

LEADING LOG RADIATIVE  
CORRECTIONS  
AT HERA

J. BLÜMLEIN - IFH ZEUTHEN / BERLIN 5/89

1. STATUS : CALCULATION OF RC AT HERA
2. LEADING LOG FORMALISM
3. ELIMINATION OF QUARK MASS TERMS
4. NC & CC CROSS SECTIONS  $O(\alpha/2\pi)$
5. CONCLUSIONS

1. STATUS: RADIATIVE CORRECTIONS  
AT HERA

$$e^{\pm} p \rightarrow \begin{cases} C^{\pm} X(\gamma) \\ \bar{C}^{\pm} X(\gamma) \end{cases} \quad O\left(\frac{\alpha}{2\pi}\right)$$

COMPLETE CALCULATIONS:

- BARDIN et al. 1981 (NC), 1987-89 (NC, CC)
- BÖHM, SPIESBERG 1987/89 (NC, CC)

APPROXIMATE CALCULATIONS:

- KRIPFGANZ, KÖHLING (NC, CC) 1987/88
- VAN NEERVEN et al. 1987 (1988) (RINGBERG/BAYAR) April 1989
- THIS NOTE (NC, CC)

- THE CALCULATION BARDIN et al. 1987/89 CONFIRMED BARDIN et al. 1981 (INDEP. CALC.)
- NUMERICAL DIFFERENCES: BARDIN et al. / BÖHM, SPIESBERG  
esp.: LOW  $y$ , HIGHER  $x$  (NC)

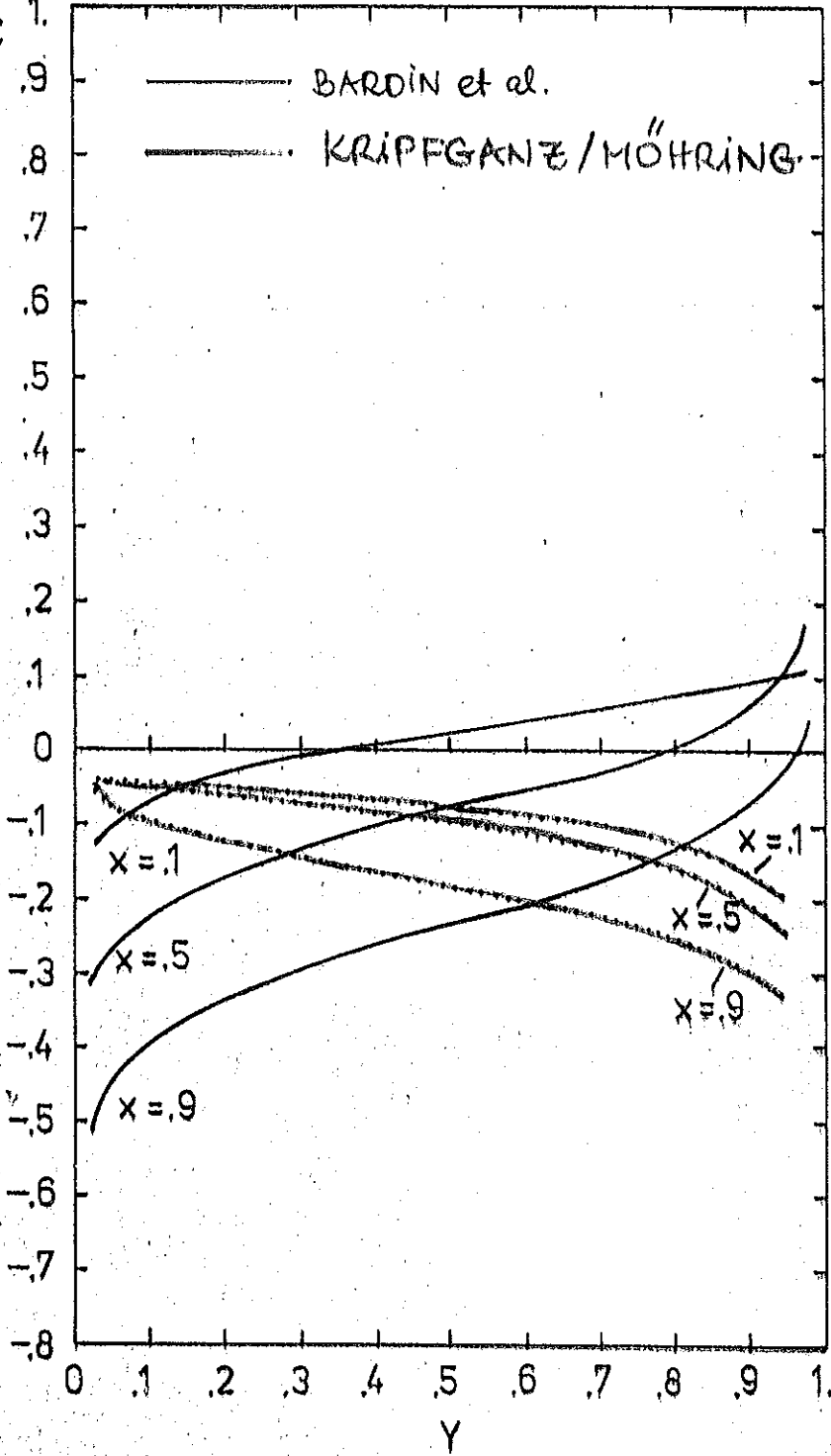
↑  
A LARGE PART OF THE STATISTICS IS  
LOCATED AT LOW  $y$ !

