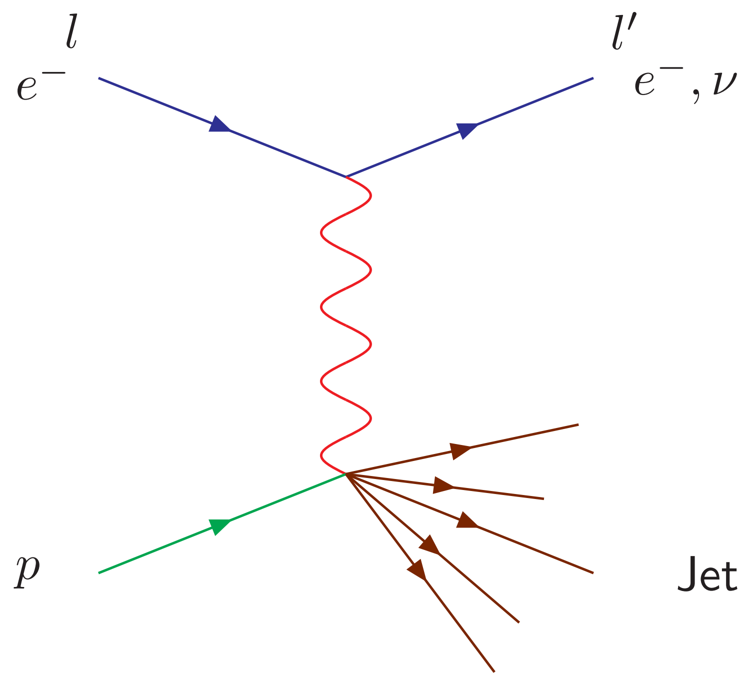
Status of Deeply Inelastic Parton Distributions

Johannes Blümlein
DESY

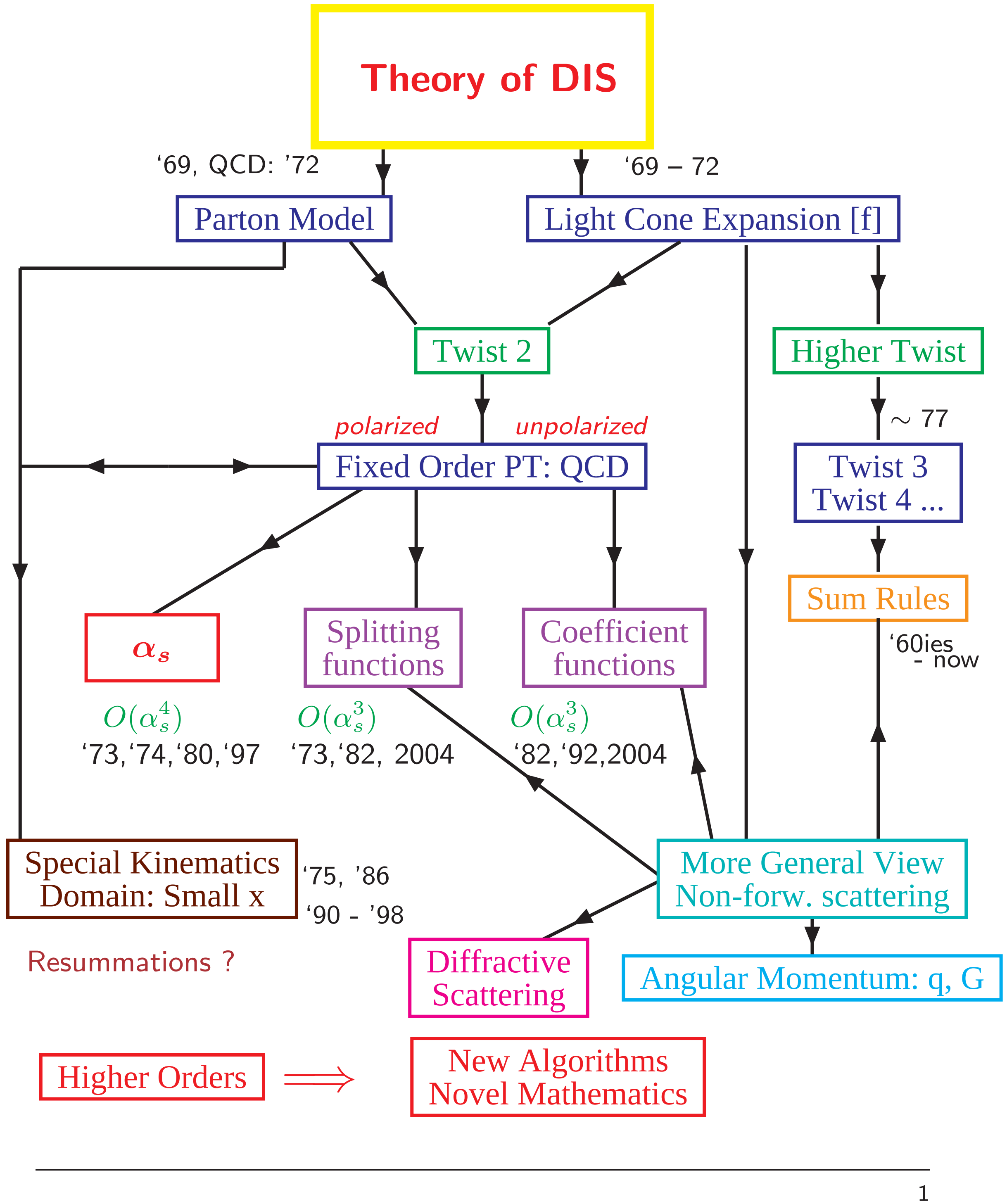
- DIS Theory Status
- QCD Analysis of Unpolarized Structure Functions
- Polarized Structure Functions
- Moments of Parton Densities
- $\Lambda_{\rm QCD}$ and $\alpha_s(M_Z^2)$
- Outlook

Deeply Inelastic Scattering



$$\textbf{space} - \textbf{like process} \;:\quad q^2 = (l-l')^2 = -Q^2 < 0 \qquad W^2 = (p+q)^2 \geq M_p^2$$

$$x = \frac{Q^2}{2p.q}, \qquad\qquad y = \frac{p.q}{p.l} \qquad\qquad 0 \leq x, y \leq 1$$


Theory of DIS
'69, QCD: '72
'69 – 72
Parton Model
Light Cone Expansion [f]
Twist 2
Higher Twist
polarized
unpolarized
~ 77
Fixed Order PT: QCD
Twist 3
Twist 4 ...
Sum Rules
α_s
Splitting functions
Coefficient functions
'60ies - now
$O(\alpha_s^4)$
'73,'74,'80,'97
$O(\alpha_s^3)$
'73,'82, 2004
$O(\alpha_s^3)$
'82,'92,2004
Special Kinematics Domain: Small x
'75, '86
'90 - '98
More General View Non-forw. scattering
Resummations ?
Diffractive Scattering
Angular Momentum: q, G
Higher Orders
New Algorithms Novel Mathematics
1

Status of Highest Order Calculations

- Running α_s: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$

  Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;

  $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;

  Kumano, Miyama; Vogelsang; 1997; Gracey 2006
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993

  Fast Mellin Space code: Blümlein & Alekhin, 2003
- Pol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^1)$ Watson 1982
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_L: $O(\alpha_s^3)$

  Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,

  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient F_2: $O(\alpha_s^2 \varepsilon)$: all operators (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: First contributions to the moments of the operator matrix elements, Bierenbaum, Blümlein, Klein, 2008

DIS Structure Functions @ Twist 2

$$F_j(x,Q^2) = \hat{f}_i(x,\mu^2) \otimes \sigma_j^i\left(\alpha_s, \frac{Q^2}{\mu^2}, x\right)$$

$$\uparrow \text{bare pdf} \qquad \uparrow \text{sub} - \text{system cross} - \text{sect.}$$

$$= \underbrace{\hat{f}_i(x,\mu^2) \otimes \Gamma_k^i\left(\alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2}\right)}_{\text{finite pdf} \equiv f_k}$$

$$\otimes \underbrace{C_j^k\left(\alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x\right)}_{\text{finite Wilson coefficient}}$$

Move to Mellin space :

$$F_j(N) = \int_0^1 dx x^{N-1} F_j(x)$$

Diagonalization of the convolutions $\otimes$ into ordinary products.

Evolution Equations

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-2\gamma_\psi(g)\right]F_i(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+\gamma_\kappa^N(g)-2\gamma_\psi(g)\right]f_k(N)=0$$

$$\left[M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}-\gamma_\kappa^N(g)\right]C_j^k(N)=0$$

CALLAN–SYMNANZIK equations for mass factorization

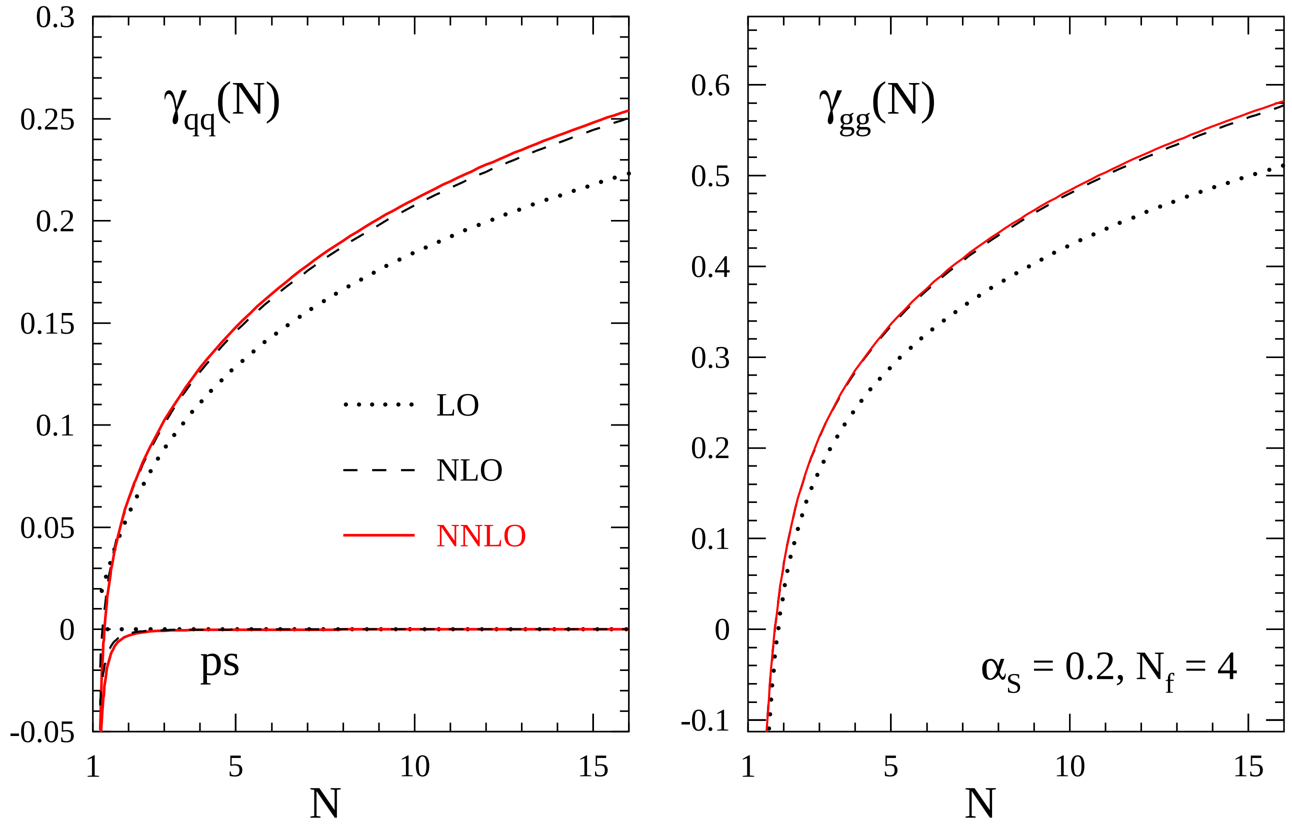
$\equiv$ ALTARELLI–PARISI evolution equations

x-space :

$$\frac{d}{d\log(\mu^2)}\begin{pmatrix} q^+(x,Q^2)\\ G(x,Q^2)\end{pmatrix}=\frac{\alpha_s}{2\pi}\boldsymbol{P}(x,\alpha_s)\otimes\begin{pmatrix} q^+(x,Q^2)\\ G(x,Q^2)\end{pmatrix}$$

$$\boldsymbol{P}(x,\alpha_s)=\boldsymbol{P}^{(0)}(x)+\frac{\alpha_s}{2\pi}\boldsymbol{P}^{(1)}(x)+\left(\frac{\alpha_s}{2\pi}\right)^2\boldsymbol{P}^{(2)}(x)+\dots$$

Anomalous Dimensions and Wilson Coefficients



Vermaseren, Moch, Vogt 2004

The Basic Functions of massless QCD to w=5:$\equiv$ 3 Loops

Representative : $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight w=3 :

$$F_1(N) = \mathbf{M}\left[\frac{\ln(1+x)}{1+x}\right](N)$$

$$F_2(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2(x)}{1+x}\right](N), \qquad F_3(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_2(x)}{1-x}\right)_+\right](N)$$

Yndurain et al., 1981: $F_2(N)$

Weight w=4 :

$$F_4(N) = \mathbf{M}\left[\frac{S_{1,2}(x)}{1+x}\right](N), \qquad F_5(N) := \mathbf{M}\left[\left(\frac{S_{1,2}(x)}{1-x}\right)_+\right](N)$$

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran ,2004

Weight w=5 :

$$F_{6,7}(N) = \mathbf{M}\left[\left(\frac{\mathrm{Li}_4(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_8(N) = \mathbf{M}\left[\frac{S_{1,3}(x)}{1+x}\right](N),$$

$$F_{9,10}(N) = \mathbf{M}\left[\left(\frac{S_{2,2}(x)}{1 \pm x}\right)_{(+)}\right](N), \qquad F_{11}(N) = \mathbf{M}\left[\frac{\mathrm{Li}_2^2(x)}{1+x}\right](N),$$

$$F_{12,13}(N) := \mathbf{M}\left[\left(\frac{\ln(x)S_{1,2}(-x) - \mathrm{Li}_2^2(-x)/2}{1 \pm x}\right)_{(+)}\right](N)$$

$F_6(N) - F_{13}(N)$: J.B., S. Moch, 2004.

Massless QCD to 3 Loops depends on 14 Functions.

Weight w=6 :

$\Rightarrow$ Representation for 3 Loop Wilson Coefficients: 35 Functions, J.B., 2008.

Complex Analysis of these Functions

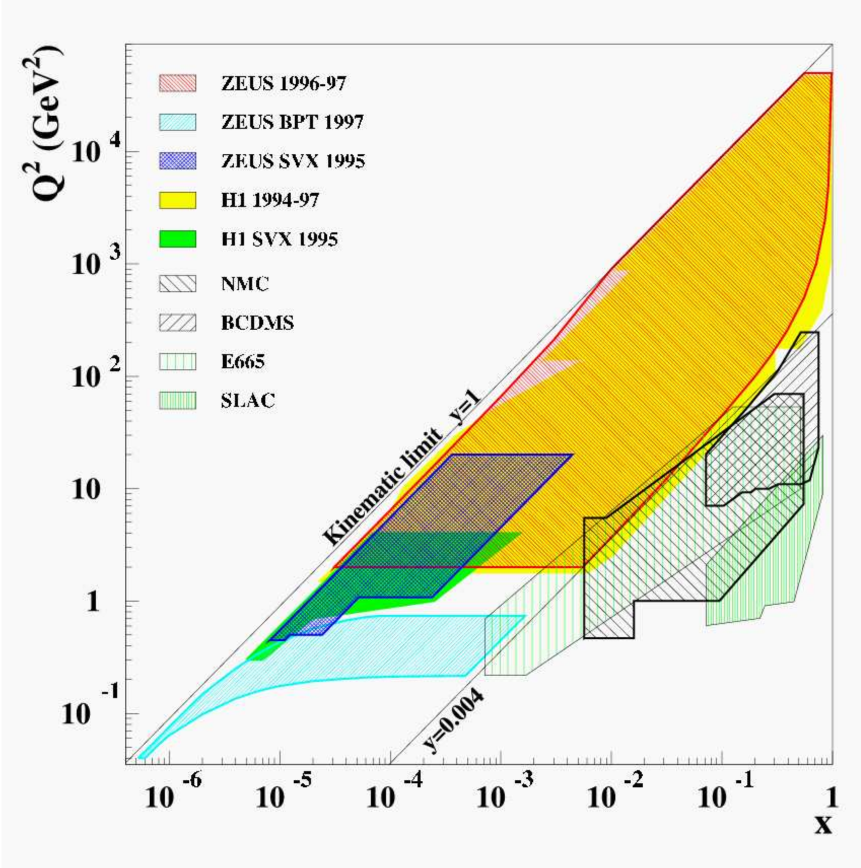
- Construct exact analytic continuations to complex N
- The functions are meromorphic
(up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \to z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all Q^2
- Include the heavy flavor Wilson coefficients in Mellin Space
- Perform a single fast, numerical Mellin inversion
(at high precision)

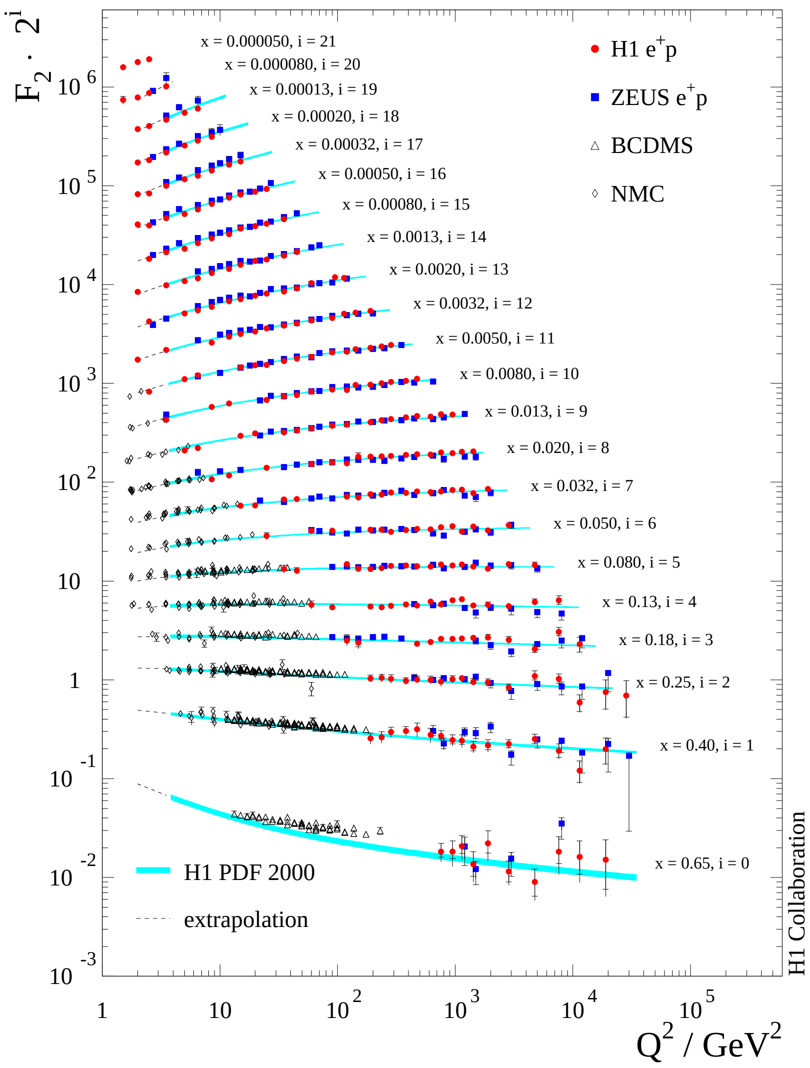
$\Longrightarrow$ **Fastest and most Precise Way of Analysis**

