Deep Inelastic Scattering Structure Functions in QCD

Mitteldeutsche Physik Combo 1996

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Universität Leipzig,
June 1996

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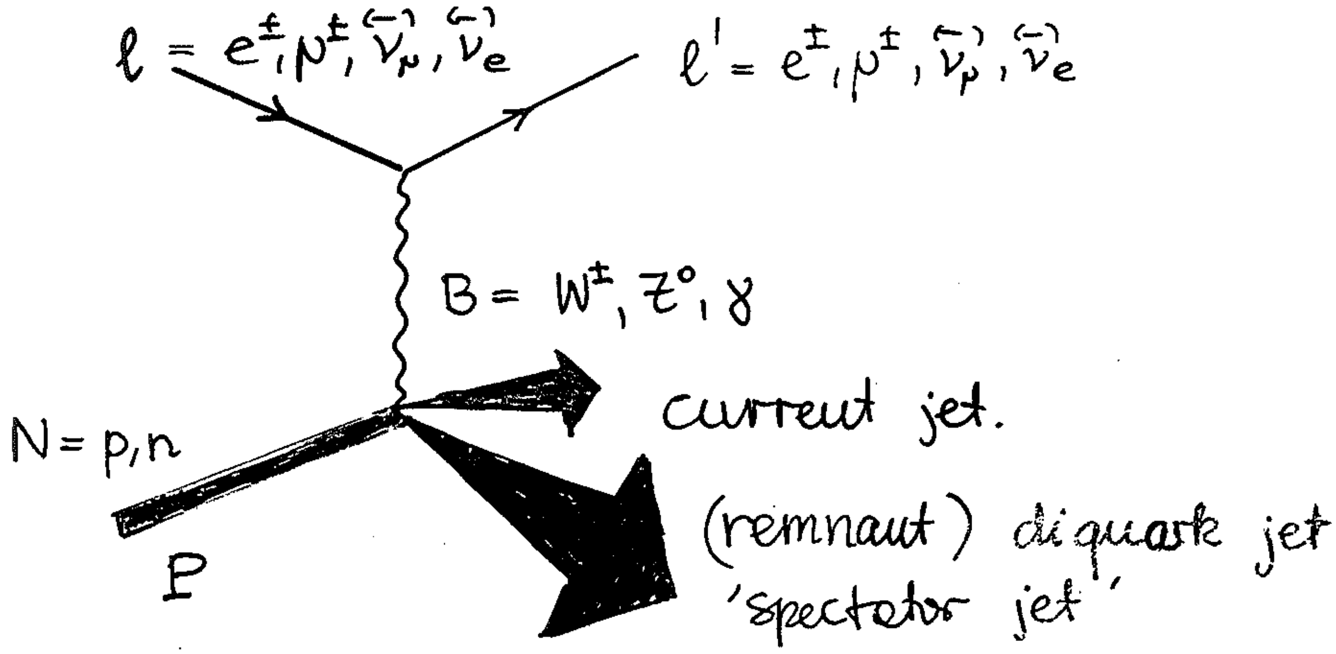
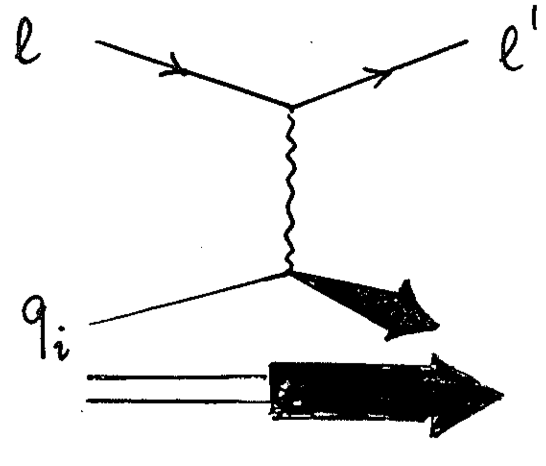
DEEP INELASTIC SCATTERING - BASIC ISSUES



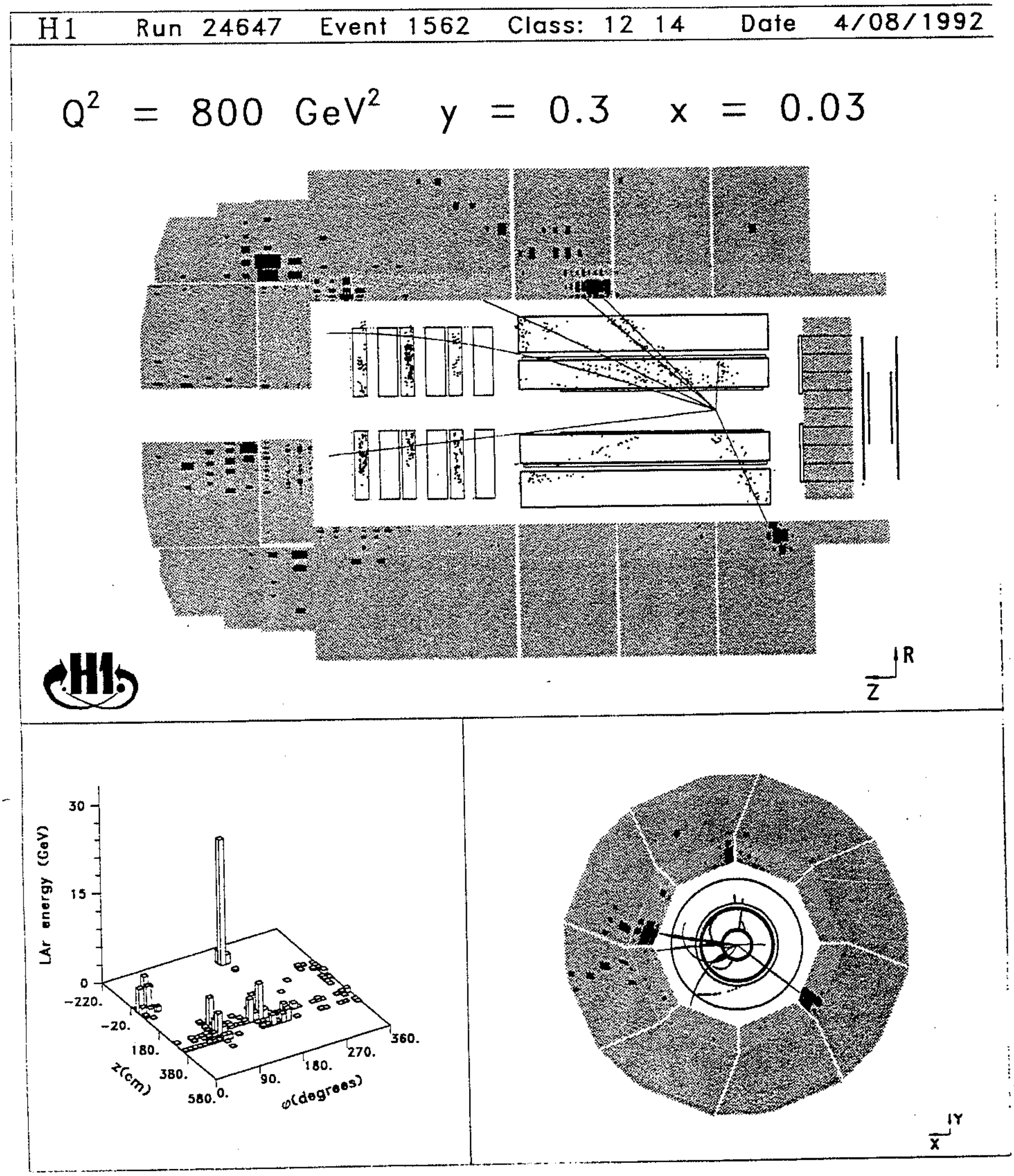


VARIABLES :

$Q^2 = -(\ell - \ell')^2 = -(p_{q_i} - p_{q_f})^2 \geqq 0 \quad , q = \ell - \ell'$

$x_B \equiv x = \frac{Q^2}{2Pq} \qquad y_B \equiv y = \frac{qP}{\ell P}.$

$s = (P + \ell)^2.$


H1 Run 24647 Event 1562 Class: 12 14 Date 4/08/1992
Q^2 = 800 GeV^2 y = 0.3 x = 0.03
R
Z
LAr energy (GeV)
30
15
0
-220.
-20.
180.
380.
580.
z(cm)
0.
90.
180.
270.
360.
φ(degrees)
Y
X

THE BORN CROSS SECTIONS

CHARGED LEPTONS :

NC :

$$\frac{d^2\sigma}{dx\,dQ^2} = 2\pi\alpha^2 \frac{M_N s}{(s-M^2)^2} \frac{1}{Q^4} L^{\mu\nu}\tilde{W}_{\mu\nu}$$

$e^\pm N \to e^\pm X$ $(\mu^\pm N \to \mu^\pm X)$

<u>pure photon exchange:</u>

$$L_{\mu\nu} = 2\left[l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu}\, l.l' \right]$$

$$W_{\mu\nu} = \frac{1}{4\pi}\sum_n \langle P | J^{em\,\dagger}_\mu(0) | n\rangle \langle n | J^{em}_\nu(0) | P\rangle (2\pi)^4\, \delta^{(4)}(P+q-p_n)$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(x,Q^2) + \frac{1}{M^2}\left[\left(P_\mu - \frac{Pq}{q^2}q_\mu\right)\left(P_\nu - \frac{Pq}{q^2}q_\nu\right)\right.$$

$$\cdot\, W_2(x,Q$$

$$F_2(x,Q^2) = x\left(-g_{\mu\nu} + \frac{12x^2}{Q^2}P_\mu P_\nu\right) W^{\mu\nu}$$

$$F_L(x,Q^2) = \frac{8x^3}{Q^2} P_\mu P_\nu W^{\mu\nu}$$

$(\mathcal{O}(\alpha_s))$

2 STRUCTUREFCT. $\quad F_L = F_2 - 2xF_1$

$\mathcal{O}(\alpha_s^0)$ 1 STRCT. FCT.

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4} Y_+ F_2(x,Q^2)$$

$(F_L \equiv 0)$
lowest order

$$Y_\pm = 1 \pm (1-y)^2$$

INCLUSION OF ℓ-BEAM POLARIZATION

& Z EXCHANGE:

$$\frac{d^2\sigma^\pm}{dx\,dQ^2} = \frac{2\pi\alpha^2}{x Q^4}\left\{ Y_+ \mathbb{F}_2^\pm(x,Q^2) + Y_- \, x\mathbb{F}_3^\pm(x,Q^2) \right\}$$

$$\mathbb{F}_2^\pm(x,Q^2) = F_2(x,Q^2) + K_Z(Q^2)\,(-v \mp \lambda a)\,G_2(x,Q^2) + K_Z^2(Q^2)\,(v^2+a^2 \pm 2\lambda v a)\,H_2(x,Q^2)$$

$$x\mathbb{F}_3^\pm(x,Q^2) = K_Z(Q^2)\,(\pm a + \lambda v) \times G_3(x,Q^2) + K_Z^2(Q^2)\,(\mp 2va - \lambda(v^2+a^2)) \times H_3(x,Q^2)$$

5 Structure fct. (without longitud.)

+3 longitudinal Structfct.

$$K_Z(Q^2) = \frac{1}{4\sin^2\theta_W \cos^2\theta_W}\,\frac{Q^2}{Q^2+M_Z^2}$$

$$a \equiv a_e = -\frac{1}{2}$$

$$v \equiv v_e = -\frac{1}{2} + 2\sin^2\theta_W$$

B

CC :

$e^{\pm}(\mu^{\pm})N \to \overset{(-)}{\nu}_{e(\mu)} X$

$$\frac{d^2\sigma^{\pm}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{x\,Q^4}\,\kappa_W^2(Q^2)\left(\frac{1\pm\lambda}{2}\right)\cdot\left\{Y_+ W_2^{\pm}(x,Q^2) \pm Y_- x W_3^{\pm}(x,Q^2)\right.$$

$$\kappa_W(Q^2) = \frac{Q^2}{Q^2+M_W^2}\cdot\frac{1}{4\sin^2\theta_W}$$

4 structure fct.

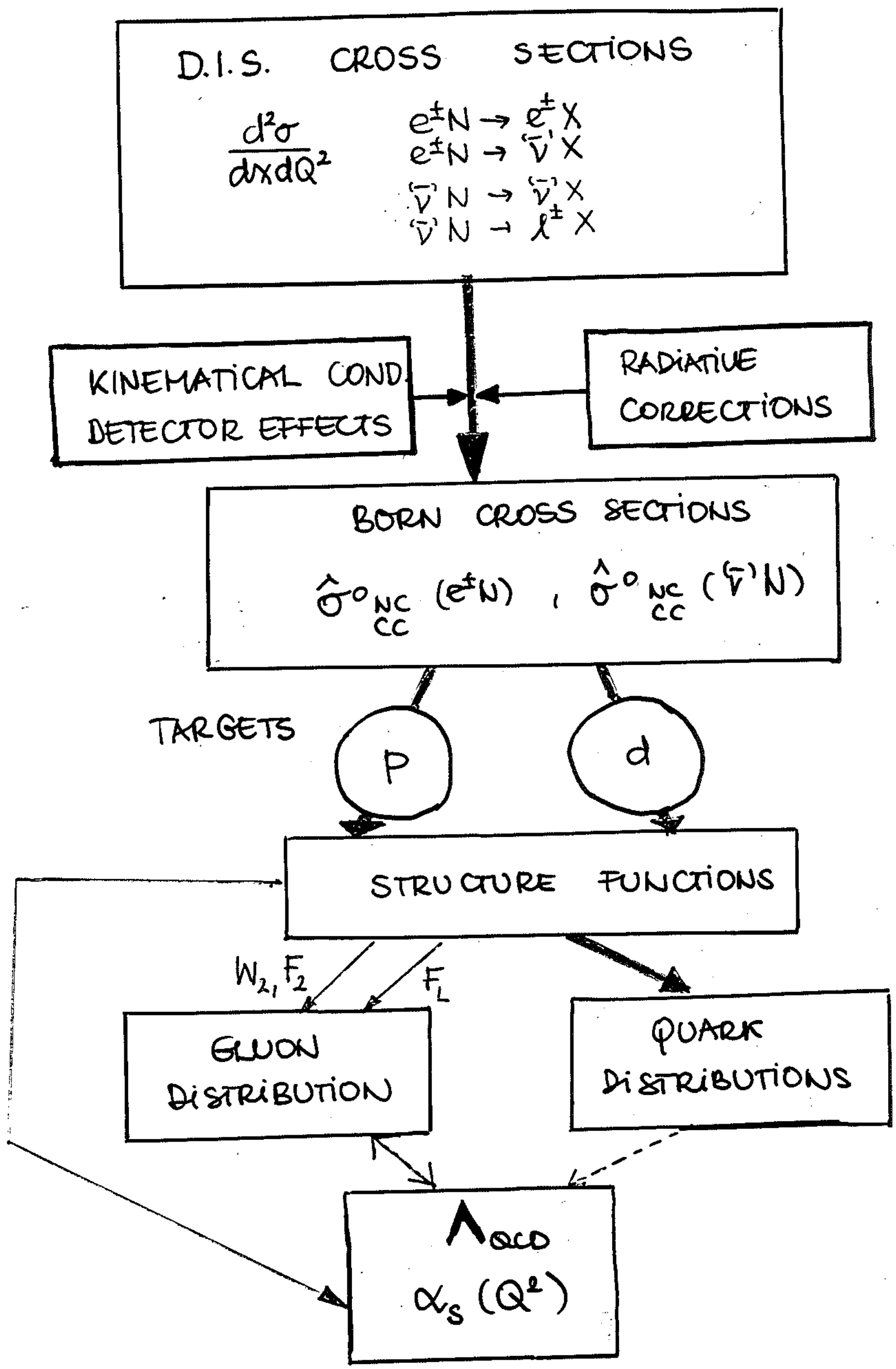
+ 2 long. structurefct.

BORN: $e^{\pm}p$ 14 structure functions !

$e^{\pm}d$ + ——— " ———

(composed out of : u, d, s, c, b
$\bar{u}, \bar{d}, \bar{s}, c=\bar{c}, b=\bar{b}$ & g

$\leq$ 10 parton densities)



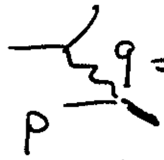
D.I.S. CROSS SECTIONS
$\frac{d^2\sigma}{dx\,dQ^2}$
$e^{\pm}N \to e^{\pm}X$
$e^{\pm}N \to \overset{(-)}{\nu}X$
$\overset{(-)}{\nu}N \to \overset{(-)}{\nu}X$
$\overset{(-)}{\nu}N \to \ell^{\pm}X$
KINEMATICAL COND.
DETECTOR EFFECTS
RADIATIVE CORRECTIONS
BORN CROSS SECTIONS
$\hat{\sigma}^{0}_{NC,CC}(e^{\pm}N)$, $\hat{\sigma}^{0}_{NC,CC}(\overset{(-)}{\nu}N)$
TARGETS
p
d
STRUCTURE FUNCTIONS
W_2, F_2
F_L
GLUON DISTRIBUTION
QUARK DISTRIBUTIONS
Λ_{QCD}
$\alpha_s(Q^2)$

REMARK ON THE PARTON MODEL

WHEN CAN WE USE THIS DESCRIPTION?
(DRELL, YAN).

INFINITE MOMENTUM FRAME:

$$P_\nu = \left(P + \frac{M^2}{2P}\,;\, 0, 0, P\right)_{P \to \infty}$$



$P \quad q = (q_0; q_3, \vec{q}_\perp)$

SATISFY :

$$M\nu = q \cdot P = (q_0 - q_3)P + \frac{M^2}{2P} q_0$$

$$-q^2 = Q^2 = (q_3 + q_0)(q_3 - q_0) + q_\perp^2 .$$

ep: CMS $\qquad q_0 - q_3 = \frac{u}{P}\,; \quad q_0 = \nu P$

$$q_0 = \frac{2M\nu + q^2}{4P} \qquad q_3 = -\frac{2M\nu - q^2}{4P} \qquad -q^2 \Rightarrow q_\perp^2 .$$

$$\tau_{int} \sim \frac{1}{q_0} = \frac{4P}{2M\nu - Q^2}$$

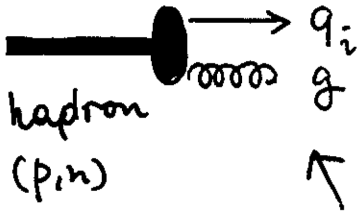
$$\tau_{life}^{part.} \sim \frac{2P}{\sum_{hadr.} \frac{k_{\perp i}^2 + m_i^2}{x_i} - M^2}$$

∃ Parton model: iff $\boxed{\tau_{life} \gg \tau_{int}}$

$$\tau_{life} \sim P \frac{2}{\left[k_\perp^2 / x(1-x)\right]} \qquad Q^2 = 10\,\text{GeV}^2,\ x = 10^{-4}, \quad k_\perp^2 = 1\,\text{GeV}^2$$

$$\tau_{int} \sim P \frac{4x}{Q^2(1-x)} \qquad \tau_{life} / \tau_{int} \sim 5.$$

PARTON MODEL

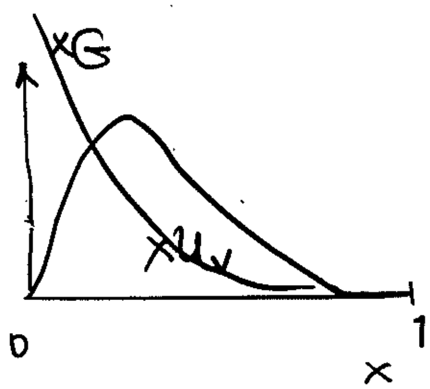


QUASIFREE : FERMIONS q_i
GLUONS g
(massless vectors)

THESE PARTICLES HAVE A PROBABILITY DISTRIBUTION (@ TWIST 2)

PARTON DENSITY: $0 \leq x = \frac{Q^2}{2Pq} \leq 1$.

eg.



THESE DISTRIBUTIONS ARE NON-PERTURBATIVE QUANTITIES.

PARTON MODEL AND FLAVOUR CONTENTS OF STRUCTURE FUNCTIONS

P

CHARGED LEPTON (BORN) STRUCTURE FCT. :

$$F_2(x,Q^2) = x \sum_q e_q^2 \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]$$

$$G_2(x,Q^2) = x \sum_q 2 e_q v_q \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]$$

$$H_2(x,Q^2) = x \sum_q (v_q^2 + a_q^2) \left[q(x,Q^2) + \bar{q}(x,Q^2) \right]$$

$$x G_3(x,Q^2) = 2x \sum_q e_q a_q \left[q(x,Q^2) - \bar{q}(x,Q^2) \right]$$

$$x H_3(x,Q^2) = 2x \sum_q v_q a_q \left[q(x,Q^2) - \bar{q}(x,Q^2) \right]$$

$$W_2^+(x,Q^2) = 2x \sum_i \left[d_i(x,Q^2) + \bar{u}_i(x,Q^2) \right]$$

$$W_2^-(x,Q^2) = 2x \sum_i \left[u_i(x,Q^2) + \bar{d}_i(x,Q^2) \right]$$

$$x W_3^+(x,Q^2) = 2x \sum_i \left[u_i(x,Q^2) - \bar{d}_i(x,Q^2) \right]$$

$$x W_3^-(x,Q^2) = 2x \sum_i \left[d_i(x,Q^2) - \bar{u}_i(x,Q^2) \right]$$

$$\overset{(-)}{u}_i = (\overset{(-)}{u}, \overset{(-)}{c}, \overset{(-)}{t})$$

$$\overset{(-)}{d}_i = (\overset{(-)}{d}, \overset{(-)}{s}, \overset{(-)}{b})$$

NEUTRINO (BORN) STRUCTURE FCT.:

P

$$F_2^{\nu}(x,Q^2) = 2x\left[a_{21}\sum_i(u_i+\bar{u}_i) + a_{22}\sum_i(d_i+\bar{d}_i)\right]$$
$$\equiv F_2^{\bar{\nu}}(x,Q^2)$$
$$xF_3^{\nu}(x,Q^2) = 2x\left[a_{31}\sum_i(u_i-\bar{u}_i) + a_{32}\sum_i(d_i-\bar{d}_i)\right]$$
$$\equiv -xF_3^{\bar{\nu}}(x,Q^2).$$

$$a_{21} = \frac{1}{4} - e_u \sin^2\theta_W + 2e_u^2 \sin^4\theta_W$$
$$a_{22} = \frac{1}{4} + e_d \sin^2\theta_W + 2e_d^2 \sin^4\theta_W$$
$$a_{31} = \frac{1}{4} - e_u \sin^2\theta_W$$
$$a_{32} = \frac{1}{4} + e_d \sin^2\theta_W$$

$$W_2^{\nu}(x,Q^2) = 2x\sum_i(d_i+\bar{u}_i)$$
$$xW_3^{\nu}(x,Q^2) = 2x\sum_i(d_i-\bar{u}_i)$$
$$W_2^{\bar{\nu}}(x,Q^2) = 2x\sum_i(u_i+\bar{d}_i)$$
$$xW_3^{\nu}(x,Q^2) = 2x\sum_i(u_i-\bar{d}_i)$$

DEUTERONS & ISOSCALAR NUCLEI

d's at colliders: $s \to s/2$!

quark contents:

$$\overset{(-)}{u}_1, \overset{(-)}{d}_1 \equiv \overset{(-)}{u}, \overset{(-)}{d} \to \frac{1}{2}\overset{(\longrightarrow)}{(u+d)}$$

⟶ previous formulae modify accordingly.

EXAMPLES:

$e^{\pm}d$

$$F_2^{ed} = \frac{5}{18}x(u_v + d_v) + \frac{10}{9} x u_s + \frac{2}{9} xs + \frac{8}{9} xc + \frac{2}{9} xb$$

$$x G_3^{ed} = \frac{1}{2} x (u_v + d_v) = \frac{1}{2} V$$

$$W_2^{e^{\pm}d} = x(u_v + d_v) + 4x u_s + 2xs + 2xc + 2xb \equiv \Sigma$$

$$x W_3^{e^{\pm}d} = x(u_v + d_v) \pm 2x\underline{(s-c)}$$

$$W_2^{\overset{(-)}{\nu} d} = \sum_i x \left[q_i(x, Q^2) + \bar{q}_i(x, Q^2) \right] \equiv \Sigma$$

$$\frac{1}{2} \left[x W_3^{\nu d} + x W_3^{\bar{\nu} d} \right] \underset{Df}{=} x W_3^d = x (u_v + d_v) \equiv V$$

WAYS TO UNFOLD PARTON DENSITIES

$e^{\pm}p$

4 CROSS SECTIONS $\sigma_{NC}^{\pm}, \sigma_{CC}^{\pm}$ $\hookrightarrow$ 4 COMBINATIONS OF PARTON DENSITIES

LINEAR MAPPING: $\overset{(-)}{U} = \sum_i x \overset{(-)}{u}_i$; $\overset{(-)}{D} = \sum_i x \overset{(-)}{d}_i$

$$\begin{pmatrix} U \\ \bar{U} \\ D \\ \bar{D} \end{pmatrix} = (\mathbb{A}_{ij}) \begin{pmatrix} \sigma_{NC}^{-} \\ \sigma_{NC}^{+} \\ \sigma_{CC}^{-} \\ \sigma_{CC}^{+} \end{pmatrix}$$

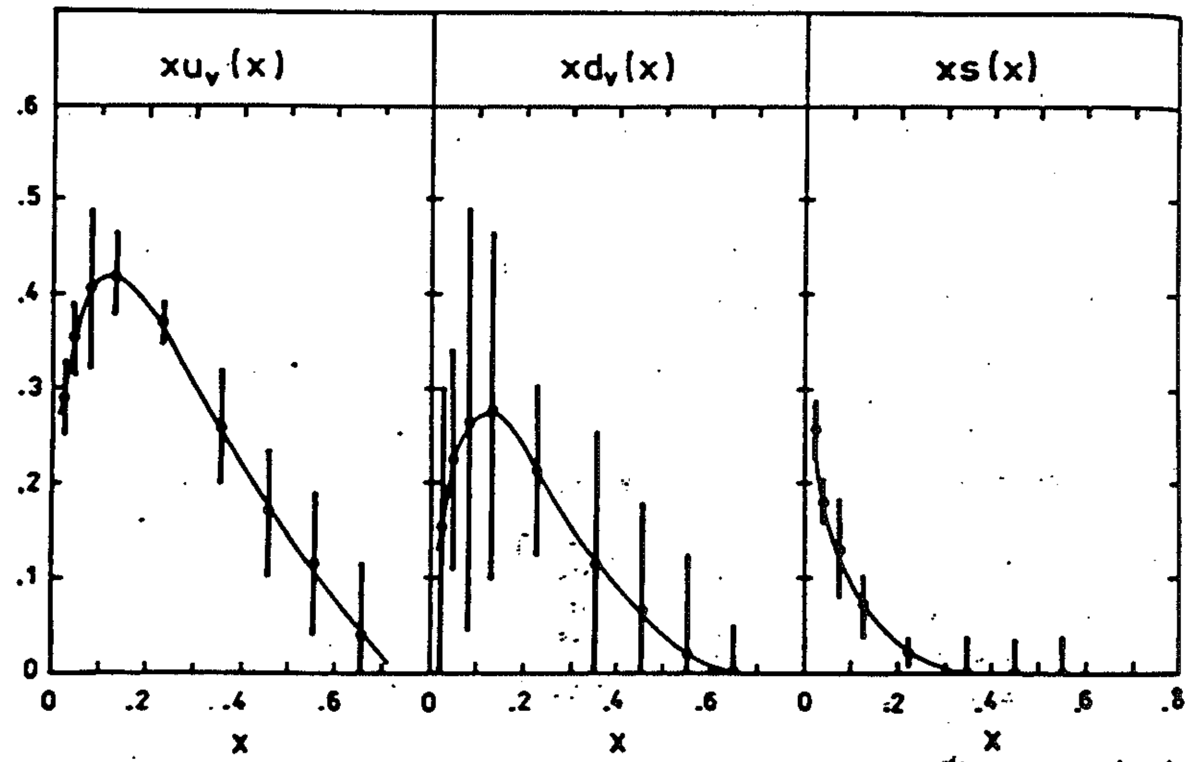
$$\det{}_4 \mathbb{A}_{ij} \sim \left\{ \kappa_Z(Q^2) \left[1-(1-y)^4\right] \right\}^{-1}$$

$(\mathbb{A}_{ij})$ becomes singular (degenerate) both for: $Q^2 \ll M_Z^2$, $y \ll 1$

CONSIDER e.g. (WITH ASSUMPTIONS ON SEA-QUARKS)

