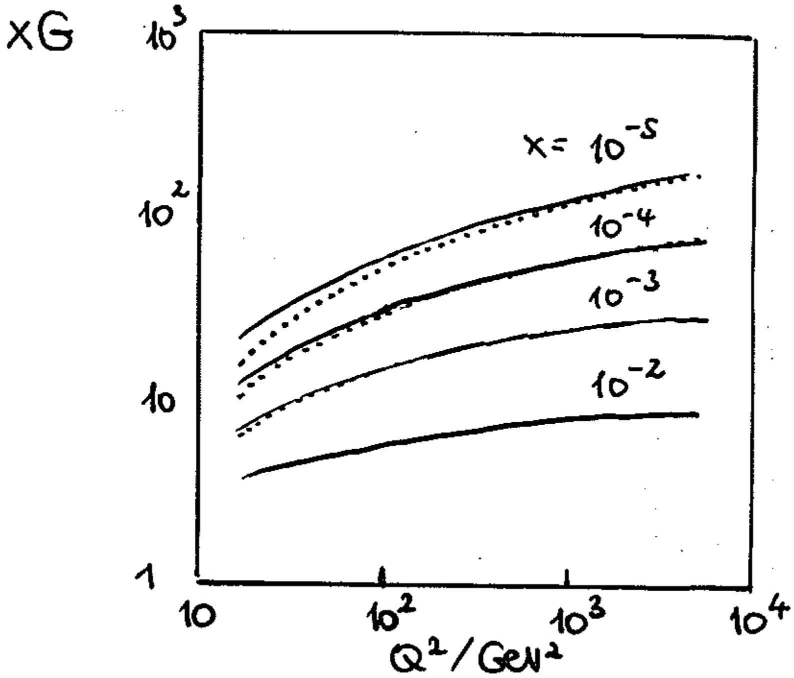
$$\left. + A_{gg}(n)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2} f_n(k'^2) + 2N_f A_{gq}(n)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2} h_n(k'^2)\right)$$

$\longrightarrow$ 'REGGEIZED' GLUON.

$$A_{gg}(n) = \frac{4}{9}\langle P_{gg}\rangle_n - \frac{1}{n-1}$$

$$A_{ij}(n) = \frac{4}{9}\langle P_{ij}\rangle_n$$

f_0 non perturbative input.



effect $\leq 5\%$!

4. FAN DIAGRAMS (Levin)

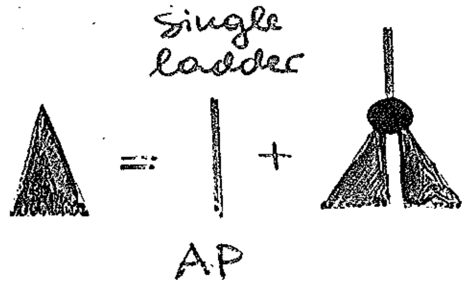
SO FAR SINGLE LADDERS WERE CONSIDERED.

$\Longrightarrow$ PICTURE OF "<u>ISOLATED</u>" PARTONS INSIDE THE NUCLEON : NO RECOMBI- NATION

FOR LOWER AND LOWER x PARTONS "OVERLAP". (HIGHER NUMBER- DENSITY)

A RECOMBINATION OF 2 (...) PARTONS IN ONE IS EXPECTED.

LEADING TERMS ? $\Longrightarrow$ Gribov, Levin, Ryskin



3 Ladder coupling

$\longrightarrow$ SUM ACROSS ALL HIGHER TWISTS TAKING LEADING TERMS

$$\bullet \equiv \frac{3}{4}\,\frac{1}{\beta_0}\left(\frac{\hat{Q}_0^2}{Q^2}\right)\pi^2\,\frac{1}{\ln(Q^2/\Lambda^2)} \qquad \text{(DLA)}$$

$$\text{GLR:} \quad \frac{\partial^2 F}{\partial \xi\, \partial y} = \underbrace{\frac{1}{2} F}_{\text{AP}} - \underbrace{C \exp\left[-(e^{\xi} + \xi)\right] F^2}_{\text{SCREENING}}$$

global properties: $F \leq F_0$ (AP case)

$$\lim_{x \to 0} F(y(x), \xi)\Big|_{\xi = \text{const}} = \frac{1}{2C} \exp\left[e^{\xi} + \xi \right]$$

Solution: (consider the integral form):

$$F(y, \xi) = \tilde{G}(y, \xi = \xi_0) + \int_{\xi_0}^{\xi} d\xi' \int_0^y dy' \cdot F(y', \xi')$$

$$\cdot \left[\frac{1}{2} - C \exp\left[e^{\xi'} - \xi' \right] F(y', \xi') \right]$$

$\uparrow$

initial distribution.