2. QCD Analysis of Unpolarized Structure Functions

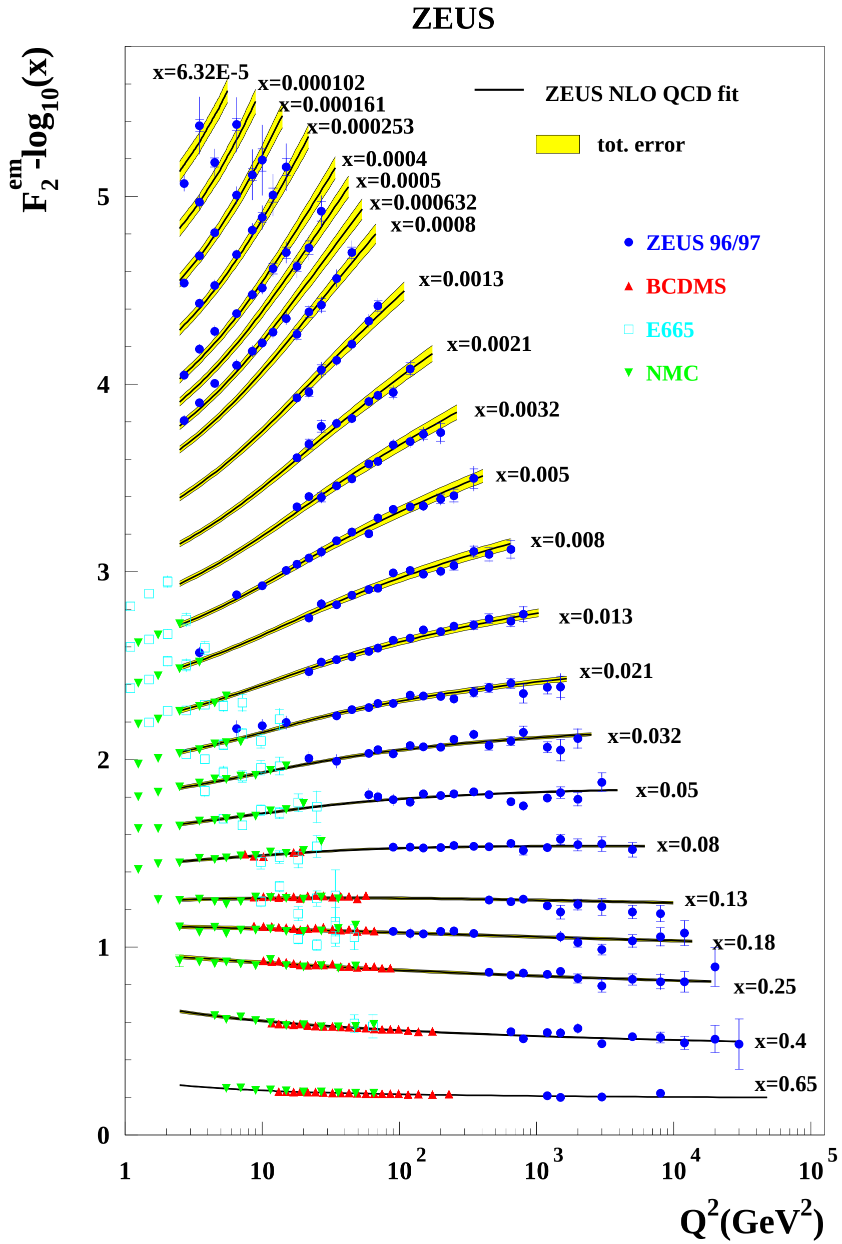
DIS range
Nucleon structure:

$10^{-5} < x < 0.9,$
$1 < Q^2 < 50.000 \mathrm{GeV}^2$



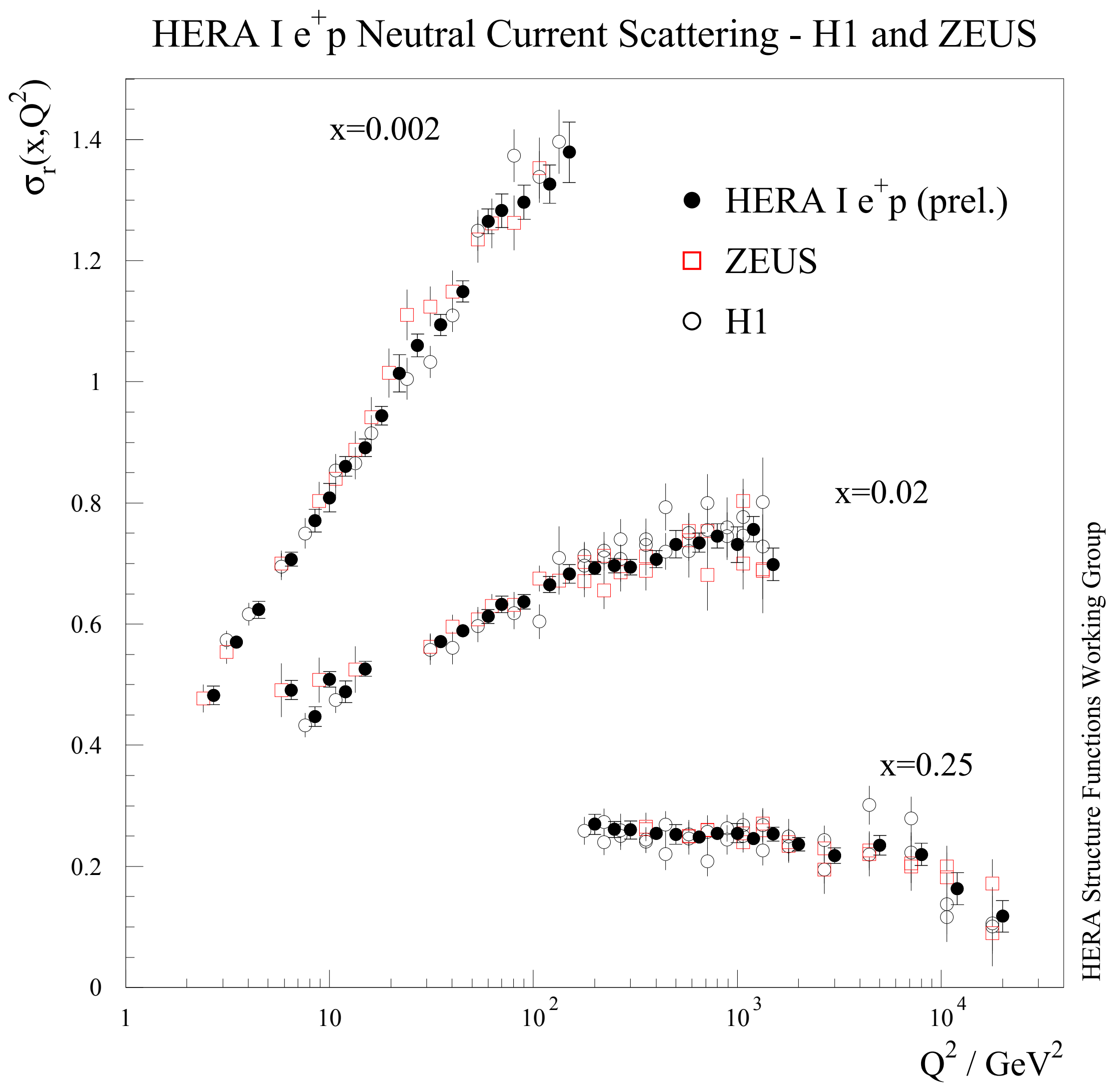


$F_2 \cdot 2^i$
x = 0.000050, i = 21
x = 0.000080, i = 20
x = 0.00013, i = 19
x = 0.00020, i = 18
x = 0.00032, i = 17
x = 0.00050, i = 16
x = 0.00080, i = 15
x = 0.0013, i = 14
x = 0.0020, i = 13
x = 0.0032, i = 12
x = 0.0050, i = 11
x = 0.0080, i = 10
x = 0.013, i = 9
x = 0.020, i = 8
x = 0.032, i = 7
x = 0.050, i = 6
x = 0.080, i = 5
x = 0.13, i = 4
x = 0.18, i = 3
x = 0.25, i = 2
x = 0.40, i = 1
x = 0.65, i = 0
H1 e^+p
ZEUS e^+p
BCDMS
NMC
H1 PDF 2000
extrapolation
H1 Collaboration
Q^2 / GeV^2



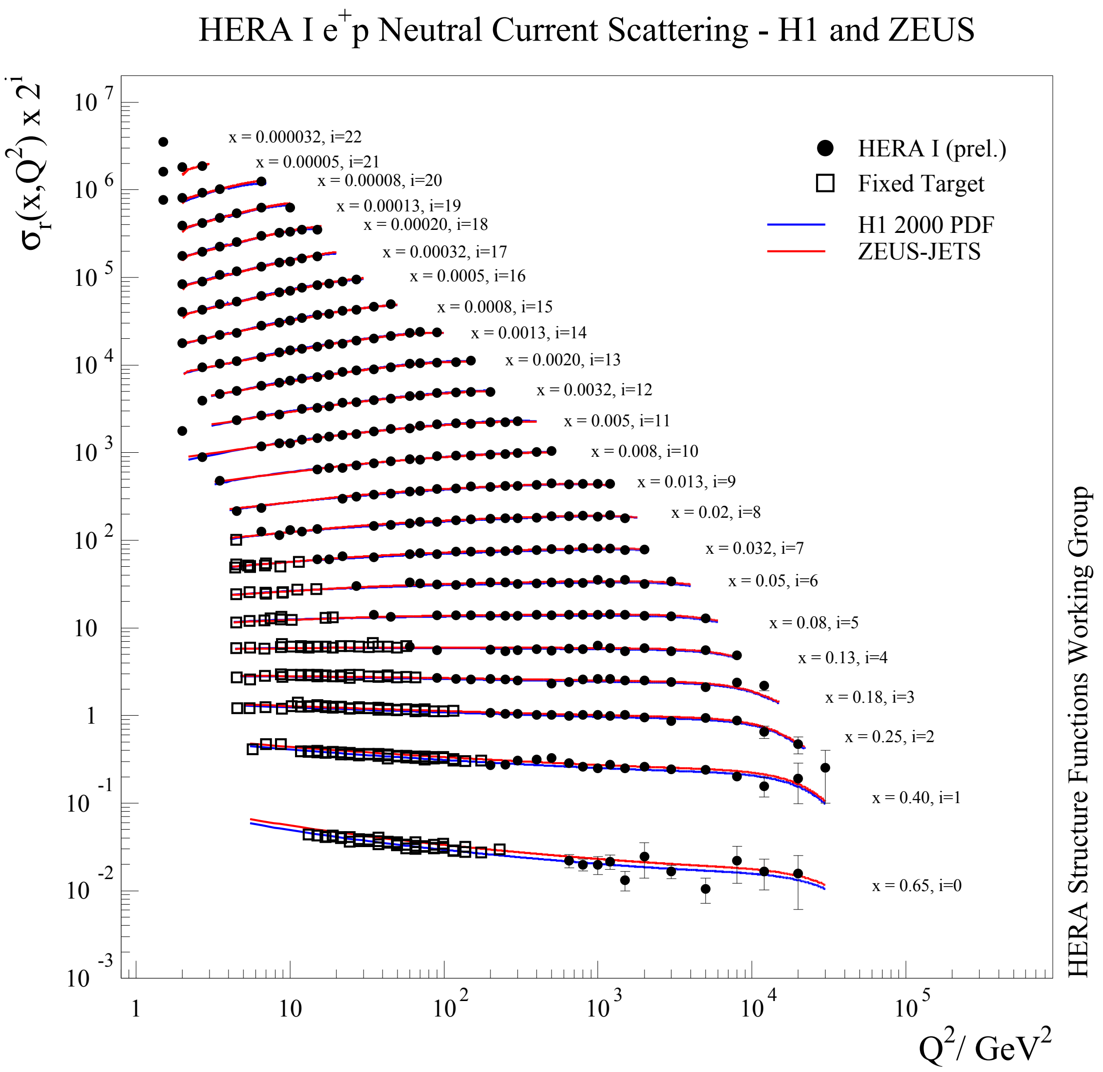
ZEUS
F_2^{em}-log$_{10}$(x)
x=6.32E-5
x=0.000102
x=0.000161
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.0013
x=0.0021
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
ZEUS NLO QCD fit
tot. error
ZEUS 96/97
BCDMS
E665
NMC
Q^2(GeV^2)

New ZEUS + H1 averaged $F_2(x, Q^2)$



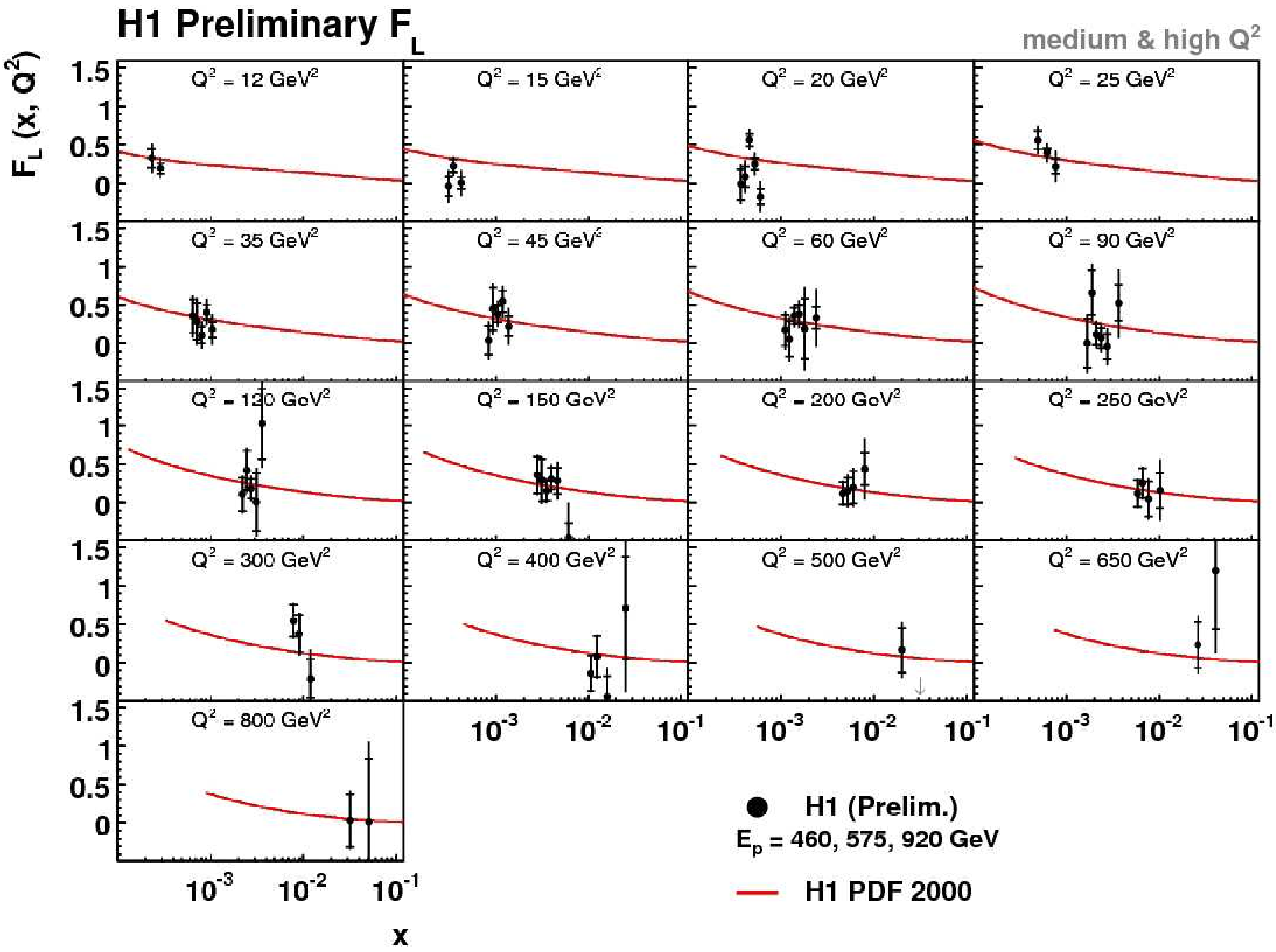
DIS08 Joël Feltesse

New ZEUS + H1 averaged $F_2(x, Q^2)$

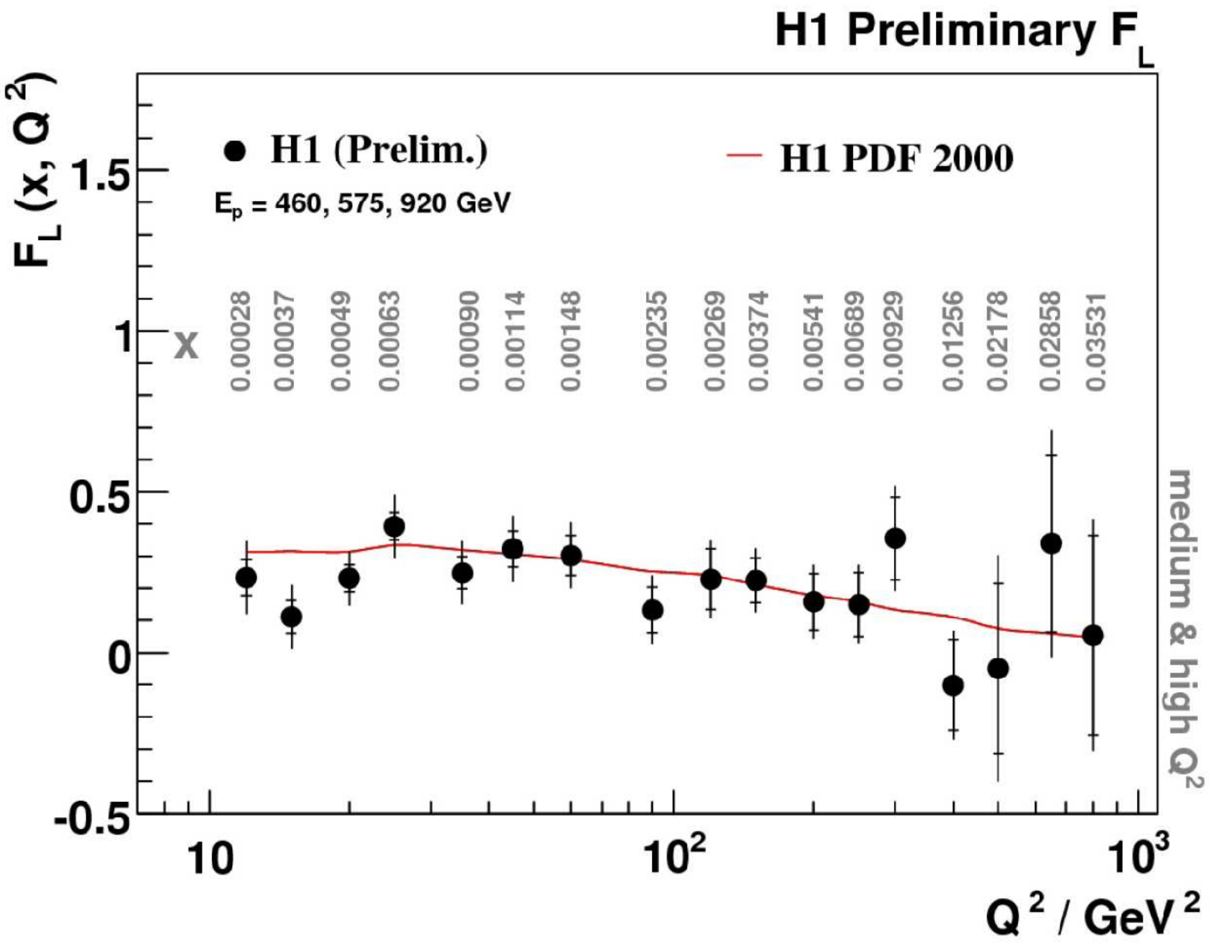


DIS08 Joël Feltesse

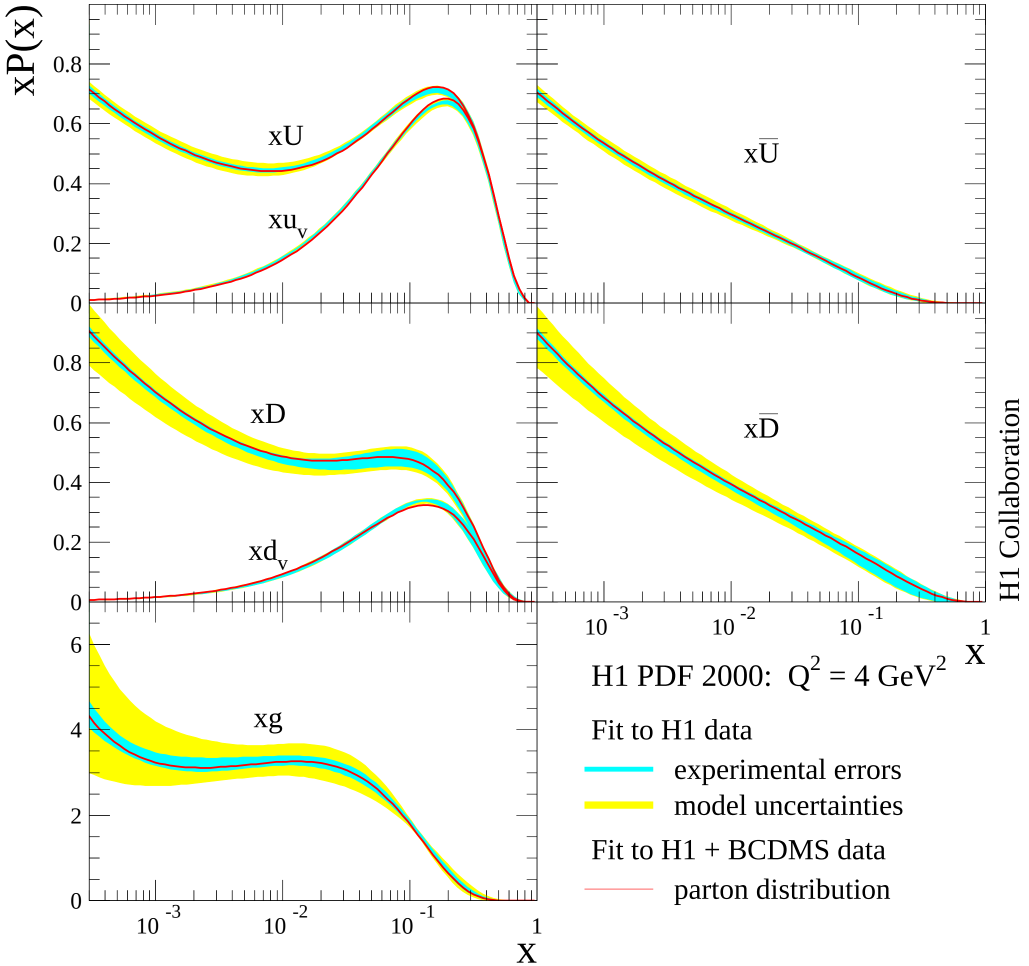
Direct $F_L(x, Q^2)$ Measurement at HERA