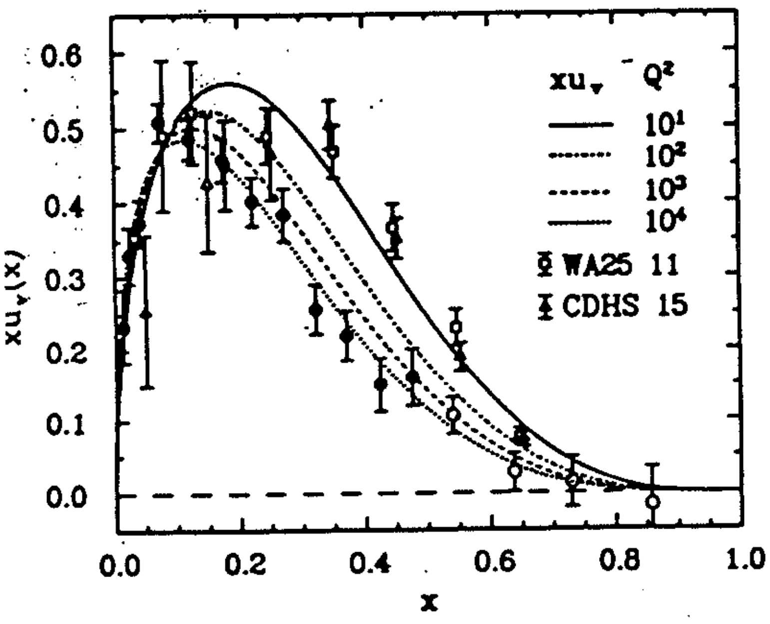
$$\begin{pmatrix} x u_v \\ x d_v \\ x s \end{pmatrix} = (\mathbb{B}_{ij}) \begin{pmatrix} \sigma_{NC}^{-} \\ \sigma_{NC}^{+} \\ \sigma_{CC}^{-} \end{pmatrix}$$

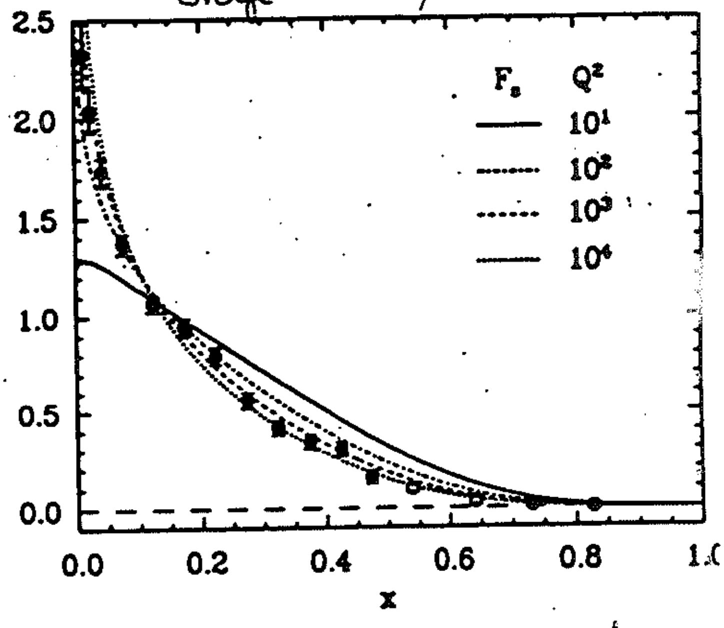
'1TP6



$\mathcal{L} = 100\,pb^{-1}$ / per beam

Ingelman / Rückl

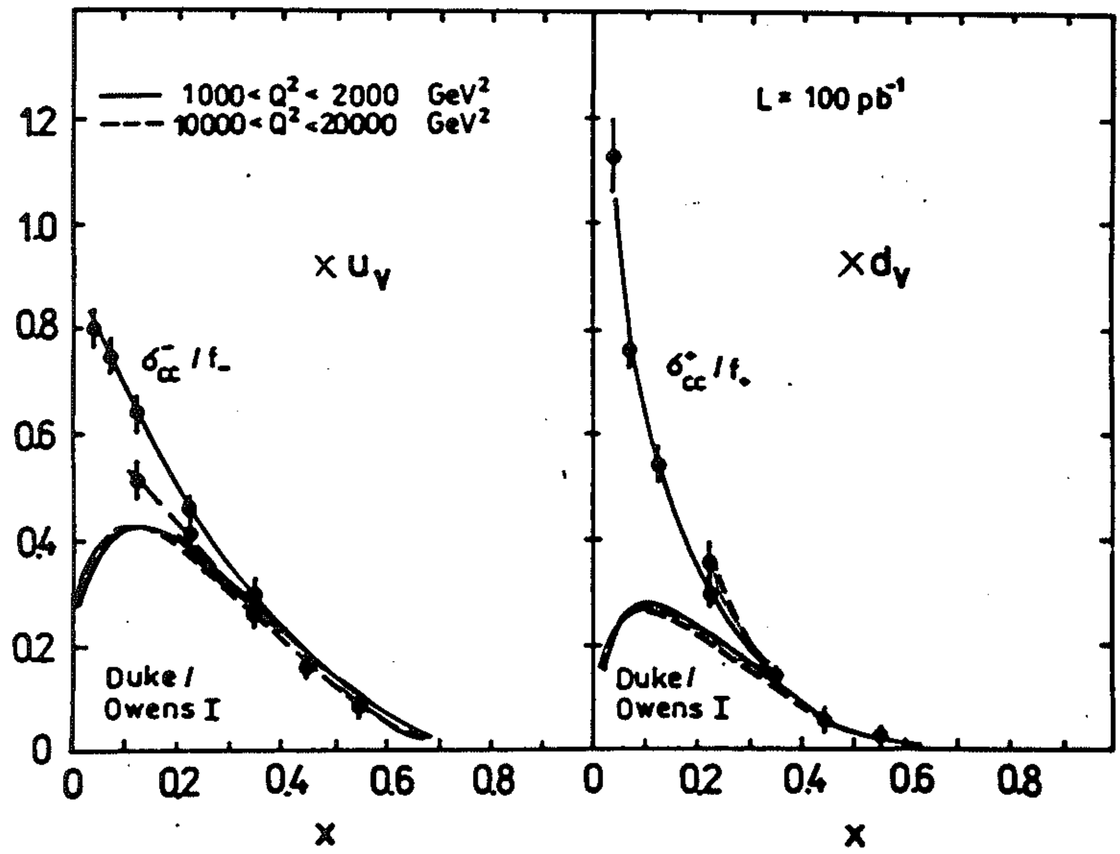




$\sum \mathcal{L} = 400\,pb^{-1}$

APPROXIMATE REPRESENTATIONS

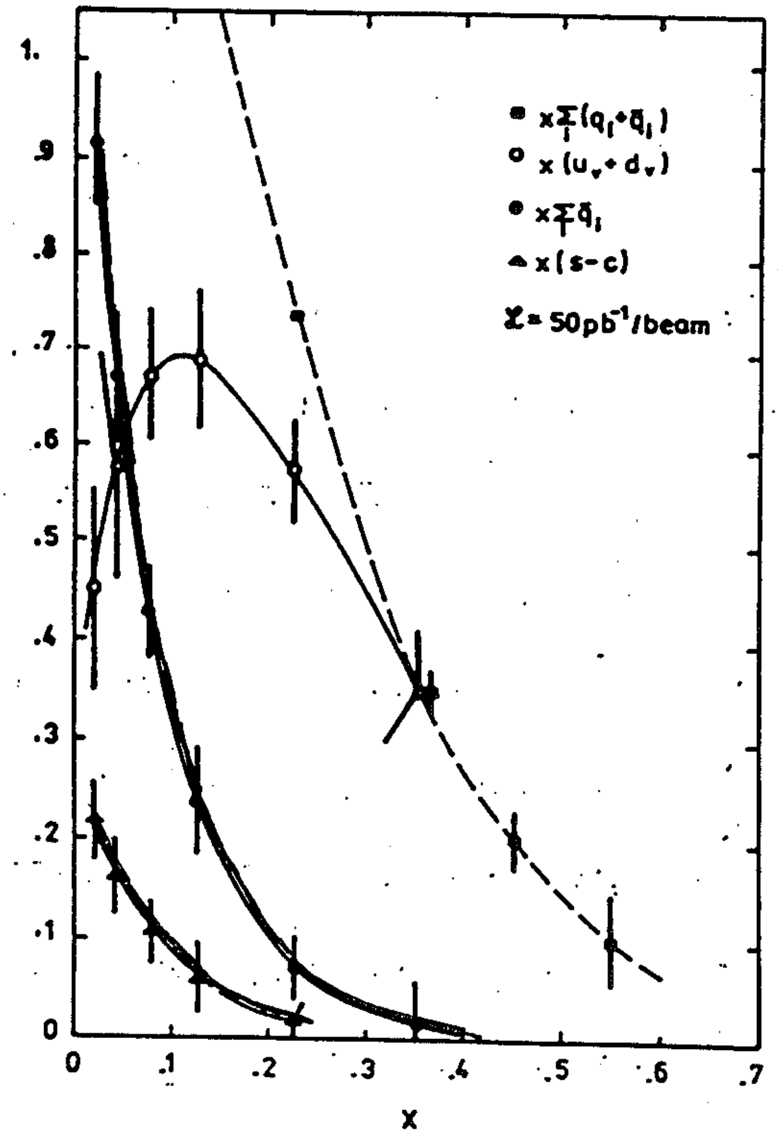
VALENCE RANGE



$$f_{\pm} = \frac{1}{2}\left(Y_{+} \mp Y_{-}\right) K_W^2$$

$\mathcal{L} = 100\,pb^{-1}$

$e^{\pm}p$ & $e^{\pm}d$

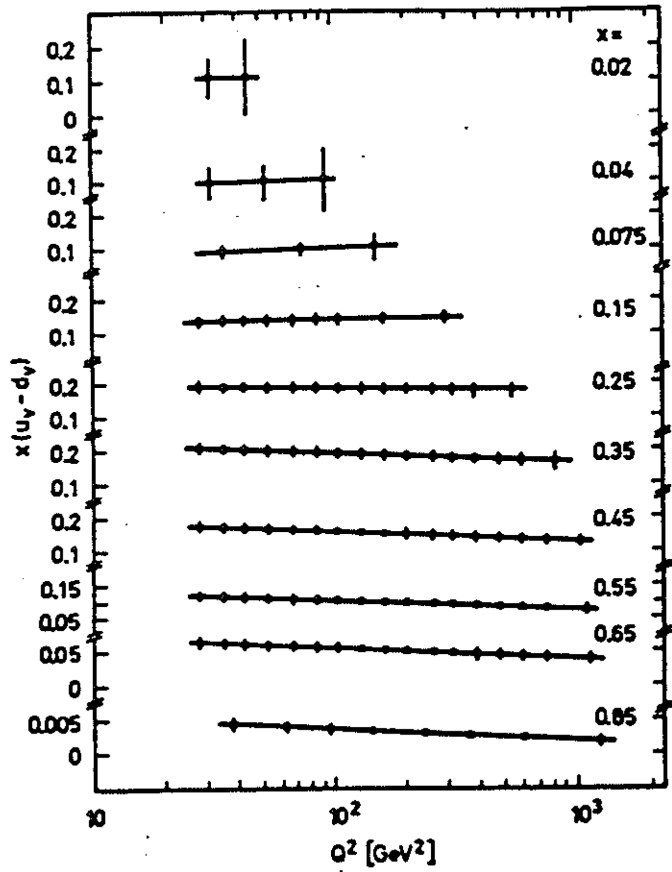


$$\bar{Q} = \frac{1}{2}\left(W_2^{eN} - x W_3^{eN}\right)$$

$$x(s-c) = \frac{5}{18} W_2^{eN} - F_2^{eN} \quad , \quad b \approx 0$$

$\mathcal{L} = 50\,pb^{-1}/\text{beam}$

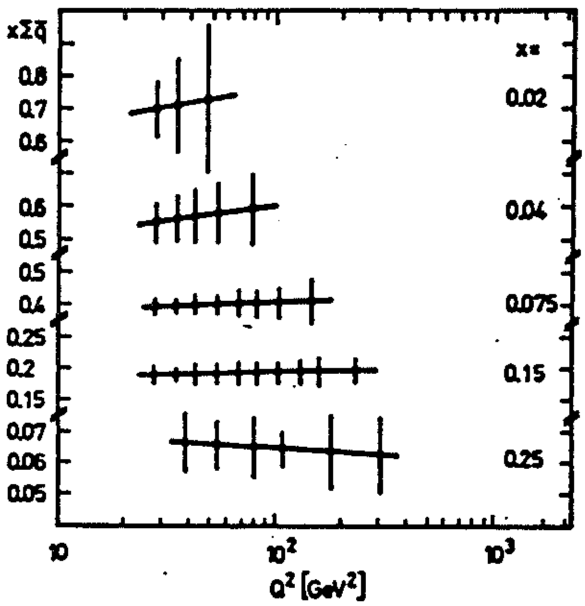
ν's: $$x(u_v - d_v) = \frac{4\pi x}{G_F^2} \frac{(M_W^2 + Q^2)^2}{M_W^4} \frac{1}{Y_+ + Y_-} \left[\frac{1}{2}\sigma^{\nu d} - \sigma^{\nu p}\right]$$



$x(u_v - d_v)$

Fig. 12. Statistical precision of a measurement of $x(u_v - d_v)$ using Eq. (7.1)

$$\sum_i x\bar{q}_i = \frac{\bar{Q}}{2} = \frac{2\pi x}{G_F^2} \frac{(M_W^2 + Q^2)^2}{M_W^4} \left[\sigma^{\bar{\nu} d} - \sigma^{\nu d}(1-y)^2\right] \frac{1}{Y_+ Y_-} - \frac{x(s+b-c)}{Y_+}$$



$\sum_i x\bar{q}_i$

Fig. 15. Statistical precision of a measurement of the antiquark distribution Eq. (7.3)

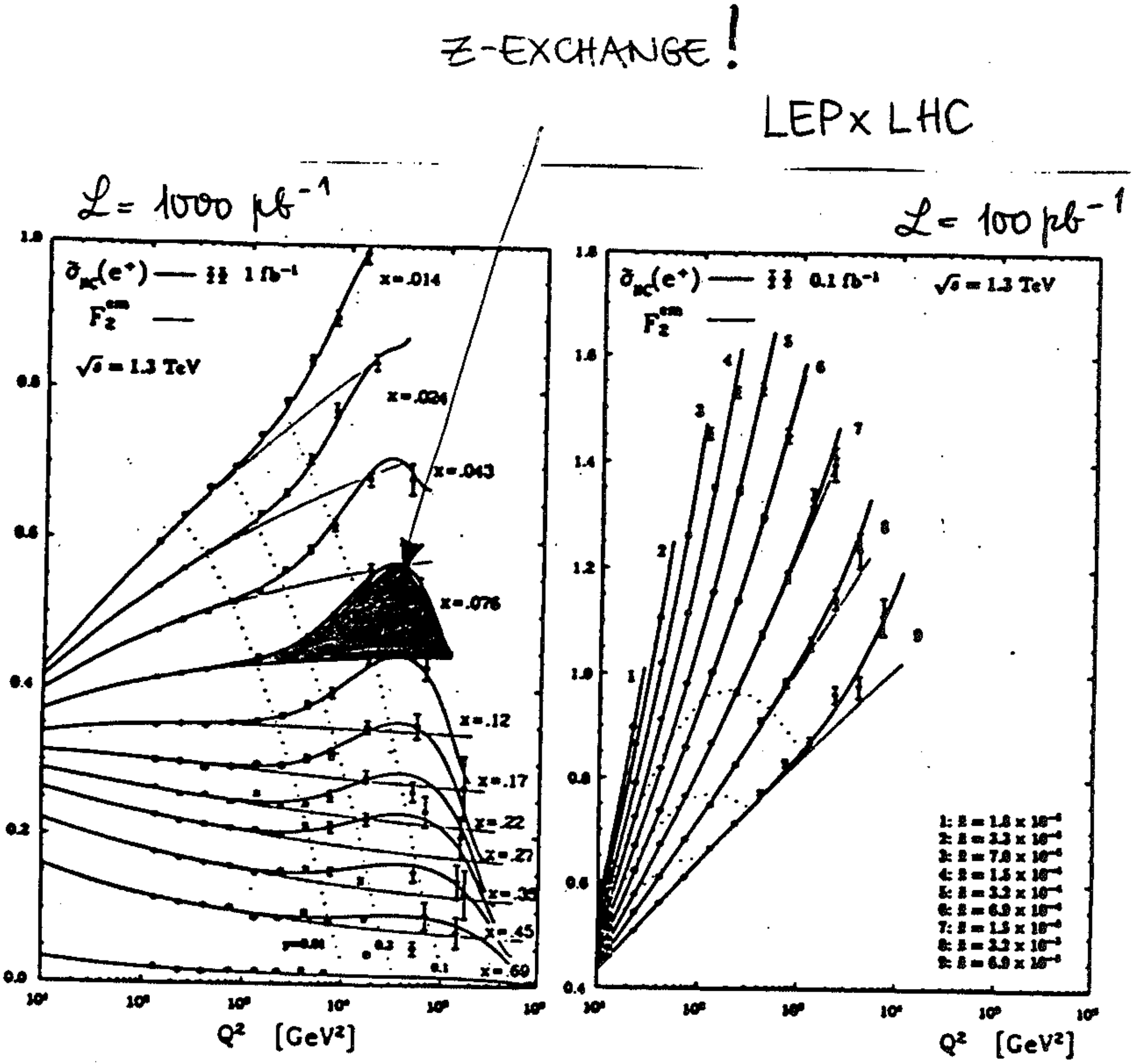


Figure 6: Q^2 dependence of the scaled differential NC e^+p cross-section at LEP+LHC for (a) $x > 10^{-3}$ and (b) $10^{-5} < x < 10^{-3}$. The full curves correspond to $\tilde{\sigma}_{NC}(e^+)$, also represented by the MC data, while the dotted curves represent F_2^{em}, i.e. pure photon exchange, and show the pure QCD scaling violations. The full (open) MC data symbols are with (without) the restriction to the experimentally acceptable phase space region shown in Fig. 5.

DEUTERON STRUCTURE FUNCTIONS

$e^{\pm}(\mu^{\pm})d$

NC : cf. $e^{\pm}p$

CC :

$$W_2^{eN} = \frac{1}{Y_+ K_W^2} \left[\frac{\sigma_{cc}^{+}}{1+\lambda_+} + \frac{\sigma_{cc}^{-}}{1-\lambda_-} \right]$$

$$xW_3^{eN} = \frac{1}{Y_- K_W^2} \left[\frac{\sigma_{cc}^{+}}{1+\lambda_+} - \frac{\sigma_{cc}^{-}}{1-\lambda_-} \right]$$

e^+ & e^- requir. $\mathcal{L}$-splitting.

$\overset{(-)}{\nu}_\mu (\overset{(-)}{\nu}_e) d$

NC : cf. $\overset{(-)}{\nu} p$ $\mapsto$ very difficult to measure in 2-dimensions (x, Q^2).

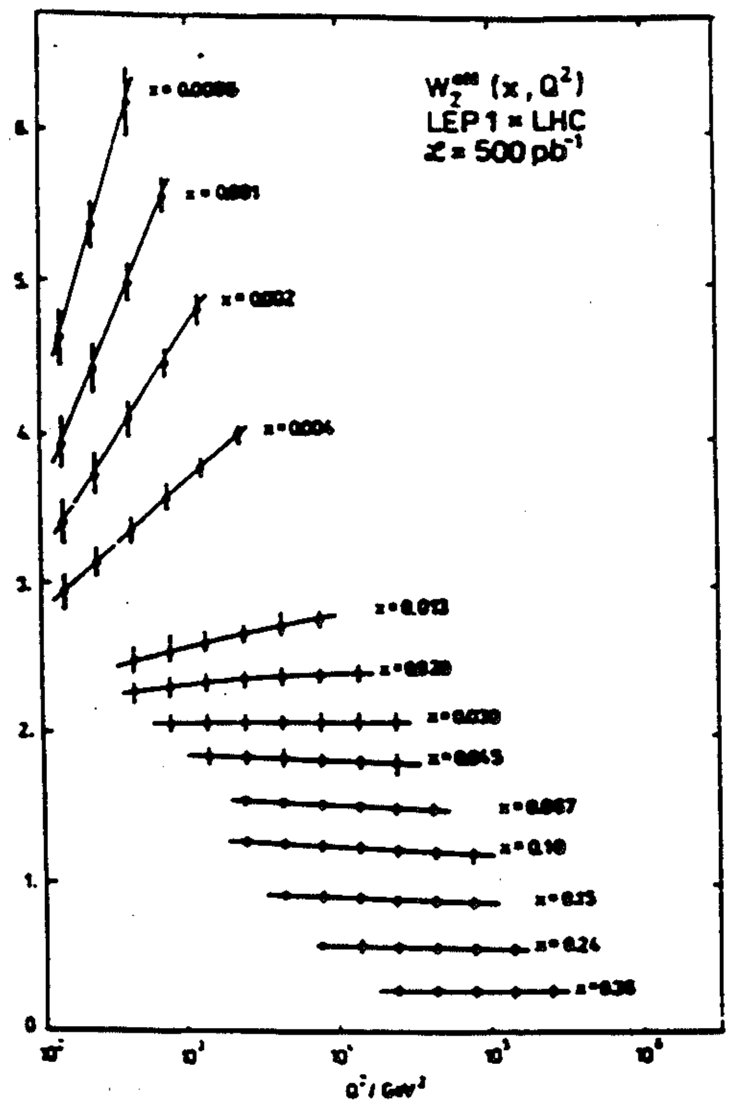
CC :

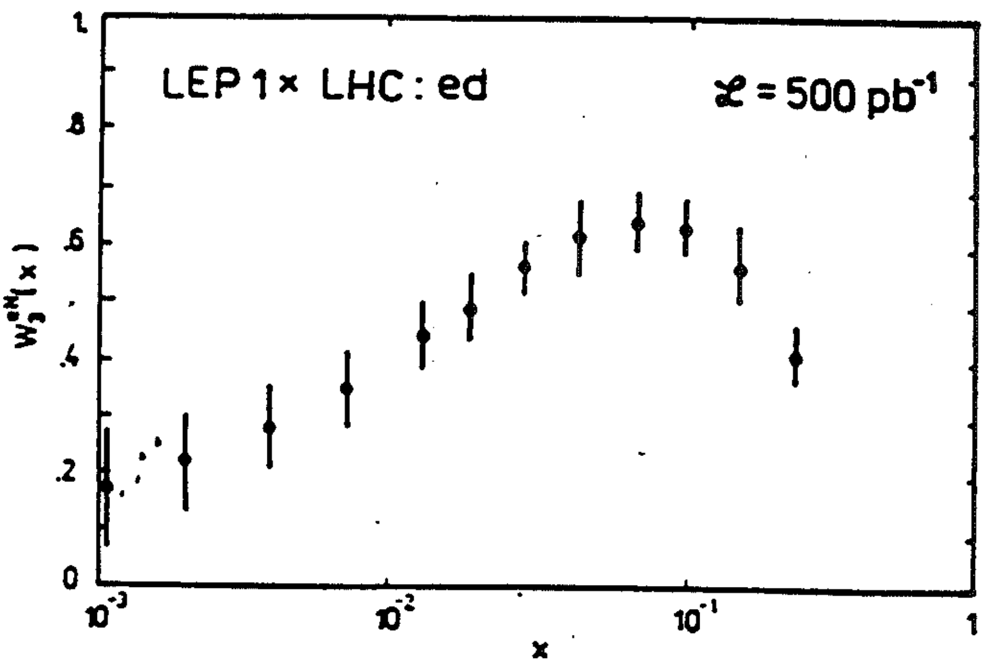
$$W_2^d = \frac{2\pi x}{G_F^2 Y_+} \frac{(M_W^2+Q^2)^2}{M_W^4} \left\{ \sigma^{\nu d} + \sigma^{\bar{\nu} d} \right\} - \frac{2x Y_-}{Y_+}(s+b-c)$$

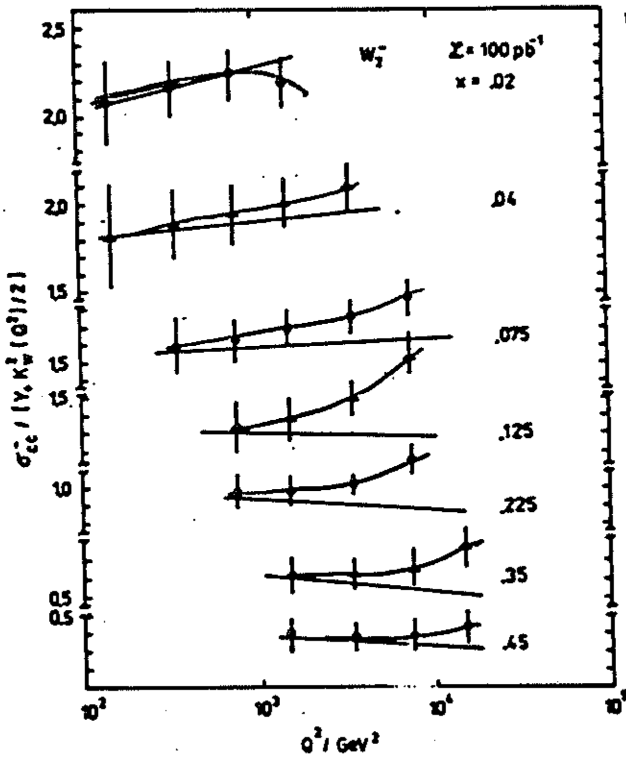
$$xW_3^d = \frac{2\pi x}{G_F^2 Y_-} \frac{(M_W^2+Q^2)^2}{M_W^4} \left\{ \sigma^{\nu d} - \sigma^{\bar{\nu} d} \right\}$$

ωS14

LEP1×LHC
$e^{\pm}d$







W_2^-, CC, ep
100pb^{-1}

$\sigma_{CC}^{e^-p} / [Y_+ k_W^2/2]$

$(Y_-) \times W_3 \lessapprox W_2$
↑
treat as correction

$W_2^d = \frac{1}{2}[W_2^{\nu d} + W_2^{\bar{\nu} d}]$ $\quad \overset{(-)}{\nu} d \quad$ $xW_3^d = \frac{1}{2}[xW_3^{\nu d} + xW_3^{\bar{\nu} d}]$

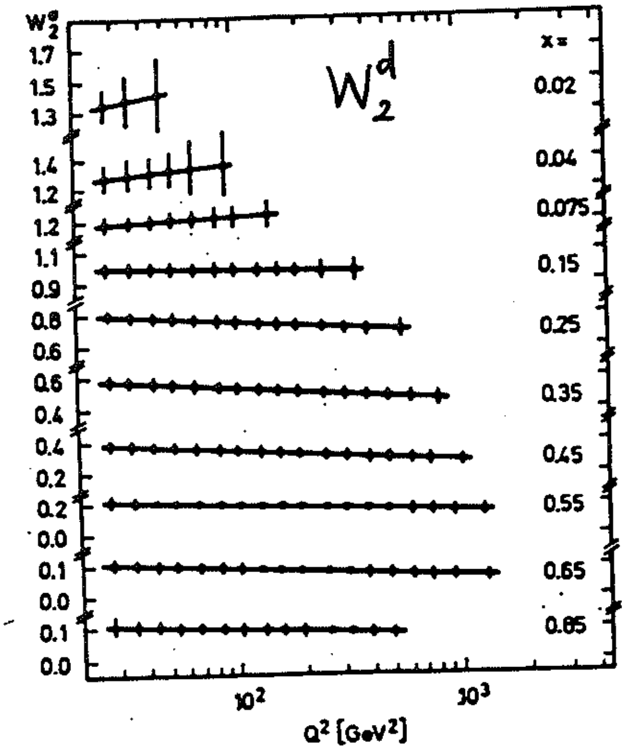


Fig. 8. Statistical preparation of W_2 in $\overset{(-)}{\nu}$-WBB's, Eq. (5.2)

UNK – WBB

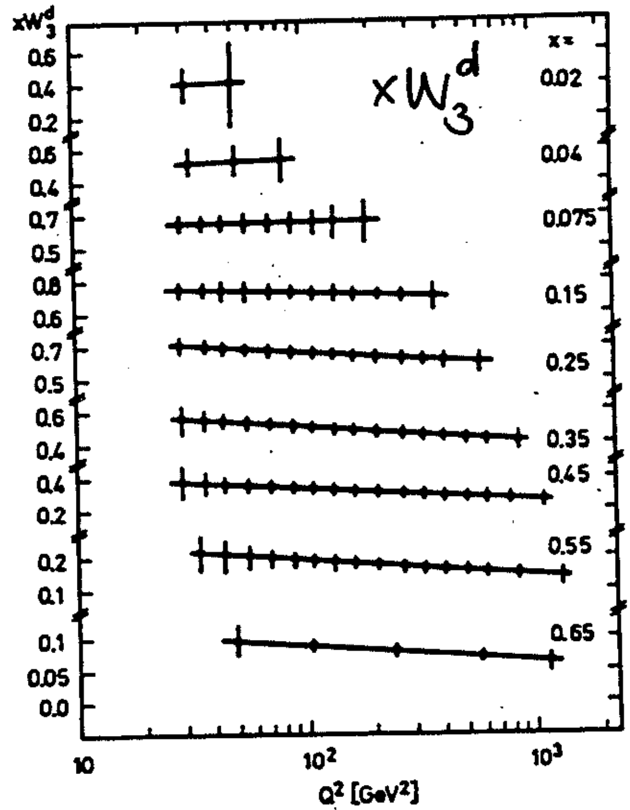


Fig. 10. Statistical preparation of xW_3 in $\overset{(-)}{\nu}$-WBB's, Eq. (5.4)

PARAMETRIZATIONS OF PARTON DISTRIBUTIONS

STIRLING

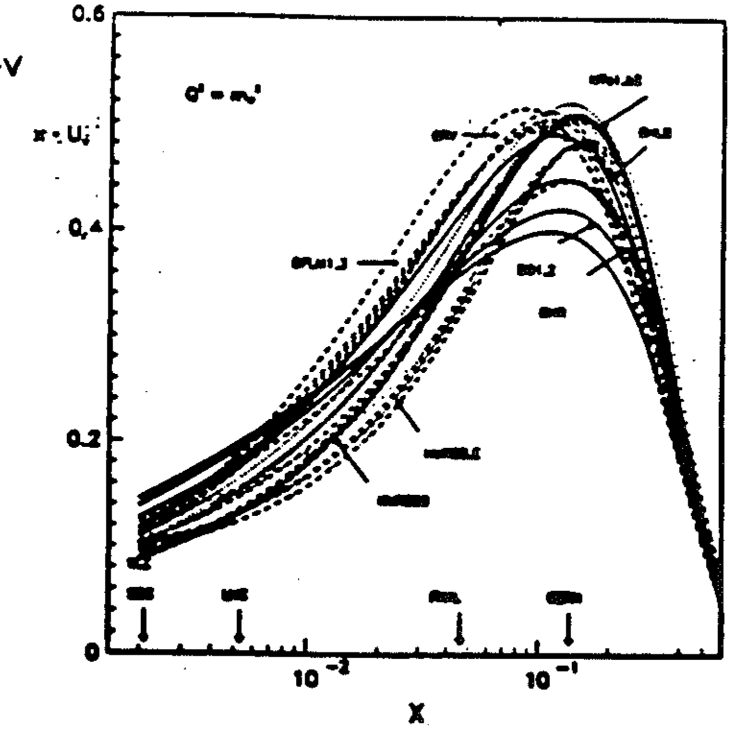


Figure 1: The valence u-quark distribution at $Q^2 = M_W^2$ [4].