- NO COMPARISON POSSIBLE FOR CC AT PRESENT.
- LARGE DIFFERENCES BOTH IN SHAPE & VALUE FOR CC BETWEEN BARDIN et al. / KRIPFGANZ & KÖHLING
- GOOD AGREEMENT BETWEEN BARDIN et al. / VAN NEERVEN et al. (NC)

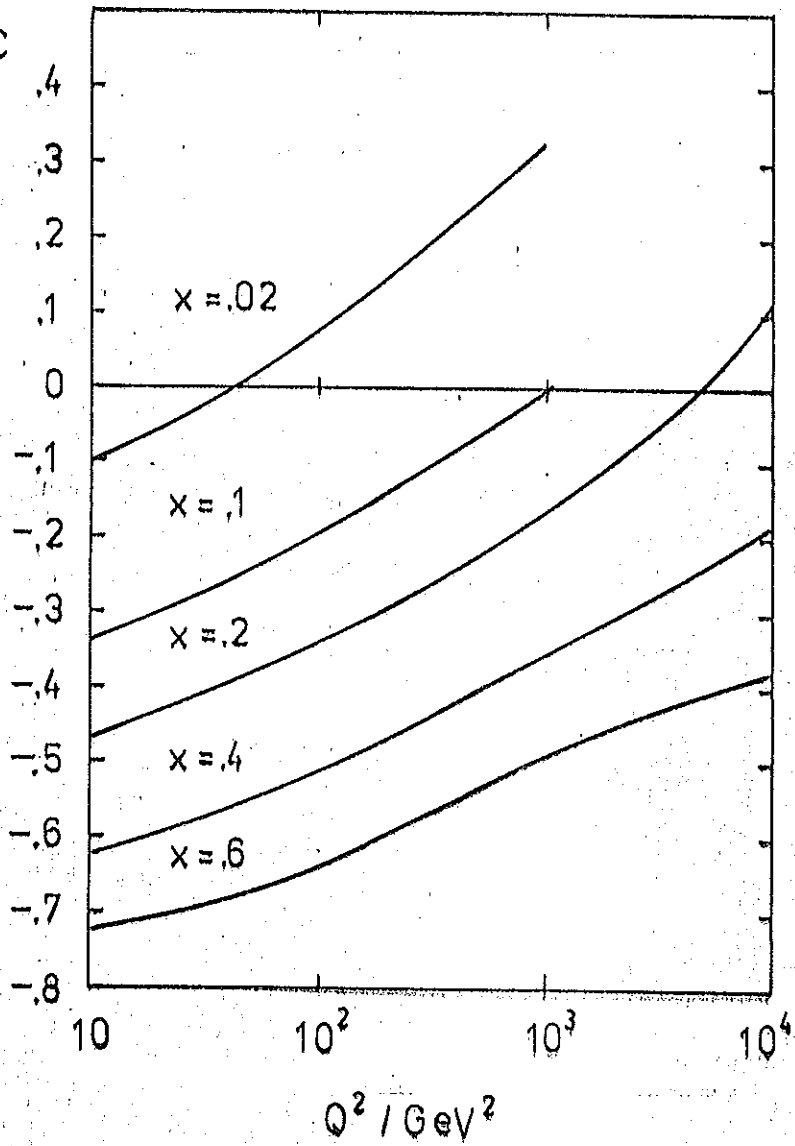


CC

$\sigma_{CC}^{e-}$



$$\frac{d^2\sigma/dx dy}{d^2\sigma_0/dx dy} - 1 = \sigma_{NC}^-$$



D.Yu. BARDIN et al.  
DUBNA E2-88-682

STRONG  
Q<sup>2</sup>-DEPENDENCE  
AT FIXED X.