Direct $F_L(x, Q^2)$ Measurement at HERA

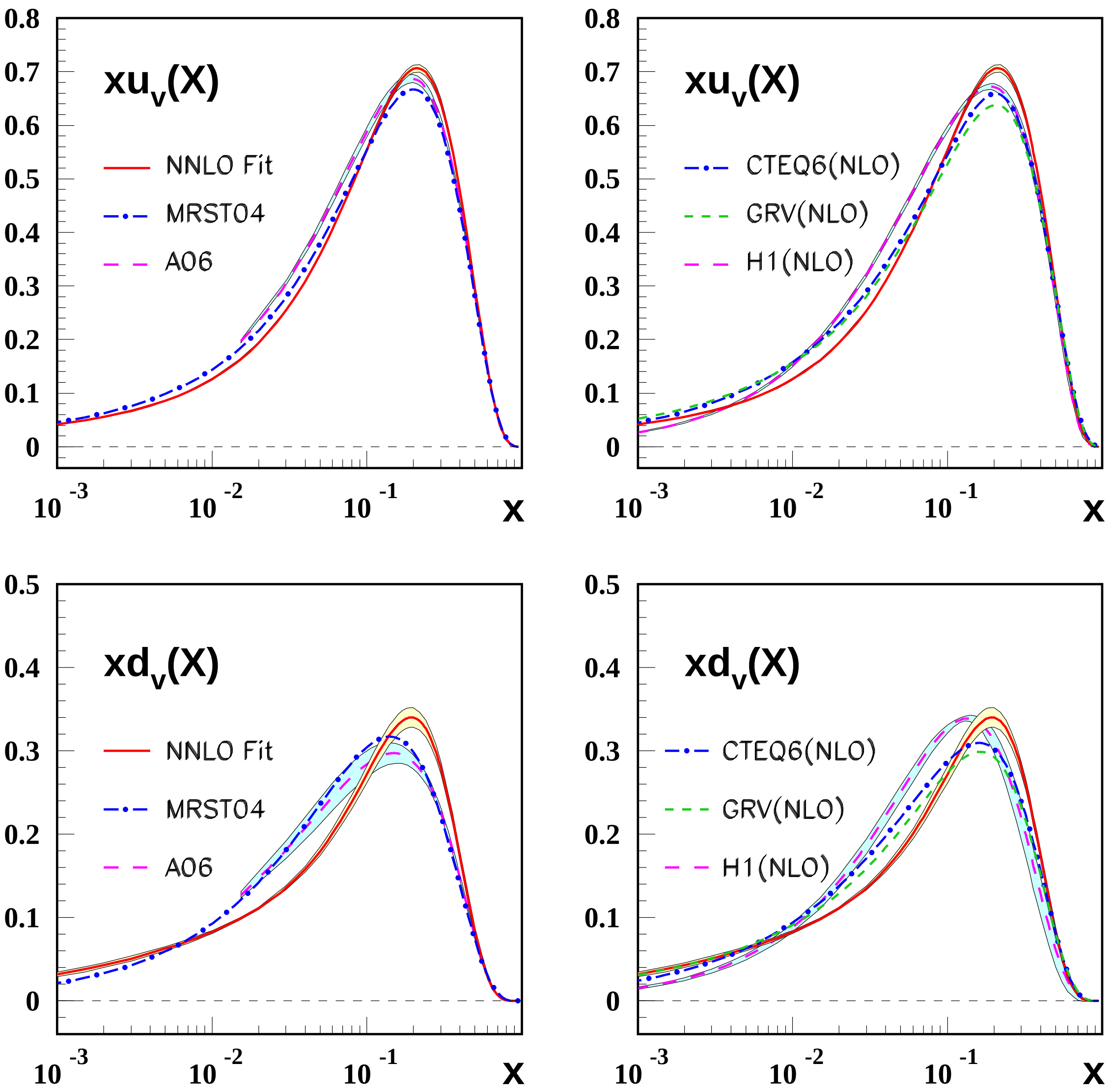


Parton Distributions: Overview



H1

World Data Analysis: Valence Distributions



World data:
NS-analysis
$W^2 > 12.5$ GeV^2, $Q^2 > 4$ GeV^2

N^{3}LO :

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

J.B., H. Böttcher,
A. Guffanti,
(hep-ph/0607200)

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\longrightarrow$ ± 2 MeV in Λ_{QCD}

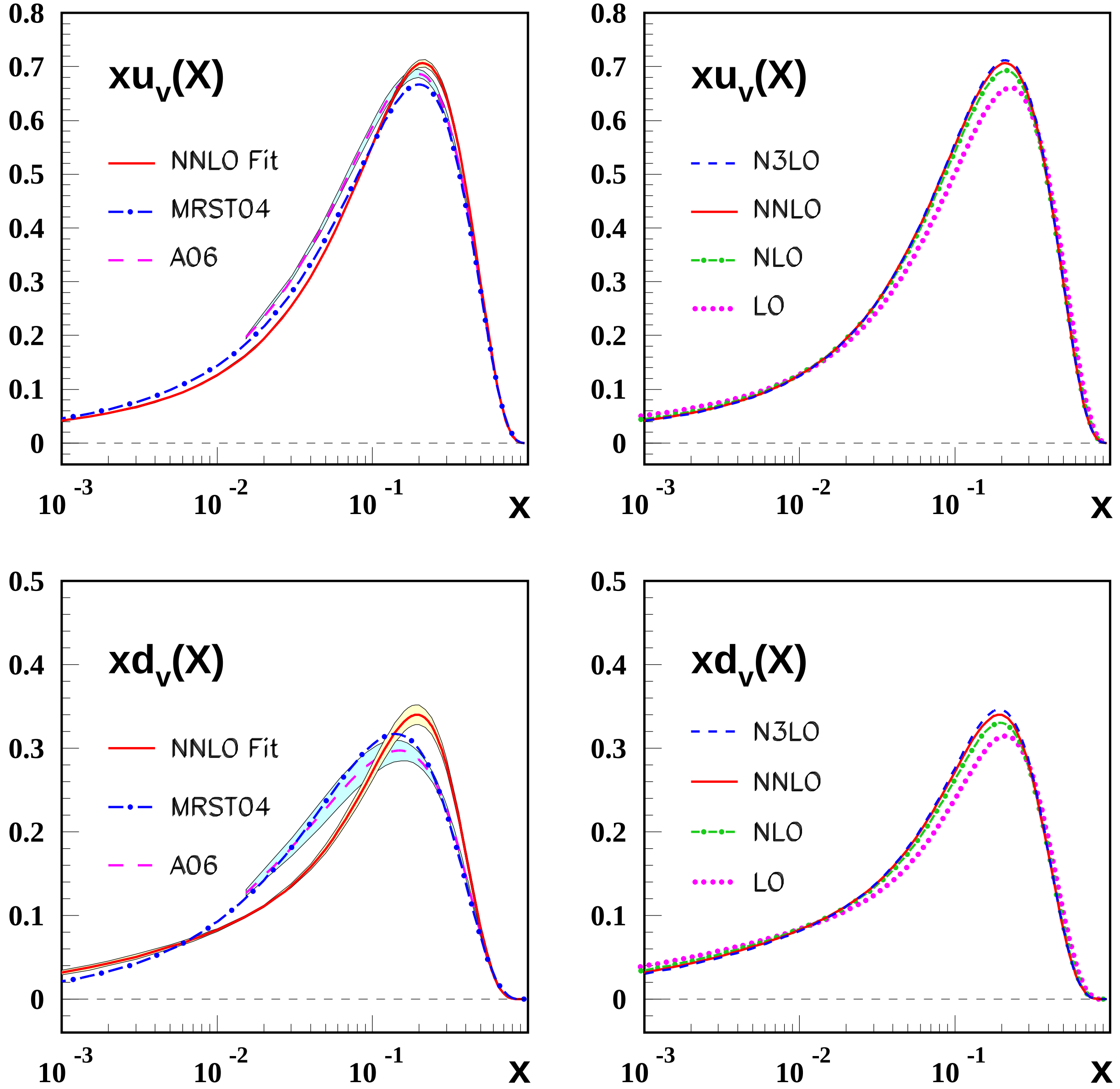
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

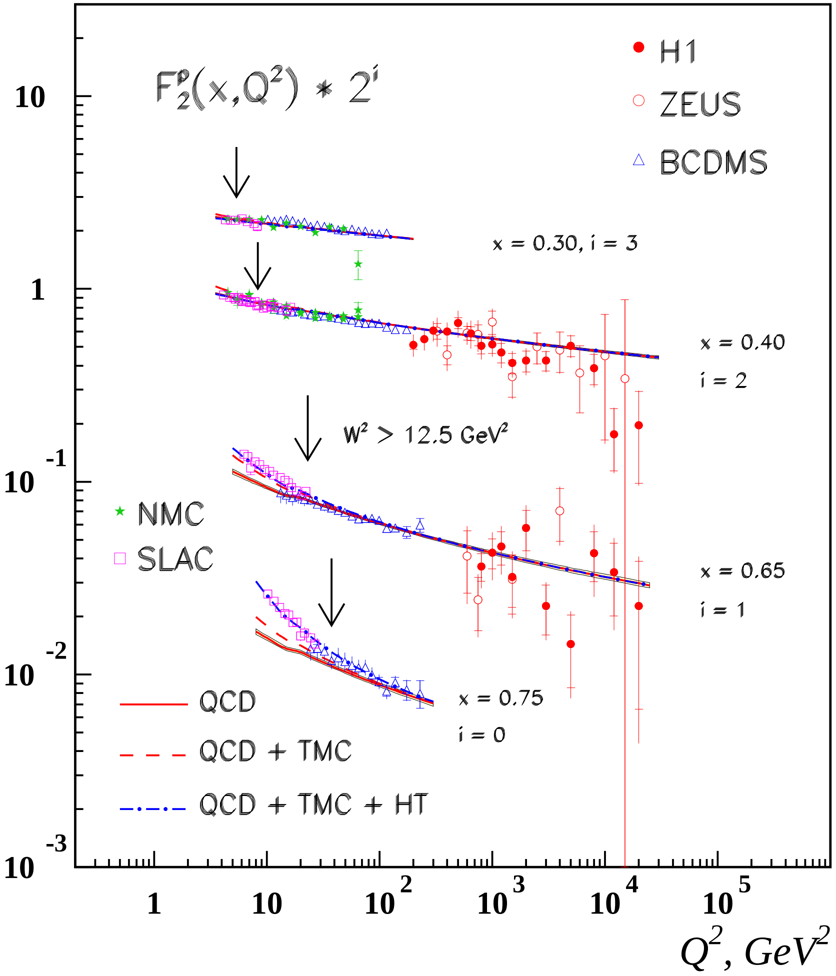
$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4$$

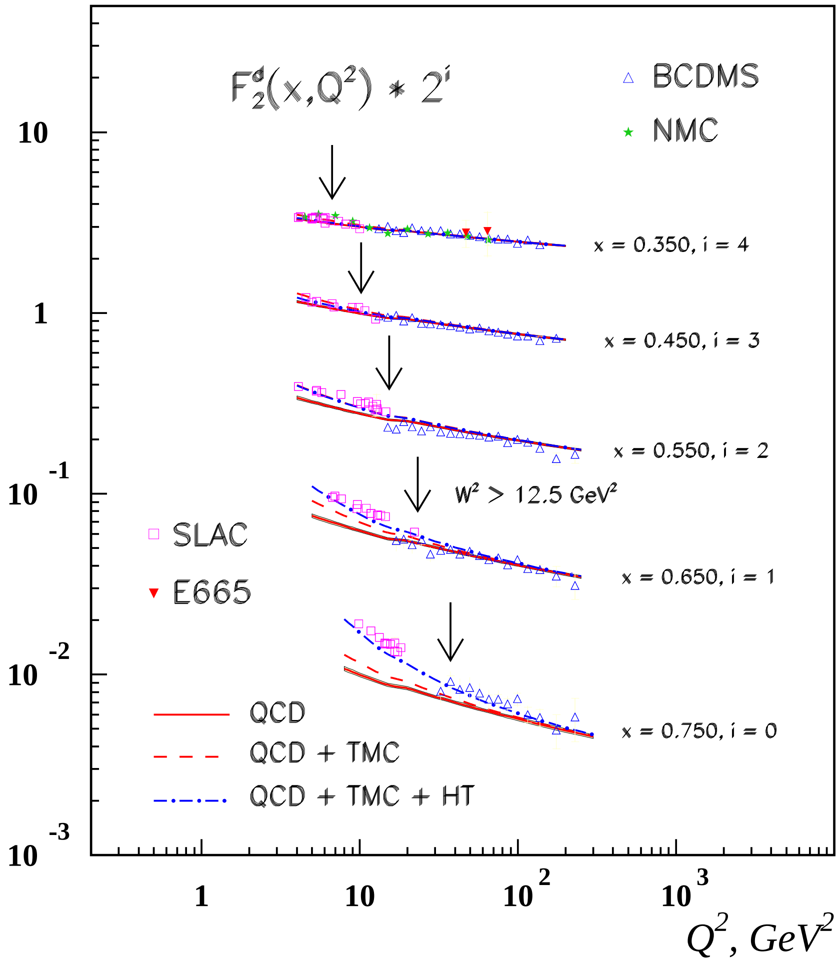
The results agree better than 20%.

Valence Distributions

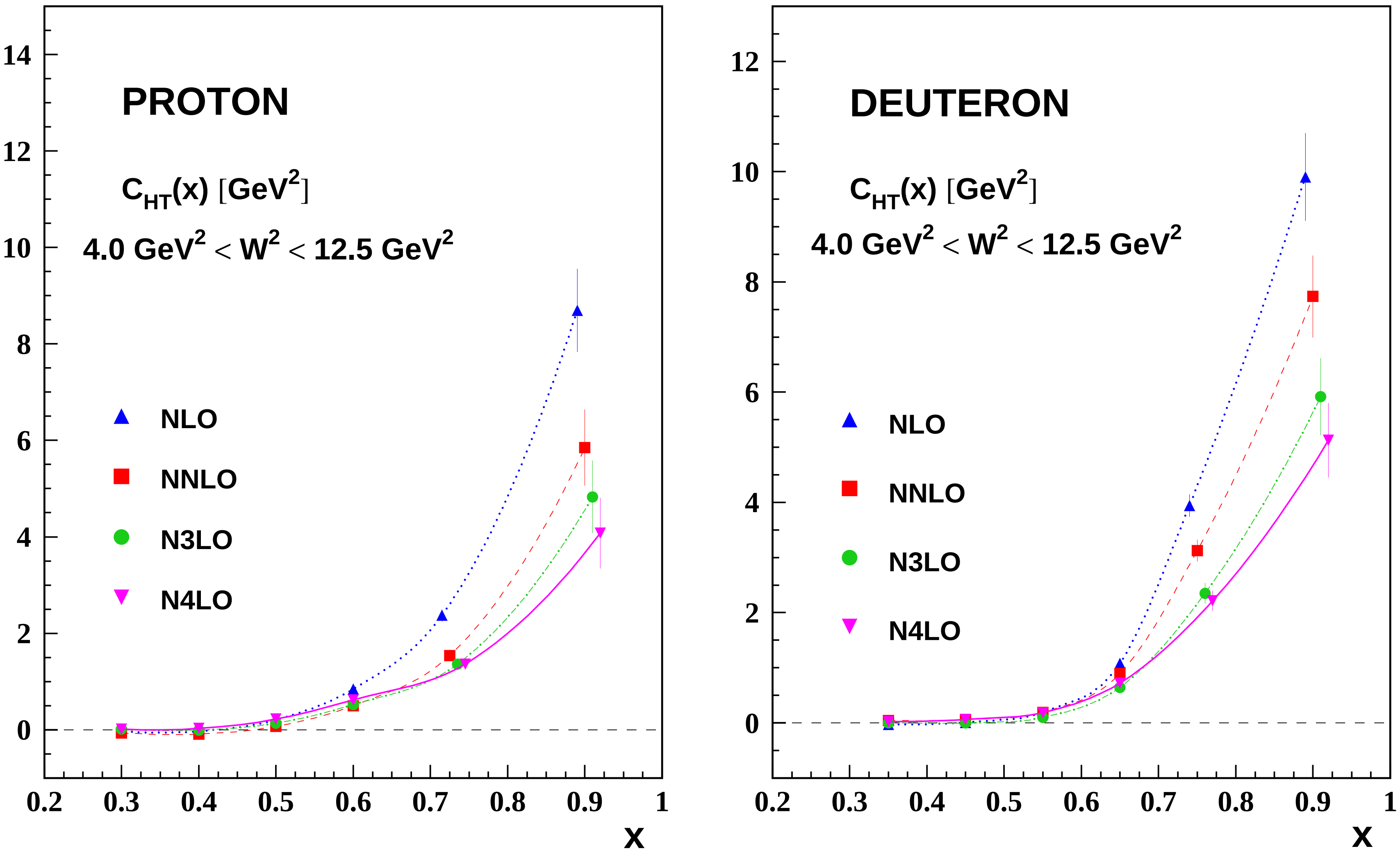


Valence Distributions



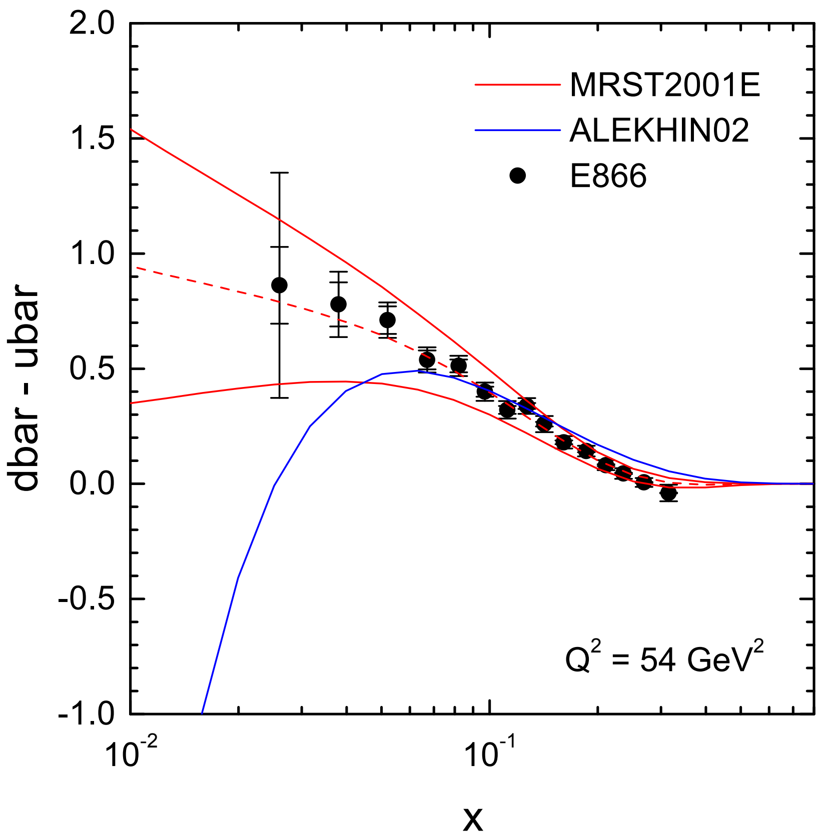


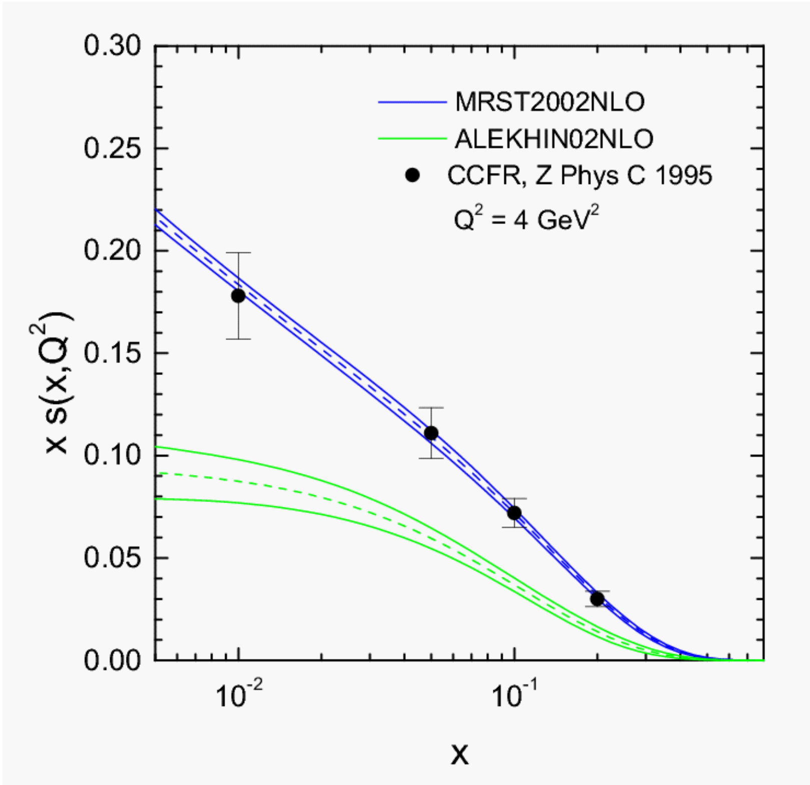
Valence Distributions: higher twist



- agreement between p and d analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