Λ in $\xi, \xi_0 \sim \ln \ln \frac{Q^2}{\Lambda^2}$

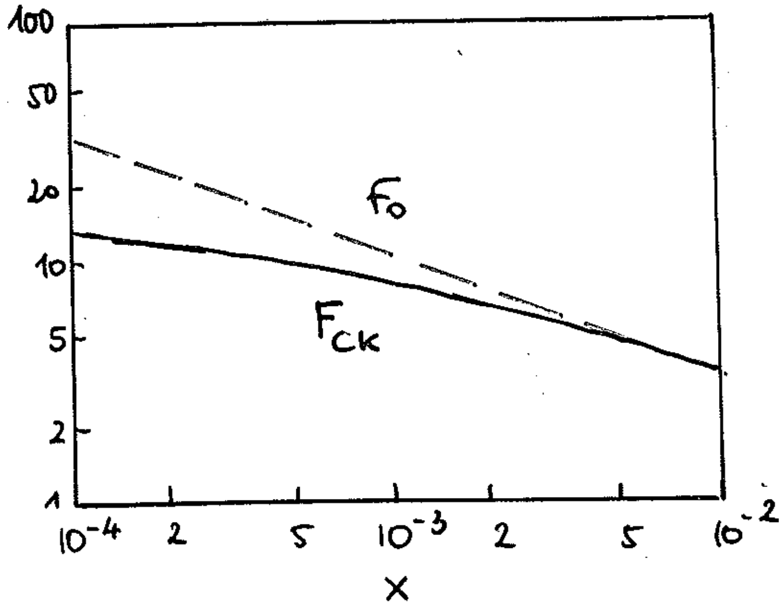
Q_0^2 in $\xi_0 = \ln \ln \frac{Q_0^2}{\Lambda^2}$

$\hat{Q}_0^2$ in $C = \frac{3}{4} \frac{1}{\beta_0} \left(\frac{\hat{Q}_0^2}{Q^2} \right) \pi^2 \frac{1}{\ln(Q^2/\Lambda^2)}$.

Kwiecinski, Collins: semiclassical solution: FAN equ.

(might be not enough)

tendency visible.



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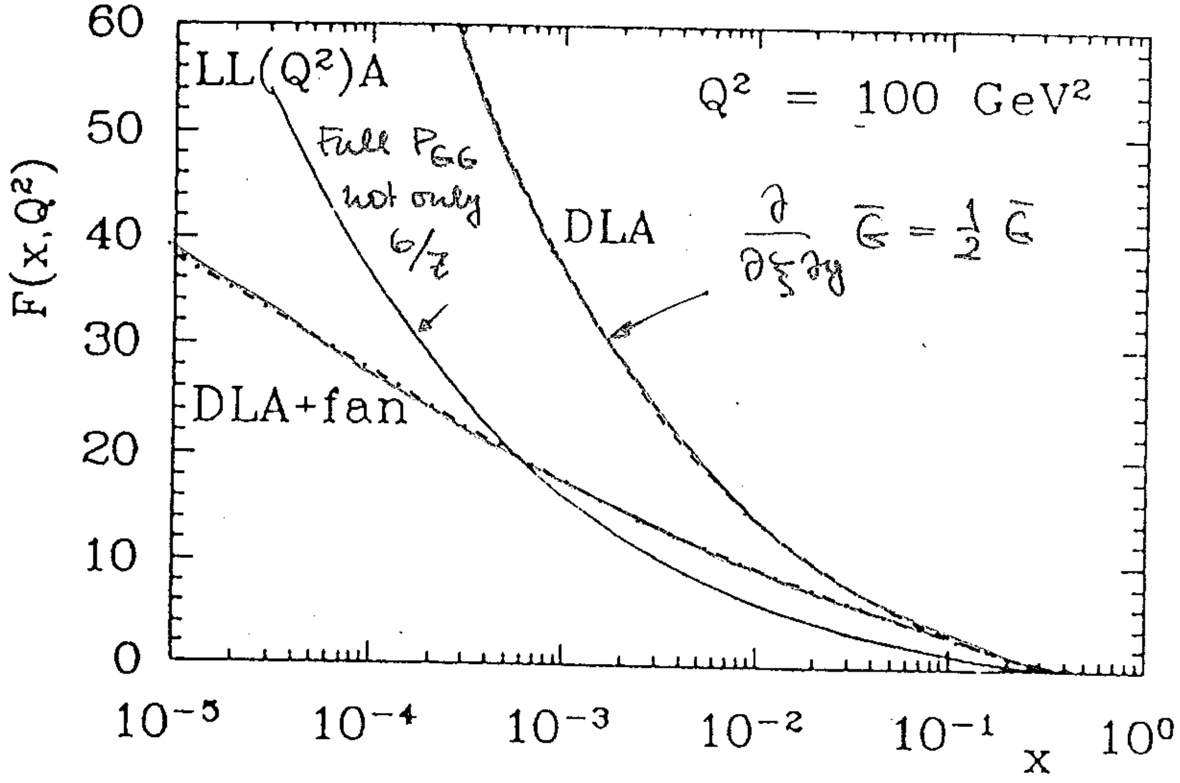

