Good Morning to Madison !

# Large $x$ Higher Twist Contributions in DIS

# Polarized Twist-2 and Twist-3 Integral Relation

Johannes Blümlein, DESY

- Non-Singlet QCD Analysis of $l^\pm N$ Data
- Extraction of Non-leading Twist Contribution
- Polarized Twist-2 and Twist-3 Integral Relations

[1 ] J.B. and H. Böttcher, arXiv:0802.0408 [hep-ph]; Phys. Lett. **B662** (2008) 336

[2 ] J.B., A. Guffanti, and H. Böttcher, Nucl.Phys.**B774** (2007) 182.

[3 ] J.B., and A. Tkabladze, Nucl.Phys.**B553** (1999) 427.

# Introduction

- Higher Twist Contributions are expected to contribute to DIS Structure Functions both

  - at large $x$

    e.g. S. Gottlieb, Nucl. Phys. **B139** (1978) 125; H.D. Politzer, Nucl. Phys. **B172** (1980) 349; R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. **B207** (1982) 1; **B212** (1983) 29

  - at small $x$

    e.g. L.V. Gribov, E.M. Ryskin, and M.G. Ryskin, Nucl. Phys. **B188** (1981) 555; A.H. Mueller and J.W. Qiu, Nucl. Phys. **B268** (1986) 427; J. C. Collins and J. Kwiecinski, Nucl. Phys. **B335** (1990) 89; J. Bartels, J. Blümlein, and G.A. Schuler, Z. Phys. **C50** (1991) 91

- In this talk we will investigate the Large $x$ Region.

- How to separate Twist-2 and Higher Twist contributions ?

    - Systematic investigation of $\ln(Q^2)$-slopes of $F_i(x, Q^2)$ cutting from large $Q^2$
    - A cut of $W^2 > 12.5$ GeV$^2$ allows the separation (see below).

- No sufficient Theoretical Description of Higher Twist Contributions is available for :

    - Anomalous Dimensions
    - Wilson Coefficients
    - HT Correlation Functions

- Fits starting with a **phenomenological Ansatz** are therefore not possible.

## Fits under Phenomenological Assumptions :

e.g. K. Varvell et al. [BEBC] Z. Phys. **C36** (1987) 1; S.I. Alekhin, A.L. Kataev, Phys. Lett. **B452** (1999) 402; M. Botje, Eur. Phys. J. **C14** (2000) 285; S. Simula, Phys. Lett. **B493** (2000) 325; A.L. Kataev, Nucl. Phys. B (Suppl. Proc.) **116** (2003) 105; A. Martin et al., Eur. Phys. J. C35 (2004) 325; S.I. Alekhin, S.A. Kulagin, S. Liuti, Phys. Rev. **D69** (2004) 114009; M. Osipenko et al., Nucl. Phys. **A766** (2006) 142;

- How to separate Twist-2 and Higher Twist contributions ?
  - Systematic investigation of $\ln(Q^2)$-slopes of $F_i(x, Q^2)$ cutting from large $Q^2$
  - A cut of $W^2 > 12.5$ GeV$^2$ allows the separation (see below).
- No sufficient Theoretical Description of Higher Twist Contributions is available for :
  - Anomalous Dimensions
  - Wilson Coefficients
  - HT Correlation Functions
- Fits starting with a **phenomenological Ansatz** are therefore not possible.

## Extraction of HT Terms :

- Twist-2 QCD Fit in the large $W^2$ region
- Application of Target Mass Corrections
- Extrapolation of the Leading Twist Results into the lower $W^2$ region:

$W^2, Q^2 \geq 4$ GeV$^2$.