Flavor distributions: light quarks



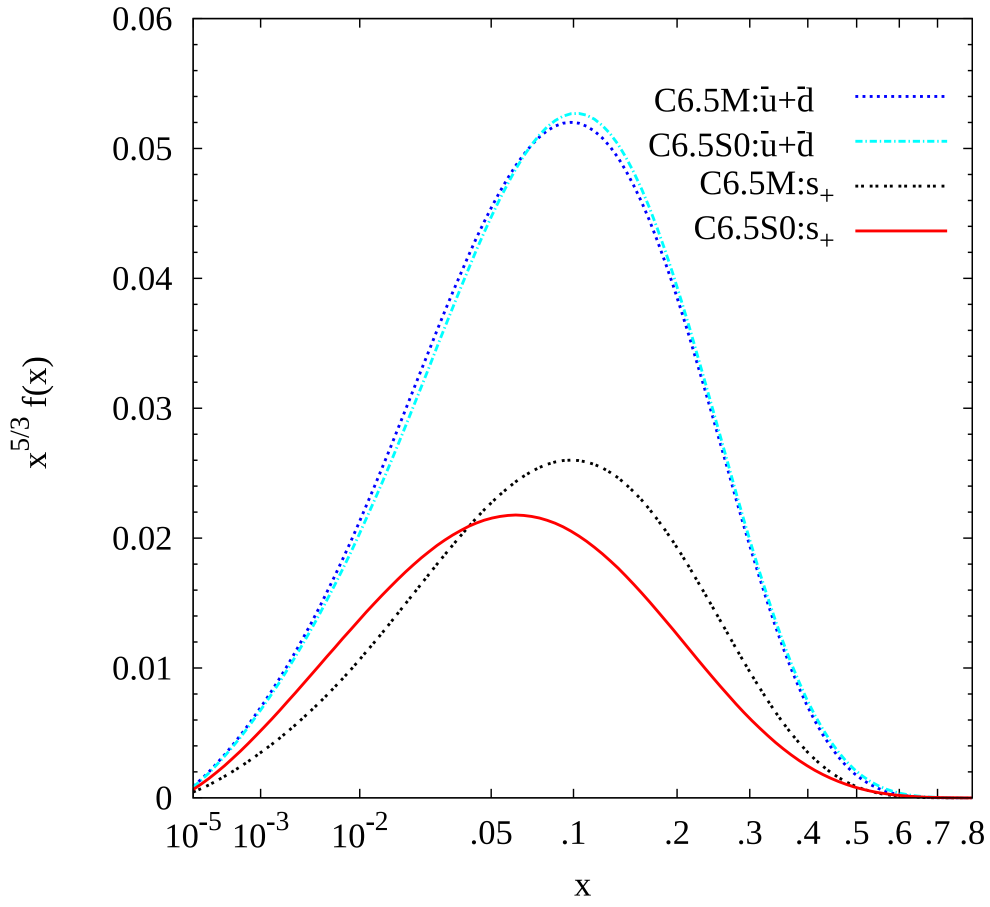


J. Stirling, 2004

More work needed.

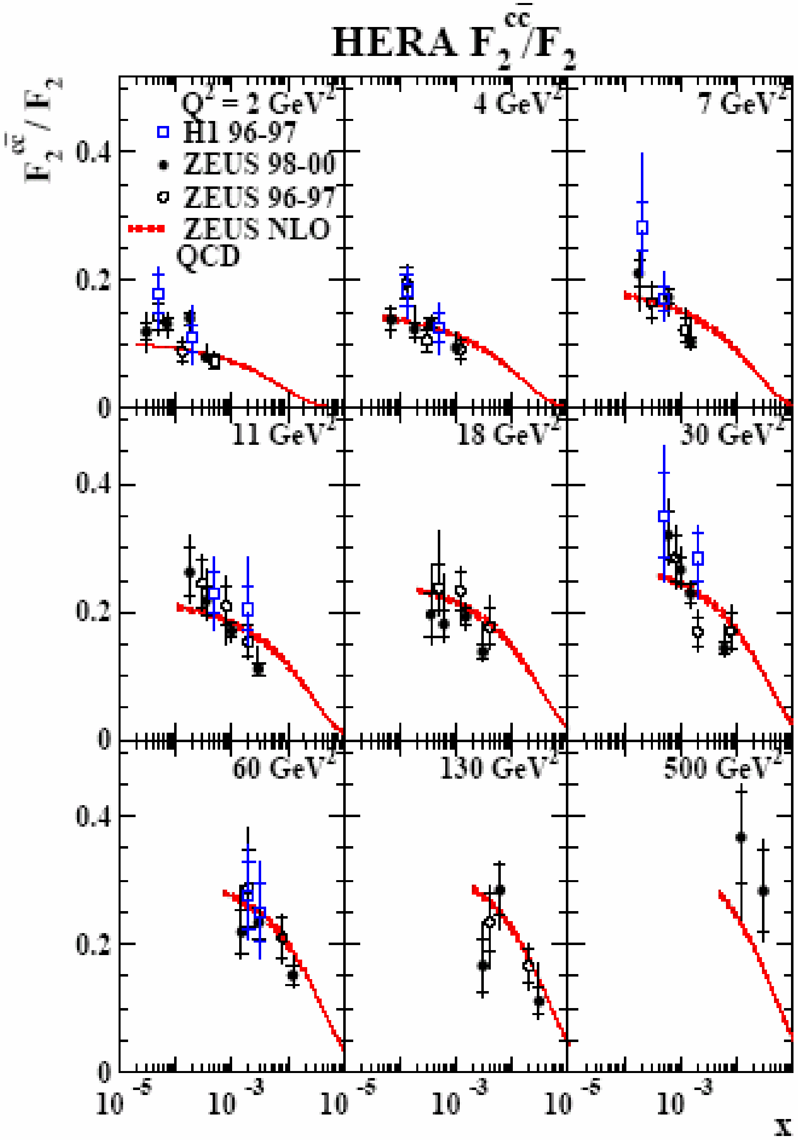
HERMES probably could measure $s(x, Q^2)$ in an independent way.

Flavor distributions: light quarks



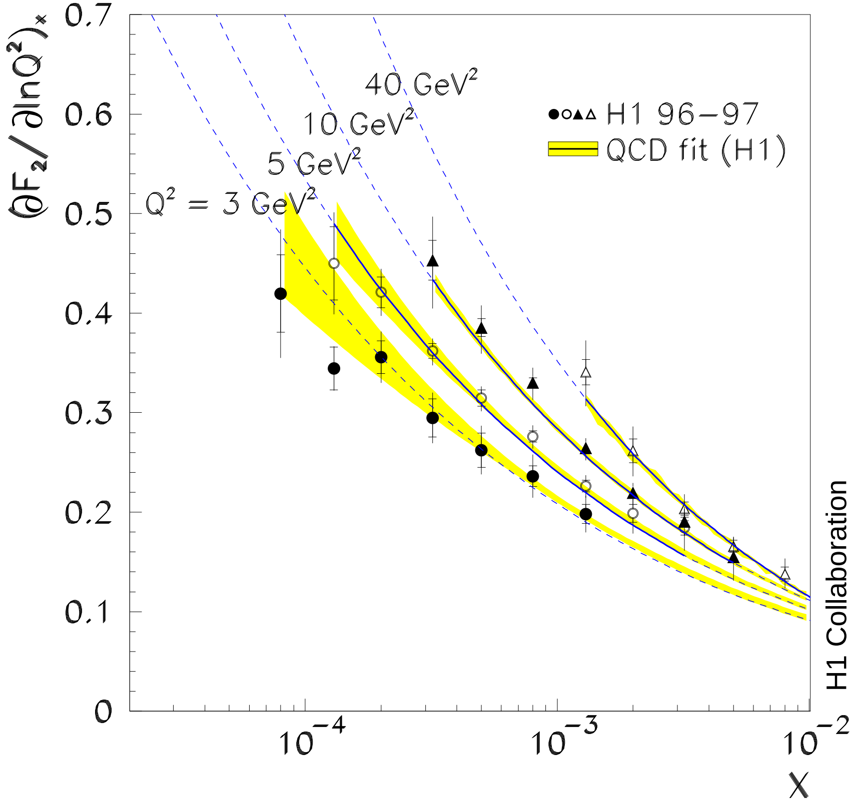
CTEQ 2007

Charm

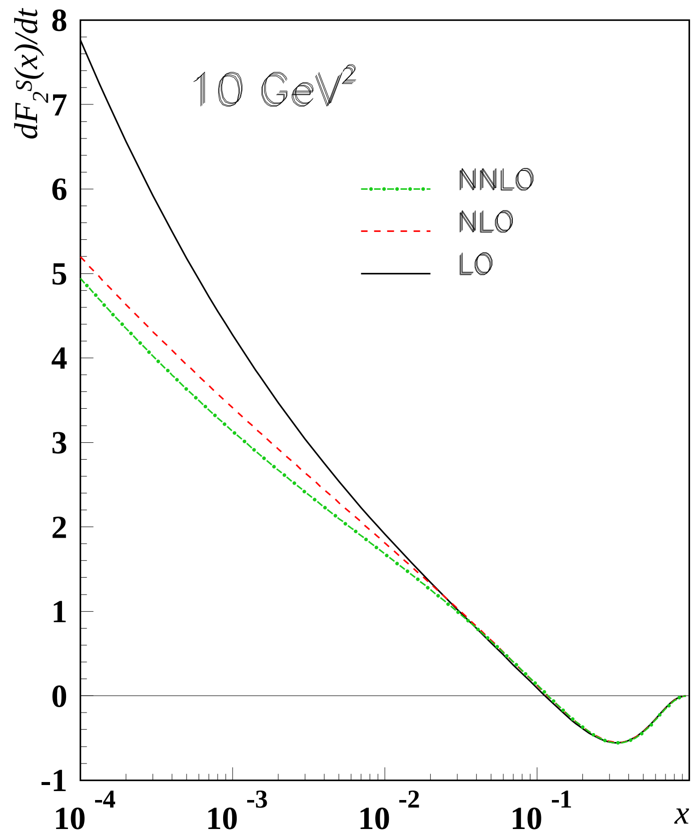


$F_2^{c\bar{c}}(x, Q^2)$ will be very well measured at HERA.

Slope of F_2 at low x



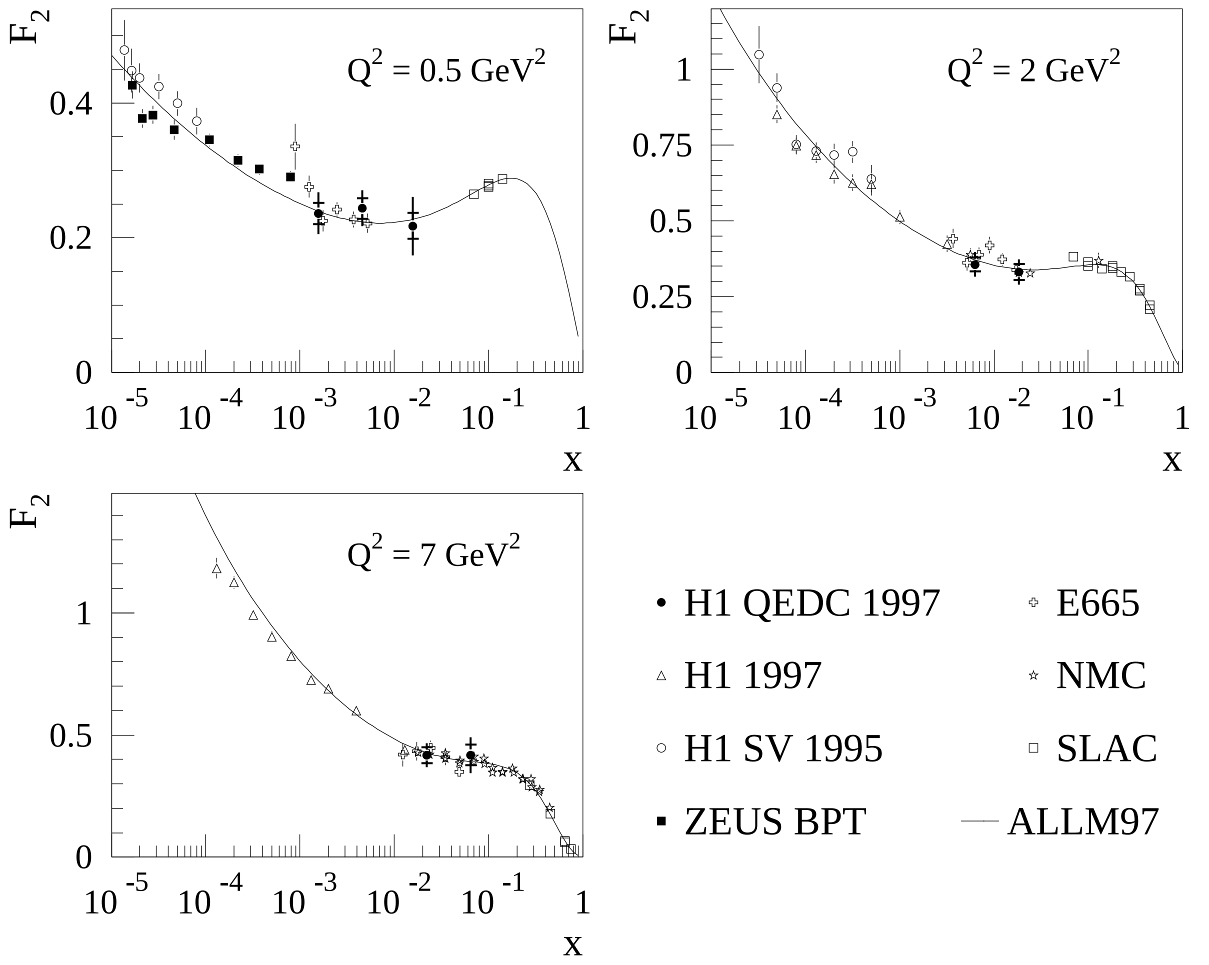
H1



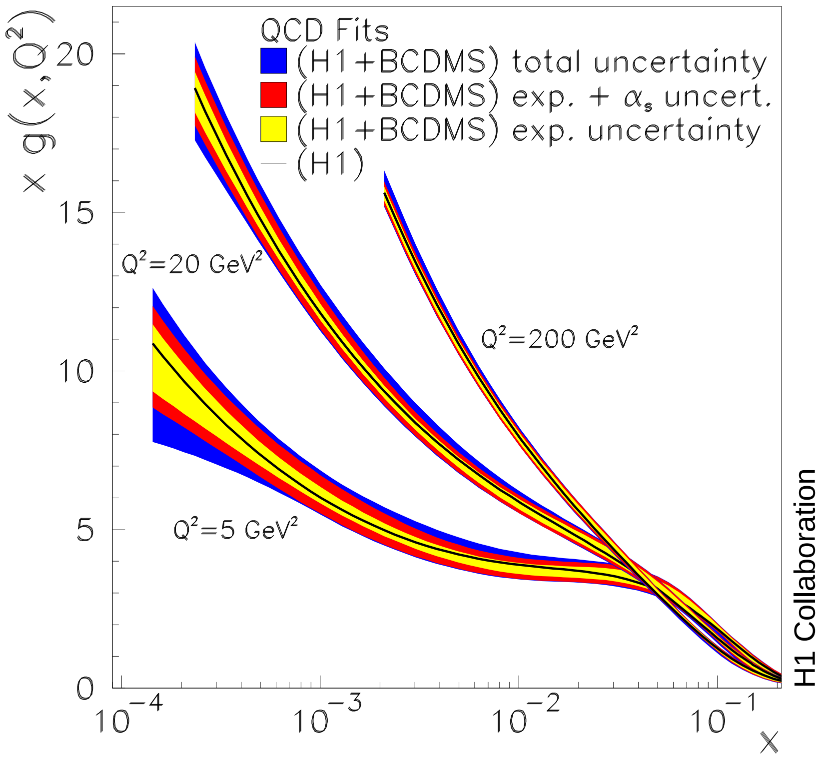
J.B., A. Guffanti 2005

Very likely, that the $\overline{\rm MS}$–gluon is remains positive!

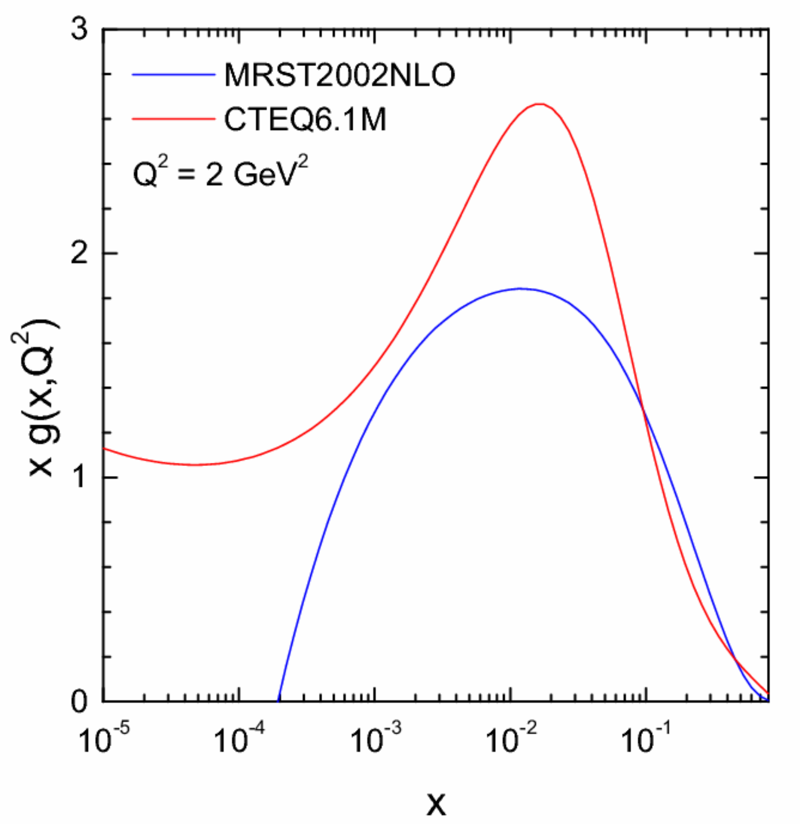
Perturbative or non-perturbative growth?



Gluon Density



H1

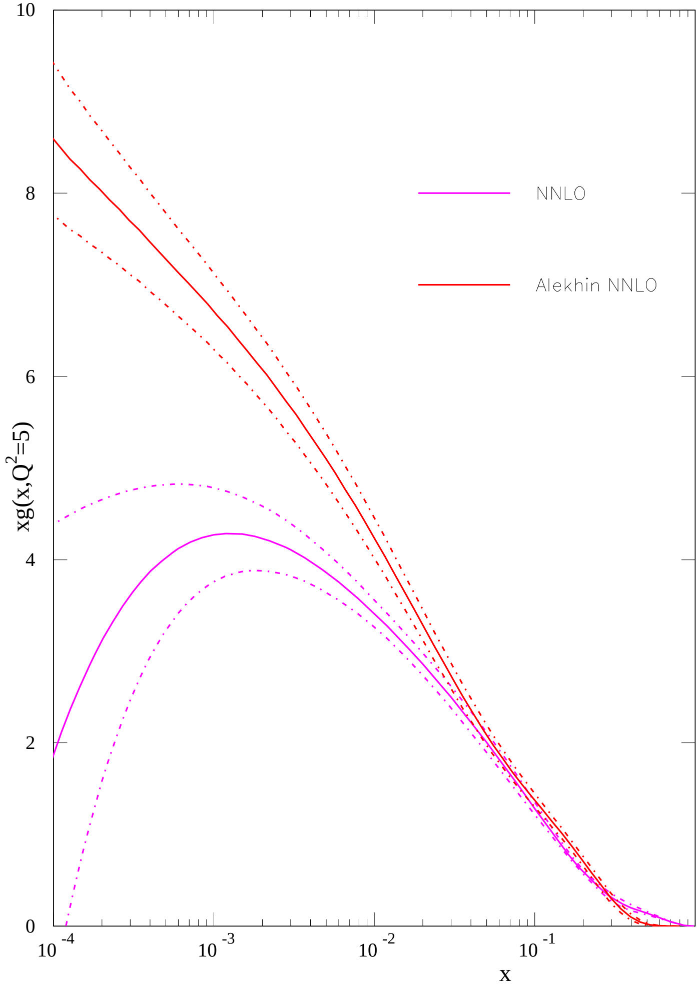


MRST 02 vs CTEQ 6

More work needed; MS– vs scheme-invariant evolution.