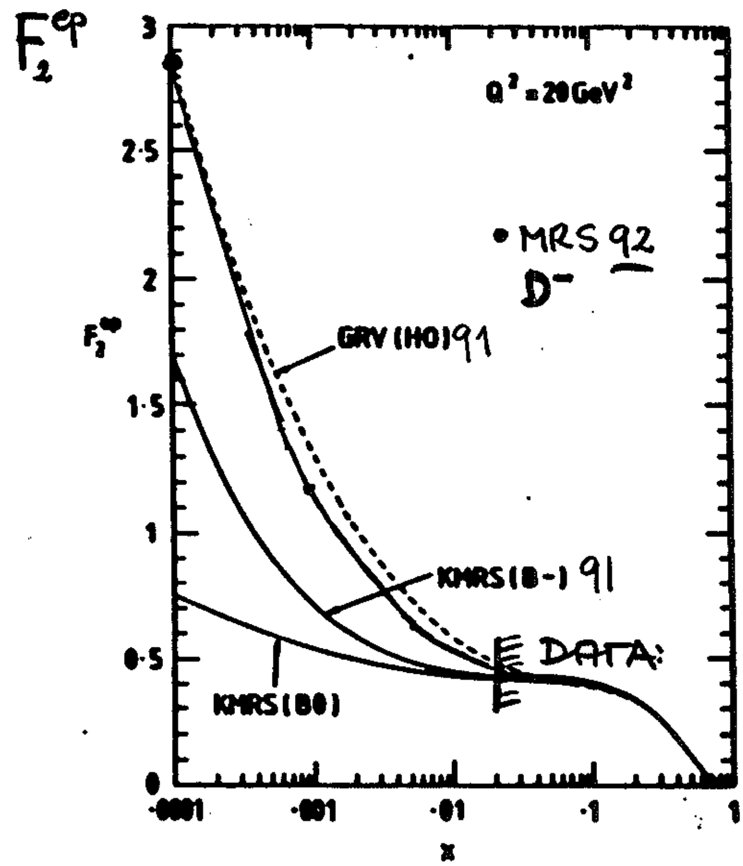


Figure 3: F_2^{ep} structure function predictions.

IDIS =1	Duke, Owens	set 1	(1984)	[12]
IDIS =2	Duke, Owens	set 2	(1984)	[12]
IDIS =3	Owens		(1991)	[13]
IDIS =4	Eichten et al.	set 1	(1984)	[14]
IDIS =5	Eichten et al.	set 2	(1984)	[14]
IDIS =6	Diemoz et al.	LO	(1988)	[15]
IDIS =7	Diemoz et al.	NTLO	(1988)	[15]
IDIS =8	Harriman et.al.	EMC	(1990)	[16]
IDIS =9	Harriman et.al.	BCDMS	(1990)	[16]
IDIS =10	Morfin,Tung	LO BCDMS+EMC SU(3) symm.sea	(1991)	[17]
IDIS =11	Morfin,Tung	DIS,BCDMS+EMC SU(3) symm.sea	(1991)	[17]
IDIS =12	Morfin,Tung	DIS,BCDMS+EMC SU(3) non-symm.sea	(1991)	[17]
IDIS =13	Morfin,Tung	DIS,BCDMS1,SU(3) symm.sea	(1991)	[17]
IDIS =14	Morfin,Tung	DIS,BCDMS2,SU(3) symm.sea	(1991)	[17]
IDIS =15	Morfin,Tung	DIS,EMC,SU(3) symm.sea	(1991)	[17]
IDIS =16	Morfin,Tung	MS,BCDMS+EMC SU(3) symm.sea	(1991)	[17]
IDIS =17	Morfin,Tung	MS,BCDMS+EMC SU(3) non-symm.sea	(1991)	[17]
IDIS =18	Morfin,Tung	MS,BCDMS1,SU(3) symm.sea	(1991)	[17]
IDIS =19	Morfin,Tung	MS,BCDMS2,SU(3) symm.sea	(1991)	[17]
IDIS =20	Morfin,Tung	MS,EMC,SU(3) symm.sea	(1991)	[17]
IDIS =21	Kwiecinski et al.	set B0	(1990)	[18]
IDIS =22	Kwiecinski et al.	set B-	(1990)	[18]
IDIS =23	Kwiecinski et al.	set B-, weak shadowing	(1990)	[18]
IDIS =24	Kwiecinski et al.	set B-, strong shadowing	(1990)	[18]
IDIS =25	Glück et al.	LO	(1991)	[19]
IDIS =26	Glück et al.	NTLO	(1991)	[19]

⋮ (1992)

⋮ etc.

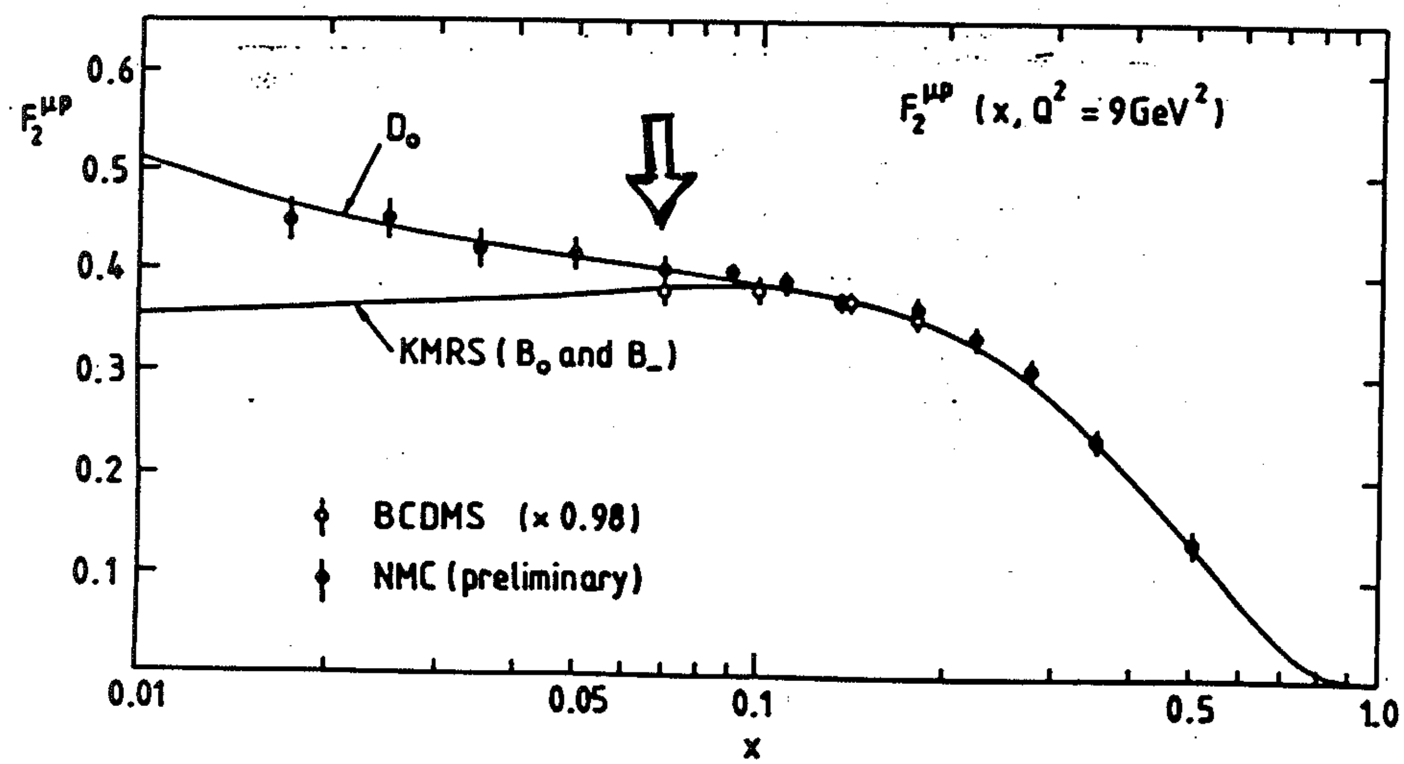
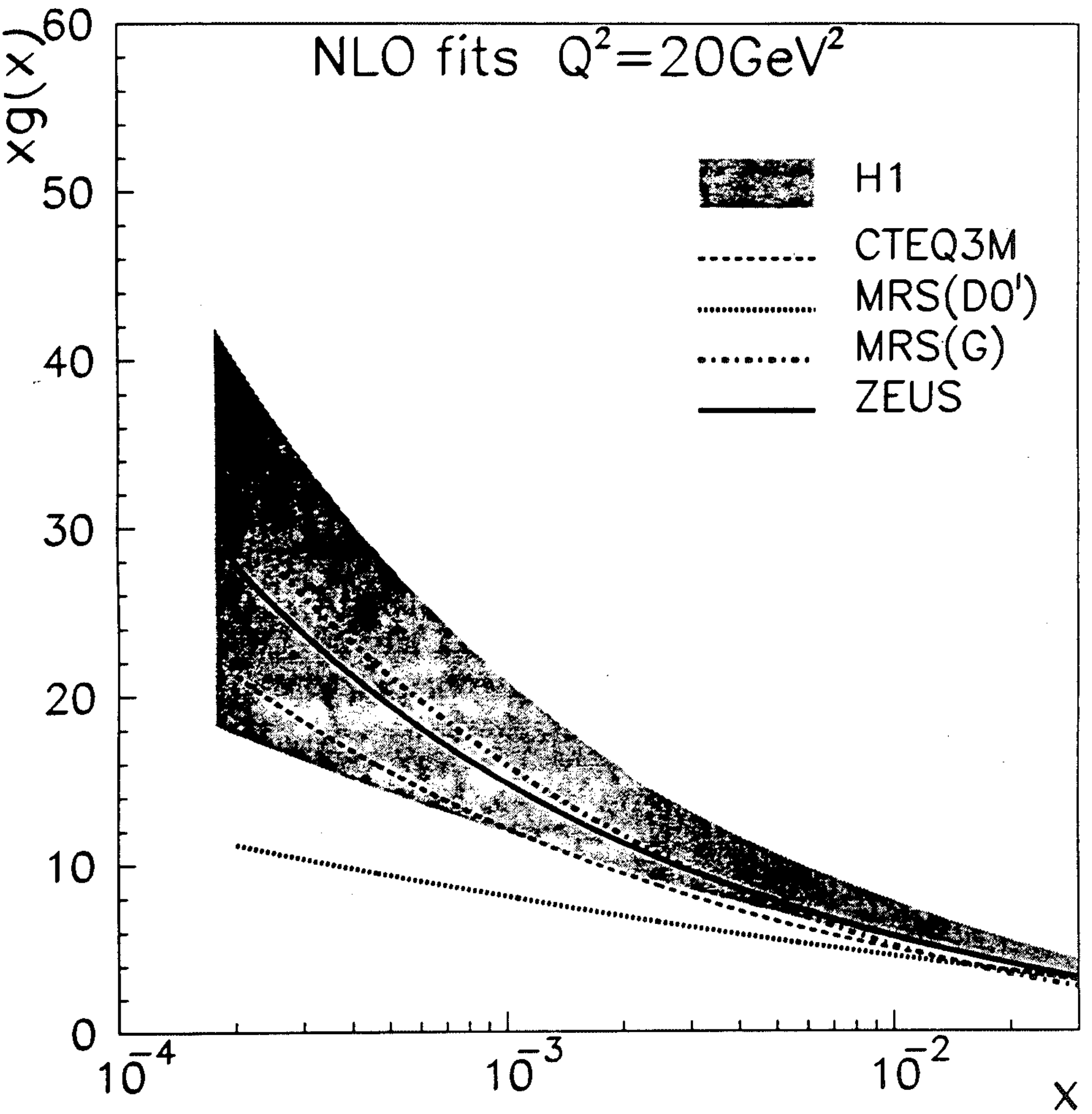


Fig. 4. $F_2(x, Q^2 = 9\text{GeV}^2)$ as measured from NMC and BCDMS, compared with the extrapolation of the earlier KMRS and with the new MRS (labelled D_0) parametrization of parton densities.



NLO fits $Q^2=20GeV^2$
xg(x)
H1
CTEQ3M
MRS(D0')
MRS(G)
ZEUS
0
10
20
30
40
50
60
10^{-4}
10^{-3}
10^{-2}
x

DIS & TARGET POLARIZATION

SO FAR : UNPOLARIZED p, N DIS WAS CONSID

WE INTRODUCE NOW TARGET POLARIZATION EXPLICITELY.

$$1^\circ, \gamma^*: \quad \frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}$$

$$L^{\mu\nu} := L^{\mu\nu}(k, s; k', s')$$

$$\equiv L_S^{\mu\nu}(k, k') + i L^A_{\mu\nu}(k, s; k') + L'^S_{\mu\nu}(k, s, k', s' \ldots$$

$$+ i L'^A_{\mu\nu}(k; k', \ldots$$

$$L^S_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(kk' - m^2)$$

$$L^A_{\mu\nu} = m\, \varepsilon_{\mu\nu\alpha\beta}\, s^\alpha (k - k')^\beta$$

$$L'^S_{\mu\nu} = (ks')(k'_\mu s_\nu + s_\mu k'_\nu - g_{\mu\nu} k'\cdot s)$$

$$- (kk' - m^2)(s_\mu s'_\nu + s'_\mu s_\nu - g_{\mu\nu} s\cdot s')$$

$$+ (k's)(s'_\mu k_\nu - k_\mu s'_\nu) - (ss')(k_\mu k'_\nu + k'_\mu k_\nu)$$

$$L'^A_{\mu\nu} = m\, \varepsilon_{\mu\nu\alpha\beta}\, s'^\alpha (k - k')^\beta$$

$$W_{\mu\nu}(q;P,S) = W^{(S)}_{\mu s}(q;P) + i\, W^{(A)}_{\mu\nu}(q;P,S)$$

$$\frac{1}{2M} W^{(S)}_{\mu\nu}(q;P) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(P.q, q^2) + \left[\left(P_\mu - q_\mu \frac{Pq}{q^2}\right)\left(P_\nu - q_\nu \frac{Pq}{q^2}\right)\right] \frac{1}{M^2} W_2(Pq, q^2)$$

$$\frac{1}{2M} W^{(A)}_{\mu\nu}(q;P,S) = \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \left\{ M S^\beta G_1(P.q, q^2) + [(P.q) S^\beta - (S q) P^\beta] \frac{G_2(P.q, q^2)}{M} \right\}$$

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2M q^4} \frac{E'}{E} \left\{ L^S_{\mu\nu} W^{\mu\nu,S} + L'^S_{\mu\nu} W^{\mu\nu,S} - L^A_{\mu\nu} W^{\mu\nu A} - L'^A_{\mu\nu} W^{\mu\nu,A} \right\}$$

$$-\sum_{s'} \left[\frac{d^2\sigma}{d\Omega dE'}(S) - \frac{d^2\sigma}{d\Omega dE'}(-S) \right] = \frac{2\alpha^2}{M q^4} \frac{E'}{E} L^A_{\mu\nu} W^{\mu\nu,}$$

SYST. ERRORS → MEASURE RATIOS.

2 OBSERVABLES: (γ^* ONLY)

$$A_{\parallel} \equiv \frac{d^2\sigma^{\rightrightarrows} - d^2\sigma^{\rightleftarrows}}{d^2\sigma^{\rightrightarrows} + d^2\sigma^{\rightleftarrows}}$$

$$A_{\perp} \equiv \frac{d^2\sigma^{\rightarrow\downarrow} - d^2\sigma^{\rightarrow\uparrow}}{d^2\sigma^{\rightarrow\downarrow} + d^2\sigma^{\rightarrow\uparrow}}$$

$$A_{\parallel} = \frac{Q^2}{2EE'} \frac{\left[(E + E'\cos\theta) M G_1 - Q^2 G_2\right]}{\left[2W_1 \sin^2(\theta/2) + W^2\cos^2(\theta/2)\right]}$$

$$A_{\perp} = \frac{Q^2 \sin\theta \left[M G_1 + 2E G_2\right]}{2E\left[2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)\right]} \cos\phi$$

$k' = E'(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$s = (\sin\alpha \cos\beta, \sin\alpha\sin\beta, \cos\alpha)$

$\phi := \beta - \varphi$

BJORKEN LIMIT:

$$\lim_{Bj} M W_1 (P.q, Q^2) = F_1(x)$$

$$\lim_{Bj} \nu W_2 (P.q, Q^2) = F_2(x)$$

$$\lim_{Bj} \frac{(P.q)^2}{\nu} G_1 (P.q, Q^2) = g_1(x)$$

$$\lim_{Bj} \nu (P.q) G_2 (P.q, Q^2) = g_2(x)$$

$$Q^2 \equiv -q^2 \to \infty$$

$$\nu = |E - E'| \to \infty$$

$$x = Q^2 / 2M\nu = \text{fixed}$$

$$Pq = \frac{Q^2}{2x} \; ; \quad \nu = \frac{Q^2}{2Mx} \; ;$$

$$\frac{(Pq)^2}{\nu} \equiv \frac{Q^4}{4x^2} \frac{2Mx}{Q^2} = \frac{Q^2 M}{2x} \; ; \quad \nu (Pq) = \frac{Q^4}{4Mx^2} .$$

$$\frac{d^2 \sigma_{nc}^{\rightrightarrows}}{dx\,dy} - \frac{d^2 \sigma_{nc}^{\rightleftarrows}}{dx\,dy} \approx -16\pi M E \frac{\alpha^2}{Q^4} x y (2-y) g_1^{\gamma}(x, Q^2)$$

$$\frac{d^2 \vec{\sigma}_{nc}^{\uparrow}}{dx\,dy\,d\varphi} - \frac{d^2 \vec{\sigma}_{nc}^{\downarrow}}{dx\,dy\,d\varphi} \approx -8M \frac{\alpha^2}{Q^4} \cos\phi \sqrt{2x(1-y) ME}\; x \cdot \left[y\, g_1^{\gamma}(x, Q^2) + 2 g_2^{\gamma}(x, Q^2) \right]$$

γZ, Z^2 & W^2 TERMS

ANSELMINO
EFREMOV
LEADER

NC:

$$\left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} + \left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+}$$

$$+\left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} + \left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+} \approx$$

$$\approx 64\pi ME\frac{\alpha^2}{Q^4}\left\{(1-y)\left[g_V\eta^{\gamma Z}(\underline{g_3^{\gamma Z}} - \underline{g_4^{\gamma Z}}) + g_A^2\eta^Z(\underline{g_3^Z} - \underline{g_4^Z})\right]\right.$$
$$\left. + xy^2\left[g_V\eta^{\gamma Z}\,\underline{g_5^{\gamma Z}} + g_A^2\eta^Z\,\underline{g_5^Z}\right]\right\}$$

$$\left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} - \left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+}$$

$$+\left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} - \left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+} \approx$$

$$\approx 64\pi ME\,\frac{\alpha^2}{Q^4}\,xy(2-y)\,g_A\eta^{\gamma Z}\,\underline{g_1^{\gamma Z}}$$

$$\left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} - \left(\frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+}$$

$$-\left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^-} + \left(\frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{nc}^{\overset{\leftarrow}{\Leftarrow}}}{dx\,dy}\right)_{\ell^+} \approx$$

$$\approx -64\pi ME\frac{\alpha^2}{Q^4}\left\{(1-y)\,g_A\eta^{\gamma Z}(\underline{g_3^{\gamma Z}} - \underline{g_4^{\gamma Z}}) + xy^2\,g_A\eta^{\gamma Z}\,\underline{g_5^{\gamma Z}}\right\}$$

CC:

$$\eta_\gamma = 1,\quad \eta_{\gamma Z} = \left(\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\right)\frac{Q^2}{Q^2+M_Z^2}$$
$$\eta_Z = \eta_{\gamma Z}^2$$

$$\frac{d^2\sigma_{cc}^{\overset{\rightarrow}{\Rightarrow}}}{dx\,dy} - \frac{d^2\sigma_{cc}^{\overset{\rightarrow}{\Leftarrow}}}{dx\,dy} \approx 64\pi ME\frac{\alpha^2}{Q^4}\eta^W$$
$$\times\left[\pm xy(2-y)\,\underline{g_1^{W\mp}} + (1-y)(\underline{g_3^{W\mp}} - \underline{g_4^{W\mp}}) + xy^2\,\underline{g_5^{W\mp}}\right]$$

$$\frac{d^3\sigma_{cc}^{\rightarrow\Uparrow}}{dx\,dy\,d\varphi} - \frac{d^3\sigma_{cc}^{\rightarrow\Downarrow}}{dx\,dy\,d\varphi} \approx 32M\frac{\alpha^2}{Q^4}\eta^W\cos\phi\,\sqrt{2xy(1-y)ME}$$
$$\times\left[\pm xy\,\underline{g_1^{W\mp}} \pm 2x\,\underline{g_2^{W\mp}} + \frac{1}{2}\,\underline{g_3^{W\mp}} + \frac{1-y}{y}\,\underline{g_4^{W\mp}} - xy\,\underline{g_5^{W\mp}}\right].$$

$$\eta_W = \frac{1}{2}\left(\frac{G_F M_W^2}{4\pi\alpha}\,\frac{Q^2}{Q^2+M_W^2}\right)^2$$

THE STRUCTURE FUNCTIONS IN THE NAIVE PARTON MODEL

γ^*:

$$F_1(x) = \sum_{q,\bar{q}} \frac{1}{2} e_q^2\, q(x) \qquad g_1(x) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\, \Delta q(x, \ldots$$

$$F_2(x) = x\sum_{q,\bar{q}} e_q^2\, q(x) = 2xF_1(x) \qquad g_2(x) = 0$$

γZ:

$$F_1^{\gamma Z} = \sum_q e_q g_V^q\, (q+\bar{q}) \qquad g_1^{\gamma Z} = \sum e_q g_V^q (\Delta q + \Delta\bar{q} \ldots$$

$$F_2^{\gamma Z} = 2x F_1^{\gamma Z} \qquad g_2^{\gamma Z} = g_4^{\gamma Z} = 0$$

$$F_3^{\gamma Z} = 2\sum_q e_q g_A^q\, (q-\bar{q}) \qquad g_3^{\gamma Z} = 2x\sum e_q g_A^q (\Delta q - \Delta\bar{q} \ldots$$

$$\equiv 2x\, g_5^{\gamma Z}$$

$|Z|^2$

$$F_1^Z = \frac{1}{2}\sum_q (g_V^{q2} + g_A^{q2})(q+\bar{q}) \qquad g_1^Z = \frac{1}{2}\sum_q (g_V^{q2} + g_A^{q2})(\Delta q + \Delta\bar{q} \ldots$$

$$F_2^Z = 2x F_1^Z \qquad g_2^Z = -\frac{1}{2}\sum g_A^{q2}(\Delta q + \Delta\bar{q})$$

$$F_3^Z = 2\sum_q g_V^q g_A^q\, (q-\bar{q}) \qquad g_3^Z = 2x\sum g_A^q g_V^q (\Delta q - \Delta \ldots$$

$$\equiv 2x\, g_5^Z$$

$$g_V^q = \begin{cases} \frac{1}{2} - \frac{4}{3}\sin^2\theta_W & (u) \\ -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W & (d) \end{cases} \qquad g_A^q = \begin{cases} \frac{1}{2} & (u) \\ -\frac{1}{2} & (d) \end{cases}$$

$|W|^2$:

$$F_1^{W^-} = U + \bar{D}$$

$$F_2^{W^-} = 2 \times F_1^{W^-}$$

$$F_3^{W^-} = 2(U - \bar{D})$$

$$g_1^{W^-} = (\Delta U + \Delta D) = -2 g_2^{W^-}$$

$$g_3^{W^-} = 2 \times (\Delta U - \Delta D) = 2 \times g_5^{W^-}$$

$$g_4^{W^-} = 0.$$

THE STRUCTURE FUNCTIONS IN THE NAIVE PARTON MODEL

γ^*:

$$F_1(x) = \sum_{q,\bar{q}} \frac{1}{2} e_q^2\, q(x) \qquad g_1(x) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2\, \Delta q(x,S)$$

$$F_2(x) = x\sum_{q,\bar{q}} e_q^2\, q(x) = 2xF_1(x) \qquad g_2(x) = 0$$

γZ:

$$F_1^{\gamma Z} = \sum_q e_q g_V^q (q+\bar{q}) \qquad g_1^{\gamma Z} = \sum e_q g_V^q (\Delta q + \Delta\bar{q})$$

$$F_2^{\gamma Z} = 2x F_1^{\gamma Z} \qquad g_2^{\gamma Z} = g_4^{\gamma Z} = 0$$

$$F_3^{\gamma Z} = 2\sum_q e_q g_A^q (q-\bar{q}) \qquad g_3^{\gamma Z} = 2x\sum e_q g_A^q (\Delta q - \Delta\bar{q}) \equiv 2x\, g_5^{\gamma Z}$$

$|Z|^2$

$$F_1^Z = \frac{1}{2}\sum_q (g_V^{q2} + g_A^{q2})(q+\bar{q}) \qquad g_1^Z = \frac{1}{2}\sum_q (g_V^{q2} + g_A^{q2})(\Delta q + \Delta\bar{q})$$

$$F_2^Z = 2x F_1^Z \qquad g_2^Z = -\frac{1}{2}\sum g_A^{q2} (\Delta q + \Delta\bar{q})$$

$$F_3^Z = 2\sum_q g_V^q g_A^q (q-\bar{q}) \qquad g_3^Z = 2x\sum g_A^q g_V^q (\Delta q - \Delta\bar{q}) \equiv 2x\, g_5^Z$$

$$g_V^q = \begin{cases} \frac{1}{2} - \frac{4}{3}\sin^2\theta_W & (u) \\ -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W & (d) \end{cases} \qquad g_A^q = \begin{cases} \frac{1}{2} & (u) \\ -\frac{1}{2} & (d) \end{cases}$$