$F(x,Q^2)$ – solution of the Fan diagram equ. of different type

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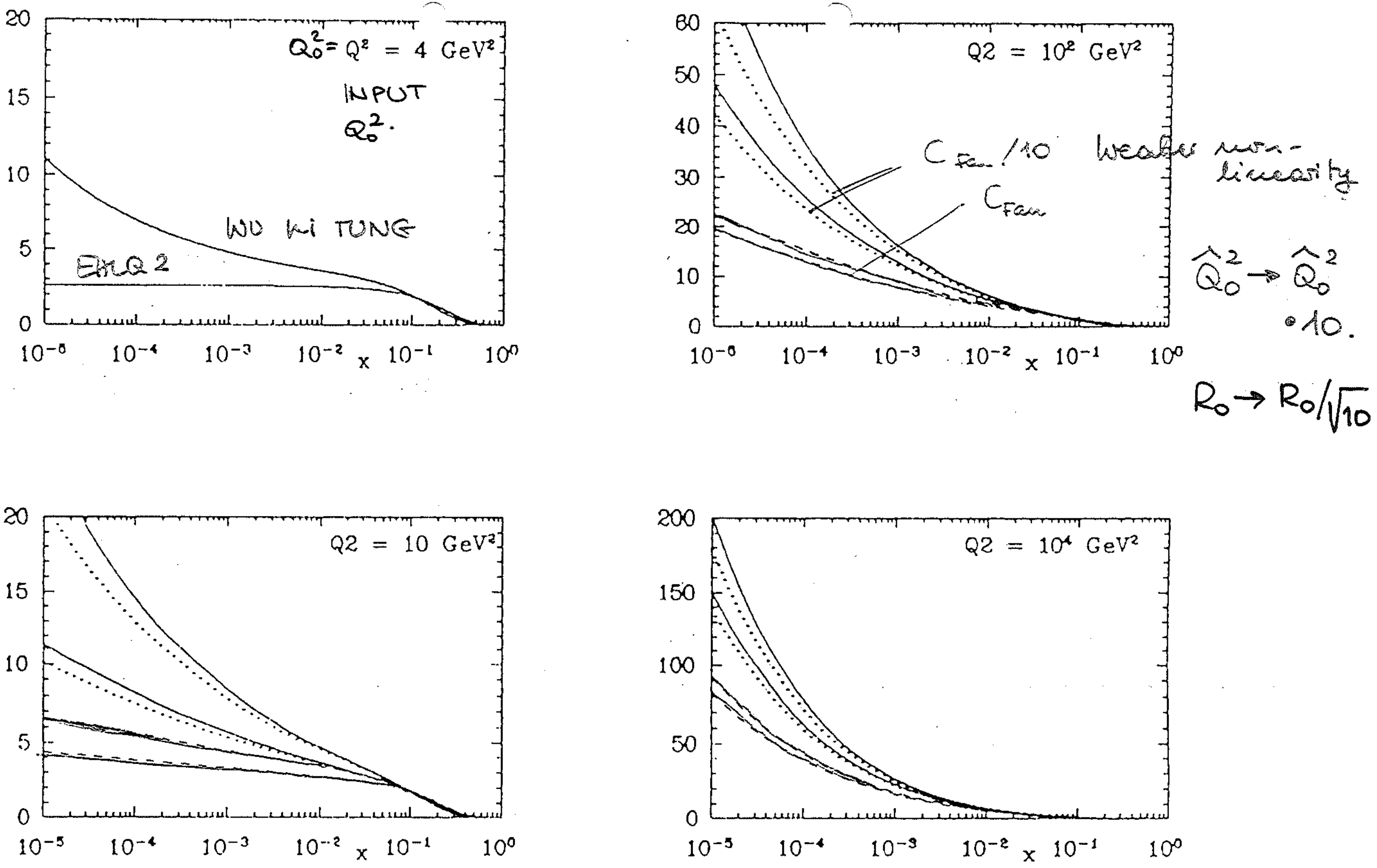