$F_L(x, Q^2)$ could be decisive.

Gluon Density



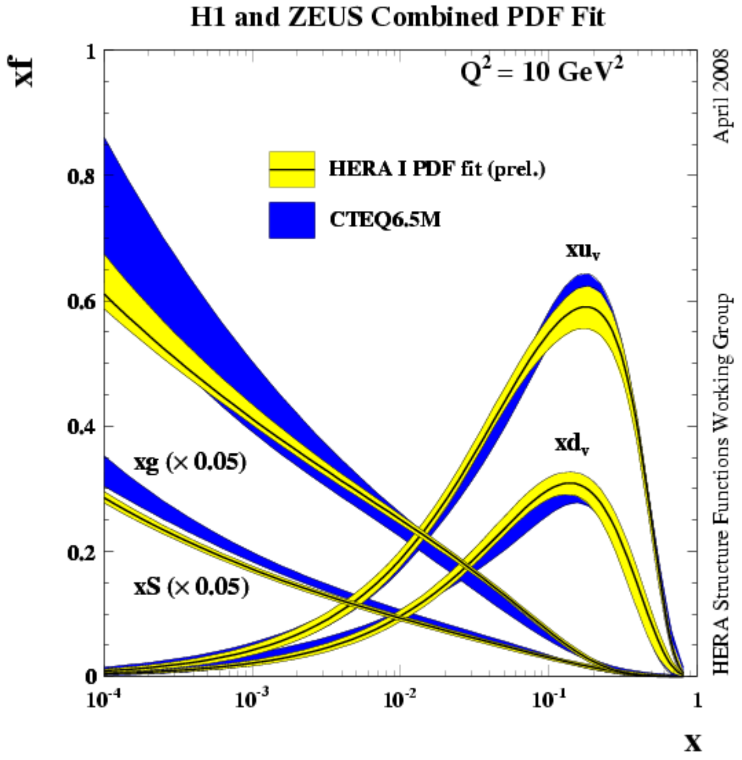
Not both distributions can be correct.

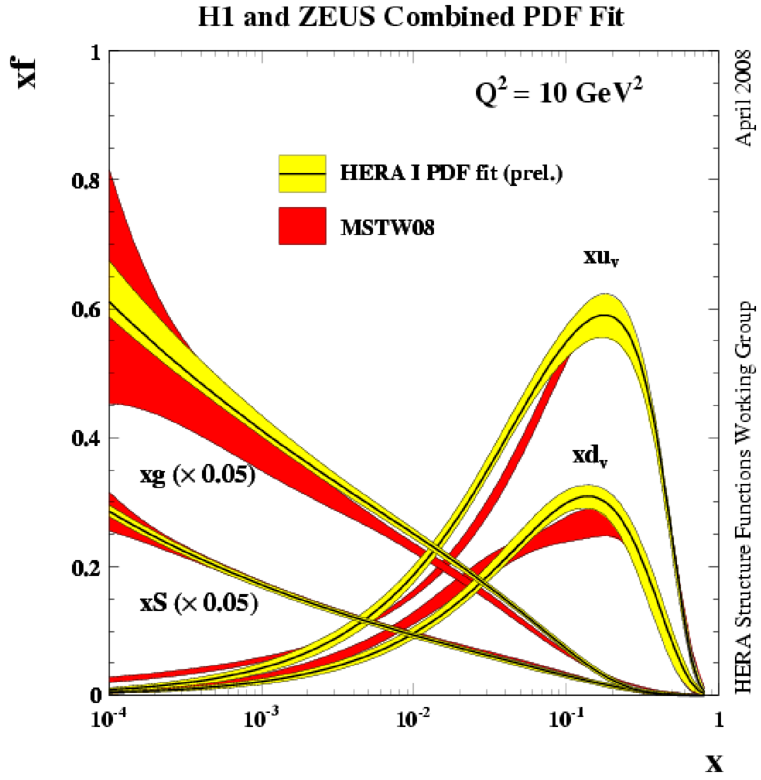
$F_L(x, Q^2)$ could be decisive.

MRST06 vs Alekhin: 2006

More work needed ! BB Analysis in progress.

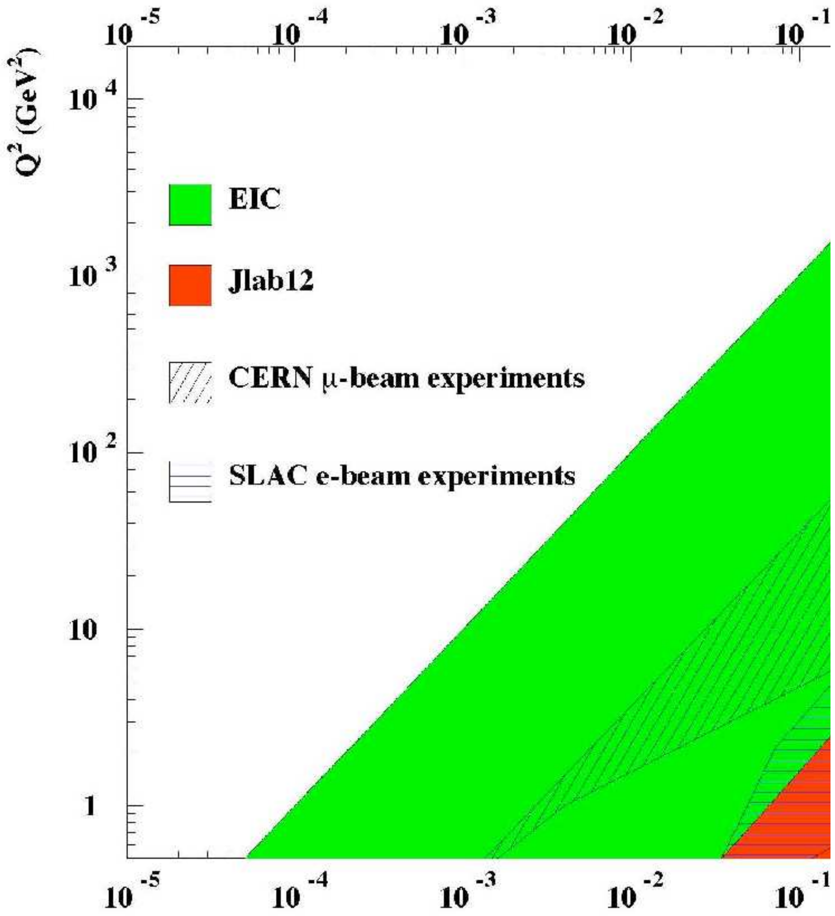
Gluon Density

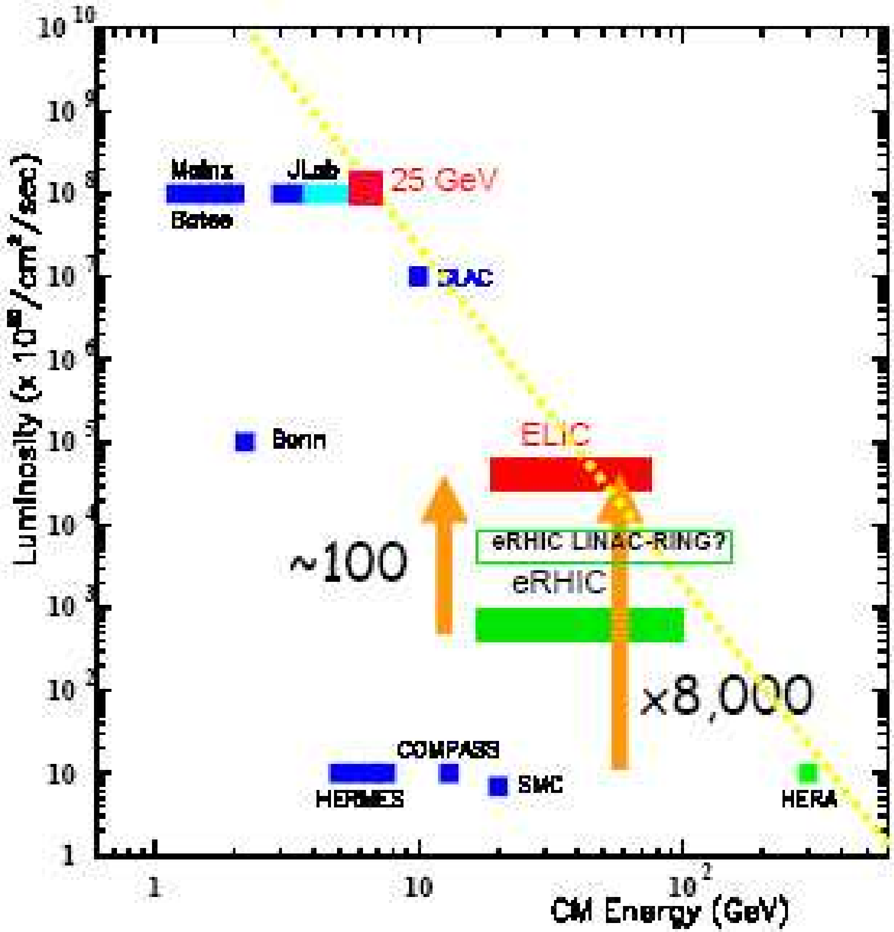




Recent Fits based on H1 + ZEUS combined data sets

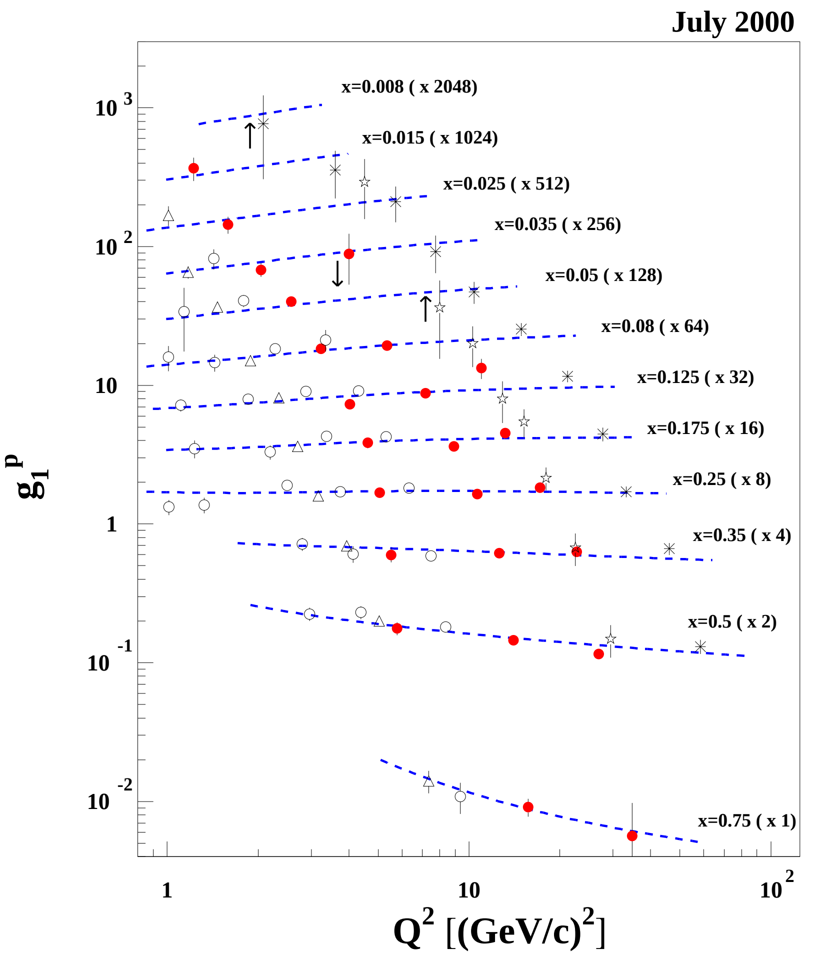
3. Polarized Structure Functions

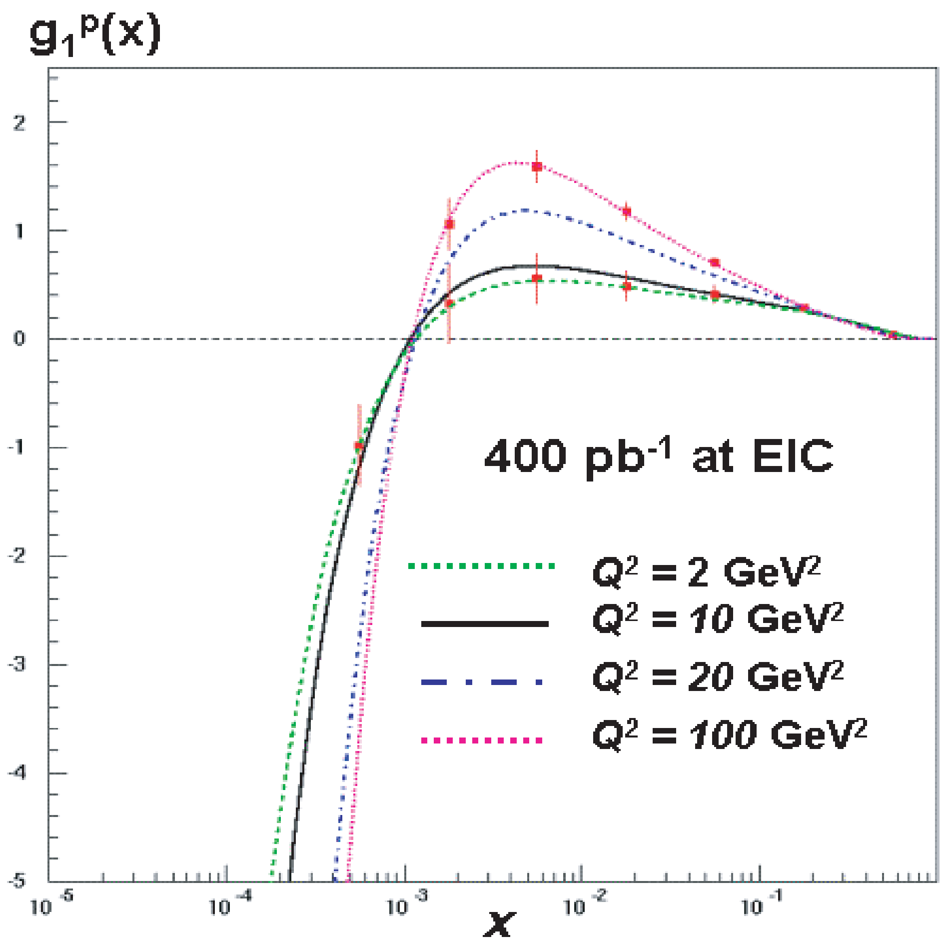




High Luminosity is most important: Various precision measurements.

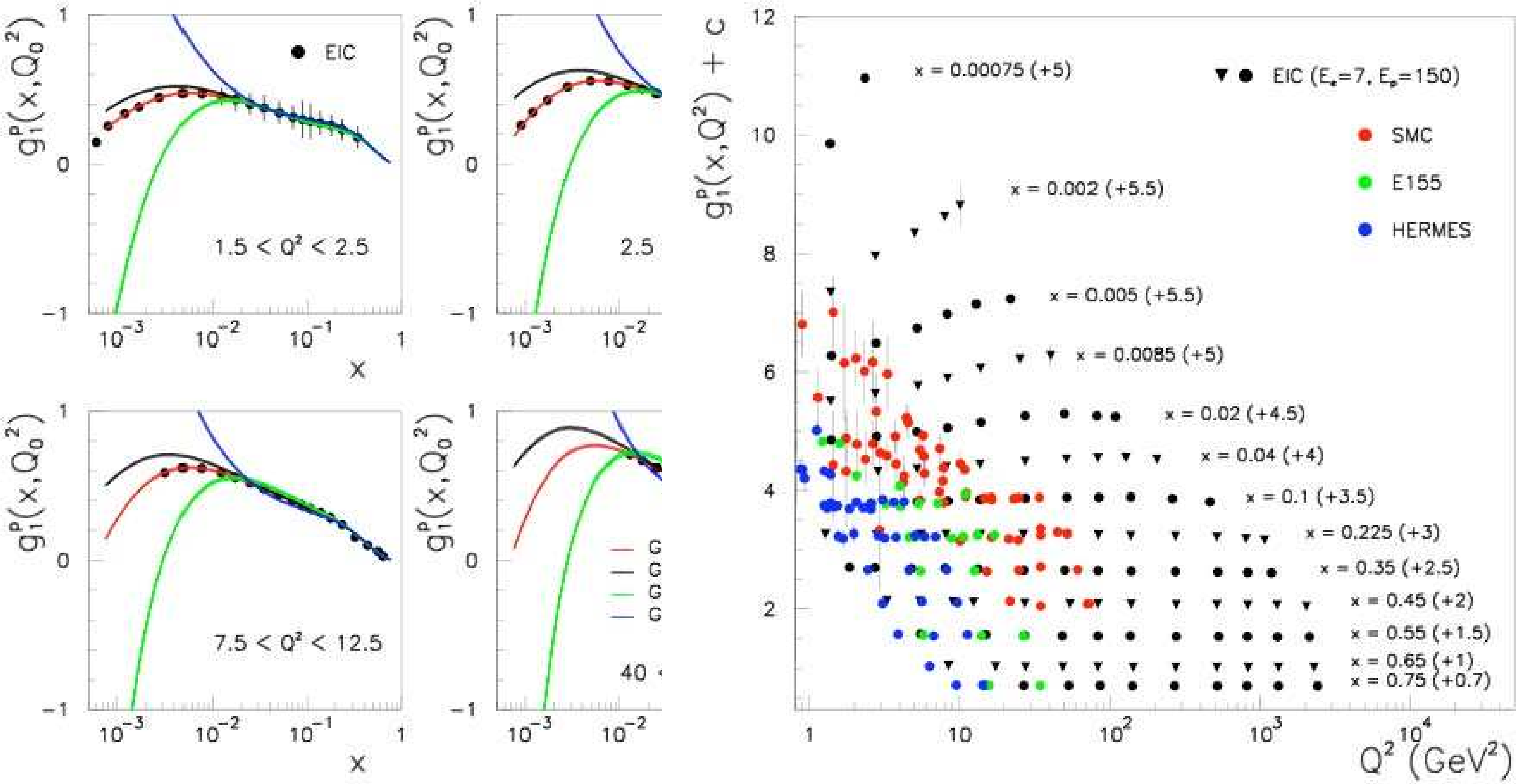
The polarized Structure Function Now & Then





Lower values of x are reached, improving accuracy in the medium and large x region.