3. The running coupling constant

- CENTRAL PARAMETER, NOT AN OBSERVABLE!
- CHARGE RENORMALIZATION IN QCD YIELDS: ($\overline{MS}$)

$$\frac{\partial \alpha_s(\mu^2)}{\partial \log \mu^2} = -\frac{\beta_0}{4\pi}\alpha_s^2 - \frac{\beta_1}{(4\pi)^2}\alpha_s^3 - \frac{\beta_2}{(4\pi)^3}\alpha_s^4 + \dots$$

$\beta_0 = 11 - \frac{2}{3}N_f$ GROSS, WILCZEK; POLITZER; T'HOOFT 1973

$\beta_1 = 102 - \frac{38}{3}N_f$ CASWELL; JONES 1974

$\beta_2 = \frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2$

TARASOV, VLADIMIROV, ZHARKOV 1980

LARIN, VERMASEREN 1993

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s(Q_0^2)} + \frac{\beta_0}{4\pi} \log\left(\frac{Q^2}{Q_0^2}\right)$$

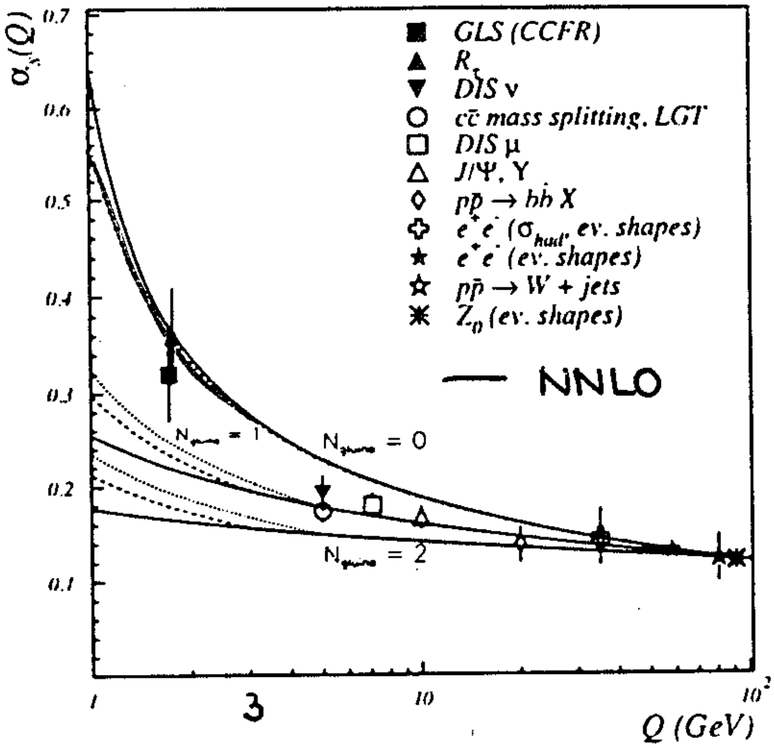
$$+ \phi^{(n)}(\alpha_s(Q^2); \beta_i) - \phi^{(n)}(\alpha_s(Q_0^2); \beta_i)$$

$$\phi_{(n)}(x; \beta_i) = -\frac{\beta_1}{8\pi\beta_0} \ln\left|\frac{16\pi^2 x^2}{16\pi^2\beta_0 + 4\beta_1\pi x + \beta_2 x^2}\right|$$

$$+ \frac{\beta_1^2 - 2\beta_0\beta_2}{8\pi\beta_0\sqrt{4\beta_2\beta_0 - \beta_1^2}} \arctan\left(\frac{2\pi\beta_1 + \beta_2 x}{2\pi\sqrt{4\beta_0\beta_2 - \beta_1^2}}\right)$$

$N_f \leqq 5 : \quad 4\beta_0\beta_2 - \beta_1^2 > 0$

$N_f = 6 : \quad 4\beta_0\beta_2 - \beta_1^2 < 0 \;!$



JB, J. BOTTS
1994

Fig. 1. Comparison of different theoretical predictions for $\alpha_s(Q^2)$ with experimental results of α_s [1]. The full curves denote the NLO solution of Eq. (2) for $N_{\tilde{g}} = 0, 1, 2$ with $m_{\tilde{g}} = 0$ taking $\alpha_s(Q_0^2) = \alpha_s(M_Z^2) = 0.122$. The dash-dotted line denotes the NNLO solution in the case of QCD. The dashed and dotted lines describe the cases $m_{\tilde{g}} = 3$ and 5 GeV, respectively.

Process	Ref.	⟨Q⟩ [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
• DIS $\nu F_2, F_3$		5	$.193^{+0.019}_{-0.018}$	.111 ± .006	.004	.004	NLO
• DIS μF_2		7.1	.180 ± .014	.113 ± .005	.003	.004	NLO
GLS (CCFR)	[15]	1.73	0.24 ± 0.047	$0.107^{+0.007}_{-0.009}$	$^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	NNLO
R_τ (CLEO)	[16]	1.78	0.302 ± 0.024	0.116 ± 0.003	0.002	0.002	NNLO
R_τ (ALEPH)	[17]	1.78	0.355 ± 0.021	0.122 ± 0.003	0.002	0.002	NNLO
R_τ (OPAL)	[17]	1.78	$0.375^{+0.032}_{-0.025}$	0.123 ± 0.003	0.002	0.002	NNLO
R_τ (Raczka)	[18]	1.78	0.333 ± 0.021	0.120 ± 0.003	0.002	0.002	NNLO
$\eta_c \to \gamma\gamma$ (CLEO)	[16]	2.98	0.187 ± 0.029	0.101 ± 0.010	0.008	0.006	NLO
$Q\overline{Q}$ states	[19]	5.0	0.188 ± 0.018	0.110 ± 0.006	0.000	0.006	q LGT
$b\overline{b}$ states	[19]	5.0	0.203 ± 0.007	0.115 ± 0.002	0.000	0.002	LGT
Υ(1S) (CLEO)	[16]	9.46	0.164 ± 0.013	0.111 ± 0.006	0.001	0.006	NLO
• $e^+e^- \to$ jets (CLEO)	[16]	10.53	0.164 ± 0.015	0.113 ± 0.006	0.002	0.006	NLO
$ep \to$ jets (H1)	[20]	5 - 60		0.123 ± 0.018	0.014	0.010	NLO
$p\overline{p} \to$ W jets (D0)	[21]	80.6	0.123 ± 0.015	0.121 ± 0.014	0.012	0.005	NLO
• $e^+e^- \to Z^0$:							
• scal. viol. (ALEPH)	[17]	91.2		0.127 ± 0.011	–	–	NLO
• ev. shapes (SLD)	[22]	91.2		0.120 ± 0.008	0.003	0.008	resum.
• $\Gamma(Z^0 \to$ had.) (LEP)	[23]	91.2		0.127 ± 0.006	0.005	$^{+0.003}_{-0.004}$	NNLO

QUARKS

Table 1. Summary of most recent measurements of α_s, presented at this conference. Abbreviations: GLS-SR = Gross-Llewellyn-Smith sum rules; (N)NLO = (next-)next-to-leading order perturbation theory; LGT = lattice gauge theory (q stands for quenched approximation); resum. = resummed next-to-leading order. Most results are still preliminary.

S. BETHKE 1995

DIS: $\overline{\alpha}_s(M_Z) = 0.112 \pm 0.004$

e^+e^-: $\overline{\alpha}_s(M_Z) = 0.121 \pm 0.004$

TWO CLUSTERS !

- LGT WITHIN BETWEEN

MORE CALCULATIONS NEEDED

→ PROPER TREATMENT OF QUARKS

Q14

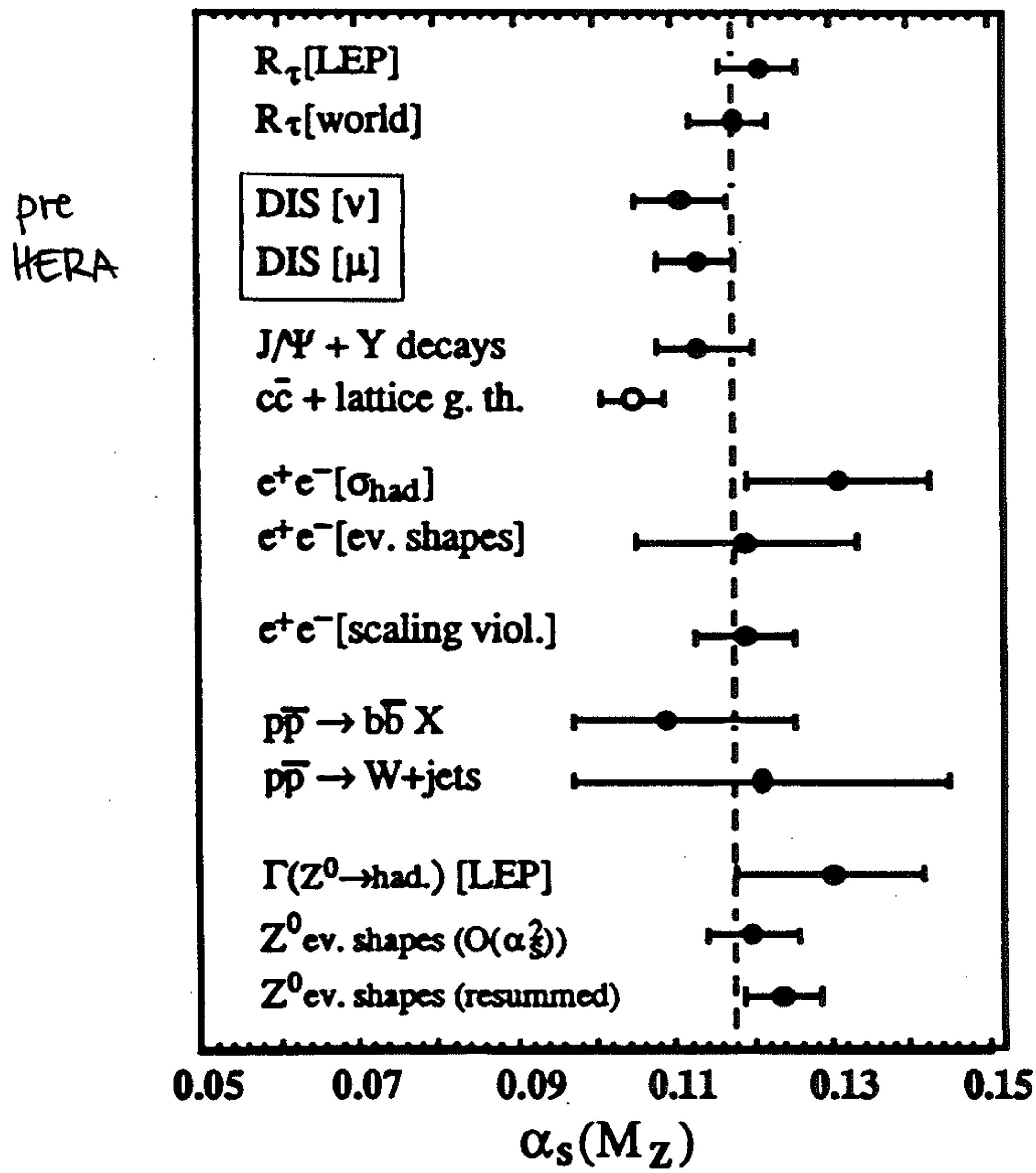


Fig. 31. Summary of measurements of $\alpha_s(M_{Z^0})$.

OTHER OBSERVABLES

S. BETHKE

Table 4. Processes and Observables from which significant determinations of α_s are derived.

Process	Observable	Theory	Caveats
e^+e^-	hadronic event shapes, jet production rates, energy correlations	NLO and re-summed NLO	hadronization corrections
	$R_Z = \frac{\Gamma(Z^0 \to \text{hadrons})}{\Gamma(Z^0 \to \text{leptons})}$	NNLO	small QCD corrections
	$R_\tau = \frac{Br(\tau \to \text{hadrons})}{Br(\tau \to e\nu)}$	NNLO	nonperturbative corrections
	scaling violations in $\frac{d\sigma}{dx}$ spectra	NLO	only through MC models
	$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to \mu^+\mu^-)}$; .... ; J/Ψ; ...	NLO	relativistic corrections
DIS	$\frac{d \ln F_2(x,Q^2)}{d \ln Q^2}$	NLO	higher twist; $g(x,Q^2)$
	$\frac{d \ln F_3(x,Q^2)}{d \ln Q^2}$	NLO	higher twist
$p\bar{p}$	$p\bar{p} \to W + \text{jets}$	NLO	statistics; k-factors
	$p\bar{p} \to b\bar{b}X$	NLO	statistics; exp. systematics
$c\bar{c}$ states	mass difference of 1s and 1p charmonium states	lattice gauge theory	quenched approximation

S.BETHKE, 1995

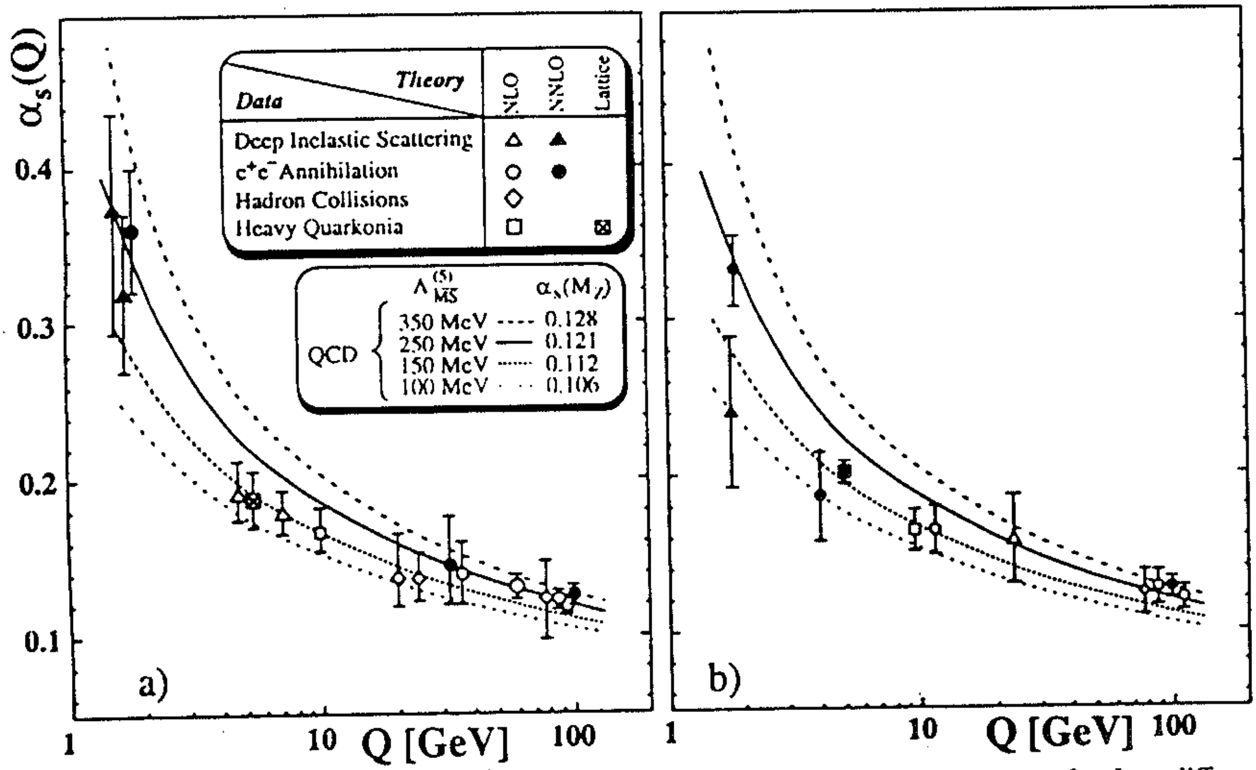


Figure 1. A Summary of measurements of α_s, compared with QCD expectations for four different values of $\Lambda_{\overline{MS}}$ which are given for $N_f = 5$ quark flavours. (a): Status before this conference. (b): Newest and mostly preliminary results, from Table 1. Curves and symbols are the same as in a).

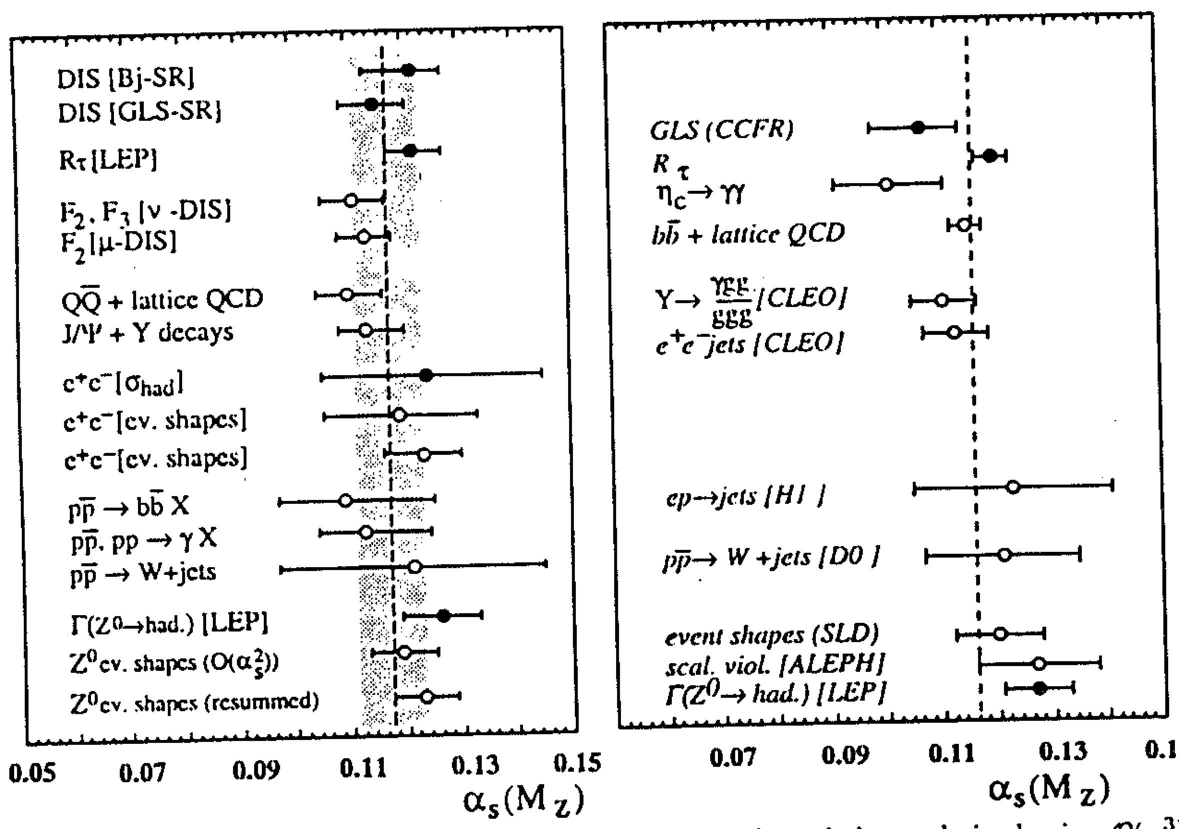
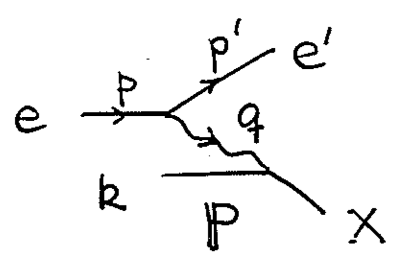


Figure 2. A Summary of measurements of $\alpha_s(M_{Z^0})$. Filled symbols are derived using $O(\alpha_s^3)$ QCD; open symbols are in $O(\alpha_s^2)$ or based on lattice calculations. (a): Status before this conference; vertical line and shaded area represent the world average of $\alpha_s(M_{Z^0}) = 0.117 \pm 0.006$. (b): Newest and mostly preliminary results, from Table 1; vertical line represents $\alpha_s(M_{Z^0}) = 0.116$.

SPLITTING FUNCTIONS AND ANOMALOUS DIMENSIONS

1) THE WEIZSÄCKER-WILLIAMS APPROXIMATION:



$$d\sigma_{ep} = \frac{1}{8\,kp}\,\frac{e^2 W_{\mu\nu} L^{\mu\nu}}{q^4}\,\frac{d^3p'}{(2\pi)^3\,2E'}$$

$$L_{\mu\nu} = 4\left[\tfrac{1}{2} q^2 g_{\mu\nu} + p_\mu p'_\nu + p_\nu p'_\mu\right] \quad ; \quad q_\mu L^{\mu\nu} = q_\nu L^{\mu\nu} = 0$$

⟶ decompose $W_{\mu\nu}$ into $W_1(q^2, k.q)$, $W_2(q^2, k.q)$

⟶ consider $\lim_{q^2 \to 0}$.

$$W_2(q^2, k.q) = W_1(0, k.q) + O(q^2)$$

$$L_{\mu\nu} W^{\mu\nu} = -4\,W_1(0, k.q)\left[2m_e^2 + q^2\,\frac{1+(1-y)^2}{y^2}\right]$$

$y = \frac{kq}{kp}$. $\frac{d^3p'}{E'} = \pi\, dq^2\, dy$

$$d\sigma_{ep} = -\frac{\alpha}{2\pi}\,\frac{W_1(0, k.q)}{4k.q}\left[\frac{2m_e^2}{q^4} + \frac{1+(1-y)^2}{y^2 q^2}\right] dq^2\, dy$$

FURTHERMORE,

$$\sigma_{\gamma p}(q, k) = -\frac{g_{\mu\nu} W^{\mu\nu}}{8k.q} = \frac{W_1(0, k.q)}{4\,k.q}.$$

TU

DEFINE NOW:

$$d\sigma^{ep} = \sigma^{\gamma p} f_{\gamma/e}(y)\, dy$$

$$f_{\gamma/e}(y) = \frac{\alpha}{2\pi}\left[2m_e^2 y \left(\frac{-1}{Q^2_{min}} + \frac{1}{Q^2_{max}}\right) + \frac{1+(1-y)^2}{y}\log\frac{Q^2_{max}}{Q^2_{min}}\right]$$

Frixione et al. '93

Q^2_{max} – process dependent! $= Q^2$

$$Q^2_{min} = \frac{m_e^2 y^2}{1-y}$$

$$f_{\gamma/e}(y) = \frac{\alpha}{2\pi}\left[-\frac{2(1-y)}{y} + \frac{2m_e^2 y}{Q^2} + \frac{1+(1-y)^2}{y}\log\left(\frac{1-y}{m_e^2 y^2}Q^2\right)\right]$$

$\ll$ small.

$$\boxed{f_{\gamma/e}(y) = \frac{\alpha}{2\pi}\left[\frac{1+(1-y)^2}{y}\right]\left[\log\frac{Q^2}{m_e^2} + \log\left(\frac{1-y}{y^2}\right)\right]}$$

QED: $P_{\gamma/e}(z) :=_{Df.} \dfrac{1+(1-z)^2}{z}$

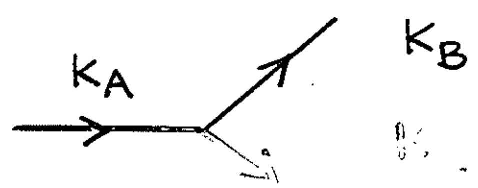
QCD: $P_{g/q}(z) = C_F \dfrac{1+(1-z)^2}{z}$, $C_F = \dfrac{\mathrm{tr}(t_a t_a)}{N_c}$

2) LO SPLITTING FUNCTIONS:

APPLY THE TECHNIQUE OF THE WWA TO MASSLESS PARTICLE RADIATION IN THE RESP. FIELD THEORY:

QCD, QED: FERMIONS, VECTORS.

(GLAPD)



INFINITE MOMENTUM FRAME: $\mathbb{P}$ - LARGE

$$\left.\begin{aligned} K_A &= (P;\, P,\, \vec{0}) \\ K_B &= \left(zP + \frac{p_\perp^2}{2zP};\, zP,\, \vec{p}_\perp\right) \\ K_C &= \left((1-z)P + \frac{p_\perp^2}{2(1-z)P};\, (1-z)P,\, -\vec{p}_\perp\right) \end{aligned}\right\} \text{KINEMATICS}$$

TO CALCULATE:

$$d\hat{P}_{BA}(z)\, dz$$

i.e. THE PROBABILITY TO FIND A PARTON B IN THE PARTON (OR PARTICLE) A.

ONE OBTAINS BY SIMILAR STEPS:

$$d\hat{P}_{BA}(z)\, dz = \frac{E_B}{E_A} g^2 \frac{|M_{A\to B+C}|^2}{(2E_B)^2 (E_B+E_C-E_A)^2} \frac{d^3 k_C}{(2\pi)^3 (2E_C)}$$

$$\frac{E_B}{E_A} = z$$

$$(2E_B)^2 (E_B+E_C-E_A)^2 = \frac{(p_\perp^2)^2}{(1-z)^2}$$

$$\frac{d^3 k_C}{(2\pi)^3 (2E_C)} = \frac{dz\, dp_\perp^2}{16\pi^2 (1-z)}.$$

$$\boxed{d\hat{P}_{BA}(z) = \frac{\alpha}{2\pi} \frac{z(1-z)}{2} \frac{1}{p_\perp^2} \overline{|M_{A\to B+C}|^2}\, d\ln p_\perp^2}$$

THE DIFFERENT SPLITTING FUNCTIONS ARE OBTAINED EVALUATING $\overline{|M_{A\to B+C}|^2}$.

$$\therefore \overline{\sum_{spins} |M_{G\to q\bar{q}}|^2} = \frac{1}{2} \mathrm{Tr}\,(\not{k}_C \gamma_\mu \not{k}_B \gamma_\nu) \frac{1}{2} \sum_{pol} \epsilon_\mu^* \epsilon_\nu$$

$$= p_\perp^2 \left(\frac{1-z}{z} + \frac{z}{1-z} \right).$$

(*) YIELDS:

$$d\hat{P}_{BA}(z) = \left(\frac{\alpha}{2\pi} \left(\frac{1}{2}\right) d\ln p_\perp^2 \right) \left[(1-z)^2 + z^2 \right]$$

$$\rightarrow \quad P_{BA}(z) = P_{qG}(z) = \frac{1}{2}\left[(1-z)^2 + z^2 \right].$$

FINALLY ONE HAS: FOR $z<1$

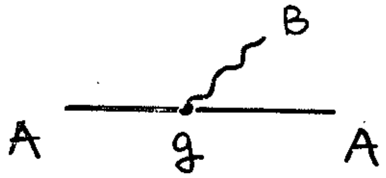
$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}$$

$$P_{qG}(z) = T(R)\left[(1-z)^2 + z^2\right]$$

$$P_{Gq}(z) = C_F \frac{1+(1-z)^2}{z}$$

$$P_{GG}(z) = 2C_G\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

THE DIAGONAL TERMS:



$$P_{AA}(z,g) = P^{(0)}_{AA}(z) + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^n P^{(n)}_{AA}(z)$$

$$\uparrow$$
$$\delta(1-z)$$

WEIERSTRASS: $\int_0^1 dz\, P^{(n)}_{AA}(z) = 0$; Λ i.e. $P^{(n)}_{AA}(z)$ IS A DISTRIBUTION

$$P_{AA}(z) = \tilde{P}_{AA}(z) - \delta(1-z)\int_0^1 dz\, \tilde{P}_{AA}(z)$$

$$\tilde{P}_{AA}(z) = P_{AA}(z)\Big|_{z<1.}$$ FOR FERMION (A) → BOSON (B)

:  ?

$$\int_0^1 dz\, z\left[2N_f P_{qG}(z) + P_{GG}(z)\right] = 0$$

CONSERVATION OF THE PROTON

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right]$$

$$P_{GG}(z) = 2\,C_G\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right]$$

$$+ \frac{1}{2}\,\beta_0\,\delta(1-z)$$

$$\int_0^1 dz\,\frac{f(z)}{(1-z)_+} = \int_0^1 dz\,\frac{f(z)-f(1)}{1-z}$$

⟹ THERE EXISTS A SERIES OF SYMMETRY RELATIONS, WHICH ALLOWS TO FIND ALL SPLITTING FUNCTIONS KNOWING ONE (QED!) & THE GROUP THEOR. FACTORS OF ~~THE~~ RESP. THEORY!