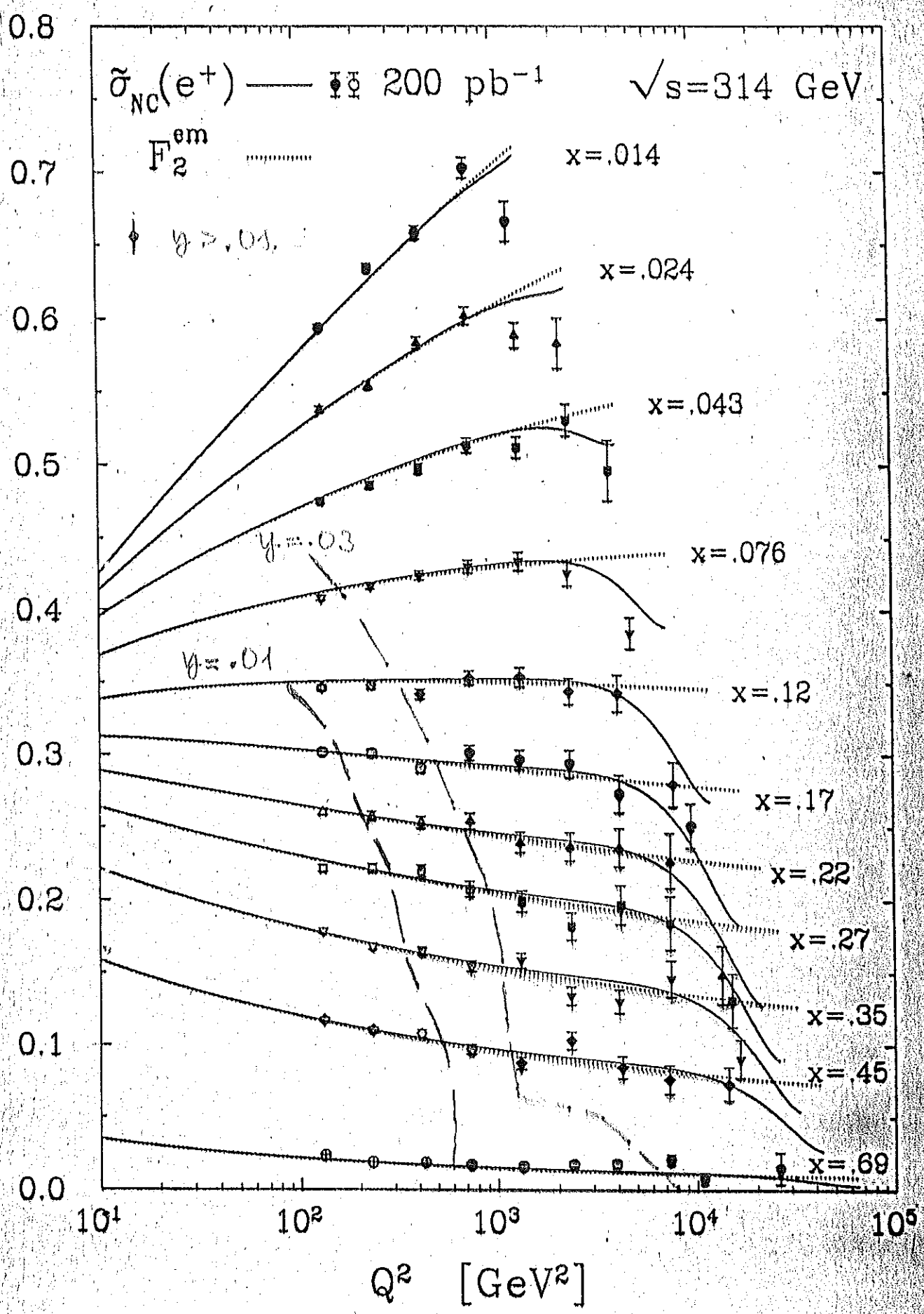
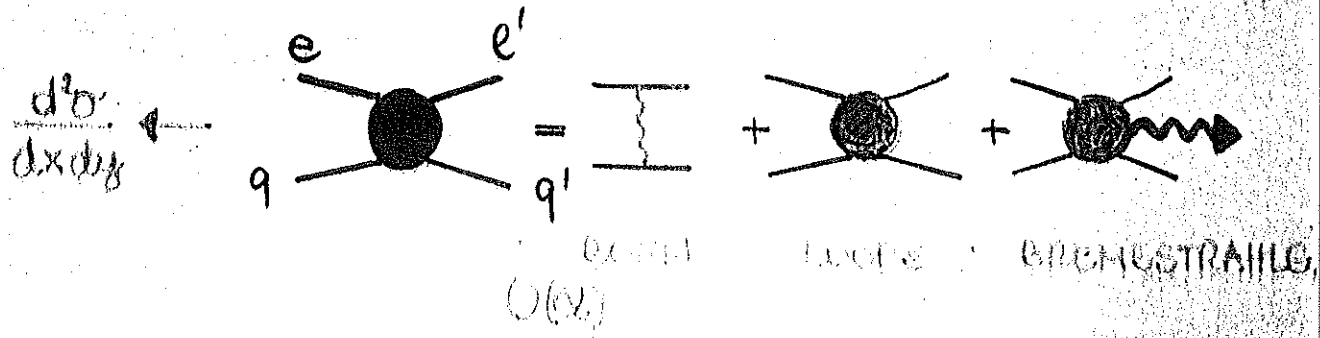


