RIKEN, 11/99.

INTEGRAL RELATIONS BETWEEN POLARIZED STRUCTURE FUNCTIONS

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1) INTRODUCTION

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3) STRUCTURE FUNCTION RELATIONS: TWIST 2
   - WHY ARE THE CG & WW RELATIONS STRUCTURALLY DIFFERENT ?

4) STRUCTURE FUNCTION RELATIONS: TWIST 3
   - A REMARK ON ELT

5) CONCLUSIONS

JB, N. KOCHELEV, NPB 448 (1997) 285

JB, A. TKABLADZE, NPB '99 hep-ph/9812... DESY 98-181

JB, B. GEYER, D. ROBASCHIK, NPB '99, DESY 99-020.

1. INTRODUCTION

HOW MANY STRUCTURE FUNCTIONS ARE DETERMINING THE DIS CROSS SECTION ?

→ EXP.

→ THY.

LORENTZ STRUCTURE :

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1 + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2$$

$$- i\varepsilon_{\mu\nu\rho\sigma} \frac{q^\rho P^\sigma}{2P\cdot q} F_3$$

POL. PART

$$\begin{cases} + i\varepsilon_{\mu\nu\rho\sigma} \dfrac{q^\rho S^\sigma}{P\cdot q}\, g_1 + i\varepsilon_{\mu\nu\rho\sigma} \dfrac{q^\rho (P\cdot q\, S^\sigma - S\cdot q\, P^\sigma)}{(P\cdot q)^2} \cdot g_2 \\ + \left(\dfrac{\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu}{2} - S\cdot q \dfrac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q}\right) g_3 \\ + S\cdot q \dfrac{\hat{P}_\mu \hat{P}_\nu}{(P\cdot q)^2}\, g_4 + \left(-g_{\mu\nu} + \dfrac{q_\mu q_\nu}{q^2}\right) \dfrac{S\cdot q}{P\cdot q}\, g_5 \end{cases}$$

+ CURRENT NONCONSERV. TERMS

$$\hat{R}_\mu = R_\mu - \frac{R\cdot q}{q^2} q_\mu$$

S - NUCLEON SPIN.

CONSIDER $m_q \to 0$: CURRENT CONSERVATION ALSO FOR WEAK TERMS.

1ST CONTRIBUTIONS : QUARKONIC OPERATORS.

NUMBER OF STRUCTURE FUNCTIONS :

A) UNPOLARIZED :

$|\gamma|^2$ 2

$|\gamma+Z|^2, |W|^2$ 3 (AND MORE FLAVOR + PROPAG. STRUCTURE)

B) POLARIZED :

$|\gamma|^2$ 2

$|\gamma+Z|^2, |W|^2$ 5 —— " ——

LORENTZ STRUCTURE.

- WHAT IMPLIES QCD (SO FAR LO) ?
- RELATIONS BETWEEN STRUCTURE FUNCTIONS

  → MINIMAL REPRESENTATIONS
- PREDICTION FOR EXPERIMENT

  ↔ EXPERIMENTAL TESTGROUND FOR THEORY !
- WHAT ARE THE CONTRIBUTIONS AT :
  - TWIST 2
  - TWIST 3 ?

2. LIGHT CONE EXPANSION

FORWARD COMPTON AMPLITUDE:

$$T_{\mu\nu}^{i} = T\left(J_\mu^{i_1 \dagger}(x)\, J_\nu^{i_2}(0)\right)$$

$$T_{\mu\nu}^{NC} \cong T_{\mu\nu}^{CC,+}$$

$$T_{\mu\nu}^{CC,\pm} = T_{\mu\nu}^{W^-} \pm T_{\mu\nu}^{W^+}$$

$$S(x) \longrightarrow \frac{2i\, \not{x}}{(2\pi)^2 (x^2 - i0)^2}$$

POLARIZED CASE:

o TWIST DECOMPOSITION

LO : • TWIST 2 CONTRIBUTIONS.