(cf. eg. DKMT)

(LO)!

QCD CORRECTIONS FOR DIS STRUCTURE FUNCTIONS

1) NTLO EVOLUTION EQUATIONS:

DEFINE COMBINATIONS OF PARTON DENSITIES:

$$q_i^- = q_i - \bar{q}_i$$
$$q_i^+ = q_i + \bar{q}_i \quad , \quad q^+ = \sum_{i=1}^{N_f} q_i^+$$
$$G$$

$$A(x) \otimes B(x) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) \, A(x_1) B(x_2)$$

$$\frac{d}{d \log Q^2} q_i^-(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} P^-(x, \alpha) \otimes q_i^-(x, Q^2)$$

$$\frac{d}{d \log Q^2} \left[q_i^+(x, Q^2) - \frac{1}{N_f} q^+(x, Q^2) \right]$$
$$= \frac{\alpha_s(Q^2)}{2\pi} P^+(x, \alpha) \otimes \left[q_i^+(x, Q^2) - \frac{1}{N_f} q^+(x, Q^2) \right]$$

$$\frac{d}{d \log Q^2} \begin{bmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{bmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \mathbb{P}(x, \alpha) \otimes \begin{bmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{bmatrix}$$

$$P^{\pm}(x,\alpha) = P_{NS}^{(0)}(x) + \frac{\alpha_s}{2\pi} P^{\pm,1}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{\pm,2}(x) + \ldots$$

$$\mathbb{P}(x,\alpha) = \mathbb{P}^{(0)}(x) + \frac{\alpha_s}{2\pi} \mathbb{P}^{(1)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathbb{P}^{(2)}(x) + \ldots$$

FACTORIZING THE PARTON DISTRIBUTIONS AT Q_0^2:

$$q_i^-(x,t) := E^-(x,t) \otimes q_i^-(x)$$

$$q_i^+(x,t) = E^+(x,t) \otimes q_i^+(x) + \frac{1}{N_f}\left[E_{11}(x,t) - E^+(x,t)\right] \otimes q^+(x)$$
$$+ \frac{1}{N_f} E_{12}(x,t) \otimes G(x).$$

$$\begin{bmatrix} q_i^+(x,t) \\ G(x,t) \end{bmatrix} = \mathbb{E}(x,t) \begin{bmatrix} q_i^+(x) \\ G(x) \end{bmatrix}$$

BOUNDARY CONDITIONS:

$$\lim_{t\to 0} E^{\pm}(x,t) = \delta(1-x)$$

$$\lim_{t\to 0} \mathbb{E}(x,t) = \mathbb{1}\cdot\delta(1-x).$$

$$t \underset{Df}{=:} -\frac{2}{\beta_0} \ln \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}$$ EVOLUTION VARIABLE.

CHANGE VARIABLES: $Q^2 \to t$

$$\frac{\alpha_s(Q^2)}{2\pi}\, d\log Q^2 = \left(1 - \frac{\beta_1}{2\beta_0}\frac{\alpha_s(Q^2)}{2\pi} + \dots\right) dt$$

EVOLUTION EQUS. FOR EVOLUTION OPERATORS:

$$\frac{d}{dt} E^{\pm}(x,t) = \left\{ P_{NS}(x) + \frac{\alpha_s(t)}{2\pi} R^{\pm}(x) + \dots \right\} \otimes E^{\pm}(x,t)$$

$$\frac{d}{dt} \mathbb{E}(x,t) = \left\{ \mathbb{P}^{0}(x) + \frac{\alpha(t)}{2\pi} \mathbb{R}(x) + \dots \right\} \otimes \mathbb{E}(x,t)$$

$$R^{\pm}(x) = P^{\pm,(1)}(x) - \frac{\beta_1}{2\beta_0} P_{NS}(x)$$

$$\mathbb{R}(x) = \mathbb{P}^{(1)}(x) - \frac{\beta_1}{2\beta_0} \mathbb{P}^{(0)}(x)$$

4.1. Splitting Functions

$O(\alpha_s)$: (LO)

$$P_{NS}^{(0)}(z) \equiv P_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z) \right]$$

$$P_{qg}(z) = T_f \left((1-z)^2 + z^2 \right)$$

$$P_{gq}(z) = C_F \, \frac{1+(1-z)^2}{z}$$

$$P_{gg}(z) = 2 C_G \left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) \right] + \frac{1}{2}\beta_0\,\delta(1-z)$$

GROSS, WILCZEK 1973
GEORGI, POLITZER 1973
GRIBOV, LIPATOV 1972
DOKSHITSER 1977
LIPATOV 1975

ALTARELLI, PARISI 1977
KIM, SCHILCHER 1977/78
et al.

$$\int_0^1 dz\; z^{N-1} P_{ab}^{(0)}(z) = -\frac{\gamma_{ab}^{0,N}}{4}$$

SPLITTING FUNCTION — ANOMALOUS DIMENSION

$O(\alpha_s^2)$ CONTR. DUE TO:

FLORATOS, D ROSS, SACHRAJDA 1977-79, BARDEEN, BURAS, DUKE, MUTA 1978
CURCI, FURMANSKI, PETRONZIO 1980
FURMANSKI, PETRONZIO 1980
GONZALEZ-ARROYO, LOPEZ, YNDURAIN 1979/80
FLORATOS, KOUNNAS, LACAZE 1981 a b c

71

NON- SINGLET :

$$P_\pm(x,\alpha) = \hat{P}_{qq}(x,\alpha) \pm \hat{P}_{q\bar{q}}(x,\alpha)$$

$\overline{MS}$

$$\hat{P}_{qq}(x,\alpha) = \left(\frac{\alpha}{2\pi}\right) C_F \left(\frac{1+x^2}{1-x}\right) + \left(\frac{\alpha}{2\pi}\right)^2 \left[C_F^2 P_F(x) + \tfrac{1}{2} C_F C_G P_G(x) + C_F N_F T_F P_{N_F}(x) \right], \tag{4.50}$$

$$\hat{P}_{q\bar{q}}(x,\alpha) = \left(\frac{\alpha}{2\pi}\right)^2 \left(C_F^2 - \tfrac{1}{2} C_F C_G\right) P_A(x), \tag{4.51}$$

$$P_F(x) = -2\frac{1+x^2}{1-x} \ln x \ln(1-x) - \left(\frac{3}{1-x} + 2x\right) \ln x - \tfrac{1}{2}(1+x)\ln^2 x - 5(1-x), \tag{4.52}$$

$$P_G(x) = \frac{1+x^2}{1-x}\left[\ln^2 x + \tfrac{11}{3}\ln x + \tfrac{67}{9} - \tfrac{1}{3}\pi^2\right] + 2(1+x)\ln x + \tfrac{40}{3}(1-x), \tag{4.53}$$

$$P_{N_F}(x) = \tfrac{2}{3}\left[\frac{1+x^2}{1-x}\left(-\ln x - \tfrac{5}{3}\right) - 2(1-x)\right], \tag{4.54}$$

$$P_A(x) = 2\frac{1+x^2}{1+x}\int_{x/(1+x)}^{1/(1+x)} \frac{\mathrm{d}z}{z} \ln\frac{1-z}{z} + 2(1+x)\ln x + 4(1-x). \tag{4.55}$$

TABLE 1

Detailed contribution of various diagrams to $\Gamma_{qq}(x, \alpha, 1/\varepsilon)$

	$\Gamma_{qq}(x, \alpha, 1/\varepsilon)$	C_F^2						$\frac{1}{2}C_F C_G$					$\frac{1}{2}N_F C_F$
							SUM					SUM	
$1/\varepsilon$ A	$\frac{1+x^2}{1-x}$	$7-\frac{2}{3}\pi^2$	$-7+\frac{2}{3}\pi^2$	0	0	0	0	$7-\frac{2}{3}\pi^2$	0	$-11+\pi^2$	$\frac{103}{9}-\frac{2}{3}\pi^2$	$\frac{67}{9}-\frac{1}{3}\pi^2$	$-\frac{10}{9}$
A	$\frac{1+x^2}{1-x}\ln^2 x$	-2	1	-1	2	0	0	-1	1	1	0	1	0
A	$(1+x)\ln x$	0	$-\frac{7}{2}$	2	-1	0	$-\frac{5}{2}$	$\frac{7}{2}$	-2	$\frac{1}{2}$	0	2	0
A	$1-x$	3	-11	0	3	0	-5	11	0	-1	$\frac{10}{3}$	$\frac{40}{3}$	$-\frac{4}{3}$
B	$(1+x)\ln^2 x$	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	0	0	0	0	0
B	$\frac{1+x^2}{1-x}\ln^2(1-x)$	0	0	0	0	0	0	0	0	2	-2	0	0
B	$\frac{1+x^2}{1-x}\ln x\ln(1-x)$	-4	2	0	0	0	-2	-2	0	6	-4	0	0
B	$\frac{1+x^2}{1-x}\ln x$	0	$-\frac{3}{2}$	0	0	0	$-\frac{3}{2}$	$\frac{3}{2}$	0	$-\frac{3}{2}$	$\frac{11}{3}$	$\frac{11}{3}$	$-\frac{2}{3}$
B	$\frac{1+x^2}{1-x}\ln(1-x)$	3	-3	4	-4	0	0	3	-4	5	-4	0	0
B	$(1-x)\ln x$	-4	2	0	3	0	1	-2	0	2	0	0	0
B	$(1-x)\ln(1-x)$	0	0	0	0	0	0	0	0	4	-4	0	0
B	$I_1\frac{1+x^2}{1-x}$	4	-4	0	0	0	0	4	0	-8	4	0	0
B	$I_0\frac{1+x^2}{1-x}$	0	0	4	-4	0	0	0	-4	8	-4	0	0
B	$I_0\frac{1+x^2}{1-x}(\ln x+\ln(1-x))$	-4	4	0	0	0	0	-4	0	8	-4	0	0
B	$I_0(1-x)$	-4	4	0	0	0	0	-4	0	8	-4	0	0

Appropriate colour factors are shown in the first line. Terms of type A satisfy the Gribov-Lipatov relation while those of type B break it.

SINGLET:

$P_{ij}^{(1)}(x)$:

$$\begin{aligned}\hat{P}^{(1,S)}_{FF} = {} & C_F^2[-1 + x + (\tfrac{1}{2} - \tfrac{3}{2}x)\ln x - \tfrac{1}{2}(1+x)\ln^2 x - (\tfrac{3}{2}\ln x + 2\ln x \ln(1-x))p_{FF}(x) + 2p_{FF}(-x)S_2(x)] \\ & + C_F C_G[\tfrac{14}{3}(1-x) + (\tfrac{11}{6}\ln x + \tfrac{1}{2}\ln^2 x + \tfrac{67}{18} - \tfrac{1}{6}\pi^2)p_{FF}(x) - p_{FF}(-x)S_2(x)] \\ & + C_F T_R N_F[-\tfrac{16}{3} + \tfrac{40}{3}x + (10x + \tfrac{16}{3}x^2 + 2)\ln x - \tfrac{112}{9}x^2 + \tfrac{40}{9}x^{-1} - 2(1+x)\ln^2 x - (\tfrac{10}{9} + \tfrac{2}{3}\ln x)p_{FF}(x)]\,,\end{aligned}$$

$$\begin{aligned}\hat{P}^{(1,S)}_{FG} = {} & C_F^2[-\tfrac{5}{2} - \tfrac{7}{2}x + (2 + \tfrac{7}{2}x)\ln x + (-1 + \tfrac{1}{2}x)\ln^2 x - 2x\ln(1-x) + (-3\ln(1-x) - \ln^2(1-x))p_{FG}(x)] \\ & + C_F C_G[\tfrac{28}{9} + \tfrac{65}{18}x + \tfrac{44}{9}x^2 + (-12 - 5x - \tfrac{8}{3}x^2)\ln x + (4+x)\ln^2 x + 2x\ln(1-x) + (-2\ln x\ln(1-x) \\ & + \tfrac{1}{2}\ln^2 x + \tfrac{11}{3}\ln(1-x) + \ln^2(1-x) - \tfrac{1}{6}\pi^2 + \tfrac{1}{2})p_{FG}(x) + p_{FG}(-x)S_2(x)] \\ & + C_F T_R N_F[-\tfrac{4}{3}x - (\tfrac{20}{9} + \tfrac{4}{3}\ln(1-x))p_{FG}(x)]\,,\end{aligned}$$

$$\begin{aligned}\hat{P}^{(1,S)}_{GF} = {} & C_F T_R N_F[4 - 9x + (-1+4x)\ln x + (-1+2x)\ln^2 x + 4\ln(1-x) \\ & + (-4\ln x\ln(1-x) + 4\ln x + 2\ln^2 x - 4\ln(1-x) + 2\ln^2(1-x) - \tfrac{2}{3}\pi^2 + 10)p_{GF}(x)] \\ & + C_G T_R N_F[\tfrac{182}{9} + \tfrac{14}{9}x + \tfrac{40}{9}x^{-1} + (\tfrac{136}{3}x - \tfrac{38}{3})\ln x - 4\ln(1-x) - (2+8x)\ln^2 x + (-\ln^2 x \\ & + \tfrac{44}{3}\ln x - 2\ln^2(1-x) + 4\ln(1-x) + \tfrac{1}{3}\pi^2 - \tfrac{218}{9})p_{GF}(x) + 2p_{GF}(-x)S_2(x)]\,,\end{aligned}$$

$$\begin{aligned}\hat{P}^{(1,S)}_{GG} = {} & C_F T_R N_F[-16 + 8x + \tfrac{20}{3}x^2 + \tfrac{4}{3}x^{-1} + (-6 - 10x)\ln x + (-2 - 2x)\ln^2 x] \\ & + C_G T_R N_F[2 - 2x + \tfrac{26}{9}x^2 - \tfrac{26}{9}x^{-1} - \tfrac{4}{3}(1+x)\ln x - \tfrac{20}{9}p_{GG}(x)] \\ & + C_G^2[\tfrac{27}{2}(1-x) + \tfrac{67}{9}(x^2 - x^{-1}) + (-\tfrac{25}{3} + \tfrac{11}{3}x - \tfrac{44}{3}x^2)\ln x + 4(1+x)\ln^2 x + (\tfrac{67}{9} - 4\ln x\ln(1-x) \\ & + \ln^2 x - \tfrac{1}{3}\pi^2)p_{GG}(x) + 2p_{GG}(-x)S_2(x)]\,.\end{aligned}$$

$$S_2(x) \equiv \int_{(1+x)/x}^{1/(1+x)} \frac{dz}{z}\ln\left(\frac{1-z}{z}\right); \quad S_1(x) \equiv \int_0^{1-x} \frac{dz}{z}\ln(1-z)\,.$$

4.2. Coefficient Functions

$O(\alpha_s)$:

$$C_{F2}^{(1)} = C_F \left[\frac{1+x^2}{1-x} \left(\ln \frac{1-x}{x} - \frac{3}{4} \right) + \frac{1}{4}(9+5x) \right]_+$$

$$C_{F1}^{(1)} = C_{F2}^{(1)} - 2x\, C_F$$

$$C_{F3}^{(1)} = C_{F2}^{(1)} - C_F (1+x)$$

$$C_{G2}^{(1)} = 2N_f T_R \left[(x^2+(1-x)^2) \ln\left(\frac{1-x}{x}\right) - 1 + 8x(1-x) \right]$$

$$C_{G1}^{(1)} = C_{G2}^{(1)} - 2N_f T_R\, 4x(1-x)$$

cf. FURMANSKI, PETRONZIO 1982 and refs. therein.

$O(\alpha_s^2)$:

F_2, F_L, xF_3 :

ZIJLSTRA, VAN NEERVEN, 1991 abc, 1992
LARIN, VERMASEREN (moments) 1991 (93).

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GRV '91

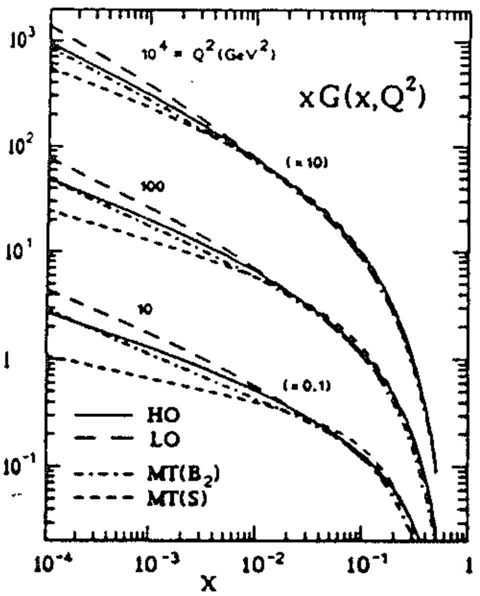


Fig. 9. The detailed small-x behavior of our radiatively generated gluon distributions in LO and HO at fixed values of Q^2, compared with the MT(S) and MT(B_2) fits [16]. The KMRS(B_0) [3] and MT(B_1) parametrizations are similar to MT(S), although slightly flatter at $Q^2 = 10\,\mathrm{GeV}^2$. The 'steep' gluon distributions [our HO, MT(B_2), KMRS(B_-) unshadowed] differ very little in the kinematic region shown

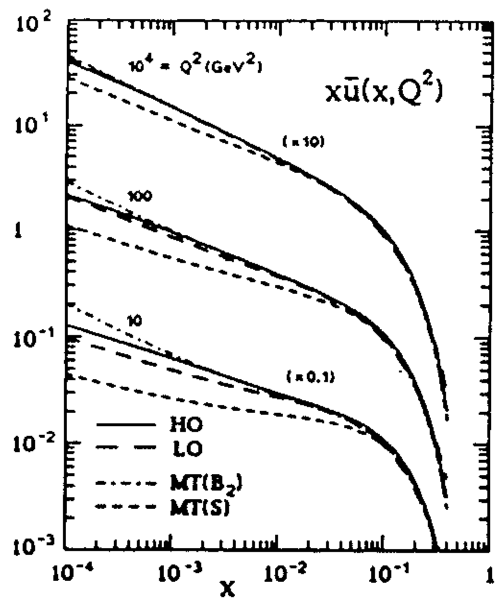


Fig. 10. The detailed small-x behavior of our radiatively generated sea distributions $\bar{u} = \bar{d}$ in LO and HO, compared with the MT(S) and MT(B_2) fits [16]. For $x < 10^{-2}$ the MT(B_1) and KMRS(B_0) [3] parametrizations are significantly ($\lesssim 30\%$) below the MT(S) fit. KMRS($B\ $) lies between MT($S$) and our results

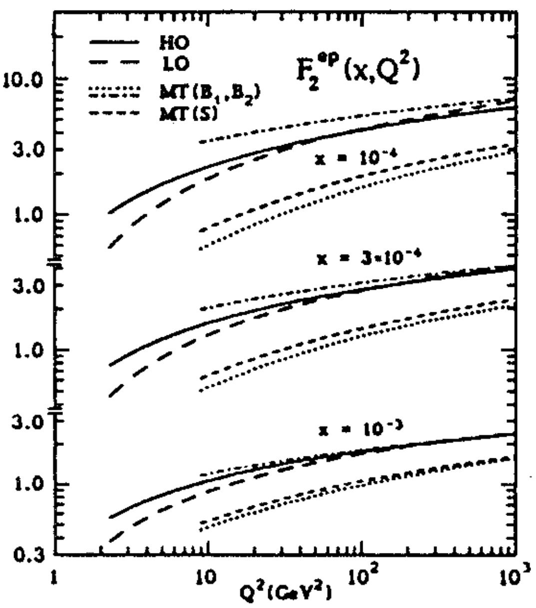


Fig. 12. Radiative LO and HO predictions for F_2^{ep} in the small-x region. For comparison we show expectations from conventional fit approaches MT [16], extrapolated to the experimentally not yet available $x < 10^{-2}$ region

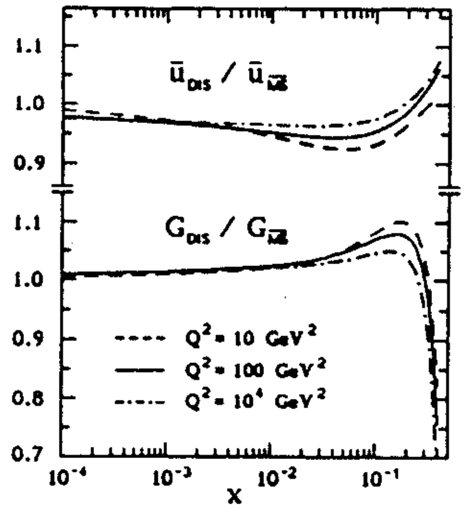
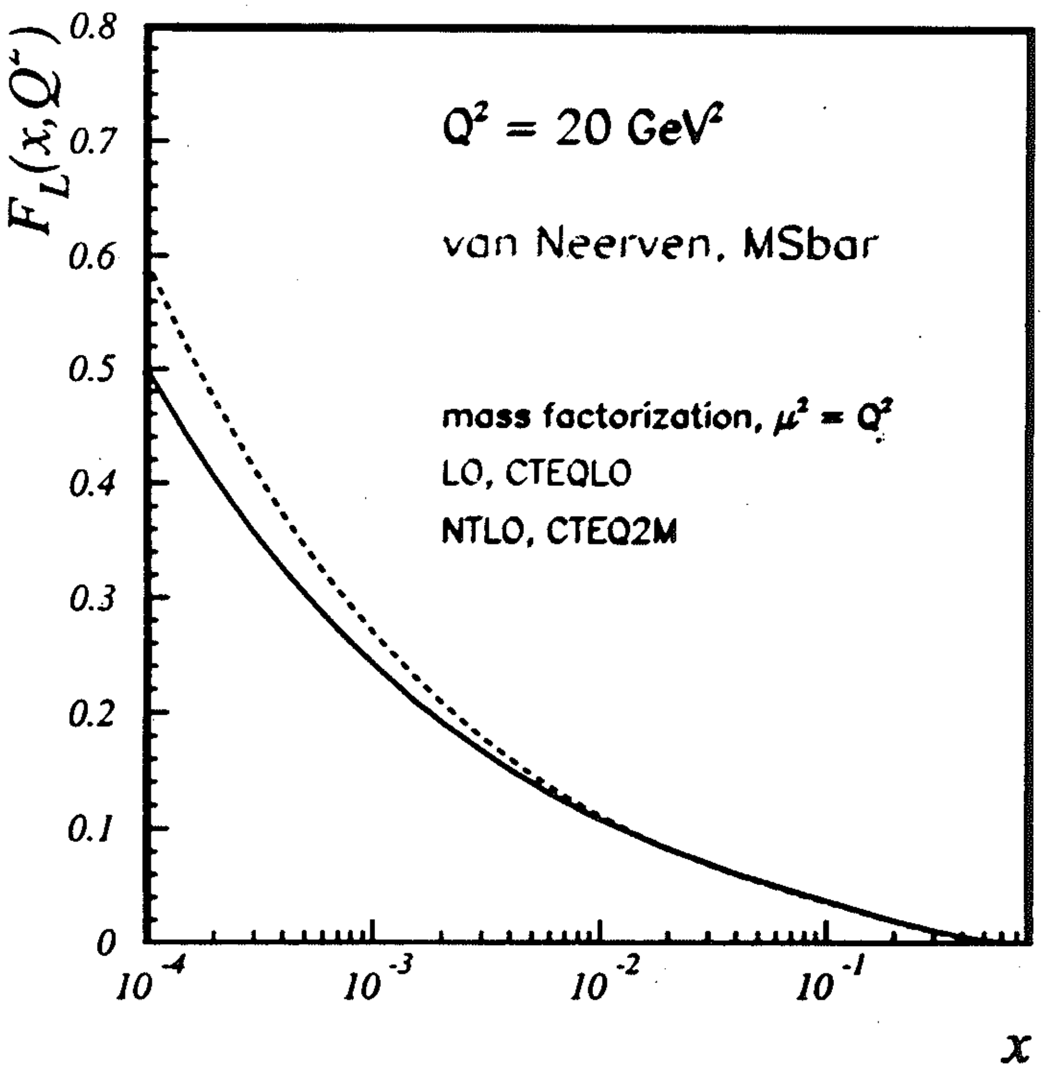


Fig. 13. Comparison of our radiative MS results with the ones transformed to the DIS factorization scheme



$F_L(x,Q^2)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$Q^2 = 20$ GeV^2
van Neerven, MSbar
mass factorization, $\mu^2 = Q^2$
LO, CTEQLO
NTLO, CTEQ2M
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

4.3. $O(\alpha_s^3)$ corrections

SUM RULES:

$$\int_0^1 dx\,(F_1^{\bar{\nu}p} - F_1^{\nu p}) = 1 - \frac{2}{3}\frac{\alpha_s}{\pi} - 2.3519\left(\frac{\alpha_s}{\pi}\right)^2 - 8.4852\left(\frac{\alpha_s}{\pi}\right)^3$$

LARIN, TKACHOV, VERMASEREN 1991

$$\int_0^1 dx\,(F_3^{\bar{\nu}p} + F_3^{\nu p}) = 6\Big[1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\left(-\frac{55}{12} + \frac{1}{3}N_f\right)$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\Big[-\frac{13841}{216} - \frac{44}{9}\zeta_3 + \frac{55}{2}\zeta_5$$
$$+ N_f\left(\underline{\frac{10009}{1296}} + \underline{\frac{91}{54}\zeta_3} - \frac{5}{3}\zeta_5\right)$$
$$- \frac{115}{648}N_f^2\Big]\Big]$$

$$\int_0^1 dx\,(g_1^{ep} - g_1^{en}) = \frac{1}{3}\left|\frac{g_A}{g_V}\right|\Big\{1 - \frac{\alpha_s}{\pi} \ldots\ldots$$
$$+ \left(\frac{\alpha_s}{\pi}\right)^3\Big[+ N_f\left(\underline{\frac{10339}{1296}} + \underline{\frac{61}{54}\zeta_3} - \ldots\right)\ldots\Big]\Big\}$$

LARIN, VERMASEREN, 1991

LARIN, RITBERGEN, VERMASEREN[16]
1994.

NS:

8th moment of $G_L(x)$

$$
\begin{aligned}
C_{L,8}(1,a_s) = {} & a_s C_F \cdot \tfrac{4}{9} \\
& + a_s^2\Big[C_F C_A\big(\tfrac{14741729}{1190700} - \tfrac{16}{3}\zeta_3\big) + C_F n_f\big(-\tfrac{14234}{8505}\big) \\
& + C_F^2\big(-\tfrac{21694349}{3572100} + \tfrac{32}{3}\zeta_3\big)\Big] \\
& + a_s^3 C_F C_A n_f\big(-\tfrac{2315308364152 9}{198037224000} + \tfrac{18459136}{363825}\zeta_3\big) \\
& + a_s^3 C_F C_A^2\big(\tfrac{7653142193467}{1800333840 00} - \tfrac{9508318}{19845}\zeta_3 + \tfrac{2240}{9}\zeta_5\big) + a_s^3 C_F n_f^2 \cdot \tfrac{1435876}{229635} \\
& + a_s^3 C_F^2 C_A\big(-\tfrac{7667007621900089}{27725211360000} + \tfrac{2476982549}{2182950}\zeta_3 - \tfrac{2720}{3}\zeta_5\big) \\
& + a_s^3 C_F^2 n_f\big(\tfrac{1184464440428 9}{198037224000} - \tfrac{914992}{10395}\zeta_3\big) \\
& + a_s^3 C_F^3\big(-\tfrac{86167166469418457}{499053804480000} - \tfrac{111668693}{218295}\zeta_3 + \tfrac{7360}{9}\zeta_5\big) \\
& + a_s^3 \cdot 3\left(\sum_{f=1}^{n_f} q_f\right)\frac{d^{abc}d^{abc}}{n_c}\big(-\tfrac{35555777437}{3000564000} - \tfrac{85378}{14175}\zeta_3 + \tfrac{160}{9}\zeta_5\big) \\
= {} & a_s \cdot 0.5925925926 + a_s^2(35.87664404 - 2.231471683 n_f) \\
& + a_s^3\left(2215.210878 - 305.4730331 n_f + 8.337149534 n_f^2 \right. \\
& \left. - 8.741107731 \sum_{f=1}^{n_f} q_f\right).
\end{aligned}
$$

NS ANOM. DIM. (F_2)

$$
\begin{aligned}
\gamma_8(a_s) = {} & a_s C_F \cdot \tfrac{9883}{1260} + a_s^2\Big[C_F C_A \cdot \tfrac{25870049}{762048} + C_F n_f\big(-\tfrac{36241943}{4762800}\big) \\
& + C_F^2\big(-\tfrac{27040578211}{4000752000}\big)\Big] + a_s^3 C_F C_A n_f\big(-\tfrac{1578915745223}{72013536000} - \tfrac{19766}{315}\zeta_3\big) \\
& + a_s^3 C_F C_A^2\big(\tfrac{8101059985033}{41150592000} + \tfrac{2510407}{132300}\zeta_3\big) + a_s^3 C_F n_f^2\big(-\tfrac{38920977797}{18003384000}\big) \\
& + a_s^3 C_F^2 C_A\big(-\tfrac{3662576699059}{112021056000} - \tfrac{2510407}{44100}\zeta_3\big) \\
& + a_s^3 C_F^2 n_f\big(-\tfrac{91675209372043}{16803158400000} + \tfrac{19766}{315}\zeta_3\big) \\
& + a_s^3 C_F^3\big(-\tfrac{1093087100974 37993}{63515938752000000} + \tfrac{2510407}{66150}\zeta_3\big) \\
= {} & a_s \cdot 10.45820106 + a_s^2(123.7764525 - 10.14583662 n_f) \\
& + a_s^3(2164.091836 - 352.3116596 n_f - 2.882493484 n_f^2).
\end{aligned}
$$

NS: MOMENTS F_2.