Fig. 3

# 2. THE LEADING LOG FORMALISM



WE WILL DISCUSS:  $\gamma$ -BREMSSTRAHLUNG + VERTEX CORR. (IR. FINITE)

- HUGE CONTRIBUTION:

SPS:  $\ln\left(\frac{Q^2 - 100 \text{ GeV}^2}{m_p^2}\right)$

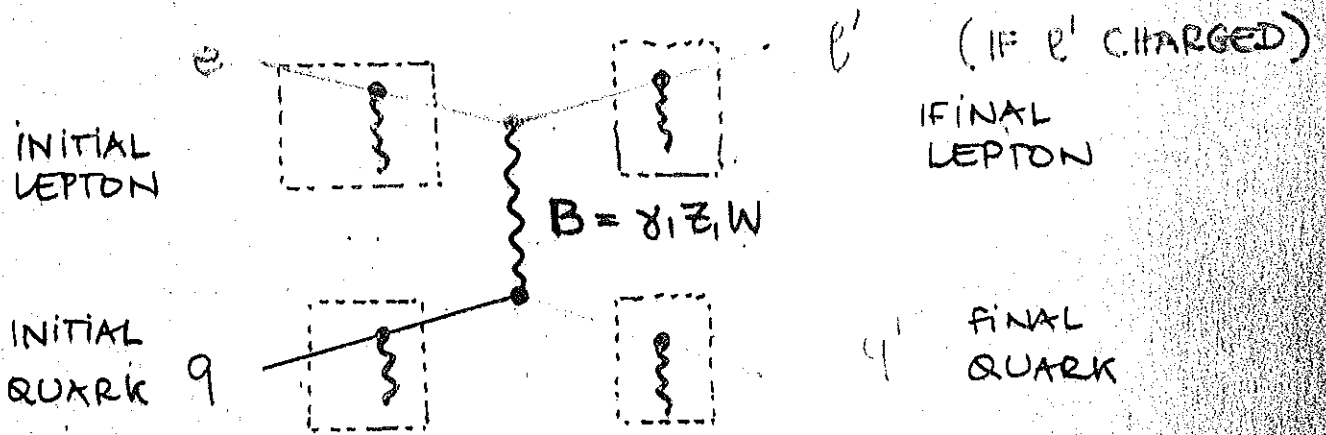


x 2.5 ... 3.0

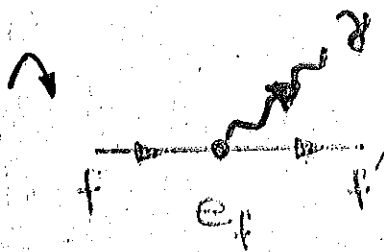
HERA:  $\ln\left(\frac{Q^2 - 10^3 \dots 10^4 \text{ GeV}^2}{m_e^2}\right)$

i) CHOOSE A PHYSICAL GAUGE.

ii) THE LEADING LOGS FACTORIZE FOR PHOTON BREMSSTRAHLUNG (AS E.G. IN QCD) THE BREMSSTRAHLUNG LOGS ARE:



iii) CALCULATE  $4 |\omega_i|^2 / s$ ; TREAT  $p_f \parallel k$  IN THE NUMERATOR OF  $\omega_i$ .



$$\begin{aligned}
 & \sim \frac{1}{s} \left( \frac{p_f \cdot k}{p_f \cdot p_f} \right) \cdot p_{ff'}(z) \cdot e_f^2 \\
 & \sim O(Q^2) \\
 & \sim \frac{1}{s} \ln(1-z)
 \end{aligned}$$

iv) FINAL QUARK BREMSSTRAHLUNG WILL NOT CONTRIBUTE AFTER SUMMATION OVER THE HADRONIC FINAL STATE: KINOSHITA-LEE-NAUENBERG THEOREM.



v) BREMSSTRAHLUNG FORMULA:

USE:  $\hat{S} \approx Q^2 = S \times y.$

$$\frac{d^2\sigma_0}{dx dy} = \sum_{i=q, \bar{q}} C_i(x, y) \frac{d^2\sigma_{0,i}}{dx dy}$$

$\frac{d^2\sigma_{i,f}^{Brem.}}{dx dy}$  IS A MAP OF  $C_i(x, y) \frac{d^2\sigma_{0,i}}{dx dy}$  (QUARKS)

OR EVEN OF  $\frac{d^2\sigma_0}{dx dy}$  (ELECTRONS) !

THIS MAP IS WIDELY INDEPENDENT OF THE BORN CROSS SECTION. IT IS DETERMINED BY THE RADIATING LEGS.