• TWIST 3

TWIST 2 : $\longleftrightarrow$ 'PARTON MODEL' (COVARIANT) $\cong$ LCE

JACKSON, ROBERTS, ROSS

BLÜMLEIN, KOCHELEV

TWIST 3 ?

TECHNICAL STEPS :

1) LIGHT CONE EXPANSION (LCE)

2) DISPERSION RELATIONS

3) LOCAL LCE

4) RELATIONS BETWEEN

STRUCTURE FUNCTIONS

EQUATING THE NONPERTURBATIVE
OPERATOR MATRIX ELEMENTS

$a_n^{\pm}$, $d_n^{\pm}$ resp.

3. RELATIONS BETWEEN STRUCTURE FUNCTIONS: TWIST 2

5 STRUCTURE FUNCTIONS

$\longleftrightarrow$

2 BASIC OPERATORS @ LEADING ORDER (ME)

$\Longrightarrow$ 3 RELATIONS BETWEEN THE TWIST 2 PARTS OF POLARIZED STRUCTURE FUNCTIONS

OPERATOR: (SYMMETRIC)

$$\Theta_S^{\pm\beta\{\mu_1\cdots\mu_n\}} = \frac{1}{n+1}\left[\Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} + \ldots + \Theta^{\pm\mu_n\{\mu_1\cdots\beta\}}\right]$$

$(g_V^q)^2 + (g_A^q)^2$ $\quad$ g_1, g_2

COUPLINGS

$2 g_V^q g_A^q$ $\quad$ g_3, g_4, g_5

3 RELATIONS @ TWIST 2 (JB, N. KOCHELEV 1996)

$$g_4^i(x,Q^2) = 2x\, g_5^i(x,Q^2)$$ DICUS '72

$$g_2^i(x,Q^2) = -g_1^i(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^i(y,Q^2)$$ WANDZURA - WILCZEK '77

$$g_3^i(x,Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4^i(x,Q^2)$$
$$= 4x^2 \int_x^1 \frac{dy}{y^2} g_5^i(x,Q^2)$$

JB, KOCHELEV '96

→ STABLE, NON-DESTRUCTIVE $M \to 0$ LIMIT (NUCLEON MASS)

BASIS: $g_1^i(x,Q^2)$, $g_5^i(x,Q^2)$

$M^2/Q^2 \not\ll 1$: JB, A. TKABLADZE DESY 98-181 NPB IN PR.

→ • CORRECTION TERM TO DICUS RELATION

→ • COMMUTATION OF TARGET MASS RESUMMATION & WW-RELATION, BK-RELATION RESP.

↑ ALSO INCLUDING QUARK MASSES.

WHY ARE THE WANDZURA-WILCZEK AND CALLEN-GROSS RELATION STRUCTURALLY DIFFERENT ?

CONSIDER THE COMPTON AMPLITUDE FOR NON-FORWARD SCATTERING.

$$T_{\mu\nu}(q, p_+, p_-) \xrightarrow[\lim\limits_{p_- \to 0}]{} T_{\mu\nu}^{\text{forw.}}(p, q)$$

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$$T_{\mu\nu}(q, p_+, p_-) = T_{\mu\nu}^{\text{symm}} + T_{\mu\nu}^{\text{asymm}}$$

$$T_{\mu\nu}^{\text{symm}} = \left(g_{\mu\nu} - \frac{p_\mu q_\nu + p_\nu q_\mu}{p.q}\right) \int Dz$$

$$\left\{ \left(\frac{1}{\xi + z_+ - i\varepsilon} - \frac{1}{\xi - z_+ - i\varepsilon} \right) \right.$$

$$\left. - z_+ \left[\frac{1}{(\xi + z_+ + i\varepsilon)^2} + \frac{1}{(\xi - z_+ - i\varepsilon)^2} \right] \right\} G(z_+, z_-)$$

$$\int_{-1+|z_+|}^{+1-|z_+|} dz_- \, G(z_+, z_-) = \int_{z_+}^{\mathrm{sign}(z_+)} \frac{dt}{t} \, \hat{g}(t)$$

$\uparrow$ partonic interpretation.