$M_{2,N}$ for $n_f = 5$:

$$M_{2,2}(n_f=5) = a_s^{32/69}(1 + 2.348059464 a_s - 6.052509330 a_s^2) A_2(\mu^2),$$

$$M_{2,4}(n_f=5) = a_s^{314/345}(1 + 8.457076895 a_s + 73.59702078 a_s^2) A_4(\mu^2),$$

$$M_{2,6}(n_f=5) = a_s^{2836/2415}(1 + 13.71561575 a_s + 192.6174600 a_s^2) A_6(\mu^2),$$

$$M_{2,8}(n_f=5) = a_s^{9883/7245}(1 + 18.17792372 a_s + 324.5935524 a_s^2) A_8(\mu^2).$$

The calculation of the 8th non-singlet moment took the equivalent of more than 600 CPU hours on an SGI Challenge workstation with a 100 MHZ MIPS 4400 chip.

8. QCD corrections to polarized structure functions

$g_1(x, Q^2)$

$\underline{O(\alpha_s)}$: ANOMALOUS DIMENSIONS / SPLITTING FCT.:

$$P_{NS,qq} \equiv P_{qq,S} = C_F \left[8 \left(\frac{1}{1-x} \right)_+ - 4(1-x) + 6\,\delta(1-x) \right]$$

$$P_{qg,S} = T_f \left[16x - 8 \right]$$

$$P_{gq,S} = C_F \left[8 - 4x \right]$$

$$P_{gg,S} = C_A \left[8 \left(\frac{1}{1-x} \right)_+ + 8 - 16x + \frac{22}{3}\delta(1-x) \right] - T_f \left[\frac{8}{3}\,\delta(1-x) \right]$$

K. SASAKI 1975

M. AHMED, G. ROSS 1975/76

G. ALTARELLI, G. PARISI 1977

- NO TERMS $\propto \frac{1}{x}$.

LO EVOLUTION EQUATIONS

ALTARELLI, PARISI 1977.

$$\Delta \overset{(-)}{q}_i = \overset{(-)}{q}_{i\uparrow} - \overset{(-)}{q}_{i\downarrow}$$

$$\Delta G = G_\uparrow - G_\downarrow$$

$$\Delta V_{ij} = \Delta q_i - \Delta \bar{q}_j$$

$$\frac{d}{d\log Q^2} \Delta V_{ij}(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \Delta P_{NS}(x) \otimes \Delta V_{ij}(x,Q^2)$$

$$\frac{d}{d\log Q^2} \Delta \overset{(-)}{q}_i = \frac{\alpha_s(Q^2)}{2\pi} \left\{ \Delta P_{qq} \otimes \Delta \overset{(-)}{q}_i + \Delta P_{qG} \otimes \Delta G \right.$$

$$\frac{d}{d\log Q^2} \Delta G = \frac{\alpha_s(Q^2)}{2\pi} \left\{ \sum_{i=1}^{2N_f} \Delta P_{Gq} \otimes \Delta q_i + \Delta P_{GG} \otimes \Delta G \right.$$

$$\Delta P_{NS} \equiv \Delta P_{qq} = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right]$$

$$\Delta P_{qG} = \frac{1}{2} \left[z^2 - (1-z)^2 \right]$$

$$\Delta P_{Gq} = C_F \frac{1-(1-z)^2}{z}$$

$$\Delta P_{GG} = C_G \left[(1+z^4)\left(\frac{1}{z} + \frac{1}{(1-z)_+} \right) - \frac{(1-z)^3}{z} + \left(\frac{11}{6} - \frac{2T_R}{3C_G} \right) \delta(1 \ldots \right.$$

$$C_F = \frac{4}{3}, \quad C_G = 3, \quad T_R = \frac{N_f}{2}.$$

GEHRMANN, STIRLING

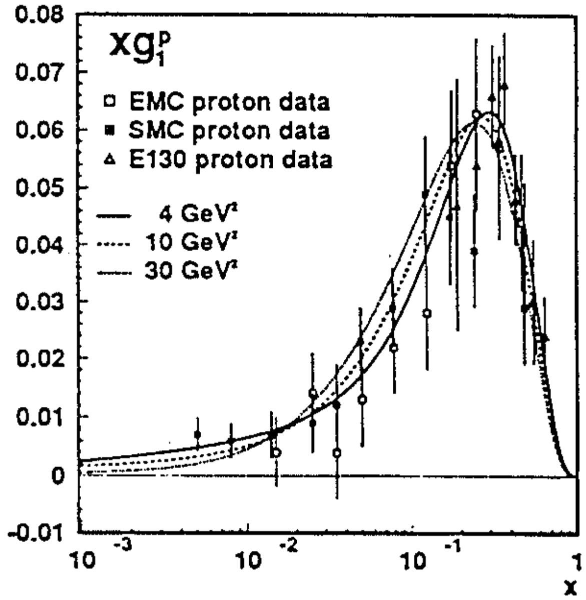


Figure 2: Fit to the g_1^p structure function with set A gluon

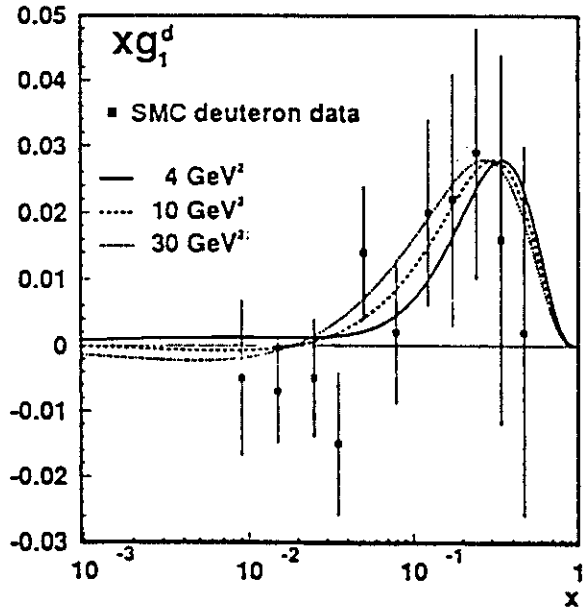


Figure 3: Prediction for the g_1^d structure function with the set A gluon

PARAMETRIZATIONS FOR $g_1(x, Q^2)$:

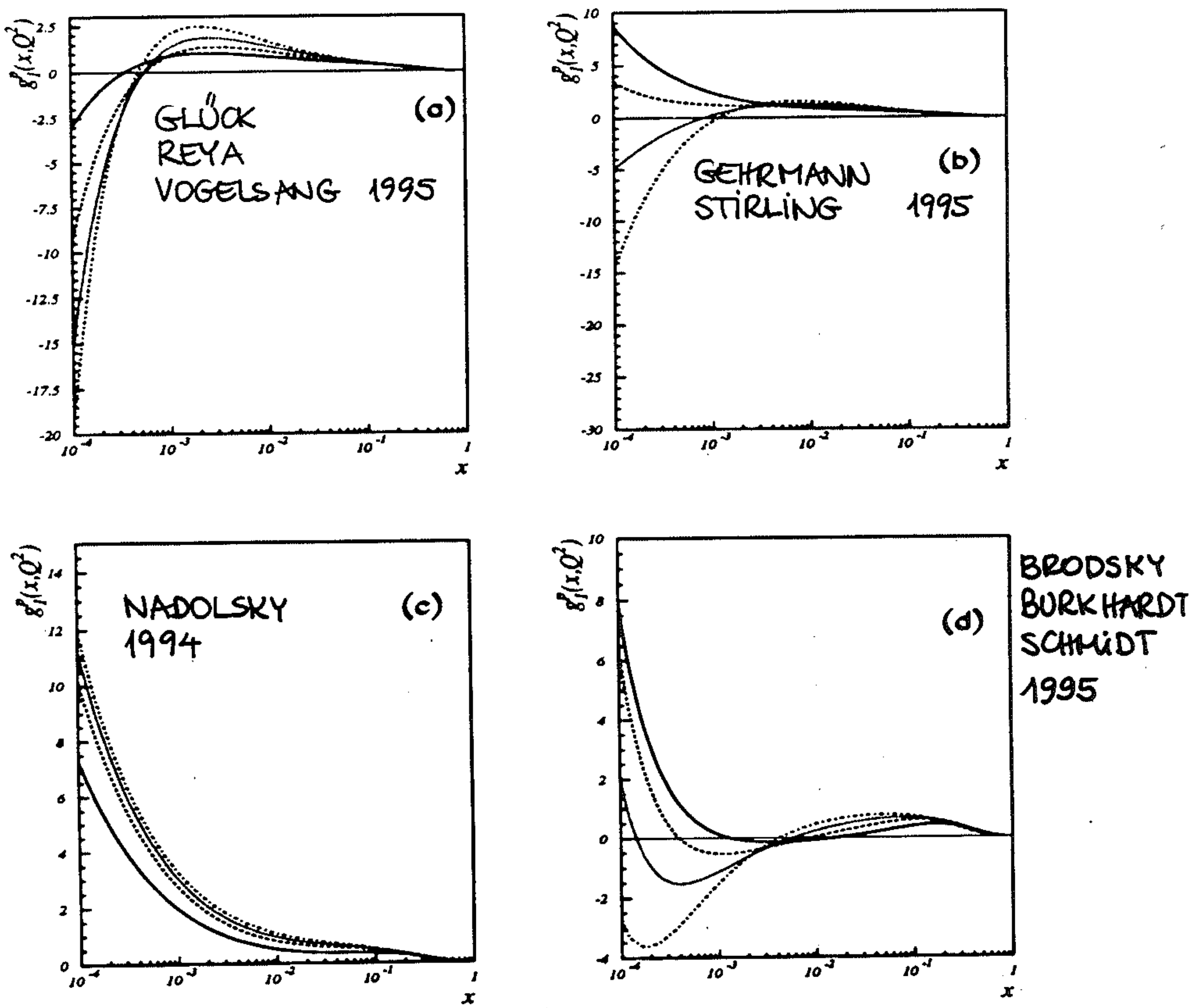


Figure 1: The structure function $g_1^p(x, Q^2)$ in the range $x > 10^{-4}$. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 10^2\,\mathrm{GeV}^2$, dotted line: $Q^2 = 10^3\,\mathrm{GeV}^2$, dash–dotted line: $Q^2 = 10^4\,\mathrm{GeV}^2$. The parametrizations are: (a) ref. [5], (b) ref. [6], (c) ref. [7], (d) ref. [8].

COEFFICIENT FUNCTIONS:

$O(\alpha_s)$: $\quad M^2 \equiv Q^2$

$$C_q^{NS} = \delta(1-z) + \frac{\alpha_s}{4\pi} C_F \left\{ 4\left(\frac{\ln(1-z)}{1-z}\right)_+ - 3\left(\frac{1}{1-z}\right)_+ - 2(1+z)\ln(1-z) - 2\frac{1+z^2}{1-z}\ln z + 4 + 2z + \delta(1-z)\left(-4\zeta(2) - 9\right) \right\}$$

ALTARELLI, ELLIS, MARTINELLI 1979
HUMPERT, VAN NEERVEN 1981

$$C_g = \frac{\alpha_s}{4\pi} N_f T_f \left\{ 4(2z-1)\left(\ln(1-z) - \ln z\right) + 4(3-4z) \right\}$$

BODWIN, QIU 1990

$O(\alpha_s^2)$

ZIJLSTRA, VAN NEERVEN 1994, ALSO $M^2 \neq Q^2$.

- NTLO ANALYSES ARE POSSIBLE NOW
  → NEED MORE PRECISE DATA STILL! IN A WIDER Q^2 RANGE.

RESUMMATION OF $\alpha_s \ln^2 x$ TERMS:

- BARTELS, ERMOLAEV, RYSKIN
- J.B.

$O(\alpha_s^2)$:

$$
\begin{aligned}
\Delta P^{1,\pm}_{qq,NS} &= P^{1,\mp}_{qq,NS} \ , \\
\Delta P^{1}_{qq,PS}(x) &= C_F T_R N_f \Big[2(1-x) - 2(1-3x)\ln x - 2(1+x)\ln^2 x \Big] \ , \\
\Delta P^{1}_{qg}(x) &= C_F T_R N_f \Big[-22 + 27x - 9\ln x + 8(1-x)\ln(1-x) \\
&\quad + \frac{1}{2}\delta p_{qg}(x)\left(4\ln^2(1-x) - 8\ln(1-x)\ln x + 2\ln^2 x - 8\zeta(2)\right) \Big] \\
&\quad + C_A T_R N_f \Big[2(12-11x) - 8(1-x)\ln(1-x) + 2(1+8x)\ln x \\
&\quad - 2\left(\ln^2(1-x) - \zeta(2)\right)\delta p_{qg}(x) - \left(2I_x - 3\ln^2 x\right)\delta p_{qg}(-x) \Big] \ , \\
\Delta P^{1}_{gq}(x) &= C_F T_R N_f \left[-\frac{4}{9}(x+4) - \frac{4}{3}\delta p_{gq}(x)\ln(1-x) \right] \\
&\quad + C_F^2 \Big[-\frac{1}{2} - \frac{1}{2}(4-x)\ln x - \delta p_{gq}(-x)\ln(1-x) \\
&\quad + \left(-4 - \ln^2(1-x) + \frac{1}{2}\ln^2 x\right)\delta p_{gq}(x) \Big] \\
&\quad + C_A C_F \Big[(4-13x)\ln x + \frac{1}{3}(10+x)\ln(1-x) + \frac{1}{9}(41+35x) \\
&\quad + \frac{1}{2}\left(-2I_x + 3\ln^2 x\right)\delta p_{gq}(-x) + \left(\ln^2(1-x) - 2\ln(1-x)\ln x - \zeta(2)\right)\delta p_{gq}(x) \Big] \\
\Delta P^{1}_{gg}(x) &= -C_A T_R N_f \left[4(1-x) + \frac{4}{3}(1+x)\ln x + \frac{20}{9}\delta p_{gg}(x) + \frac{4}{3}\delta(1-x) \right] \\
&\quad - C_F T_R N_f \Big[10(1-x) + 2(5-x)\ln x + 2(1+x)\ln^2 x + \delta(1-x) \Big] \\
&\quad + C_A^2 \Big[\frac{1}{3}(29-67x)\ln x - \frac{19}{2}(1-x) + 4(1+x)\ln^2 x - 2I_x\delta p_{gg}(-x) \\
&\quad + \left(\frac{67}{9} - 4\ln(1-x)\ln x + \ln^2 x - 2\zeta(2)\right)\delta p_{gg}(x) + \left(3\zeta(3) + \frac{8}{3}\right)\delta(1-x) \Big]
\end{aligned}
$$

where, as mentioned above, the unpolarized NS pieces $P^{1,\pm}_{qq,NS}$ can be found in [12] and [39]

ZIJLSTRA, VAN NEER 94

HERTIG, VAN NEERIJ 95

VOGELSANG 95

$$
\begin{aligned}
\delta p_{qg}(x) &\equiv 2x - 1 \ , \\
\delta p_{gq}(x) &\equiv 2 - x \ , \\
\delta p_{gg}(x) &\equiv \frac{1}{(1-x)_+} - 2x + 1 \ .
\end{aligned}
$$

Furthermore we have in eqs. (26-30) $\zeta(2) = \pi^2/6$, $\zeta(3) \approx 1.202057$ and

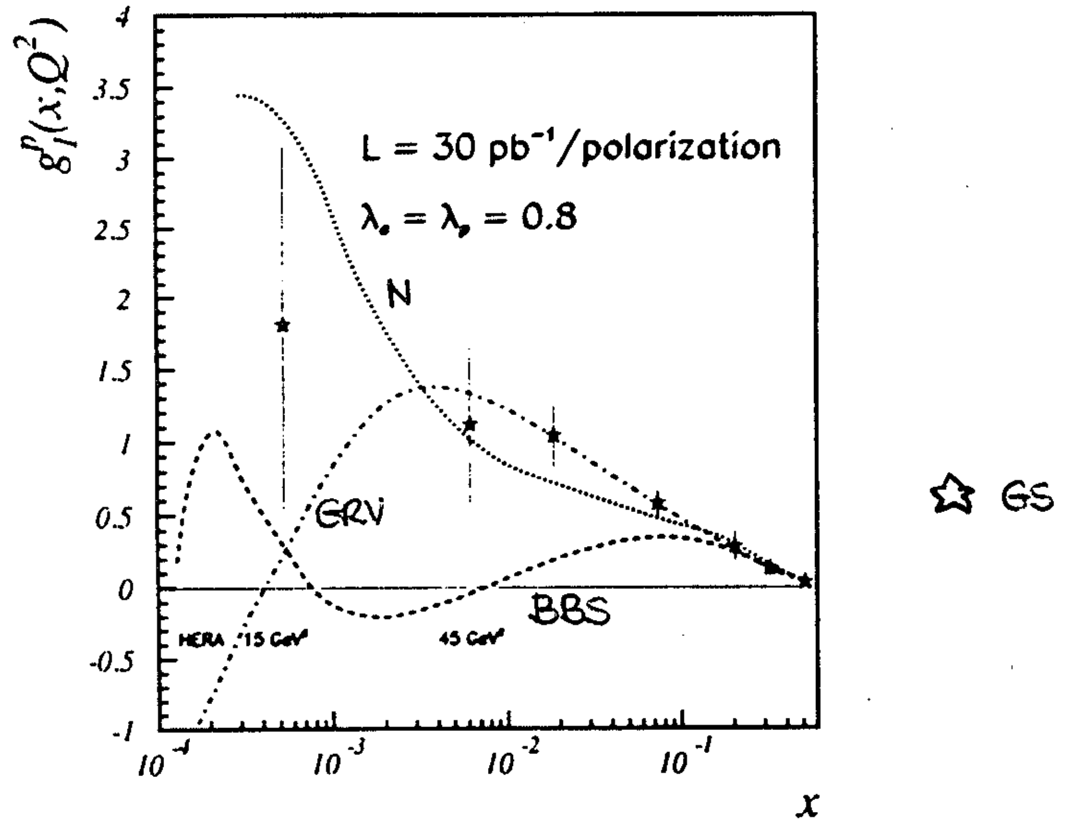
$$I_x \equiv \int_{x/(1+x)}^{1/(1+x)} \frac{dz}{z} \ln\left(\frac{1-z}{z}\right) \ .$$

For relating our results to those of [8] the relation

$$I_x = -2Li_2(-x) - 2\ln x \ln(1+x) + \frac{1}{2}\ln^2 x - \zeta(2)$$

LARGEST SINGULARITY: $\propto \alpha_s^2 \ln^2 x$ for $x \to 0$.

POSSIBLE FUTURE MEASUREMENT AT HERA:
POL. e & POL. p (820 GeV).



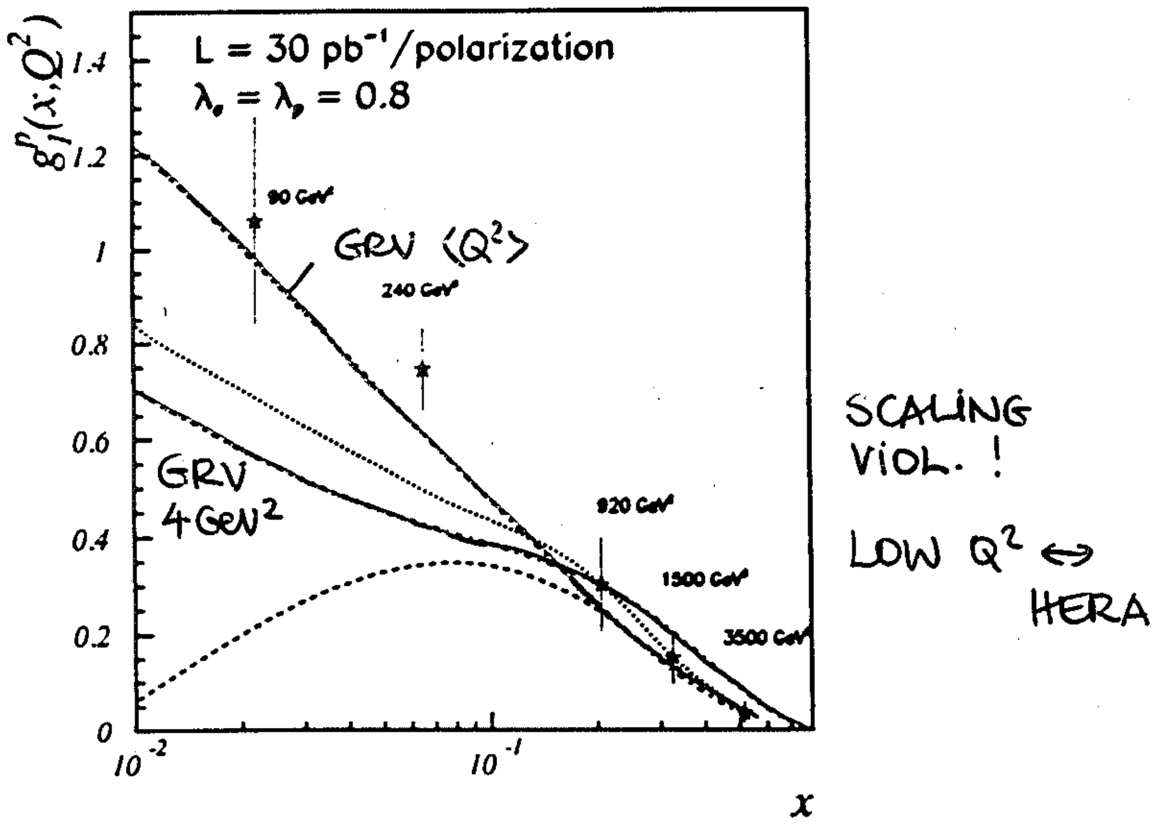


Figure 6: Statistical precision of a measurement of $g_1^p(x,Q^2)$ in the kinematical domain of HERA at larger values of x. The data points represent averages over the accessible Q^2 range and were calculated using the parametrization [6]. The dashed, dotted, and upper dash-dotted line correspond to the values of $g_1^p(x,\langle Q^2\rangle)$ for the parametrizations [8], [7], and [5], respectively. The lower dash-dotted line shows $g_1^p(x,Q_0^2)$ for $Q_0^2 = 4\,\mathrm{GeV}^2$ for parametrization [5].

QCD – ANALYSIS:

νN – SCATTERING

ONLY $\overset{(-)}{\nu}_{e,\mu} N \rightarrow e^{\pm}(\mu^{\pm}) X$ REACTIONS MAY BE MEASURED TO THE REQUIRED PRECISION FOR A QCD TEST.

1) NON-SINGLET ANALYSIS:

OBSERVABLE : $xW_3^d(x,Q^2) = \frac{1}{2}\left[xW_3^{\nu d}(x,Q^2) + xW_3^{\bar{\nu} d}(x,Q^2)\right]$.

$$\frac{\partial\, xW_3^d(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} P_{NS}(x) \otimes xW_3^d(x,Q^2)$$

$$\chi^2 \underset{MIN}{:=} \sum_{bins} \left[\frac{xW_3^{exp}(x,Q^2) - E_{NS}(\Lambda,x,Q^2,Q_0^2) \otimes xW_3(x,Q_0^2)}{\delta xW_3^{exp}(x,Q^2)}\right]^2$$

with: $xW_3(x,Q^2) = E_{NS}(x,Q^2) \otimes xW_3(x,Q_0^2)$

$\Lambda_{QCD}^{NS} = \Lambda_{QCD}$, NO CORREL. TO $xG(x,Q^2)$.

2) COMBINED SINGLET & NON-SINGLET ANALYSIS:

OBSERVABLES: xW_3^d

$$W_2^d \equiv \Sigma$$

$$\bar{Q} = \sum_i x\bar{q}_i$$

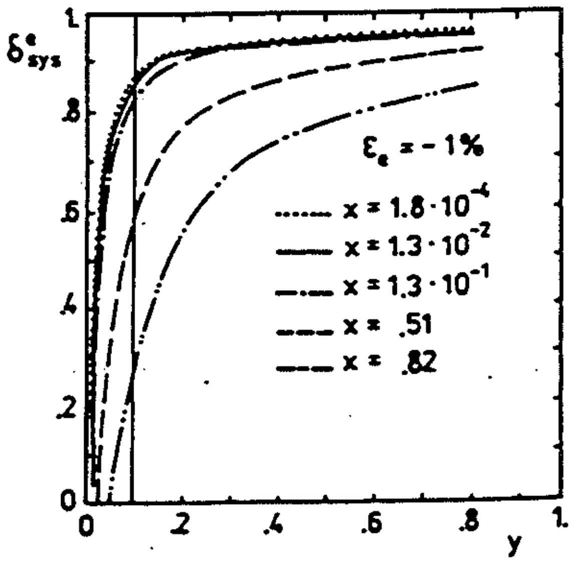
$$xW_3(x,Q^2) = E_{NS}(x,Q^2) \otimes V$$

$$W_2(x,Q^2) = E_{FF}(x,Q^2) \otimes (V+S) + E_{FG}(x,Q^2) \otimes G$$

$$\bar{Q}(x,Q^2) = (E_{FF} - E_{NS})(x,Q^2) \otimes V + E_{FF}(x,Q^2) \otimes S + E_{FG}(x,Q^2) \otimes G$$

$$V_i(S,G) = V(x,Q_0^2)\ (S(x,Q_0^2), G(x,Q_0^2))$$

NECESSITY OF CROSS CALIBRATION OF CALORIMETERS



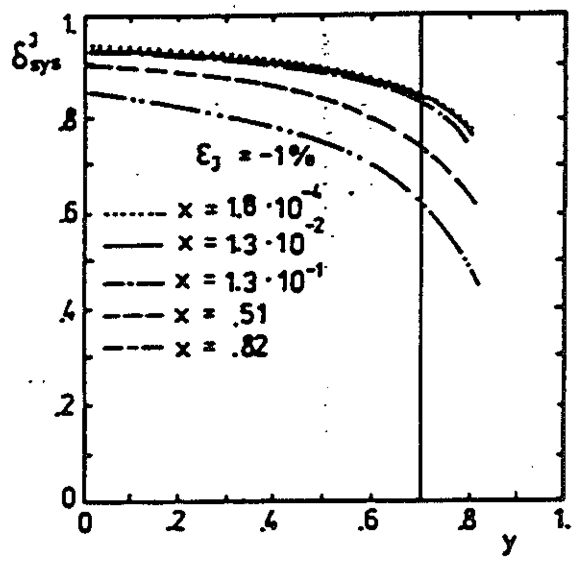


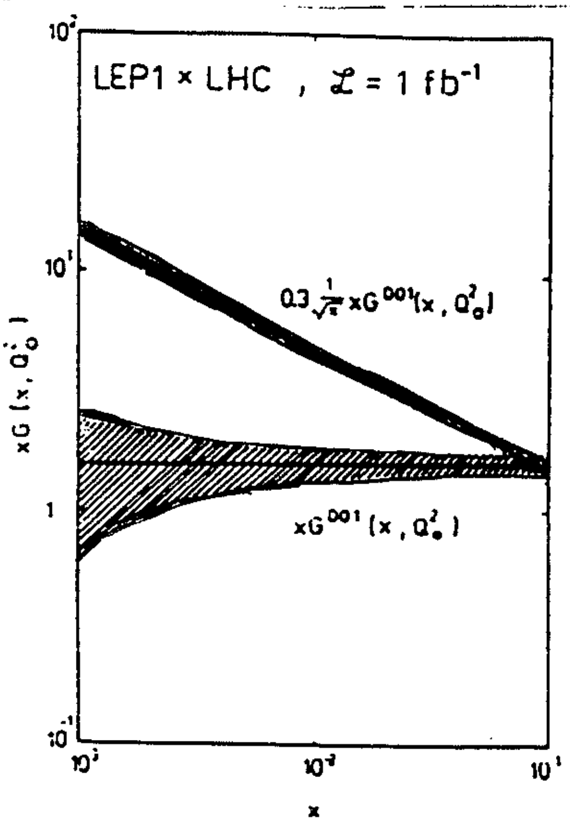
Figure 1: $\delta_{sys} = (d^2\tilde{\sigma}_{nc}/dxdy)/(d^2\sigma_{nc}/dxdy)$ for displacements of $\varepsilon_{e,J} = -1\%$ with $\frac{d^2\tilde{\sigma}}{dxdy}(\varepsilon) = \frac{d^2\sigma}{dxdy}(1+\varepsilon)$.

electromagnetic calorimeter	hadronic calorimeter	$\mathcal{L}$ in pb^{-1}	$\sqrt{s} = 314$ GeV $\delta\varepsilon_e$	$\sqrt{s} = 314$ GeV $\delta\varepsilon_J$	$\sqrt{s} = 190$ GeV $\delta\varepsilon_e$	$\sqrt{s} = 190$ GeV $\delta\varepsilon_J$
BEMC	CB	10	0.0049	0.0075	0.0050	0.0070
BBE	CB	10	0.0173	0.0220	0.0186	0.0199
CB	CB	10	0.0128	0.0097	0.0130	0.0098
CB	FB/OF	100	0.0158	0.0386	–	–
BEMC	all	10	0.0025	0.0033	0.0026	0.0033
BBE	all	10	0.0073	0.0067	0.0085	0.0068
CB	all	10	0.0031	0.0025	0.0028	0.0025
OF and IF	all	100	0.0258	0.0122	0.0762	0.0324

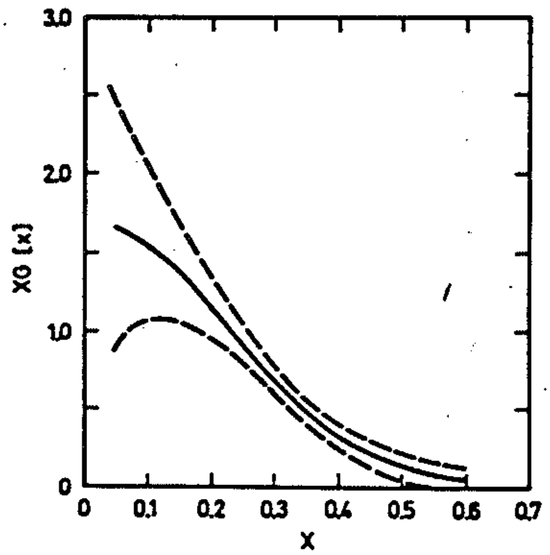
Table 2: Accuracies of ε_e and ε_J using $d^2\sigma_{nc}/dxdy$.