ONE OBTAINS:

(v) ELECTRONS:

$$\frac{\alpha}{2\pi} \cdot e_e^2 \cdot \ln\left(\frac{Q^2}{m_e^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \theta(z-z_0) \frac{y}{y} \frac{1}{z^2} \frac{d^2\sigma_0}{dx dy} \Big|_{x=\frac{z}{y}} - \frac{d^2\sigma_0}{dx dy} \right\}$$

(iv) QUARKS:

$$\frac{\alpha}{2\pi} \cdot \sum_{i=q, \bar{q}} e_q^2 \cdot \ln\left(\frac{Q^2}{m_q^2}\right) \int_0^1 dz \frac{1+z^2}{1-z} \left\{ \theta(z-z_0) \frac{y}{y} \frac{1}{z^2} C_i(\hat{x}, \hat{y}) \frac{d^2\sigma_{0,i}}{dx dy} \Big|_{x=\frac{z}{y}} - C_i(x, y) \frac{d^2\sigma_{0,i}}{dx dy} \right\}$$

	INITIAL $e^{\pm}$	FINAL $e^{\pm}$	INITIAL QUARK
$Q$	1	2	0
$z_0$	$(1-y)/(1-xy)$	$1-y+xy$	x
$\hat{x}$	$x \cdot y z / (z+y-1)$	$x \cdot y / (z+y-1)$	$x/z$
$\hat{y}$	$(z+y-1)/z$	$(z+y-1)/z$	y
$\hat{Q}^2$	$Q^2 z$	$Q^2/z$	$Q^2$

→ ANALYTICAL AGREEMENT WITH VAN NEERVEN et al.

$|g|_{NC}^2$

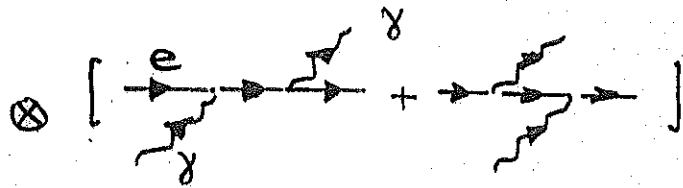
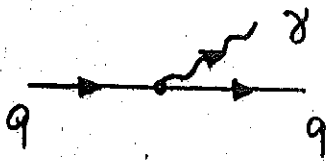
10  
 IFF  $M^2(B) \ll Q^2$  ALL LEADING LOGS ARE FOUND ALREADY. (E.G. CC-REACTIONS)

vi)  $M^2(B) \ll Q^2$  : (eg. PHOTONS)

$$\sim \ln\left(\frac{Q^2}{m_q^2}\right)$$

TERM DUE TO :

(cf. also VAN NEERVEN '89)



THE QUARK FRAGMENTS INTO A PHOTON + QUARK

THE PHOTON SCATTERS WITH THE ELECTRON IN A COMPTON PROCESS

→ THE "2nd PEAK." by HO & TSAI 1971.

$$\frac{d^2\sigma_q}{dx dy} = \frac{\alpha^3}{Sx} \ln\left(\frac{Q^2}{m_q^2}\right) \int_x^1 \frac{dz}{z^3} F_2(z) \frac{1+(1-y)^2}{1-y} \left\{ \frac{z^2 + (z-x)^2}{x} \right\}$$

THIS FORMULA MAY BE DIRECTLY DERIVED FROM THE EXACT CALCULATION OF EINSTEIN ET AL. WHERE THE INTERPRETATION OF THESE TERMS WAS NOT OBVIOUS!

$$\rightarrow \ln \frac{S}{M_p^2} \approx \ln \frac{Q^2}{m_q^2} \quad \text{TERMS}$$

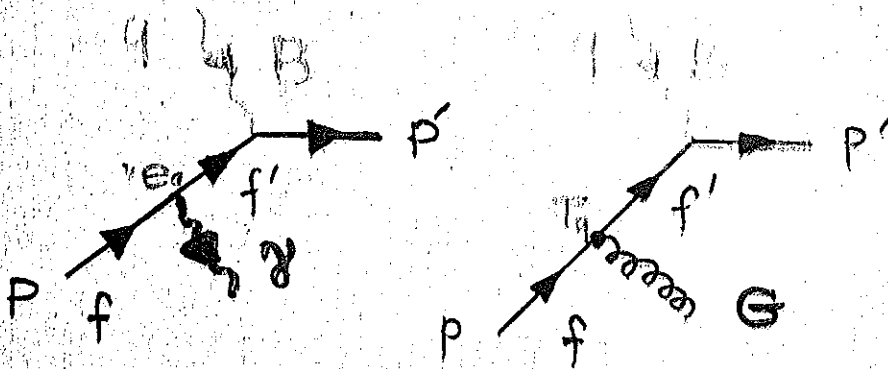
### 3. ELIMINATION OF QUARK MASS TERMS

MOST OF THE EARLY CALCULATIONS USE **SIRLIN'S SCHEME**: QUARKS ARE TREATED AS FERMIONS WITH A FIXED MASS!  
 ALSO QUARKS ARE TREATED AS FERMIONS WITH A FIXED MASS!