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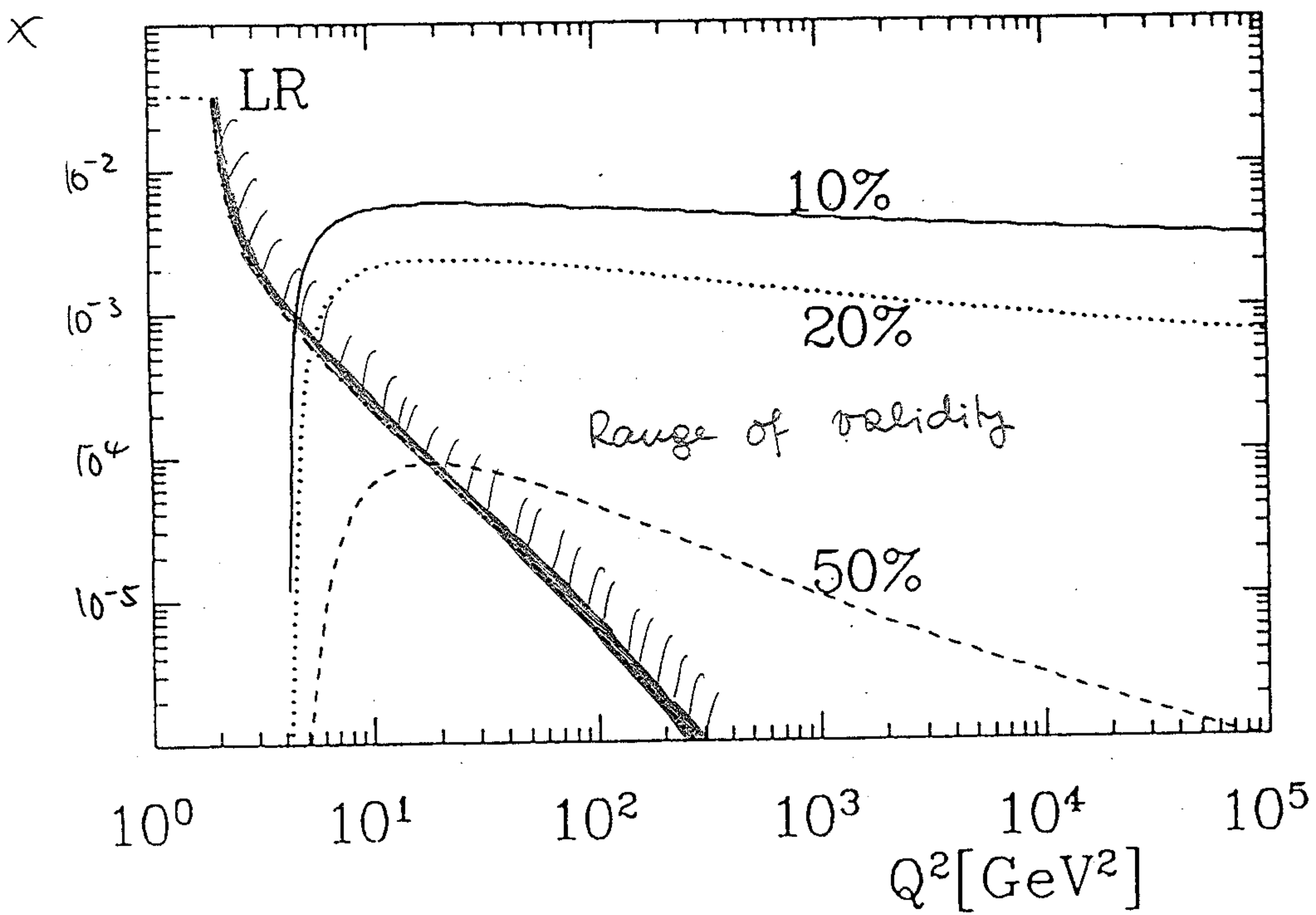
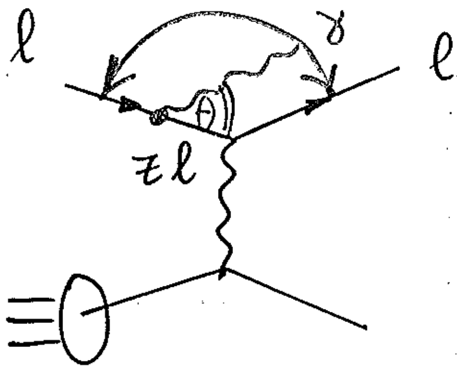