# Twist-2 Non-Singlet Analysis [2]

- Separate the kinematic region $x < 0.4, \quad x > 0.4$
- $x < 0.4$ : $F_{2\mathrm{NS}}$ from $p$ and $d$ data and $\bar{d} - \bar{u}$ (Drell-Yan process.)
- $x > 0.4$ : Valence approximation for $F_2^{p,d}$ from $p$
- 3-Loop QCD Analysis

  3L Anom. Dim. S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B688** (2004) 101
- Known Heavy Flavor contributions are taken into account (NLO)

  1% and smaller
- Effective 4-Loop QCD Analysis possible

  3L WC: J. Vermaseren, A. Vogt., S. Moch Nucl. Phys. **B724** (2005) 3

Finally:

Account for the dominant large $x$ Terms in the Wilson Coefficient to $O(\alpha_s^4)$

S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B726** (2005) 317;

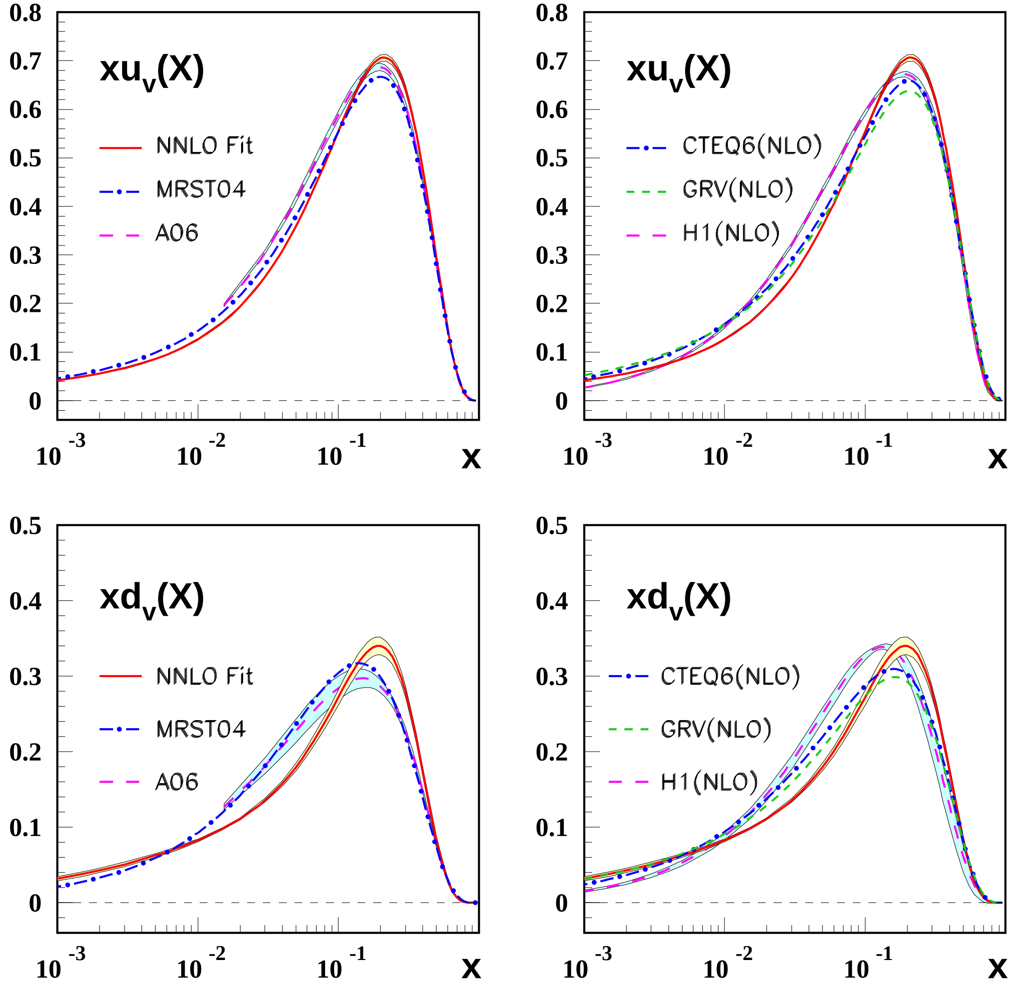
V. Ravindran, Nucl. Phys. **B752** (2006) 173