HOWEVER,  $m_q = m_q(Q^2)$  IS RUNNING (QCD).

SOLUTION:

(i) INITIAL QUARK BREMSSTRAHLUNG: A. DE RUJULA et al. (1979)



→ MODIFICATION OF NS & SINGLET QCD-EVOLUTION

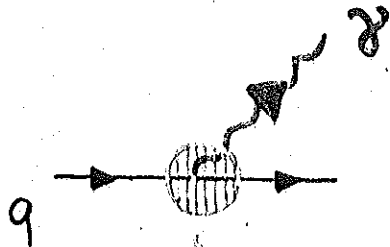
$$d \Delta_q^{NS}(Q^2, x) / d \ln Q^2 = \frac{4}{3} \left[ \frac{\alpha_s(Q^2)}{2\pi} + \frac{3}{4} \frac{\alpha_e}{2\pi} e_q^2 \right] P_{qq}(x) \otimes \Delta_q^{NS}(Q^2, x)$$

$$P_{qq}(x) = x(1-x)$$

$$\frac{2\pi}{\alpha_s} \frac{d \Delta_q^{NS}(Q^2, x) / d \ln Q^2}{\Delta_q^{NS}(Q^2, x)} = \left\{ \left[ 1 + \frac{3}{4} \frac{\alpha_e}{\alpha_s(Q^2)} \right] P_{qq}(x) + P_{qq}(x) \otimes G \right\}$$

$m_q$  IS "ABSORBED" INTO THE PARTON-DISTRIBUTIONS AT  $Q_0^2$ .