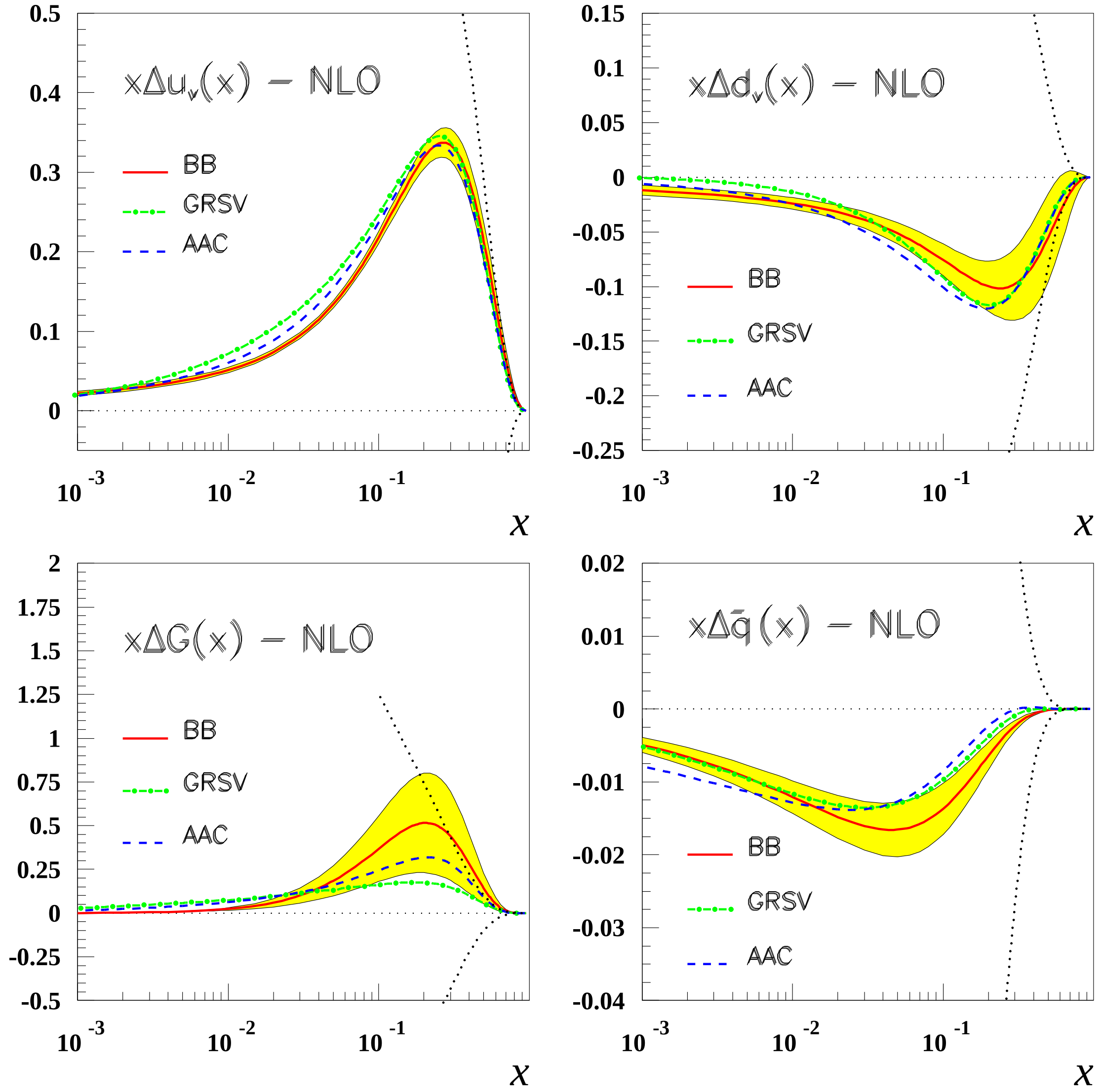
The polarized Structure Function Now & Then



How does $g_1(x, Q^2)$ behave at small x ?

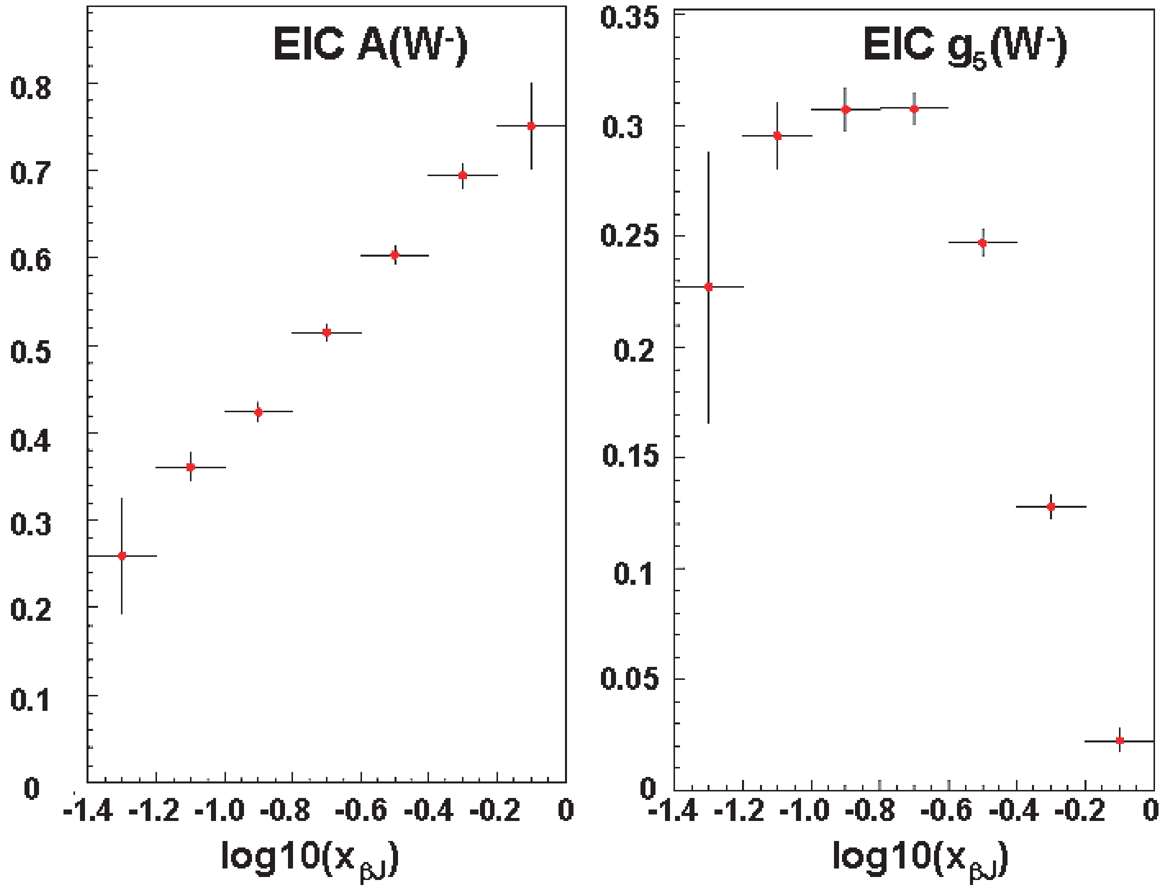
$\Longrightarrow$ Sufficiently wide slopes in $\ln(Q^2)$.

Polarized Parton Densities at Present



J.B., H. Böttcher (2002)

The W-Asymmetry A_{W^-} and $g_5(x,Q^2)$

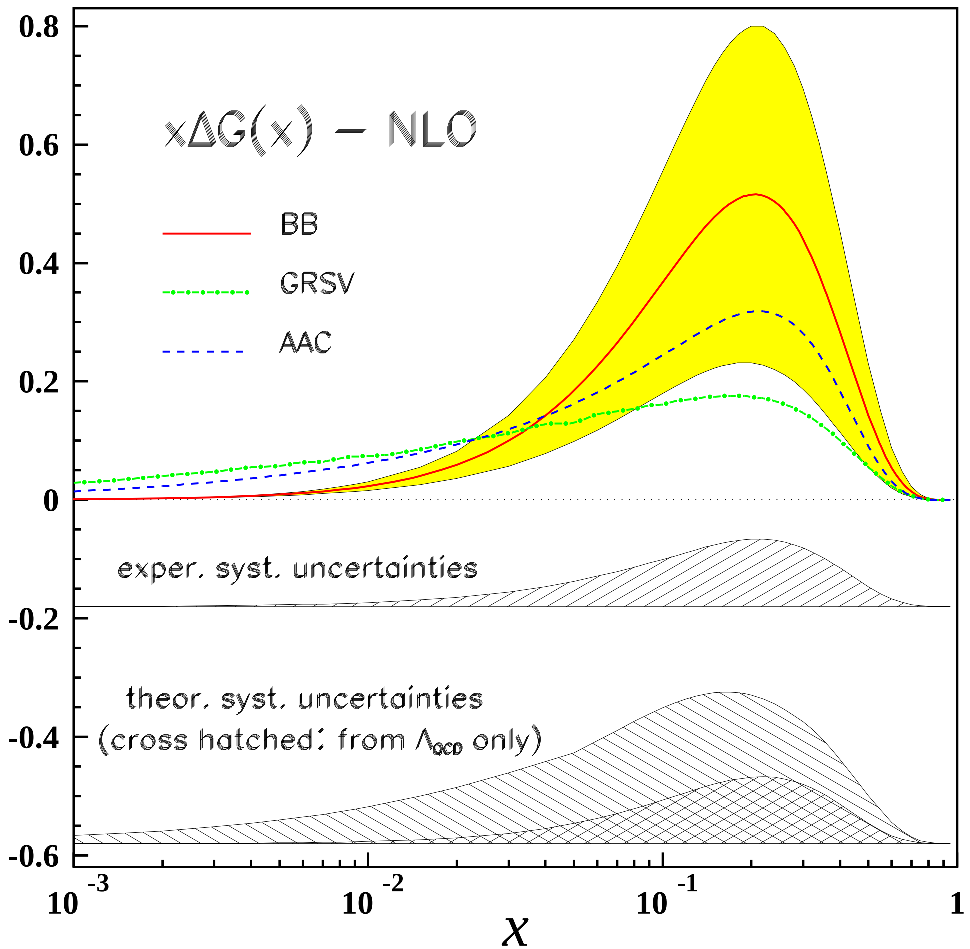


$$A^{W^-} = [2b(y)g_1^{W^-} + a(y)g_5^{W^-}]/[a(y)F_1^{W^-} + b(y)F_3^{W^-}]$$

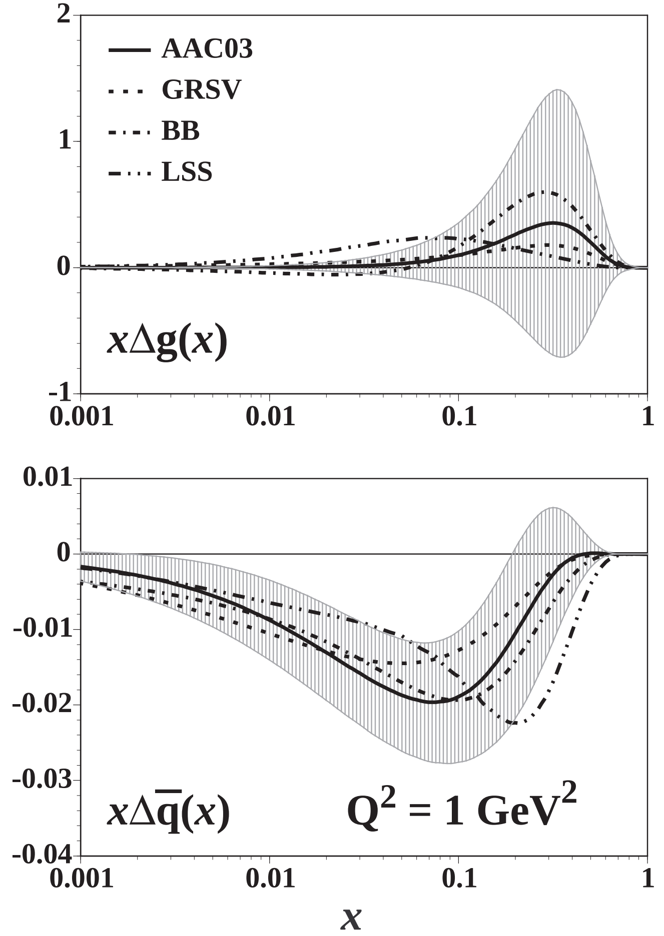
$$g_5^{W^+,p} + g_5^{W^-,p} = \Delta u_v + \Delta d_v, \qquad g_5^{W^+,p} - g_5^{W^-,n} = -[\Delta(u+\bar{u}) - \Delta(d+\bar{d})]$$

Measurement of a polarized electro-weak structure function at high Q^2.

The Polarized Gluon Distribution at Present



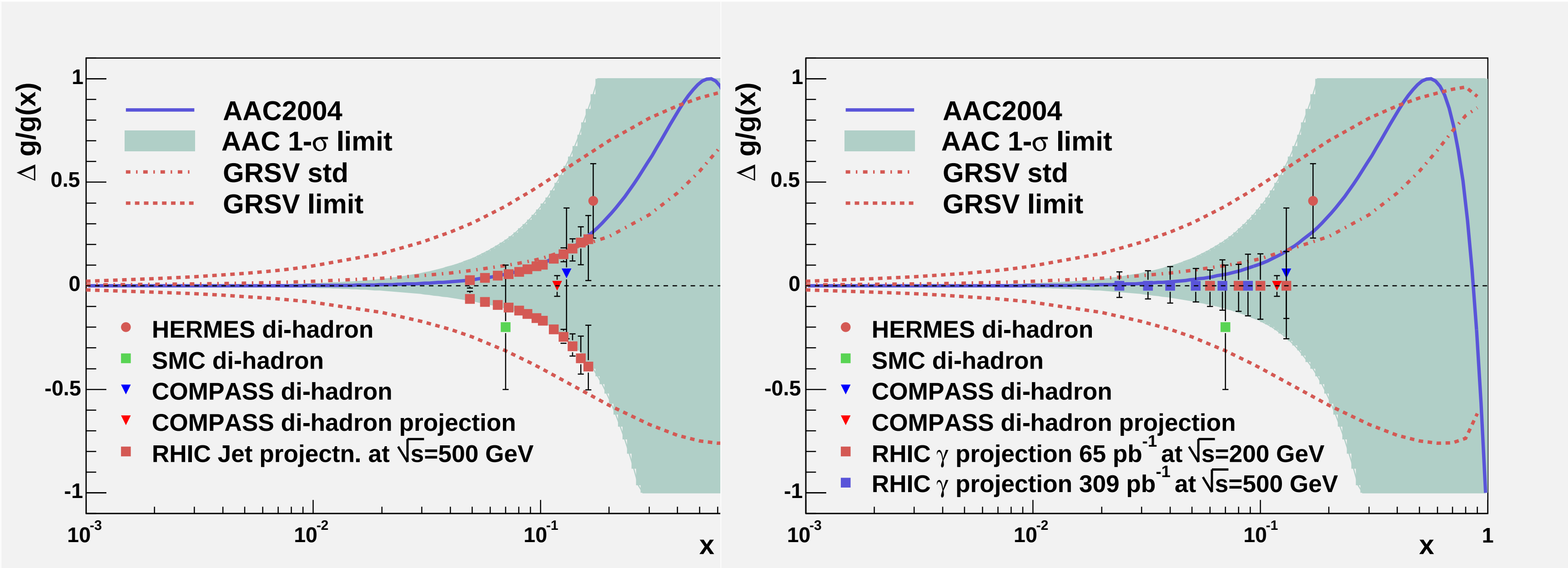
J.B., H. Böttcher (2002)



AAC

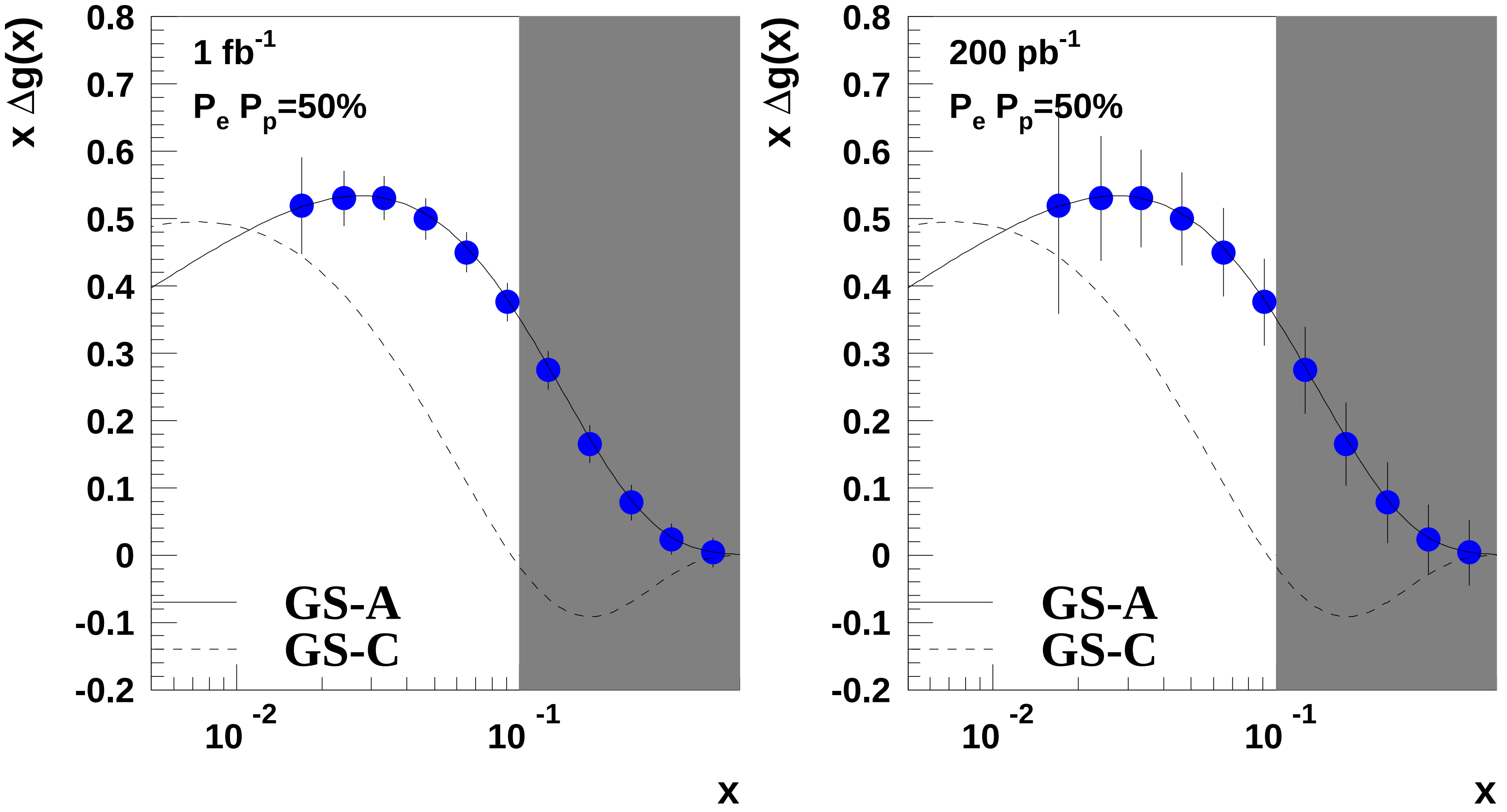
$\Longrightarrow$ Currently slight move of ΔG towards lower values

The Polarized Gluon Distribution at RHIC



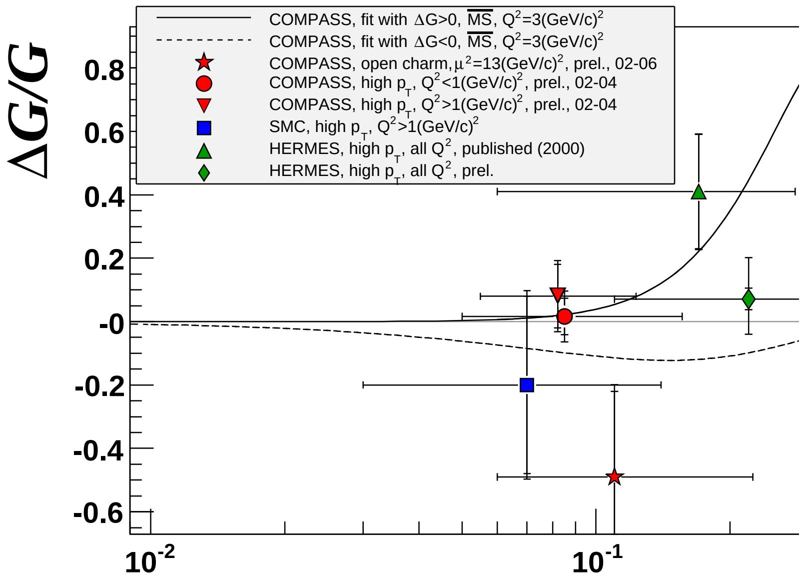
Accuracies for ΔG, which may be reached at RHIC