$\delta\varepsilon_e$ & $\delta\varepsilon_h$ COULD HAVE A SYSTEMATIC IMPACT ON $\Delta\Lambda = \pm$ 50 ... 150 MeV !

Q11

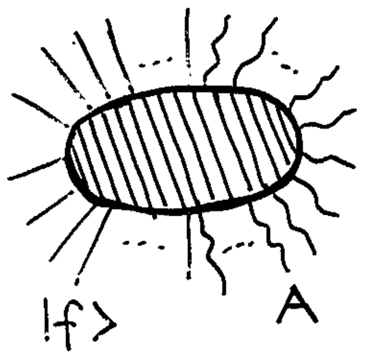


LEP × LHC



ν, UNK

RENORMALIZATION GROUP EQUATIONS



$$G^{(N_\psi, N_A)}(\ldots)$$

∴ AMPUTED, RENORMALIZED, ONE PARTICLE IRR., PROPER VERTEX FUNCTION:

$$\Gamma_R^{N_\psi, N_A} = \frac{G_R^{N_\psi, N_A}}{\prod^{N_F} G_R^{2,0} \prod^{N_A} G_R^{0,2}}$$

$$G_R^{N_\psi, N_A} = \langle 0 | T(\psi_1 \ldots \psi_{N_\psi} A_1 \ldots A_{N_A}) | 0 \rangle$$

$$\Gamma_R^{N_\psi, N_A}(p_j, g, \varepsilon, \mu) = Z_\psi^{N_\psi/2} Z_A^{N_A/2} \Gamma_u^{N_\psi, N_A}(p_j, g_0, \varepsilon)$$

$$\exists \lim_{\varepsilon \to 0} \Gamma_R^{N_\psi, N_A}(\ldots, \varepsilon, \ldots).$$

$$\boxed{\frac{d\Gamma_u^{N_\psi, N_A}}{d\mu} = 0}$$

$$\Gamma_u^{N_\psi, N_A} = Z_\psi^{-N_\psi/2} Z_A^{-N_A/2} \Gamma_R^{N_\psi, N_A}$$

$$\frac{d}{d\mu} \left[Z_\psi^{-N_\psi/2} Z_A^{-N_A/2} \Gamma_R^{N_\psi, N_A} \right] \equiv 0$$

$$\mu \frac{d}{d\mu} \equiv \mu \left.\frac{\partial}{\partial\mu}\right|_{g_R} + \mu \frac{\partial g_R}{\partial\mu} \frac{\partial}{\partial g_R} \equiv \frac{\partial}{\partial\mu} + \underline{\beta(g_R)} \frac{\partial}{\partial g}$$

$$\left[\mu \frac{\partial}{\partial\mu} + \beta(g) \frac{\partial}{\partial g} - N_\psi \gamma_\psi(g) - N_A \gamma_A(g)\right] \Gamma_R^{N_\psi, N_A} = 0$$

$$\gamma_\psi(g) = \frac{1}{2}\mu \frac{\partial}{\partial\mu} \ln Z_\psi$$

$$\gamma_A(g) = \frac{1}{2}\mu \frac{\partial}{\partial\mu} \ln Z_A$$

$$\beta(g) = \mu \frac{\partial g}{\partial\mu}$$

RGE

RGE's: QCD

NS:

$$J\cdot J = \sum_n C_n^{NS}\, O_{NS}^n$$

$$\langle NS|J\cdot J|NS\rangle = \sum_n C_n^{NS}\,\langle NS|O_{NS}^n|NS\rangle$$

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_\psi(g)\right]\langle NS|JJ|NS\rangle = 0$$

$$O_{NS}^n = \frac{O_{NS}^{0,n}}{Z_{NS}^n} \quad ; \quad |NS\rangle = Z_\psi^{1/2}\,|NS^{(0)}\rangle$$

$$\gamma_{NS}^n(g^2) = \mu\frac{\partial}{\partial\mu}\ln Z_{NS}^n.$$

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_{NS}^n(g) - 2\gamma_\psi(g)\right]\langle NS|O_{NS}^n|NS\rangle = 0$$

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - \gamma_{NS}^n(g)\right] C_n^{NS}\left(\frac{Q^2}{\mu^2}, g^2\right) = 0$$

SINGLET:

$$O_a^n = (Z^{n-1})_a^b\, O_b^{0,n} \qquad a,b=\psi, G$$

$$\langle c| = Z_c^{1/2}\,\langle c^0|$$

$$\left\{\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} - 2\gamma_c(g)\right]\delta_a^b + \gamma_a^{b,n}(g)\right\}\langle c|O_b^n|c\rangle = 0$$

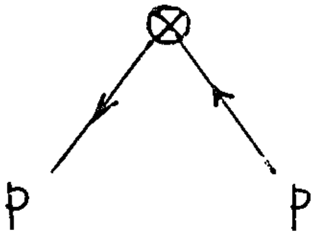
$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right] C_{a,n}\left(\frac{Q^2}{\mu^2}, g^2\right) = \gamma_a^{b,n}\, C_{b,n}\left(\frac{Q^2}{\mu^2}, g^2\right)$$

$$\gamma_a^{b,n}(g^2) = \left(\mu\frac{\partial}{\partial\mu}\ln Z^n\right)_a^b .$$

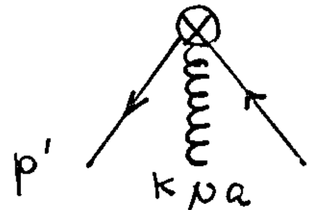
PRACTICAL EXAMPLE: γ_n^{NS}

DIAGRAMS:

BARE :



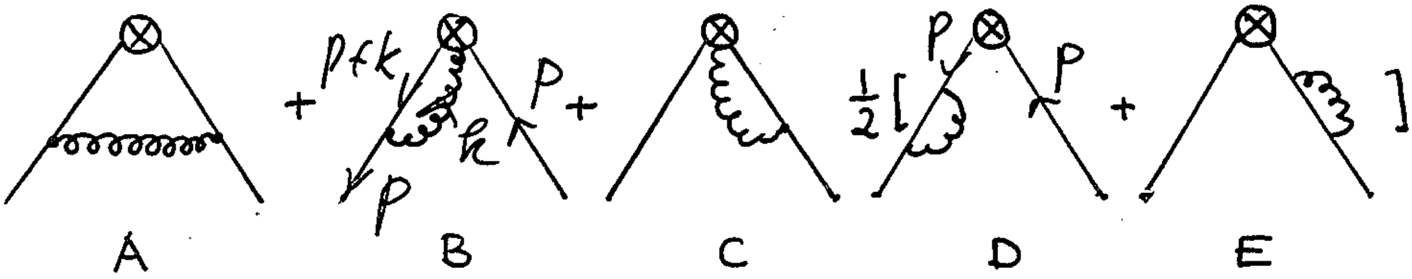
$$S\gamma_{\mu_1} p_{\mu_2} \cdots p_{\mu_n} t^i$$



$$S\left(-g T^a \gamma_{\mu_1} g_{\mu_2 \mu} p'_{\mu_3} \cdots p'_{\mu_n} + g T^a \gamma_{\mu_1} p_{\mu_2} g_{\mu_3 \mu} p'_{\mu_4} \cdots p'_{\mu_n} + \ldots (-1)^{n-1} g T^a \gamma_{\mu_1} p'_{\mu_2} \cdots p'_{\mu_{n-1}} g_{\mu_n \mu}\right) t^i$$

T^a SU_3 GENERATOR

t^i $SU(N_f)$ GENERATOR



CALCULATE POLE TERMS $\sim \frac{1}{\varepsilon}$ IN $(4-\varepsilon)$ DIM.

$A = -4C_F \frac{1}{n(n+1)}$ $\qquad$ $D = E = C_F$

$B = C = 4C_F \sum_{j=2}^{n} \frac{1}{j}$

$$\boxed{\gamma_n^{NS} = \frac{8}{3}\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n} \frac{1}{j}\right]}, \quad C_F = \frac{4}{3}.$$

THE ANOMALOUS DIMENSIONS OF THE LOCAL OPERATORS:

$$\langle NS | O^n_{NS} | NS \rangle = 1 + \frac{g^2}{16\pi^2} \frac{\lambda^n_{NS}}{2} \ln \frac{Q^2}{\mu^2} + \dots$$

$$\langle c | O^n_b | c \rangle = \delta_{bc} + \frac{g^2}{16\pi^2} \frac{\lambda^n_{bc}}{2} \ln \frac{Q^2}{\mu^2} + \dots$$

$$\gamma^{0,n}_{NS} = \lambda^n_{NS} + 2\gamma^0_\psi$$

$$\gamma^{0,n}_{ab} = \lambda^n_{ab} + 2\gamma^0_a \delta_{ab} \quad ; \quad a,b = \psi, G$$

↑ REMEMBER $\delta(1-z)$ PARTS IN P_{ab} ALSO

LO RESULTS:

$$\gamma^{0,n}_{\psi\psi} \equiv \gamma^{0,n}_{NS} = \frac{8}{3}\left[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n} \frac{1}{j}\right]$$

$$\gamma^{0,n}_{\psi G} = -4N_f \frac{(n^2+n+2)}{n(n+1)(n+2)}$$

$$\gamma^{0,n}_{G\psi} = -\left(\frac{16}{3}\right) \cdot \frac{(n^2+n+2)}{n(n^2-1)}$$

$$\gamma^{0,n}_{GG} = 6\left[\frac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+2)(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}\right] + \frac{4}{3}N_f$$

$$\Longrightarrow \quad \boxed{\int_0^1 dz\, z^{n-1} P^{(0)}_{ab}(z) = -\frac{\gamma^{0,n}_{ab}}{4}}$$

MARRIAGE OF THE FORMAL & INTUIT. APPROACH

5. Resummation of small x contributions

- AT SMALL x : DOMINANT TERMS IN P_{ab}, C_{ab}

⟶ LARGE CONTRIBUTIONS, INTEND TO RESUM THESE TERMS.

STRUCTURE FUNCTIONS :

⟶ BFKL CONTRIBUTIONS (BALITSKII, FADIN KURAEV, LIPATOV 1976-78.

CHARACTERISTIC EQU.

$$\ell - 1 = \bar{\alpha}_s \chi(\gamma_L(\ell, \alpha_s)) \quad , \bar{\alpha}_s = \frac{C_A \alpha_s}{\pi}$$
$$\chi(z) = 2\psi(1) - \psi(z) - \psi(1-z)$$

$$\gamma_L(\ell, \bar{\alpha}_s) = \frac{\bar{\alpha}_s}{\ell - 1}\left\{1 + 2\sum_{k=1}^{\infty} b_{2k+1}\gamma_L^{2k+1}(\ell, \bar{\alpha}_s)\right\}$$

$$= A + \underline{2 b_3 A^4} + 2 b_5 A^6 + 12 b_3^2 A^7 + \dots$$

$$A = \frac{\bar{\alpha}_s}{\ell - 1}. \qquad \propto \bar{\alpha}_s^4$$

Solution for $\gamma \to 1/2$:

$$\frac{1}{\rho} := \frac{j-1}{\overline{\alpha}_s} \qquad \gamma := \frac{1}{2} - \alpha$$

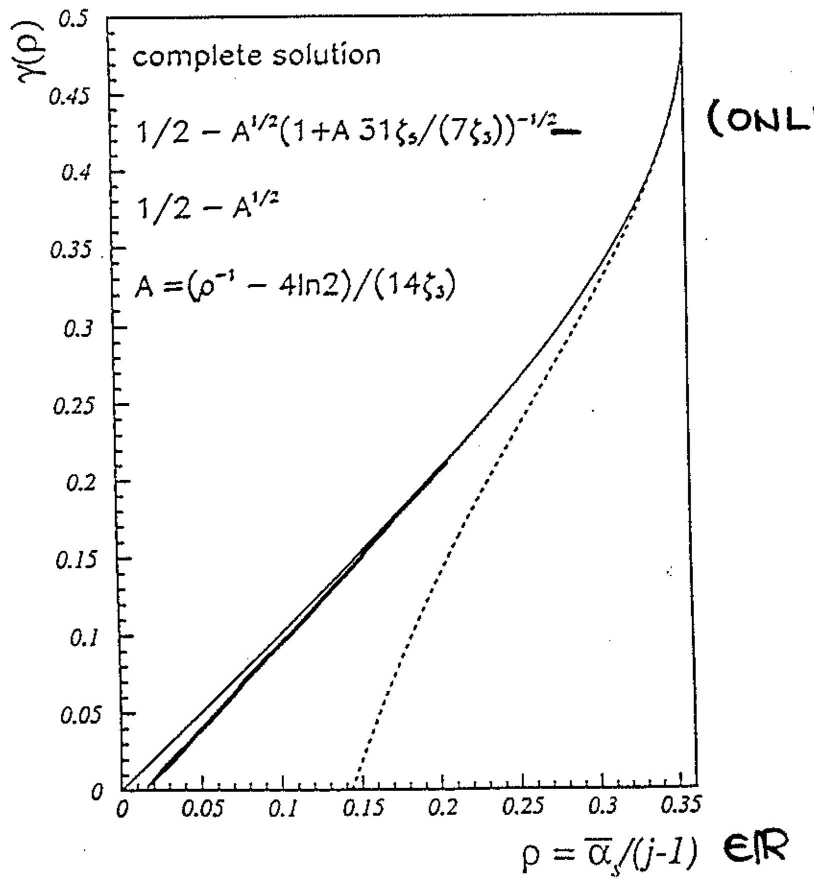
$$\frac{1}{\rho} = 2\psi(1) - \psi\left(\frac{1}{2} - \alpha\right) - \psi\left(\frac{1}{2} + \alpha\right)$$

$$\frac{1}{\rho} = 4\log 2 + \sum_{n=1}^{\infty} \zeta_{2n+1}\left(2^{2(n+1)} - 2\right)\alpha^{2n}$$

$$\alpha_{(0)} \approx 0 \qquad \gamma_c^{(0)} \approx \frac{1}{2}$$

$$\gamma_c^{(1)} \approx \frac{1}{2} - \sqrt{\left(\frac{1}{\rho} - 4\log 2\right)\frac{1}{14\zeta_3}} = \frac{1}{2} - \alpha_{(1)}$$

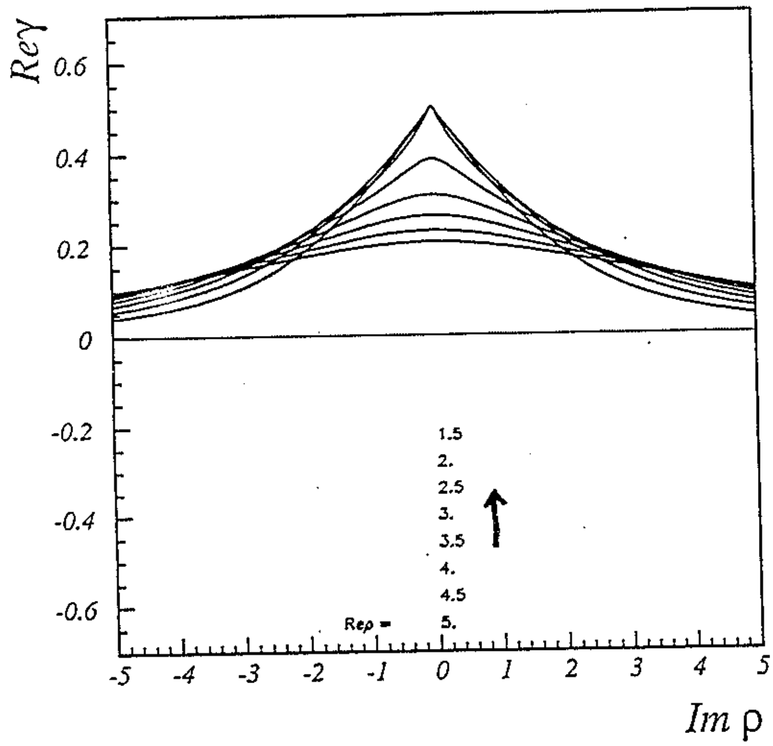
$$\gamma_c^{(2)} \approx \frac{1}{2} - \frac{\alpha_{(1)}(\rho)}{\sqrt{1 + (31\zeta_5/7\zeta_3)\alpha_{(1)}^2(\rho)}}$$

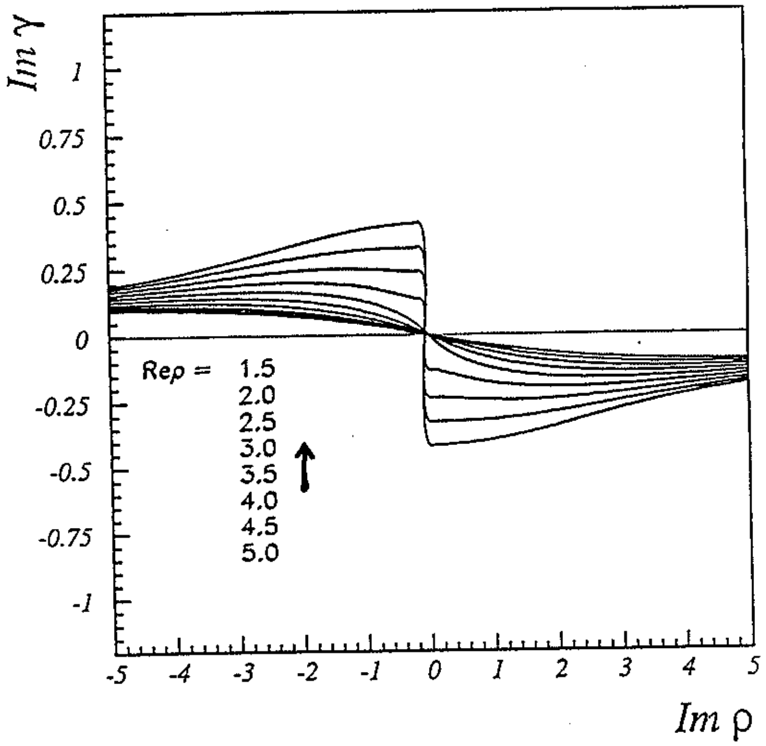


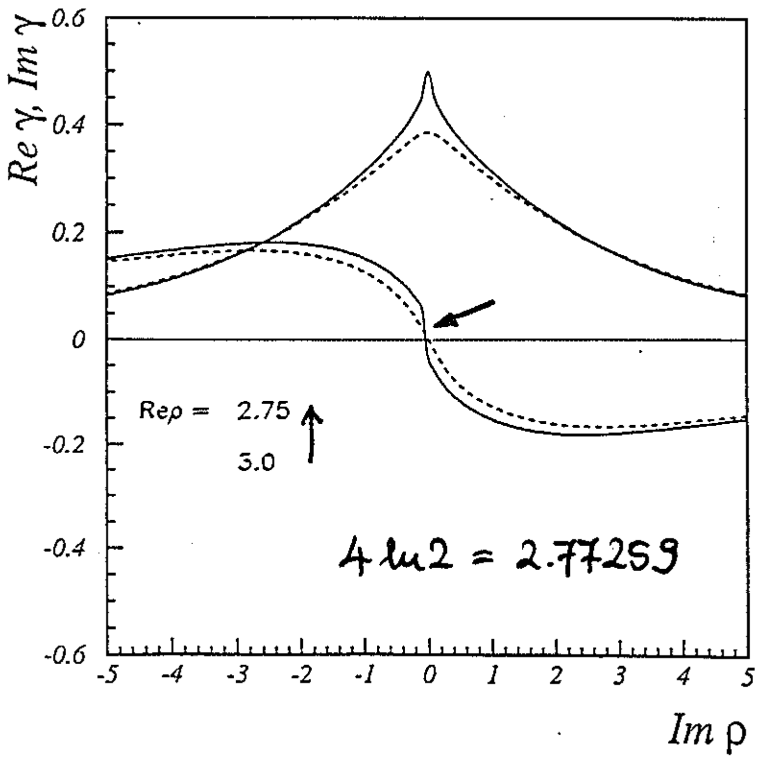
(ONLY UP TO ζ_5 ...)

The behaviour of $\gamma_c(\rho)$ for $\rho \in C$

$Re\rho \geq 1.5$

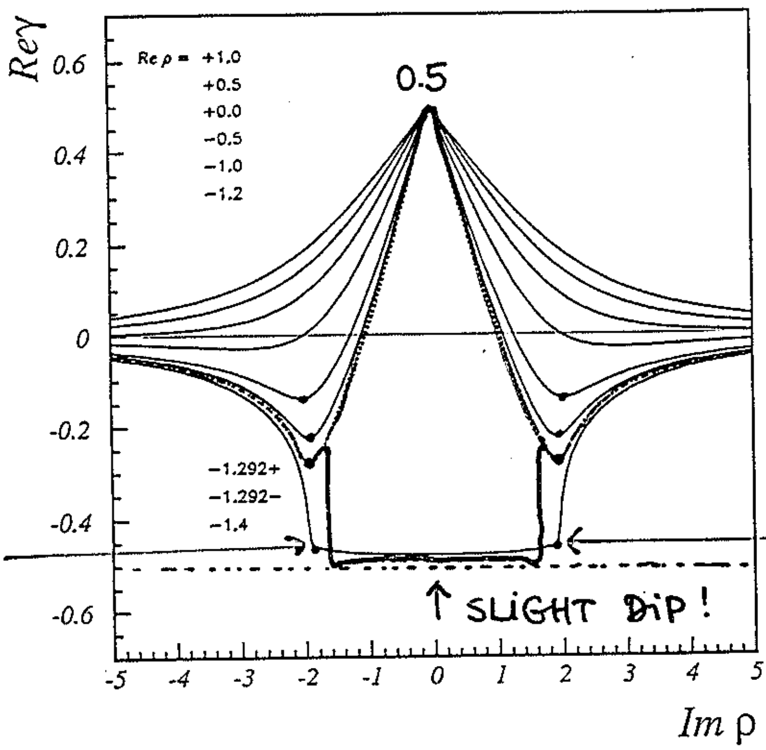






(USE:
ADAPTIVE
NEWTON
ALGORITHM).

$1.5 > Re\rho > -1.5$

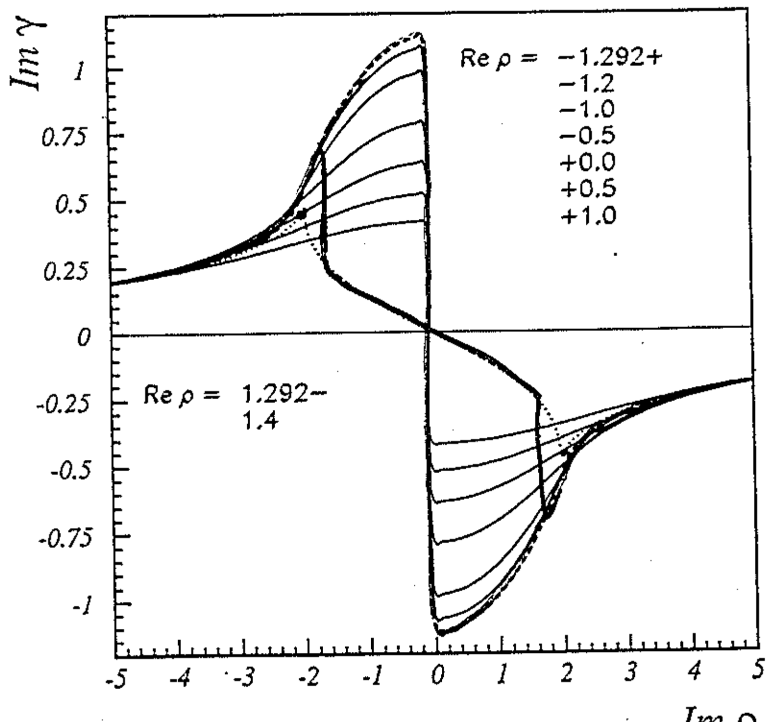


RIDGE ↓ BATH TUB

$(-1.41, \pm 1.97)$

BRANCH POINTS

√ ELLIS/HAUTMANN, WEBBER.



POSITION OF THE 'TRANSITION POINT': EXPAND AROUND $\gamma_c \sim -\frac{1}{2}$:

$$\rho = \underbrace{4(\log 2 - 1)}_{-1.22741} - \frac{8\alpha}{1-2\alpha} + \sum_{k=0}^{\infty} b_{2k+1}\left(2^{2(k+1)} - 2\right)\alpha^{2k}$$

$Im\, \alpha = 0$, $Re\, \alpha = 0.0082$ ↷ $Re\, \rho \approx -1.292$.