(ii) QUARK FRAGMENTATION INTO A PHOTON



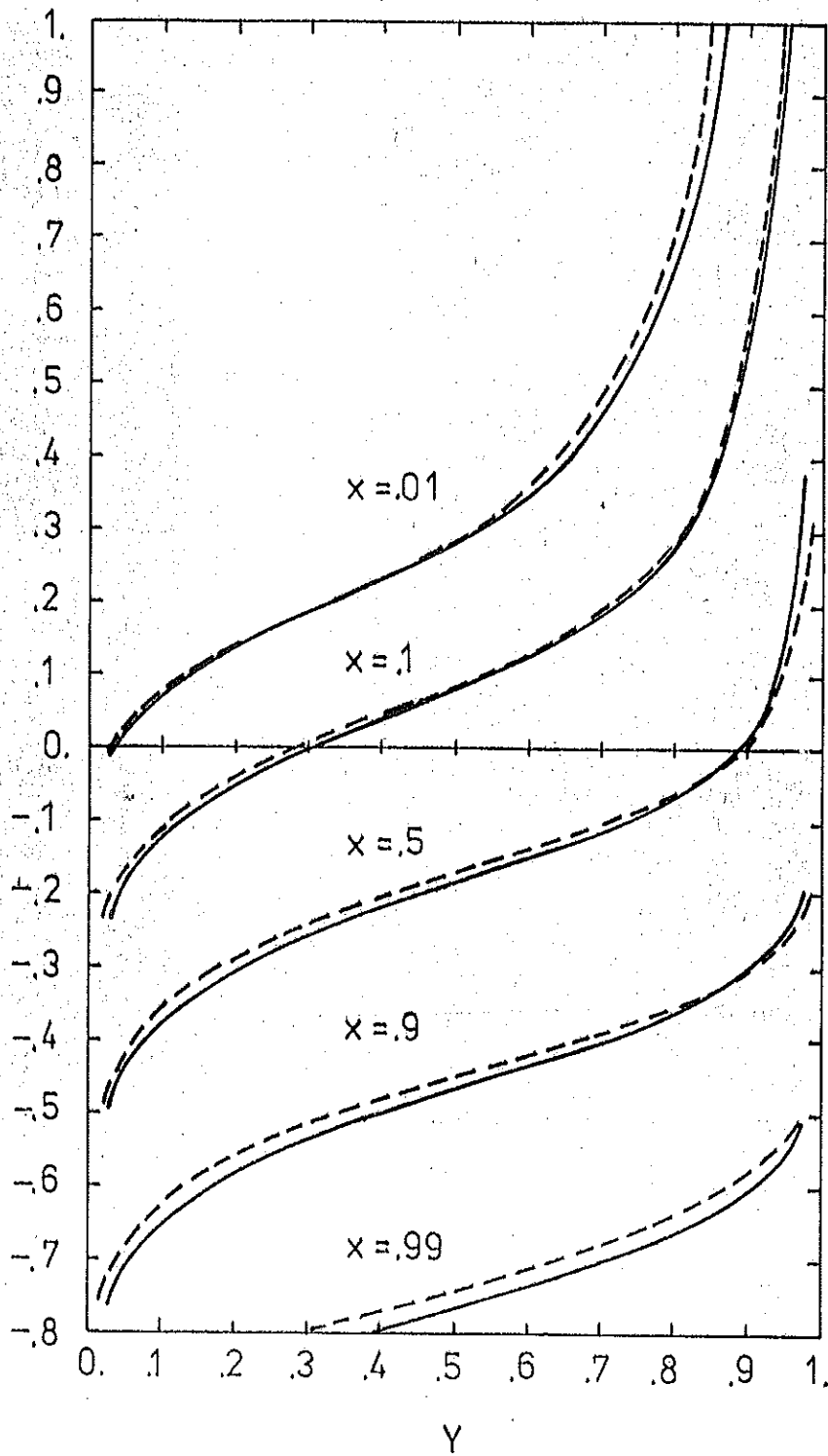
QUARK FRAGMENTATION FUNCTION

REN. GROUP

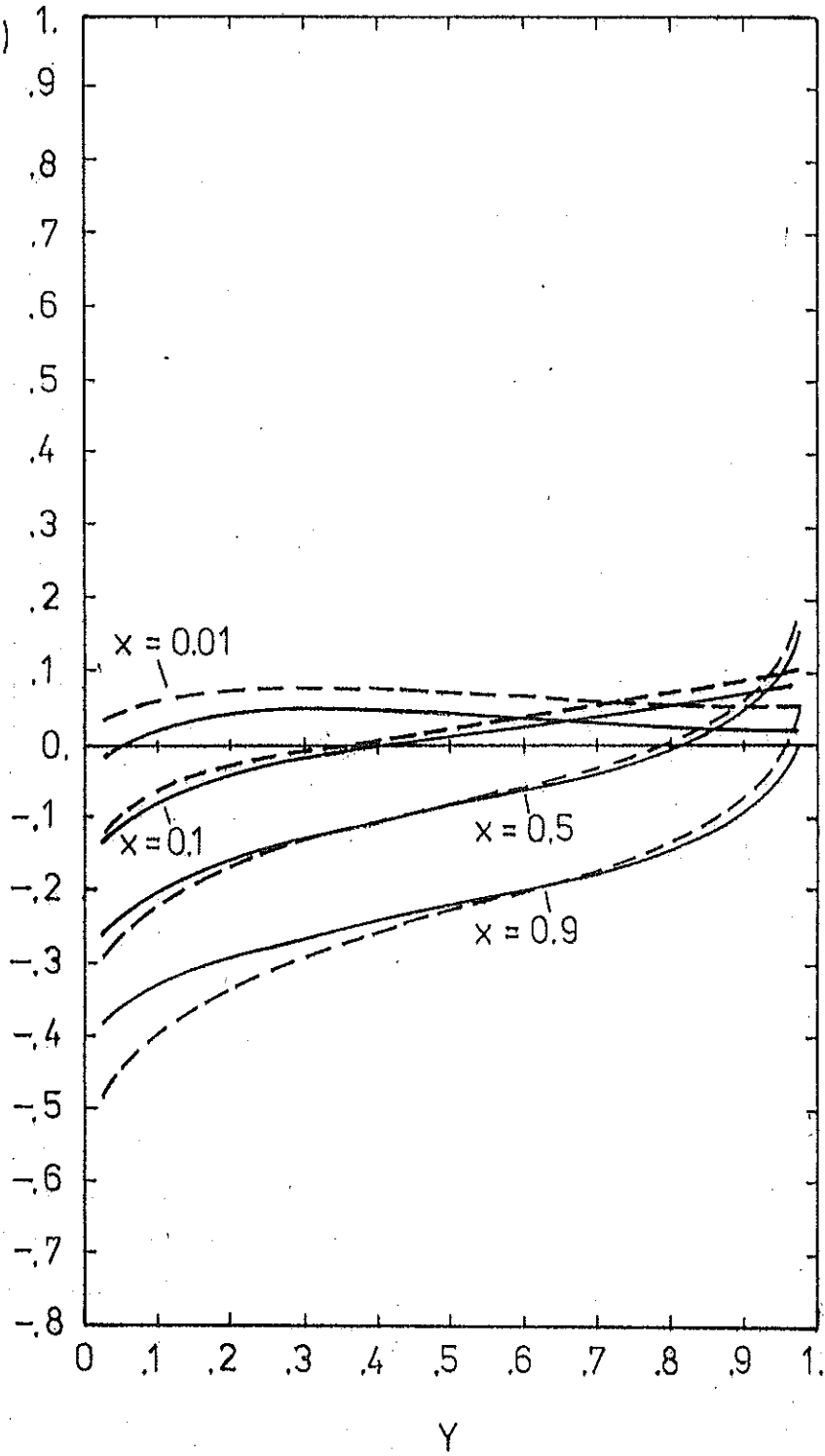
$$x D_{\gamma/q}(x, Q^2) = \frac{\alpha}{2\pi} \ln\left(\frac{Q^2}{\Lambda^2}\right) \cdot \frac{1+(1-x)^2}{x} e_q^2$$