$$\int_{-1}^{+1} dz_+ \; \frac{z_+}{(\xi \pm z_+ - i\varepsilon)^2} \int_{z_+}^{\mathrm{sign}(z_+)} \frac{dz}{z}\, \hat{g}(z) = \pm \int_{-1}^{+1} \frac{dz_+}{\xi \pm z_+ - i\varepsilon} \left[\int_{z_+}^{\mathrm{sign}(z_+)} \frac{dz}{z}\, \hat{g}(z) - \hat{g}(z) \right].$$

I.E.: THE 2nd TERM CONTAINS ‚INTEGRAL' CONTRIBUTIONS AS WELL, WHICH CANCEL IN THE FORWARD CASE.

INTEGRAL TERMS ARE NORMALLY THERE, BUT MAY CANCEL IN SPECIAL SITUATIONS.

$$T_{\mu\nu}^{asym} = i\varepsilon_{\mu\nu}{}^{\gamma\beta} \frac{q_\gamma p_\beta}{(p\cdot q)^2}\, q\cdot S \times$$

$$\int_{-1}^{+1} dz_+ \left[\frac{1}{\xi + z_+ - i\varepsilon} + \frac{1}{\xi - z_+ - i\varepsilon} \right] \left[\int_{z_+}^{\mathrm{sign}(z_+)} \frac{dz}{z}\, \hat{g}_5(z) - g_5(z) \right]$$

$$- i\varepsilon_{\mu\nu}{}^{\gamma\beta} \frac{q_\gamma S_\beta}{p\cdot q} \int_{-1}^{+1} dz_+ \left[\frac{1}{\xi + z_+ - i\varepsilon} + \frac{1}{\xi - z_+ - i\varepsilon} \right] \cdot \int_{z_+}^{\mathrm{sign}(z_+)} \frac{dz}{z}\, \hat{g}_5(z).$$

$$W_{\mu\nu} = \frac{1}{2\pi} \, \mathrm{Im} \, T_{\mu\nu}$$

$$\frac{1}{2\pi} \, \mathrm{Im} \, \frac{1}{\xi \pm z_+ - i\varepsilon} = \frac{1}{2} \, \delta(\xi \pm z_+)$$

$$\boxed{\begin{aligned} F_2(x, Q^2) &= 2x \, F_1(x, Q^2) \\ g_2^{\tau=2}(x, Q^2) &= -g_1^{\tau=2}(x, Q^2) + \int_x^1 \frac{dz}{z} \, g_1^{\tau=2}(z, Q^2) \end{aligned}}$$

4. RELATIONS BETWEEN STRUCTURE FUNCTIONS: TWIST 3

5 STRUCTURE FUNCTIONS

$\longleftrightarrow$

2 BASIC OPERATORS @LEADING ORDER (ME)

$\Longrightarrow$ 3 RELATIONS BETWEEN THE TWIST 3 PARTS OF POLARIZED STRUCTURE FUNCTIONS

OPERATOR:

$$\Theta_R^{\pm\beta\{\mu_1\cdots\mu_n\}} = \Theta^{\pm\beta\{\mu_1\cdots\mu_n\}} - \Theta_S^{\pm\beta\{\mu_1\cdots\mu_n\}}$$

TWO OPERATOR MATRIX ELEMENTS

$\propto (g_V^q)^2 + (g_A^q)^2$ $\qquad g_1, g_2$

$\propto 2 g_V^q g_A^q$ $\qquad g_3, g_4, g_5$

CRUCIAL POINT:

- $\frac{M^2}{Q^2}$ CANNOT BE NEGLECTED !
- IF SO, THEY HAVE TO BE TO RESUMMED TO ALL ORDERS TO AVOID UNPHYSICAL (NON-EXISTENT) SINGULARITIES FOR $x \to 1$

(JB, A. TKABLADZE '98)

NPB '99
hep-ph/9812478

LCE WITH TARGET MASSES.

$\longrightarrow$ ALL POLARIZED STRUCTURE FUNCTIONS RECEIVE TWIST 3 TERMS !