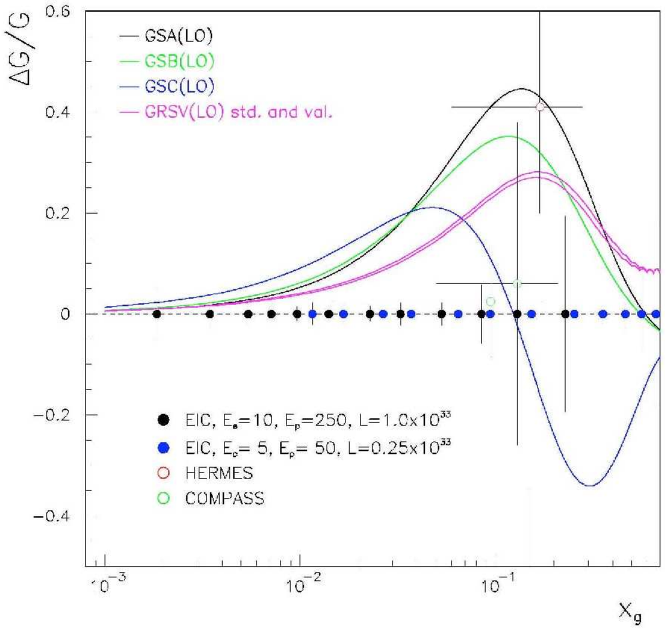
The Polarized Gluon Distribution at EIC



Excellent Resolution for ΔG at EIC

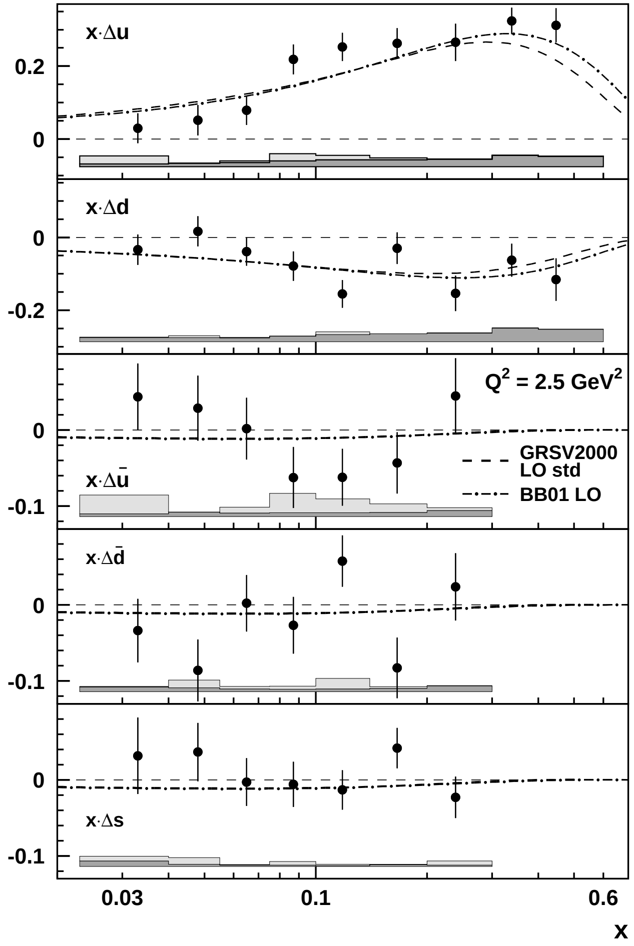
The Polarized Gluon Distribution at EIC



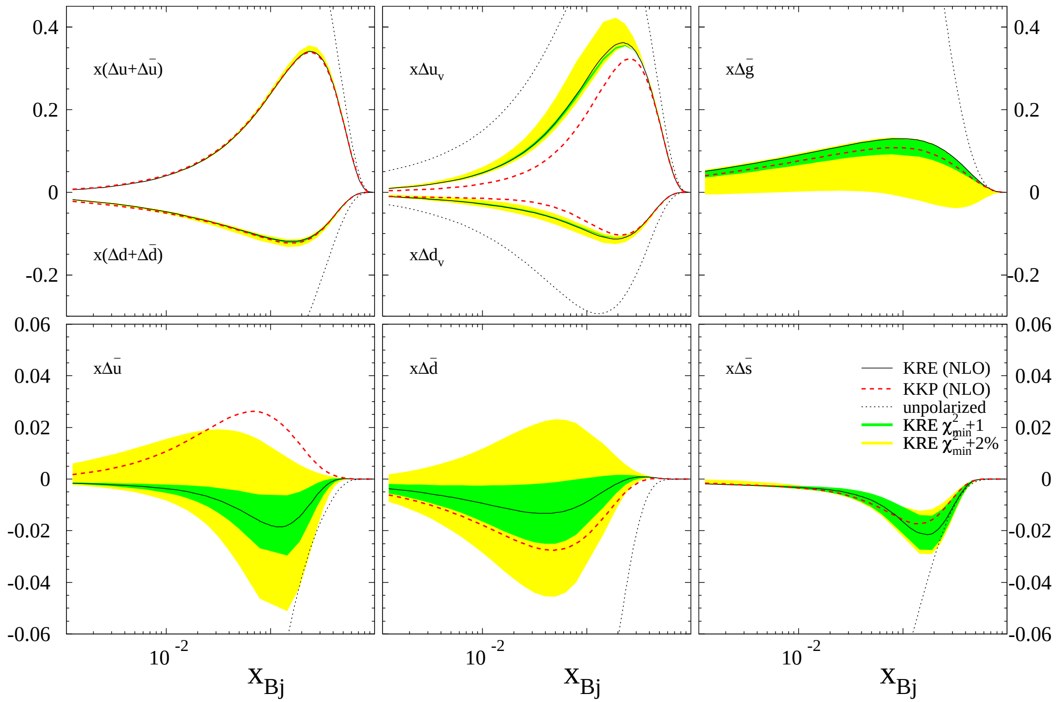


Accuracy for ΔG from Open Charm Production at EIC

The Polarized Gluon Distribution at EIC

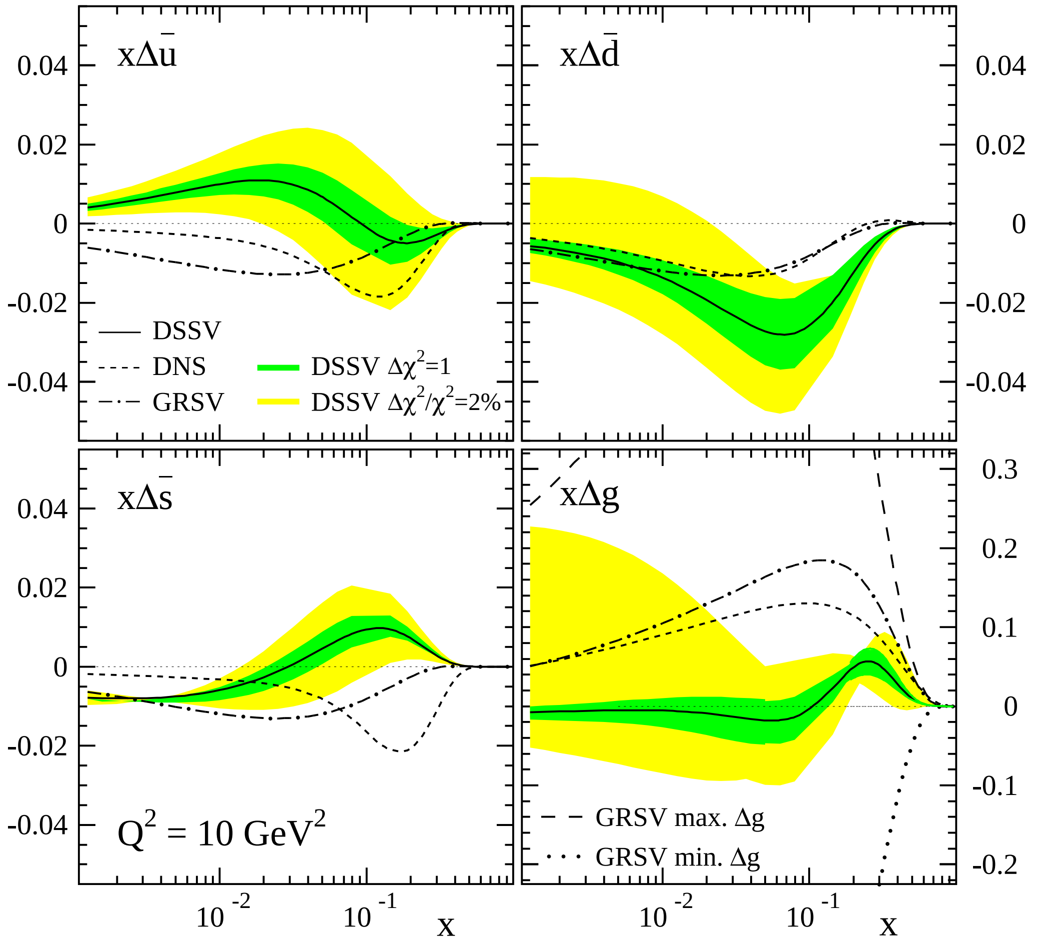


HERMES, 2004



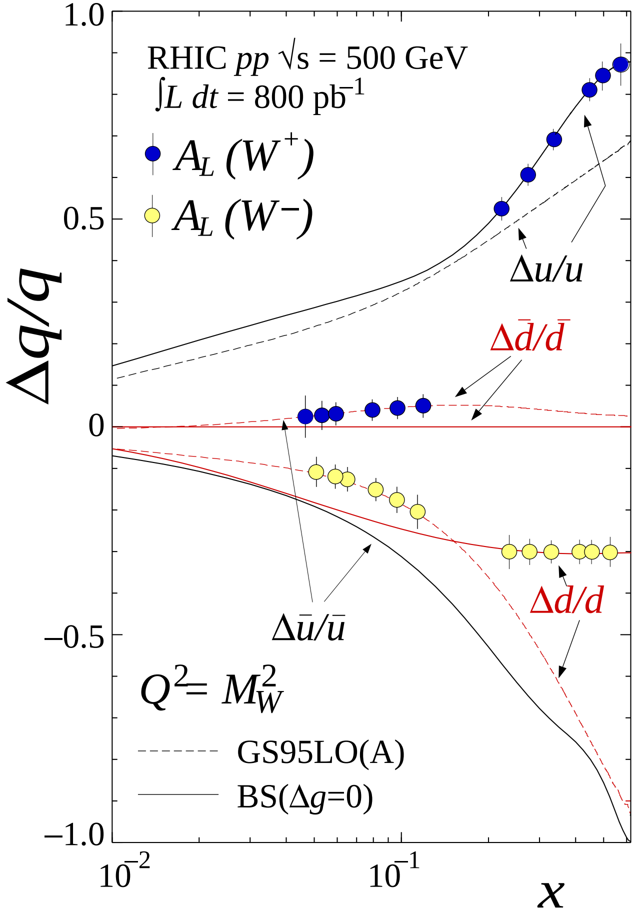
De Florian & Sassot, 2005

Unfolding the Sea Quarks

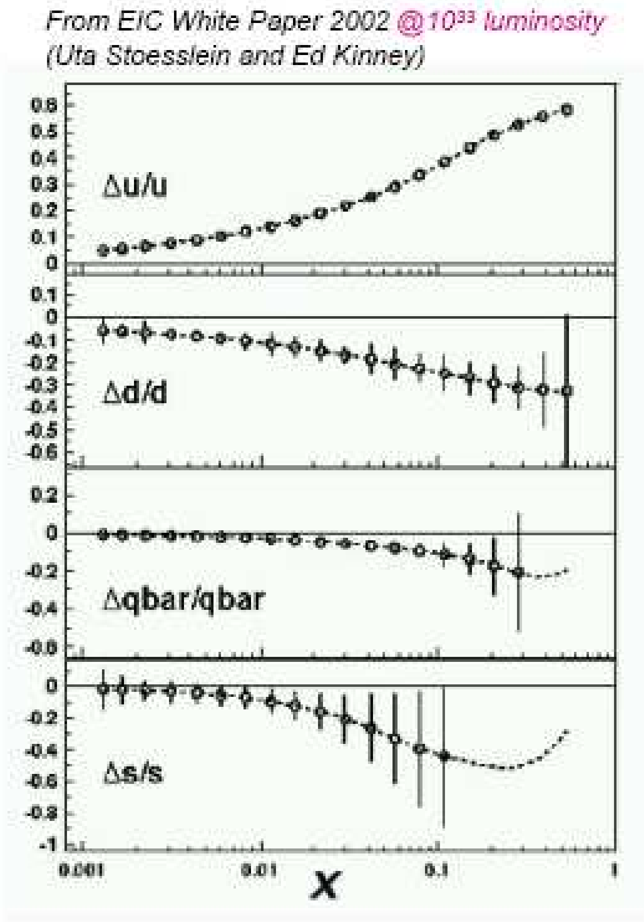


De Florian, Sassot, Stratmann, Vogelsang, 2008

Unfolding the Sea Quarks

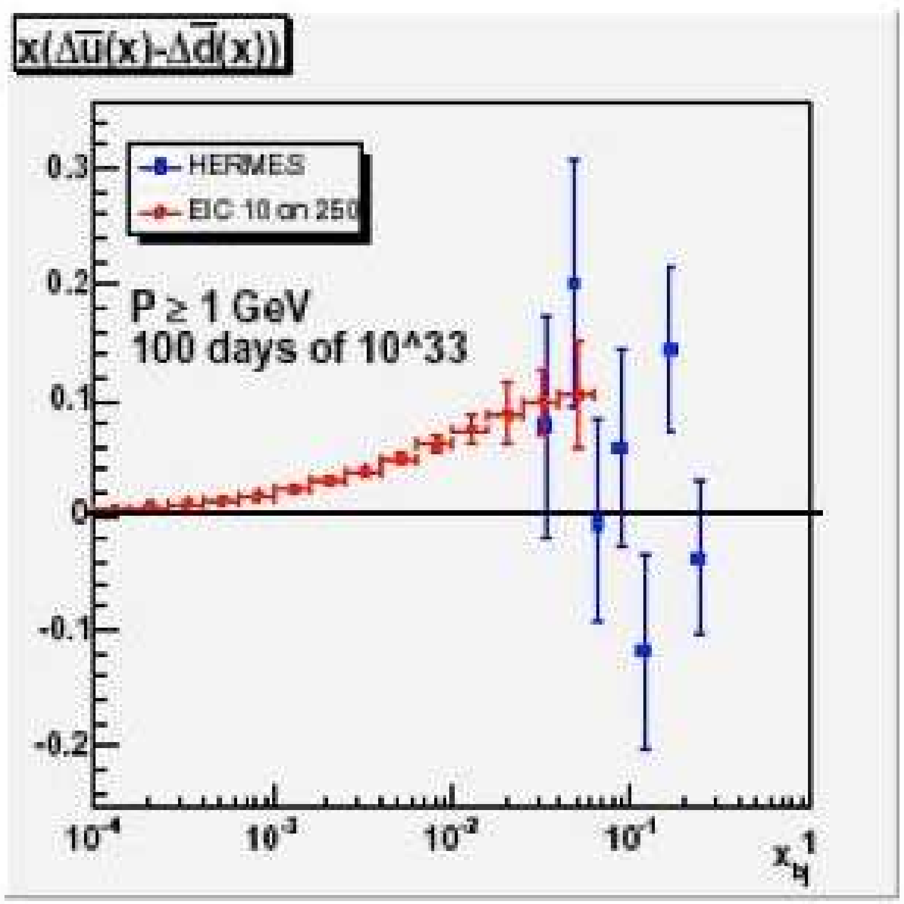


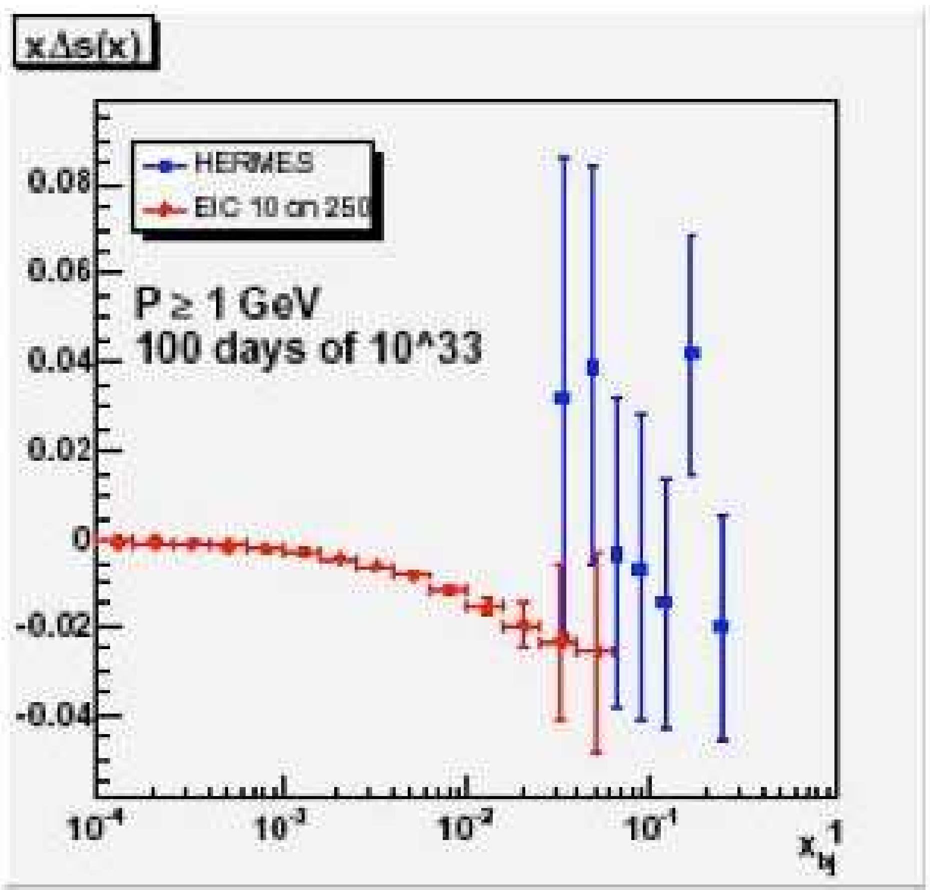
RHIC, G. Bunce et al. 2000



EIC

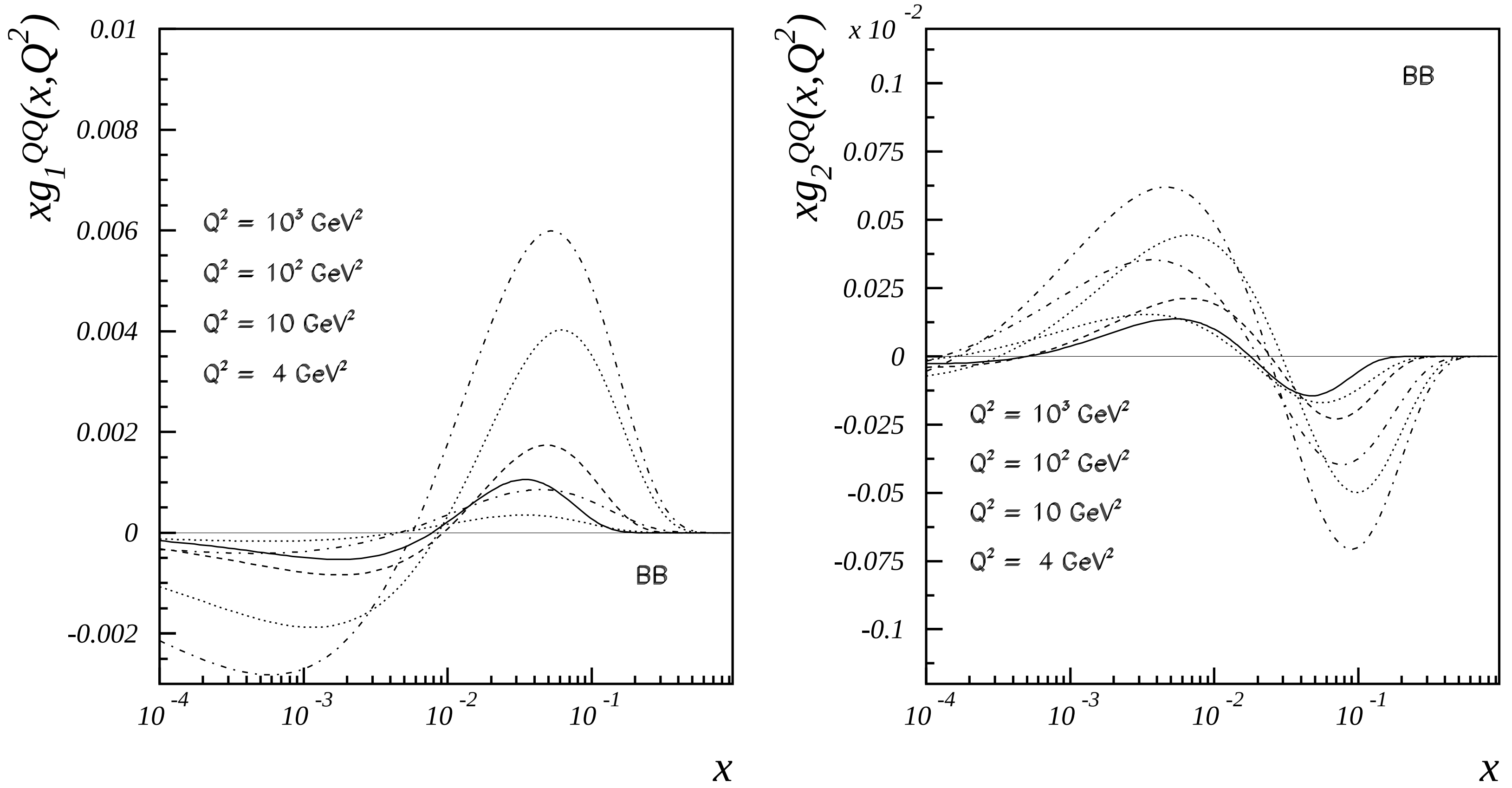
Unfolding the Sea Quarks





Comparison of the Sensitivity at HERMES and at EIC

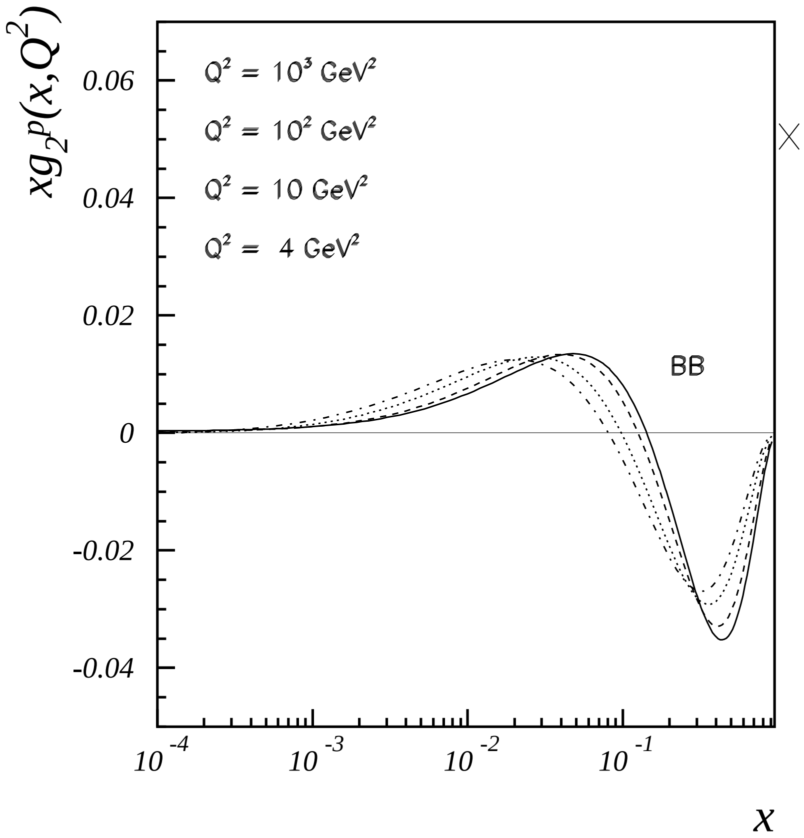
Charm Contributions



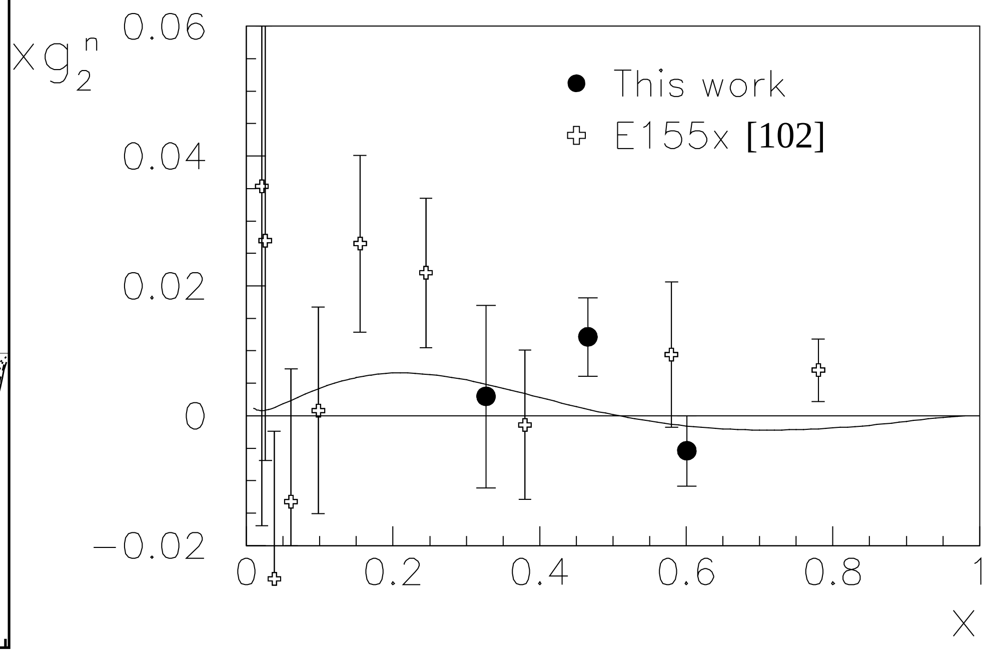
JB, Ravindran, van Neerven (2003): $g_{1,2}^{c\bar{c}}(x,Q^2)$

$\Longrightarrow$ **to be measured at EIC**

$g_2(x, Q^2)$ - a Window to Higher Twist



$g_2^{\tau=2}(x, Q^2)$ (light partons)



JLAB Hall A, 2004

Accurate measurement highly desired.

How big is the $\tau = 3$ contribution ?

4. Moments of PDF's: PT + data

f	n	This Fit N^{3}LO	MRST04 NNLO	A02 NNLO
u_v	2	0.3006 ± 0.0031	0.285	0.304
	3	0.0877 ± 0.0012	0.082	0.087
	4	0.0335 ± 0.0006	0.032	0.033
d_v	2	0.1252 ± 0.0027	0.115	0.120
	3	0.0318 ± 0.0009	0.028	0.028
	4	0.0106 ± 0.0004	0.009	0.010
$u_v - d_v$	2	0.1754 ± 0.0041	0.171	0.184
	3	0.0559 ± 0.0015	0.055	0.059
	4	0.0229 ± 0.0007	0.022	0.024

J.B., H. Böttcher, A. Guffanti, 2006

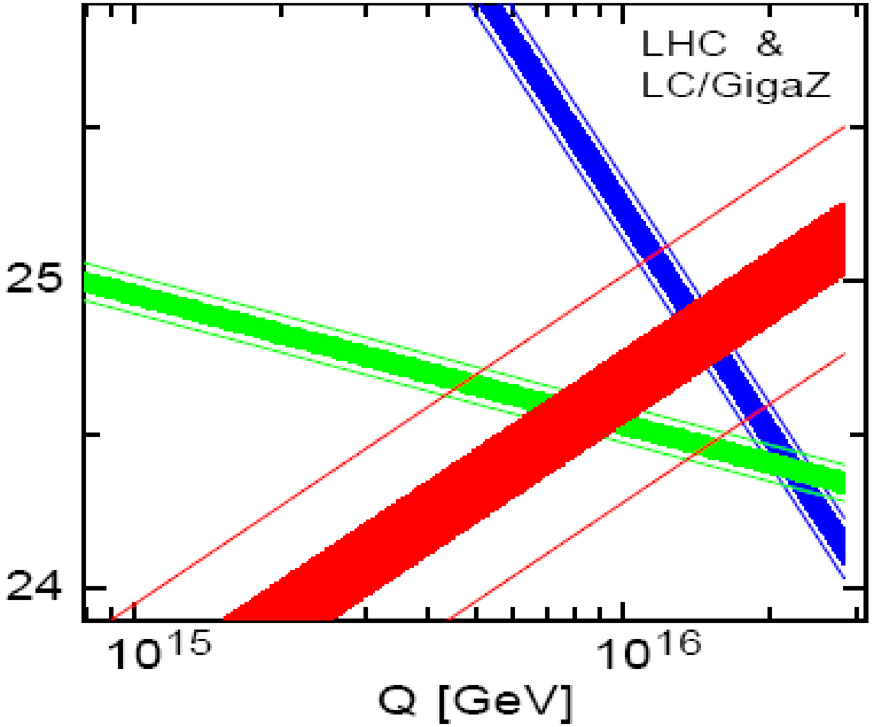
	Moment	BB, NLO
Δu_v	0	0.926
	1	0.163 ± 0.014
	2	0.055 ± 0.006
Δd_v	0	-0.341
	1	-0.047 ± 0.021
	2	-0.015 ± 0.009
$\Delta u_v - \Delta d_v$	0	1.267
	1	0.210 ± 0.025
	2	0.070 ± 0.011

J.B., H. Böttcher, 2002

Lattice Results : developping; different fermion-types studied.
Low values of m_π crucial; values approach 270 MeV now.

5. Λ_{QCD} and $\alpha_s(M_Z^2)$

$$\frac{\delta\alpha_{\rm em}(0)}{\alpha_{\rm em}(0)} \sim 3\cdot 10^{-11} \qquad \frac{\delta\alpha_{\rm weak}}{\alpha_{\rm weak}} \sim 7\cdot 10^{-4} \qquad \frac{\delta\alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2\cdot 10^{-2}$$



P. Zerwas, 2004

Overview of the Analyses

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N^{3}LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	± 0.0065		[1]
MRST03	0.1165	± 0.0020	± 0.0030	[2]
A02	0.1171	± 0.0015	± 0.0033	[3]
ZEUS	0.1166	± 0.0049		[4]
H1	0.1150	± 0.0017	± 0.0050	[5]
BCDMS	0.110	± 0.006		[6]
GRS	0.112			[10]
BBG	0.1148	± 0.0019		[9]
BB (pol)	0.113	± 0.004	$^{+0.009}_{-0.006}$	[7]

NLO

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]
N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

NNLO and N^{3}LO

BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

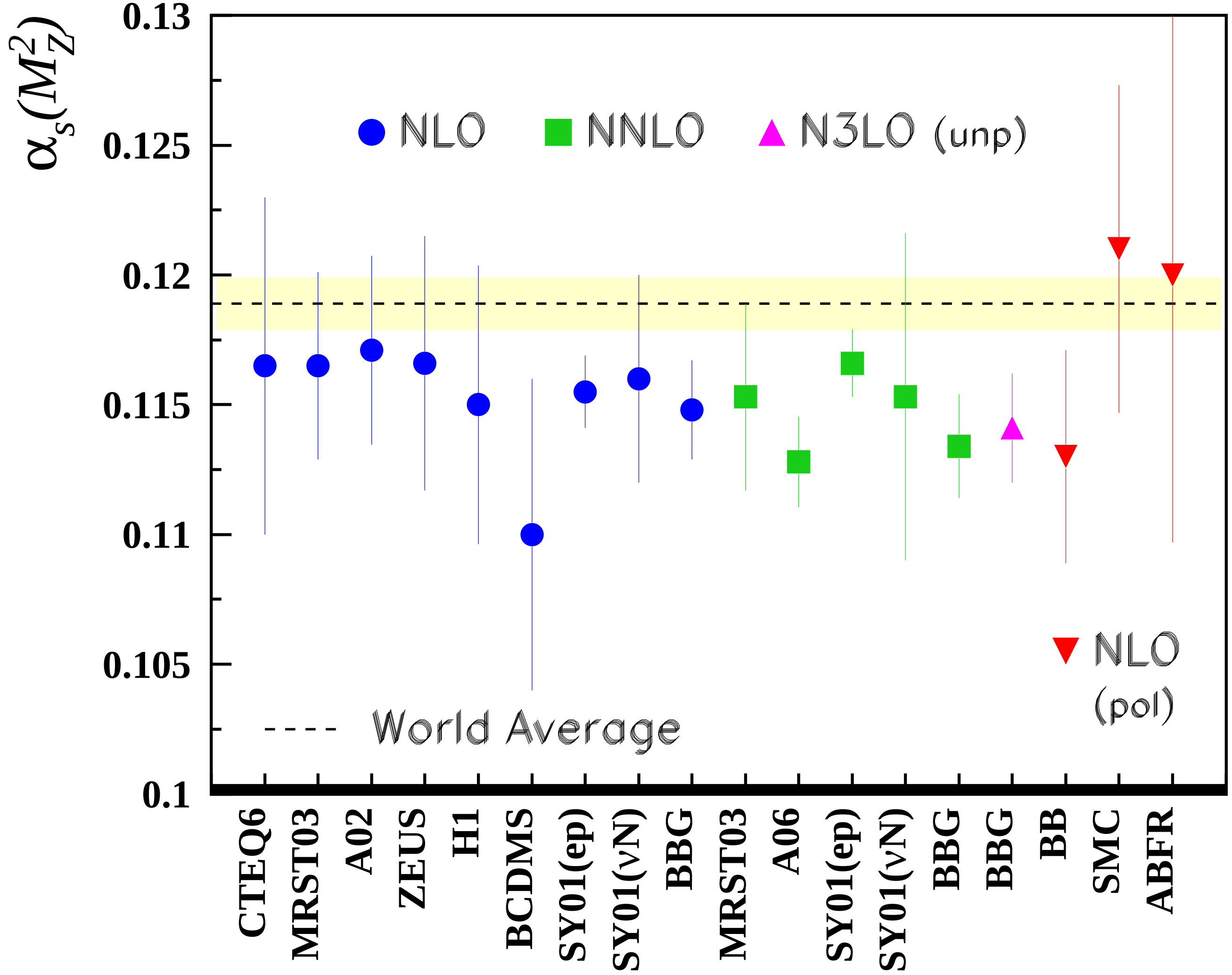
Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger Values, to be discussed.

$\alpha_s(M_Z^2)$

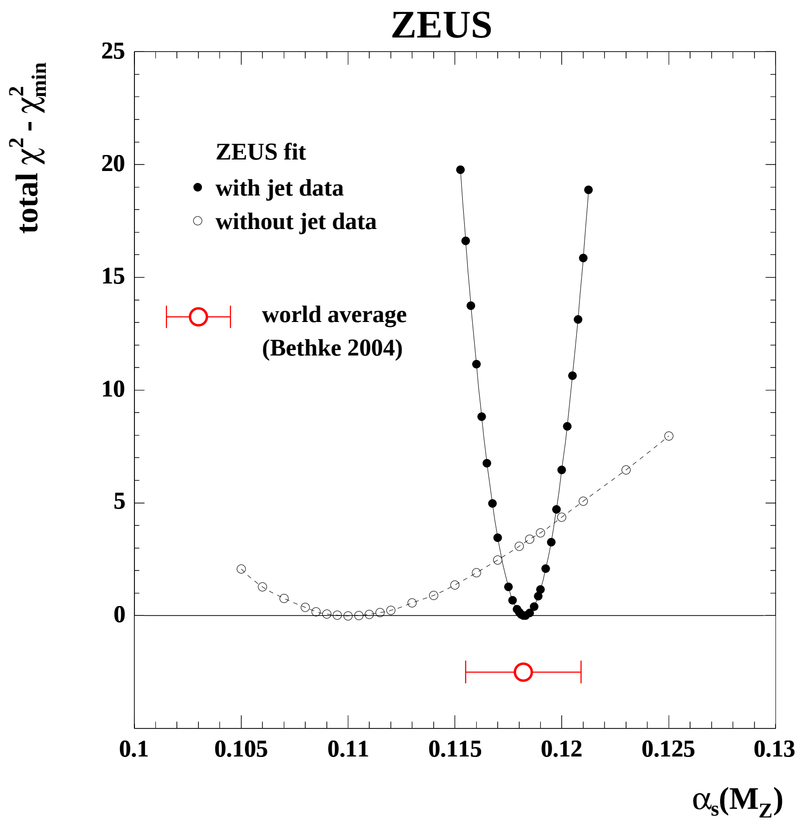


J.B., H. Böttcher, A. Guffanti, 2006

More Global Analyses

- $\alpha_s(M_Z^2)$ for different data sets included are too different !

$\Rightarrow$ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet



M. Cooper-Sarkar, 2005

6. What would we like to know?

HERA:

- Analyze complete collected luminosity for $F_2(x, Q^2)$, $F_2^{c\bar{c}}(x, Q^2)$, $g_2^{c\bar{c}}(x, Q^2)$, and measure $h_1(x, Q^2)$.

RHIC & LHC:

- Improve constraints on gluon and sea–quarks: polarized and unpolarized. DIS PDF's $\Longleftrightarrow$ Collider PDF's

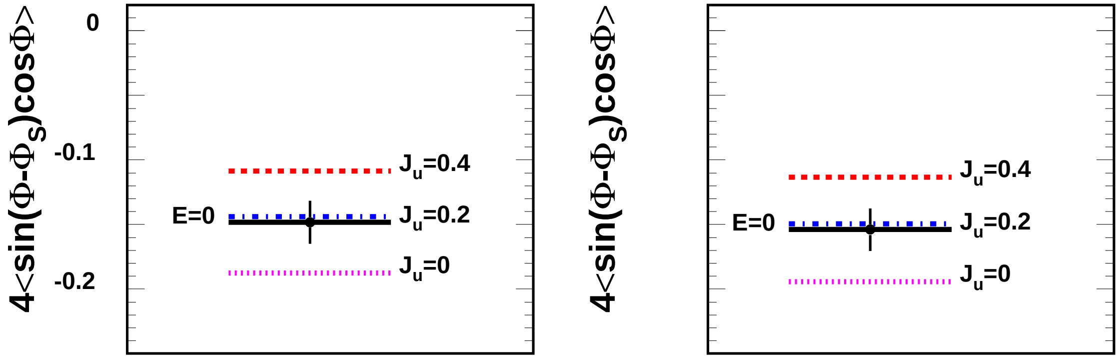
JLAB:

- High precision measurements in the large x domain at unpolarized and polarized targets; supplements HERA's high precision measurements at small x.

L_q from DVCS

- HERA and JLAB : Improve DVCS data

Theory widely developed, cf. rev. Belitsky & Radyushkin, 2005



Expected DVCS asymmetry $A_{UT}^{\sin(\phi-\phi_S)\cos\phi}$ with $b_v = 1$, $b_s = \infty$, $J_u = 0.4(0.2, 0.0)$, $J_d = 0.0$ in the Regge (left panel) and factorized (right panel) ansatz, at the average kinematics of the full measurement. $E = 0$ denotes zero effective contribution from the GPD E. The projected statistical error for 8M DIS events is shown. The systematic error is expected to not exceed the statistical one.

F. Ellinghaus et al. 2005

The measurement of L_q off data is model–dependent at the moment.
Lattice calculations at low pion masses are needed to complete the picture

Graph Resummation and Saturation

Further study of proposed mechanisms needed: RHIC, LHC for nucleus-nucleus collisions.

ep scattering: partly different mechanisms

more studies would be welcome; link to higher twist contributions in gluon–dynamics

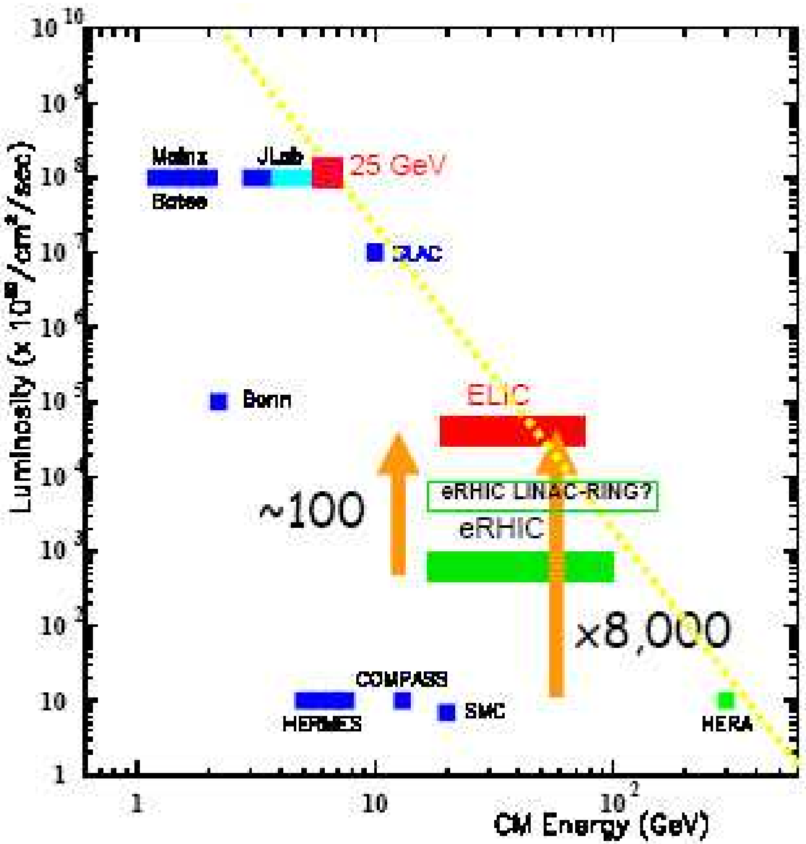
How do the non-perturbative and perturbative parts factorize ?

Conservation laws and interplay between the small x and medium x range behaviour

New DIS Machines

Where to go ?

- High energies : small x, large Q^2 desirable.
- High luminosities : ELIC/EIC: $\sqrt{s}$ between CERN and HERA energies



R. Ent,2004
high precision physics
polarized and unpolarized

Would be an important extension of the present programmes in many respects.

Enhancing Precision Further...

- What is the correct value of $\alpha_s(M_z^2)$? $\overline{\rm MS}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor. (Theory & Experiment)
- Flavor Structure of Sea-Quarks: More studies needed.(All Experiments)
- Revisit polarized data upon completion of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed. (Theory)
- QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions, Transversity, diffraction in polarized scattering (HERMES, High Precision polarized experiments, JLAB, ELIC)
- Comparison with Lattice Results: α_s, Moments of Parton Distributions, Angular Momentum.