- THE ABOVE ARGUMENTS HOLD FOR THE EXACT EW-RC CALCULATIONS TOO. (THE PROBLEM OCCURS ONLY IN THE COLLINEAR SITUATION!)
- THE  $O\left(\frac{\alpha}{2\pi}\right)$  EWRC'S DEPEND ON NO LIGHT QUARK MASS SCALES, BUT ON  $\Lambda_{QCD}$ .
- THE FINAL CONTRIBUTION OF (ii) IS BY FAR LARGER THAN THAT OF (i)!

$\delta_{NC}^e(\gamma)$



$\delta_{cc}^{e^-}(W)$



THE  $\gamma$  BEHAVIOR OF  $\nu N \rightarrow \nu X$

A. DE RUFULA  
 et al. Nucl. Phys.  
 B154 (1979) 394

A. De Rufula et al. / Radiative corrections

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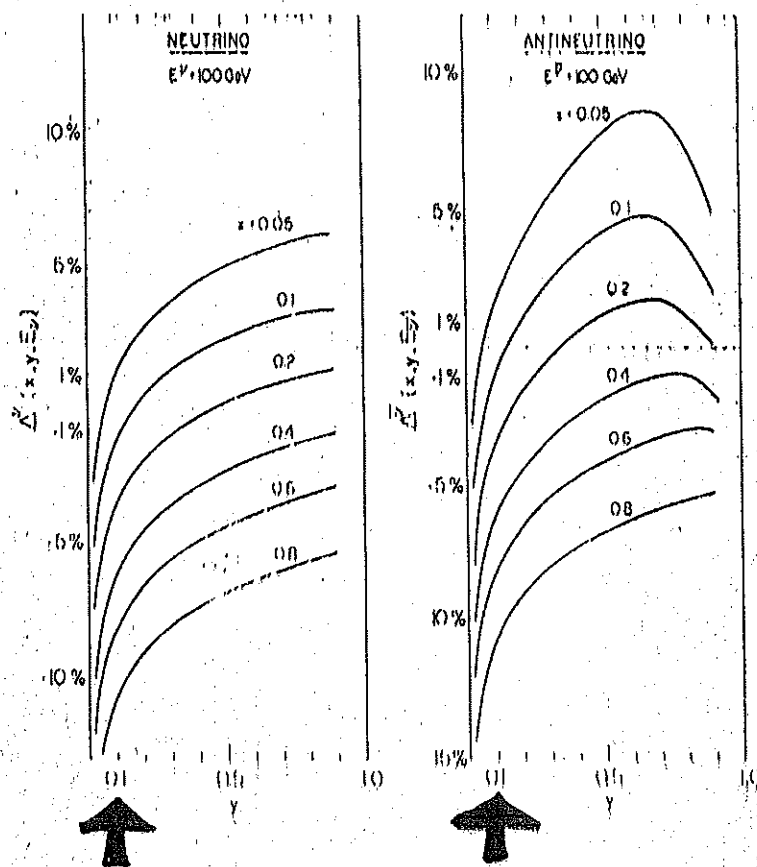


Fig. 3. The doubly differential normalized radiative correction defined in eq. (3.9): (a) for neutrino scattering; (b) for antineutrinos.

THIS IS ONLY  
 PARTIAL BREMS-  
 STRALLUNG ONLY.)



## 5. SUMMARY

- THE  $O(\frac{\alpha}{2\pi})$  BREMSSTRAHLUNG RC'S FOR

$$e^{\pm}p \rightarrow e^{\pm} X \gamma \sim |\gamma|^2$$

AND

$$e^{\pm}p \rightarrow \bar{\nu}_e X \gamma$$

HAVE BEEN CALCULATED IN THE LEADING LOG APPROXIMATION.

- WE FOUND :

- a) GOOD AGREEMENT WITH BARDIN et al. FOR  
NC & CC

FOR ALL  $x$  &  $y$

→  $y$ -PEAK AT  $y \ll 1$

→ BEHAVIOUR OF THE LINES  $x = \text{CONST.}$   
AT HIGH  $y$  FOR CC

- b) AGREEMENT WITH VAN NEERVEN, BENAKER, BETHREND  
FOR  $|\gamma|^2$  NC

- c) LIGHT QUARK MASSES DO NOT ENTER  $O(\frac{\alpha}{2\pi})$  EWRC  
AS IN THE NAIVE QPM. THE RESULTS DEPEND  
ON  $\Lambda_{\text{QCD}}$  INSTEAD : SCALING VIOLATIONS ;  
QUARK FRAGMENTATION INTO  
PHOTONS