5) LOW Q^2 AT LOW x

WHY DO WE NEED THAT FOR D.i.S.?

BORN : $Q^2 > Q_0^2$ (NOT TO LOW) (EXP. CUT)



$Q^2 = (\ell - \ell')^2$: INITIAL STATE BREMSSTRAHLUNG:

$\hat{\ell} = z\,\ell$

$\hat{Q}^2 = (z\ell - \ell')^2 \approx \underline{z\,Q^2}$!

MEASURING FROM ℓ & ℓ' A Q^2 MIGHT BE CONNECTED WITH $F_2(x, zQ^2)$!

→ small

RC : CORRECT LOW Q^2 PARAMETRIZATIO OF $\overset{(-)}{q}_i(x,Q^2)$ IS REQUIRED!

Landshoff, Donnachie:

$$V(x,Q^2) = 1.33\,x^{.56}\,(1-x)^3\left(\frac{Q^2}{Q^2+.85}\right)^{.44}$$

$$S(x,Q^2) = 0.17\,x^{-.08}\,(1-x)^5\left(\frac{Q^2}{Q^2+.36}\right)^{1.08}$$

$$c(x,Q^2) = .045\,\frac{Q^2}{Q^2+2.5^2}\,\xi^{-.086}\,(1-\xi)^7$$

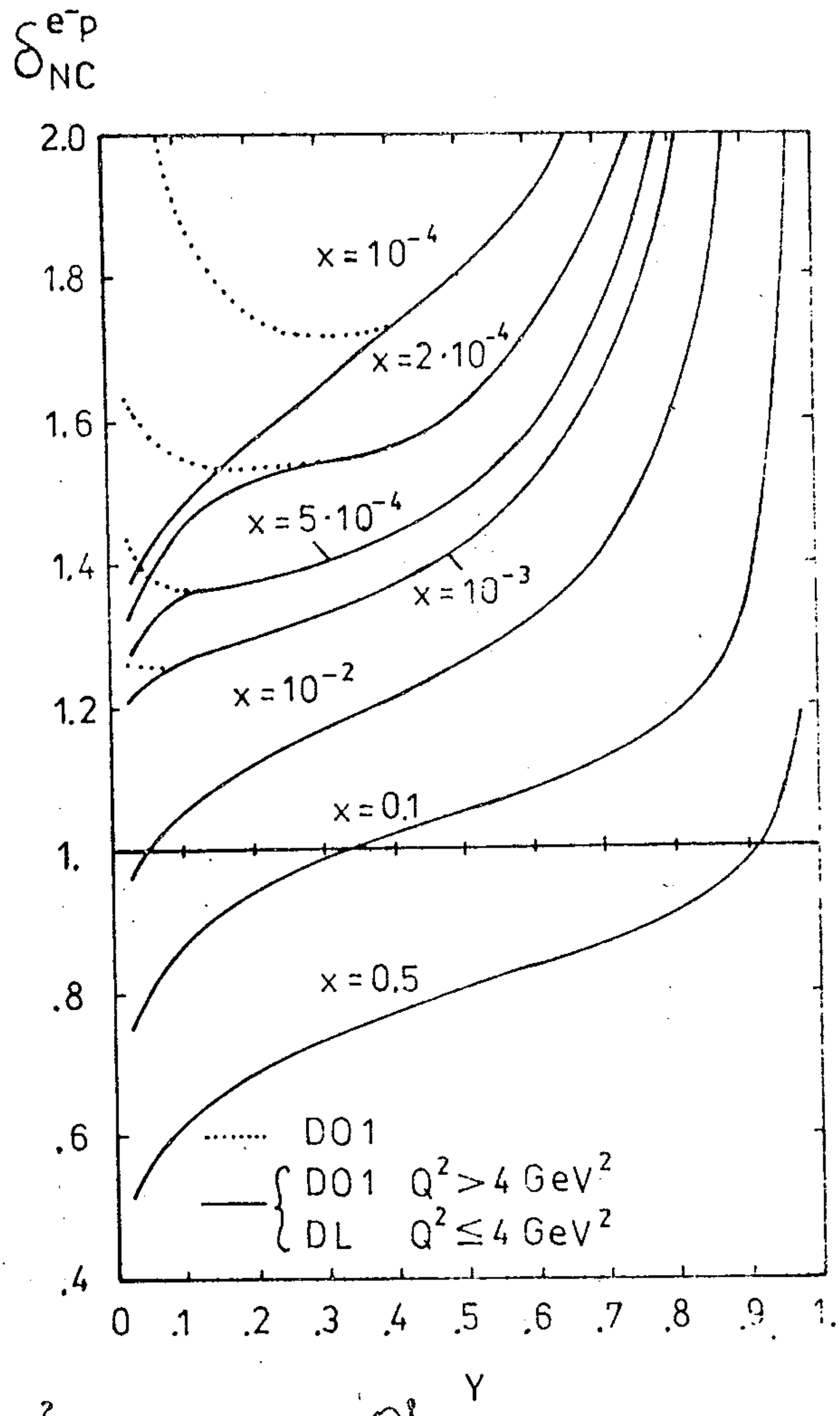
$$\xi = x\left(1+\left(\frac{4.}{Q}\right)^2\right)$$

σ vanishes as $Q^2 \to 0$.

NOT INCORPORATED IN ALMOST ALL PARAMETRIZATIONS FOR $x\overset{(-)}{q}$: ON THE MARKET!