EARLIER TREATMENT ($S_\perp$, JB, KOCHELEV
$S_\parallel$ VARIOUS AUTHORS)

$\longrightarrow$ HAD TO BE EXTENDED.

$$
\begin{aligned}
g_1^{\tau=3}(x,Q^2) = \sum_q \frac{1}{4}\left[(g_V^q)^2 + (g_A^q)^2\right] \frac{4M^2x^2}{Q^2} \Bigg\{ & \frac{D^{\pm q}(\xi)}{\left[1+4M^2x^2/Q^2\right]^{3/2}} \\
& - \frac{3}{(1+4M^2x^2/Q^2)^2} \int_\xi^1 \frac{d\xi_1}{\xi_1}\, D^{\pm q}(\xi_1) \\
& + \frac{(2-4M^2x^2/Q^2)}{(1+4M^2x^2/Q^2)^{5/2}} \int_\xi^1 \frac{d\xi_1}{\xi_1} \int_{\xi_1}^1 \frac{d\xi_2}{\xi_2}\, D^{\pm q}(\xi_2)
\end{aligned}
$$

$$
\begin{aligned}
g_2^{\tau=3}(x,Q^2) = \sum_q \frac{1}{4}\left[(g_V^q)^2 + (g_A^q)^2\right] \Bigg\{ & \frac{D^{\pm q}(\xi)}{\left[1+4M^2x^2/Q^2\right]^{3/2}} \\
& - \frac{1-8M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^2} \int_\xi^1 \frac{d\xi_1}{\xi_1}\, D^{\pm q}(\xi_1) \\
& - \frac{12M^2x^2/Q^2}{(1+4M^2x^2/Q^2)^{5/2}} \int_\xi^1 \frac{d\xi_1}{\xi_1} \int_{\xi_1}^1 \frac{d\xi_2}{\xi}\, D^{\pm q}(\xi_2)
\end{aligned}
$$

EXACT RELATIONS:

$$
g_1^{\tau=3}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2^{\tau=3}(x,Q^2) - 2\int_x^1 \frac{dy}{y}\, g_2^{\tau=3}(y,Q^2)\right]
$$

Q^2 dependence through: M^2/Q^2 $\rightarrow$ see above.

THE NEW RELATIONS :

$$g^i_{1,\tau=3}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g^i_{2,\tau=3}(x,Q^2) - 2\int_x^1 \frac{dy}{y}\, g^i_{2,\tau=3}(x,Q^2)\right]$$

$$\frac{4M^2x^2}{Q^2}\, g^i_{3,\tau=3}(x,Q^2) = g^i_{4,\tau=3}(x,Q^2)\left(1+\frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y}\, g^i_{4,\tau=3}(y,Q^2)$$

$$2x\, g^i_{5,\tau=3}(x,Q^2) = -\int_x^1 \frac{dy}{y}\, g^i_{4,\tau=3}(y,Q^2)$$

TWIST 2: $g_{1,\tau=2}$, $g_{4,\tau=2}$

DETERMINES : $g_{2,3,5;\,\tau=2}$

TWIST 3: $g_{2,4\;\tau=3}$

DETERMINES : $g_{1,3,5;\,\tau=3}$

$M^2/Q^2 \to 0$ BREAK DOWN OF THE ABOVE RELATIONS.

TWIST 3 LIVES IN THE DOMAIN $M^2/Q^2 \not\ll 1$,

HOWEVER!

IN THE ABOVE RELATIONS ALL TM' TERMS WERE RESUMMED EXACTLY.

$$g_{1,\tau=3}^{|\gamma|^2}(x,Q^2) = \frac{4M^2x^2}{Q^2}\Big[g_{2,\tau=3}^{|\gamma|^2}(x,Q^2) - 2\int_x^1 \frac{dy}{y}\, g_{2,\tau=3}^{|\gamma|^2}(x,Q^2)\Big]$$

CAN BE TESTED IN PRECISION MEASUREMENTS OF

$$g_2^{e.m}(x,Q^2)\,.$$

⟶ CEBAF, SLAC g_2-MEASUREMENT.