LOCATION OF THE BRANCH POINTS

$$\rho = \frac{\ell - 1}{\bar{\alpha}_S} = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma).$$

$$1 = \left[-\psi'(\gamma) + \psi'(1-\gamma)\right] \frac{\partial \gamma}{\partial \rho}$$

$$\frac{1}{\partial \gamma / \partial \rho} = \psi'(1-\gamma) - \psi'(\gamma) = 0$$

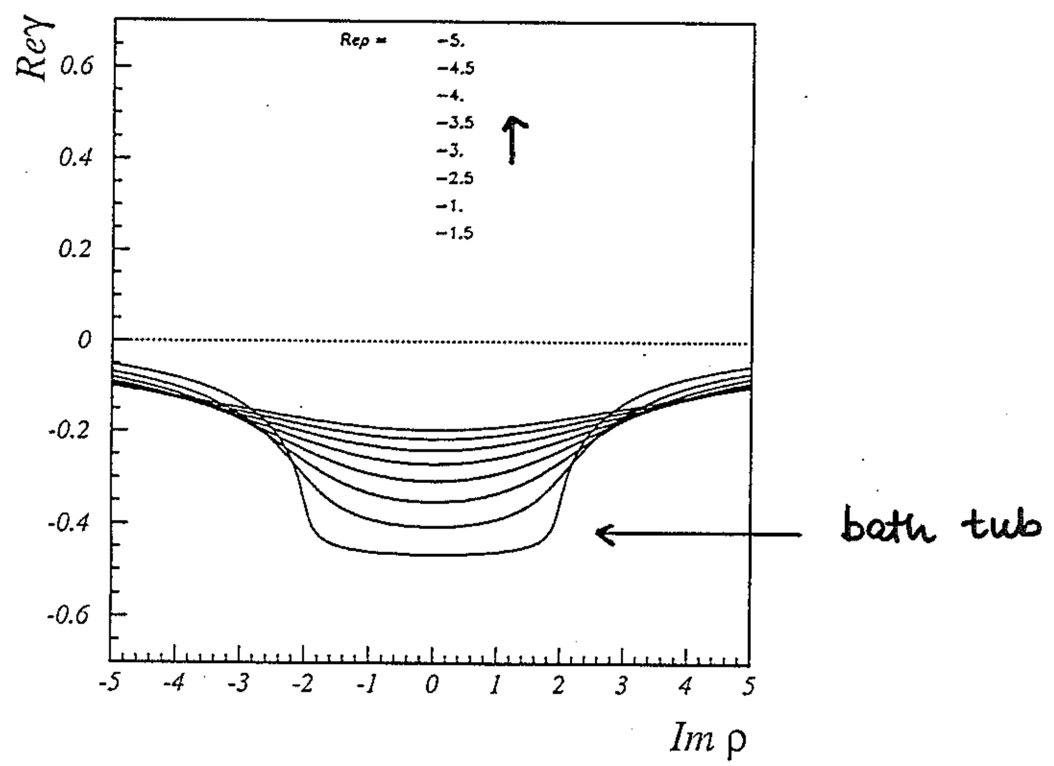
$$\boxed{\psi'(z) - \frac{\pi^2}{2} \frac{1}{\sin^2 \pi z} = 0}$$

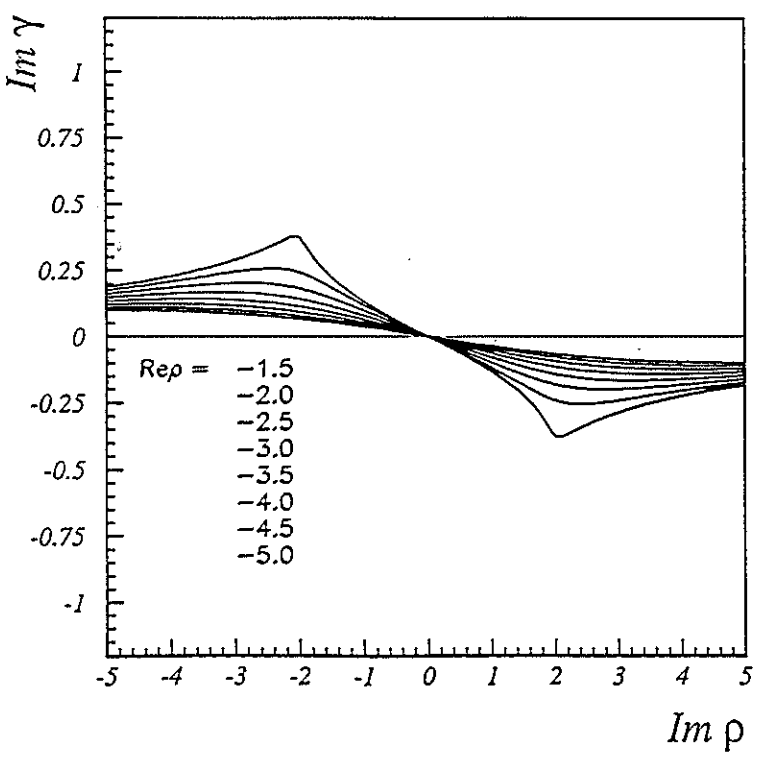
$\gamma_1 = \frac{1}{2} + 0i$ $\qquad \rho_1 = 4 \ln 2$

$\gamma_{2,3} = -0.425214 \pm i\,0.473898$

$\rho_{2,3} = -1.4105 \pm i\,1.9721.$

$Re\rho \leq -1.5$

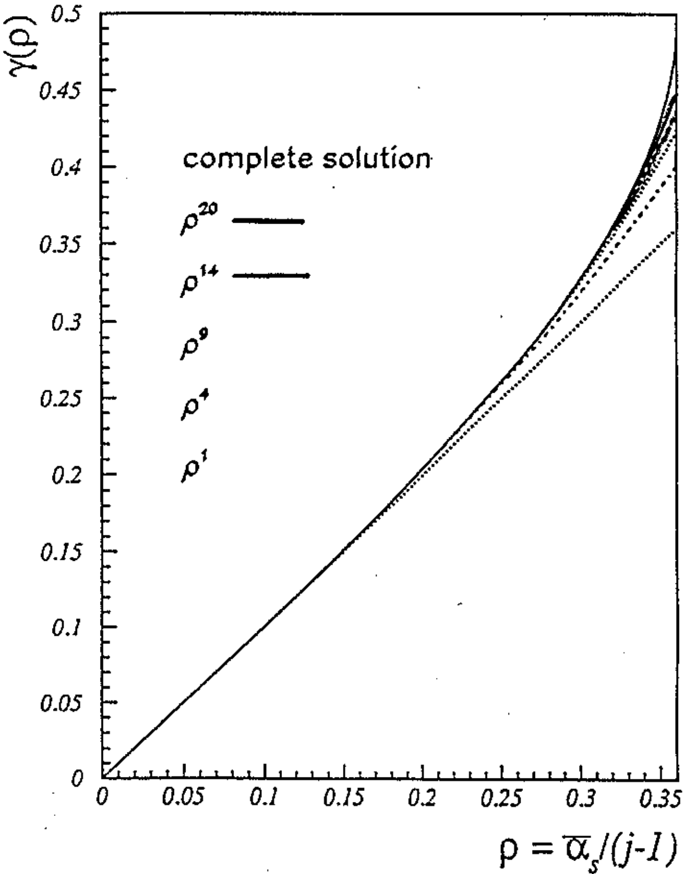

Reγ
Rep =
-5.
-4.5
-4.
-3.5
-3.
-2.5
-1.
-1.5
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-5
-4
-3
-2
-1
0
1
2
3
4
5
Im ρ
bath tub


Im γ
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
-1
Reρ =
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Im ρ

Solution for $\gamma \to 0$:

$$\gamma_c(j,\overline{\alpha}_s) = \frac{\overline{\alpha}_s}{j-1}\left\{1 + 2\sum_{k=1}^{\infty}\zeta_{2k+1}\gamma_c^{2k+1}(j,\overline{\alpha}_s)\right\}$$

$$\gamma_c(j,\overline{\alpha}_s) \equiv \gamma_c(A) = \sum_{l=1}^{\infty} g_l A^l; \qquad A = \frac{\overline{\alpha}_s}{j-1}$$



$$\begin{aligned}
g_1 &= 1\\
g_2 &= 0\\
g_3 &= 0\\
g_4 &= 2\zeta_3\\
g_5 &= 0\\
g_6 &= 2\zeta_5\\
g_7 &= 12\zeta_3^2\\
g_8 &= 2\zeta_7\\
g_9 &= 32\zeta_3\zeta_5\\
g_{10} &= 2[48\zeta_3^3 + \zeta_9]\\
g_{11} &= 2[20\zeta_3\zeta_7 + 10\zeta_5^2]\\
g_{12} &= 2[220\zeta_3^2\zeta_5 + \zeta_{11}]\\
g_{13} &= 2[440\zeta_3^4 + 24\zeta_3\zeta_9 + 24\zeta_5\zeta_7]\\
g_{14} &= 2[312\zeta_3^2\zeta_7 + 312\zeta_5^2\zeta_3 + \zeta_{13}]\\
g_{15} &= 2[2912\zeta_3^3\zeta_5 + 28\zeta_3\zeta_{11} + 28\zeta_5\zeta_9 + 14\zeta_7^2]\\
g_{16} &= 2[4368\zeta_3^5 + 420\zeta_3^2\zeta_9 + 840\zeta_3\zeta_5\zeta_7 + 140\zeta_5^3 + \zeta_{15}]\\
g_{17} &= 2[6720\zeta_3^2\zeta_5^2 + 4480\zeta_3^3\zeta_7 + 32\zeta_3\zeta_{13} + 32\zeta_5\zeta_{11} + 32\zeta_7\zeta_9]\\
g_{18} &= 2[1088\zeta_3\zeta_5\zeta_9 + 544\zeta_3\zeta_7^2 + 544\zeta_3^2\zeta_{11} + 38080\zeta_3^4\zeta_5 + 544\zeta_5^2\zeta_7 + \zeta_{17}]\\
g_{19} &= 2[6528\zeta_3\zeta_5^3 + 36\zeta_3\zeta_{15} + 19584\zeta_3^2\zeta_5\zeta_7 + 6528\zeta_3^3\zeta_9 + 45696\zeta_3^6 + 36\zeta_5\zeta_{13} + 36\zeta_7\zeta_{11} + 18\zeta_9^2]\\
g_{20} &= 2[1368\zeta_3\zeta_5\zeta_{11} + 1368\zeta_3\zeta_7\zeta_9 + 684\zeta_3^2\zeta_{13} + 124032\zeta_3^3\zeta_5^2 + 62016\zeta_3^4\zeta_7 + 684\zeta_5^2\zeta_9 + 684\zeta_5\zeta_7^2 + \zeta_{19}]
\end{aligned}$$

IARCHESINI, CATANI et al.

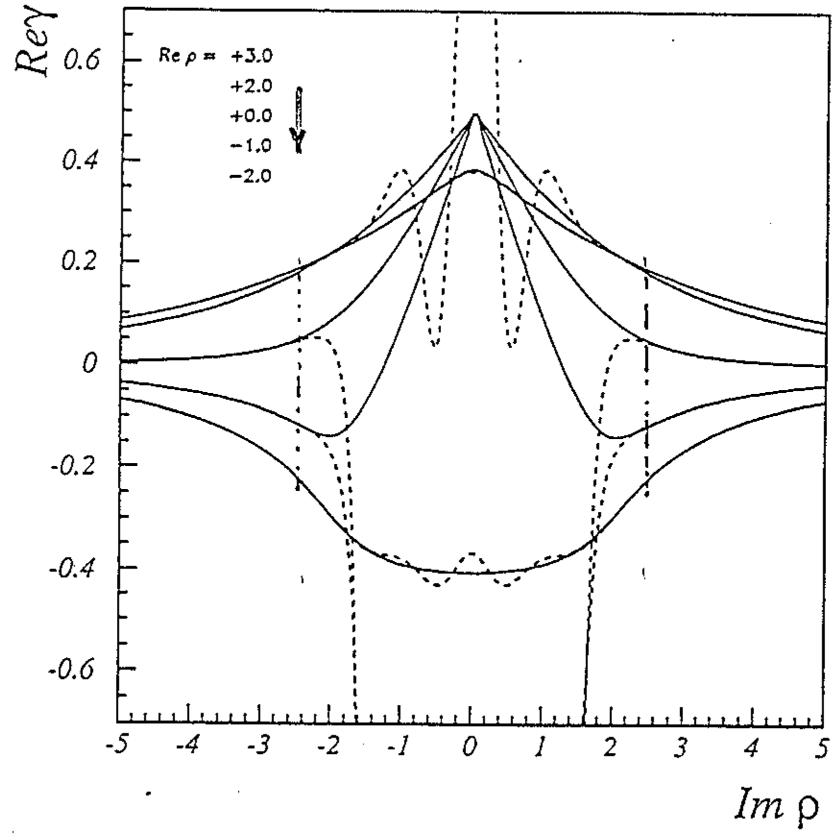
⋮ One has:

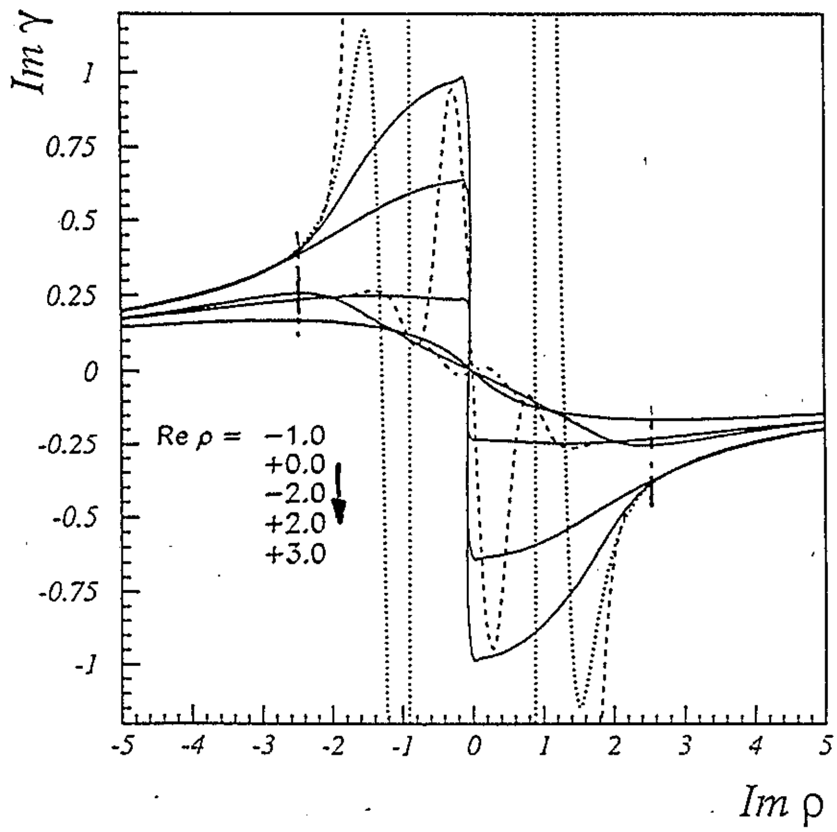
$$g_n \sim \sum_\sigma a_\sigma \left(\prod_i \zeta_{\nu_i}^{\mu_i}\right)_\sigma \Longrightarrow \sum_i \mu_i\nu_i = n-1$$

$$\gamma_c(\rho) \approx \sum_{l=1}^{N} g_l \rho^{-l}, \qquad N = 14 \qquad \text{-----}$$

vs. complete sol.

POLE STRUCT. ⟷ POWER SERIES





KERNELS: CATANI, HAUTMANN 1994
ELLIS, HAUTMANN, WEBBER
ROBERTS et al. 1995
JB, VOGT, RIEMERSMA 1996
↳ P, γ

EVOLUTION EQU.

$$\frac{d f_a(\omega,\mu)}{d \ln \mu^2} = \sum_b \gamma_{ab}(\omega, \alpha_s(\mu^2))\, f_b(\omega,\mu^2)$$

$$f_a(\omega) = \int_0^1 dx\, x^\omega f_a(x)$$

$$\gamma_{ab}(\omega,\alpha_s) = \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{\omega}\right)^k A_{ab}^{(k)} + \sum_{k=0}^{\infty} \alpha_s \left(\frac{\alpha_s}{\omega}\right)^k B_{ab}^{(k)} + O\left(\alpha_s^2 \left(\frac{\alpha_s}{\omega}\right)^k\right).$$

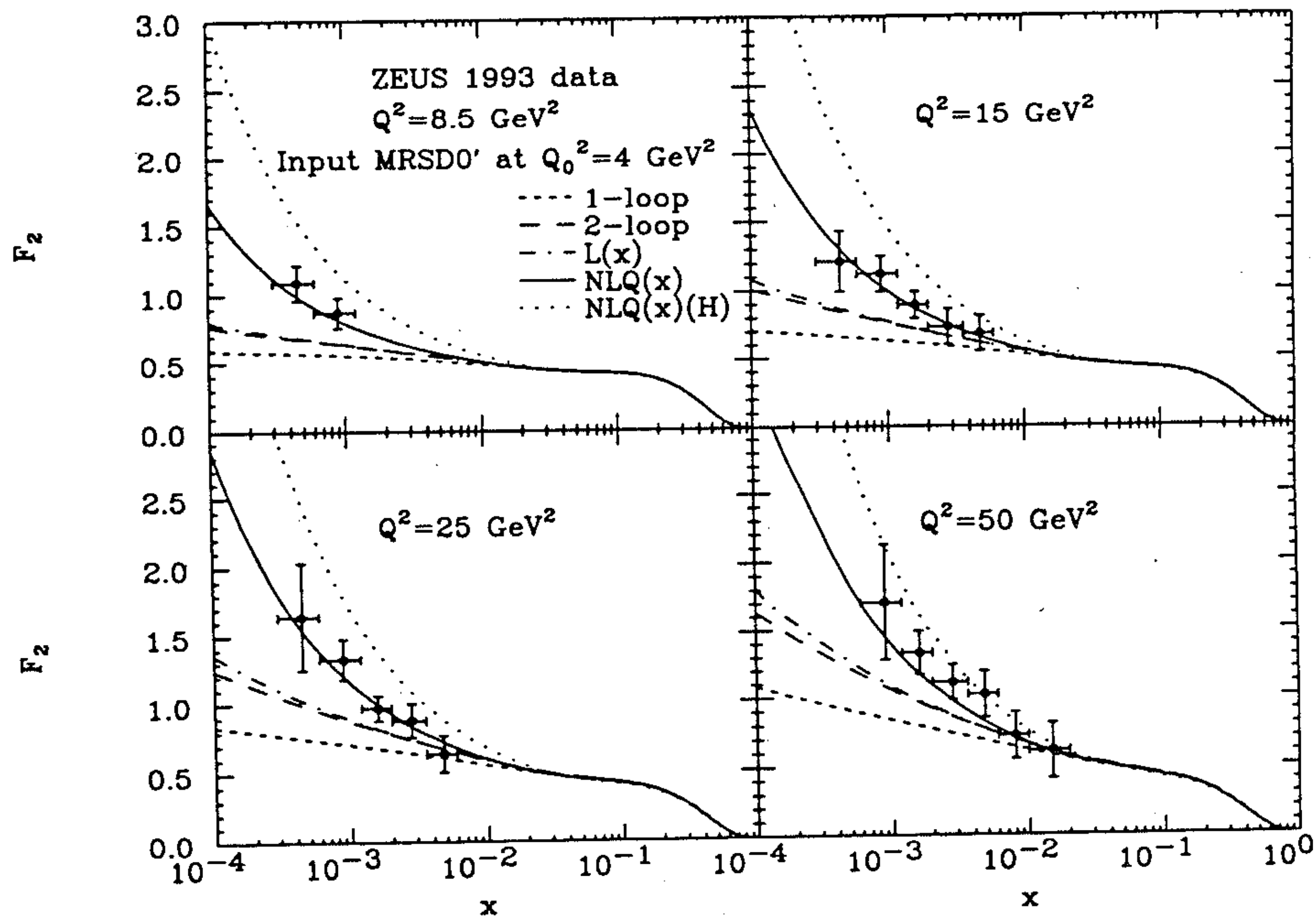
$$\frac{d}{d\mu^2}\begin{pmatrix} f_S \\ f_g \end{pmatrix} = \begin{pmatrix} \gamma_{SS} & \gamma_{SG} \\ \gamma_{GS} & \gamma_{GG} \end{pmatrix} \begin{pmatrix} f_S \\ f_G \end{pmatrix}$$

$$\gamma_L = \begin{pmatrix} 0 & 0 \\ \frac{C_F}{C_A}\gamma_L(\omega) & \gamma_L(\omega) \end{pmatrix}$$

$$\gamma_{NL} = \begin{pmatrix} \frac{C_F}{C_A}\gamma_{NL}(\omega) - \frac{2\alpha_s}{3\pi} T_f & \gamma_{NL}(\omega) \\ \gamma_\delta & \gamma_\eta \end{pmatrix} + \ldots$$

$$\gamma_{NL} \simeq \frac{2\alpha_s}{3\pi} T_f \left\{ 1 + 2.17 \frac{\alpha_s}{\omega} + 2.30 \left(\frac{\alpha_s}{\omega}\right)^2 + 8.27 \left(\frac{\alpha_s}{\omega}\right)^3 + \ldots \right\}$$

⟶ TAKE NTLO RESULTS COMPL INTO ACC.
(SUBTR. ACC. TERMS IN γ_L, γ_{NL} !)



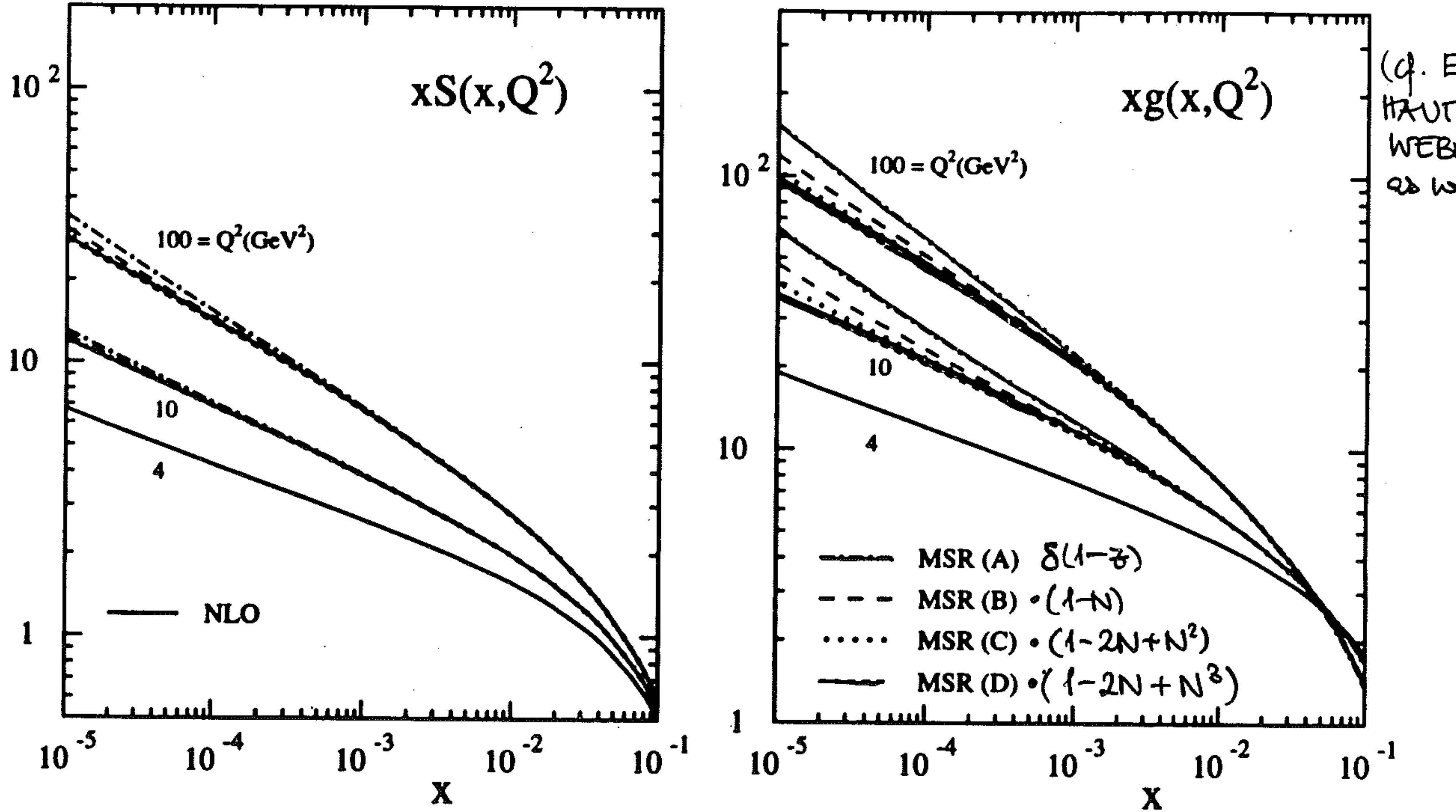
ZEUS 1993 data
Q²=8.5 GeV²
Input MRSD0' at Q₀²=4 GeV²
1-loop
2-loop
L(x)
NLQ(x)
NLQ(x)(H)
Q²=15 GeV²
Q²=25 GeV²
Q²=50 GeV²
F2
F2
x
x

DESY 96-096

LIPATOV RESUM. $O\left(\left(\frac{\alpha}{N-1}\right)^{\ell}\right)$

Toy input at $Q_0^2 = 4\ GeV^2$, f=4, NLO (DIS) + Lx

JB, A. VOGT, S. RIEMERSMA



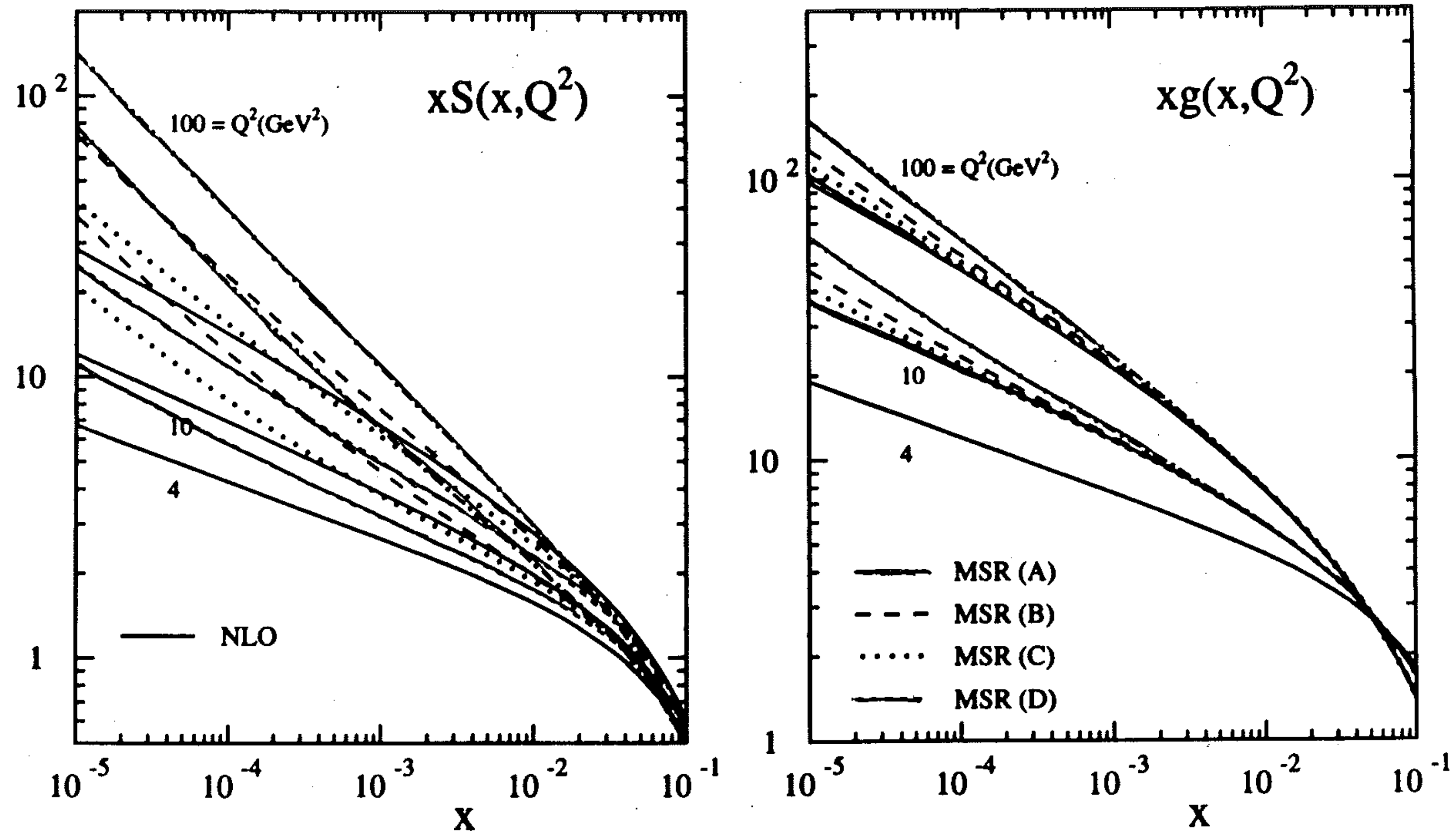
(cf. ELLIS, HAUTMANN, WEBBER as well.)

2

$$O\left(\left(\frac{\alpha}{N-1}\right)^{\ell}\right) + O\left(\alpha \left(\frac{\alpha}{N-1}\right)^{\ell}\right)$$

JB, A. VOGT, S. RIEMERSMA

Toy input at $Q_0^2 = 4\ \mathrm{GeV}^2$, f=4, NLO (DIS) + NLx



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