RC-MONTE CARLOS NEED IT : STRONG EFFECTS AT LOW x.

J.B.



DL ; $Q^2 < Q_0^2$ $\qquad F_2 \sim \frac{Q^2}{Q^2+m^2} \cdot F_2^0$

6. SUMMARY

1) AS THE SOLUTIONS OF THE AP-EQU. VIOLATE UNITARITY FOR $x \to 0$ ADDITIONAL DYNAMICAL TERMS MUST COME INTO THE PLAY AT SMALL x.

2) THERE ARE ALSO CONTRIBUTIONS DUE TO REGGEIZED GLUONS (LIPATOV EQU.) WHICH ARE SMALL ($O(\sim 5\%)$ $x \gtrsim 10^{-4}$) AT THE LINEAR TERM. THEIR CONTRIBUTION TO THE FAN-DIAGR.-EQ. HAS NOT YET BEEN CALCULATED NUMERICALLY.

3) FAN DIAGRAM CONTRIBUTIONS MAY CAUSE A SIGNIFICANT DEPLETION COMPARED TO THE AP-SOLUTION FOR xG. THIS DEPENDS ON INPUT WHICH HAS TO BE FITTED:

$$\Lambda,\ xG(Q_0^2)\ ,\ Q_0^2\ ,\ \hat{Q}_0^2 \propto R_{eff}$$

4) RC Monte Carlos REQUIRE A CORRECT LOW Q^2 PARAMETRIZATION OF PARTON DENSITIES. NEED FOR A FUSION OF NONPERTURBATIVE TERMS $\sim Q^2/(Q^2+M^2)$ AND QCD-SCALING VIOLATIONS IN xq_i & $x\bar{q}_i$.