EFREMOV-TERYAEV-LEADER SR:

$$\int_0^1 dx\, x\,\left[g_1^{val}(x,Q^2) + 2\, g_2^{val}(x,Q^2)\right] = 0$$

(TWIST 3 REL.)

INTRODUCE TARGET MASSES (TW2, TW3)

$$\int_0^1 dx\, x\,\left[g_1^{val}(x,Q^2) + 2\, g_2^{val}(x,Q^2)\right]$$

$$= \sum_q \frac{e_q^2}{2}\, \frac{m_q}{M} \int_0^1 dx\, \frac{h_{1q}(x) - \bar{h}_{1q}(x)}{\left[1 - \frac{M^2 x^2}{Q^2}\right]^2}$$

IFF: $Q^2 > M^2$ ↳ THE INTEGRAL CONVERGES. ONE MAY THEN PERFORM $m_q \to 0$!

↳ ELT SR, OTHERWISE NOT.

WHAT HAS TO BE DONE IN THE SLAC g_2 EXPERIMENTS ?

$$\frac{d^2\sigma(\pm S_L)}{dx\,dy} = \mp 4\pi \frac{\alpha^2}{Q^2}\left\{\left(2-y-\frac{2xyM^2}{S}\right)\left[g_1^{(2)}+g_1^{(3)}\right] + \frac{4xM^2}{S}\left[g_2^{(2)}+g_2^{(3)}\right]\right\}$$

$$\equiv \mathcal{L}_L\left(g_1^{(2)}, g_2^{(3)}\right)$$

$$\frac{d^3\sigma(\pm S_T)}{dx\,dy\,d\phi} = \mp 4\frac{\alpha^2}{Q^2}\sqrt{\frac{M^2}{S}}\sqrt{xy\left[1-y-\frac{xyM^2}{S}\right]}\cos(\alpha-\phi)$$

$$\left\{\left[g_1^{(2)}+g_1^{(3)}\right]-\frac{2}{y}\left[g_2^{(2)}+g_2^{(3)}\right]\right\}$$

$$\equiv \mathcal{L}_T\left(g_1^{(2)}, g_2^{(3)}\right)$$

WITH :

$$g_2^{(2)}(x) = -g_1^{(2)}(x)+\int_x^1\frac{dz}{z}\,g_1^{(2)}(z)$$

$$g_1^{(3)}(x) = \frac{4M^2x^2}{Q^2}\left[g_2^{(3)}(x)-2\int_x^1\frac{dz}{z}\,g_2^{(3)}(z)\right]$$

MEASURE : $g_1^{(2)}(x, Q^2, M^2)$

$g_2^{(3)}(x, Q^2, M^2)$

(DETERMINED ITERATIVELY).

5. CONCLUSIONS

1) IN LO IN QCD THE FIVE POLARIZED STRUCTURE FUNCTIONS RECEIVE TWIST 2 AND TWIST 3 CONTRIBUTIONS

2) THE RENORMALIZATION GROUP REQU. THE TWIST DECOMPOSITION.

3) ALL STRUCTURE FUNCTIONS HAVE TO BE REPRESENTED IN THE TWIST DECOMPOSITION

4) THE OPERATOR AND FLAVOR STRUCTURE (CURRENT PROJECTION) RESULTS INTO

   2 INDEP. TWIST 2 OME'S

   2 INDEP. TWIST 3 OME'S

5) 3 TWIST 2 AND
   3 TWIST 3 RELATIONS BETWEEN THE STRUCTURE FUNCTIONS ARE IMPLIED

6) FOR THE TWIST 3 TERMS $\forall$ M^2/Q^2 TERMS ARE NEEDED.
   THE ETL- REL.REMAINS INTACT FOR $Q^2 > M^2$.

7) THE RELATION FOR $g_{1,\tau=3}^{|\gamma|^2} \leftarrow G[g_{2,\tau=3}^{|\gamma|^2}]$ CAN BE MEASURED IN THE FUTURE !