# Formalism

$$F_2^{p,d;\mathrm{NS}}(N,Q^2) = \sum_{k=0}^{\infty} a_s^{k-1}(Q^2) C_{k-1}^{\mathrm{NS}}(N) f_2^{p,d;\mathrm{NS}}(N,Q^2) \ ,$$

$$\begin{aligned}
f_2^{p,d;\mathrm{NS}}(N,Q^2) \quad = \quad & f_2^{p,d;\mathrm{NS}}(N,Q_0^2) \left(\frac{a}{a_0}\right)^{-\hat{P}_0(N)/\beta_0} \Bigg\{ 1 - \frac{1}{\beta_0}(a-a_0) \left[\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right] \\
& - \frac{1}{2\beta_0}\left(a^2 - a_0^2\right) \left[\hat{P}_2^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_1^+(N) + \left(\frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right] \\
& + \frac{1}{2\beta_0^2}(a-a_0)^2 \left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^2 \\
& - \frac{1}{3\beta_0}\left(a^3 - a_0^3\right) \Bigg[\hat{P}_3^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_2^+(N) + \left(\frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2}\right)\hat{P}_1^+(N) \\
& + \left(\frac{\beta_1^3}{\beta_0^3} - 2\frac{\beta_1\beta_2}{\beta_0^2} + \frac{\beta_3}{\beta_0}\right)\hat{P}_0(N)\Bigg] \frac{(a-a_0)\left(a_0^2 - a^2\right)}{2\beta_0^2} \left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right) \\
& \times \left[\hat{P}_2(N) - \frac{\beta_1}{\beta_0}\hat{P}_1(N) - \left(\frac{\beta_1^2 - \beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right] \\
& - \frac{(a-a_0)^3}{6\beta_0^3}\left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^3 \Bigg\} \ .
\end{aligned}$$

# World Data Analysis: Valence Distributions




World data:

NS-analysis

$W^2 > 12.5\ \mathrm{GeV}^2, Q^2 > 4\ \mathrm{GeV}^2$

N$^3$LO :

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

J.B., H. Böttcher, A. Guffanti,

Nucl. Phys. **B774** (2007) 182

# Why an $O(\alpha_s^4)$ analysis can be performed?

Assume an $\pm 100\%$ error on the Padé approximant $\longrightarrow \pm 2$ MeV in $\Lambda_{QCD}$

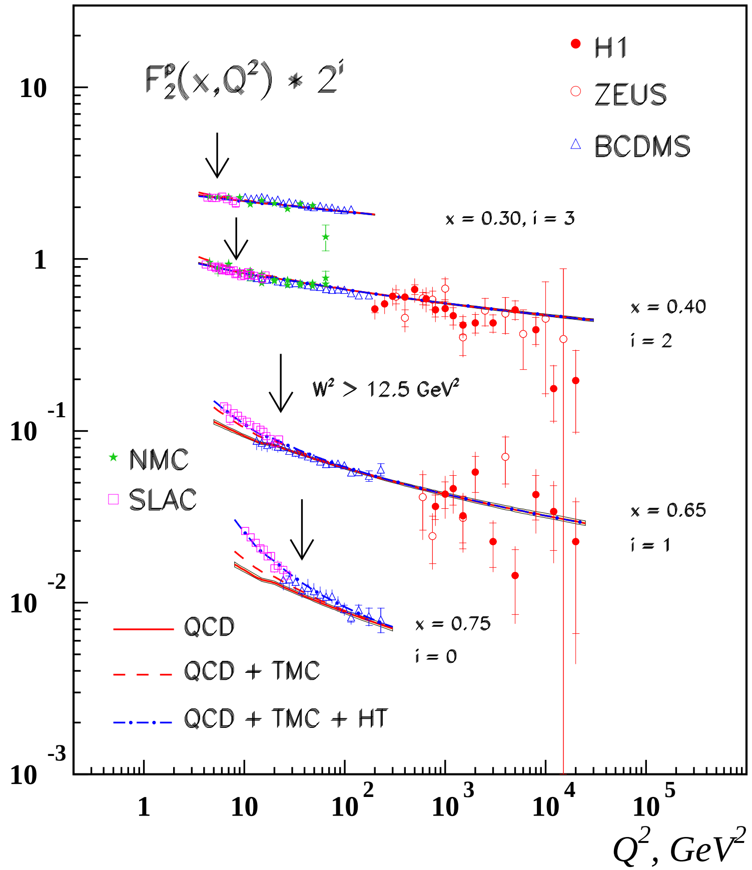
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

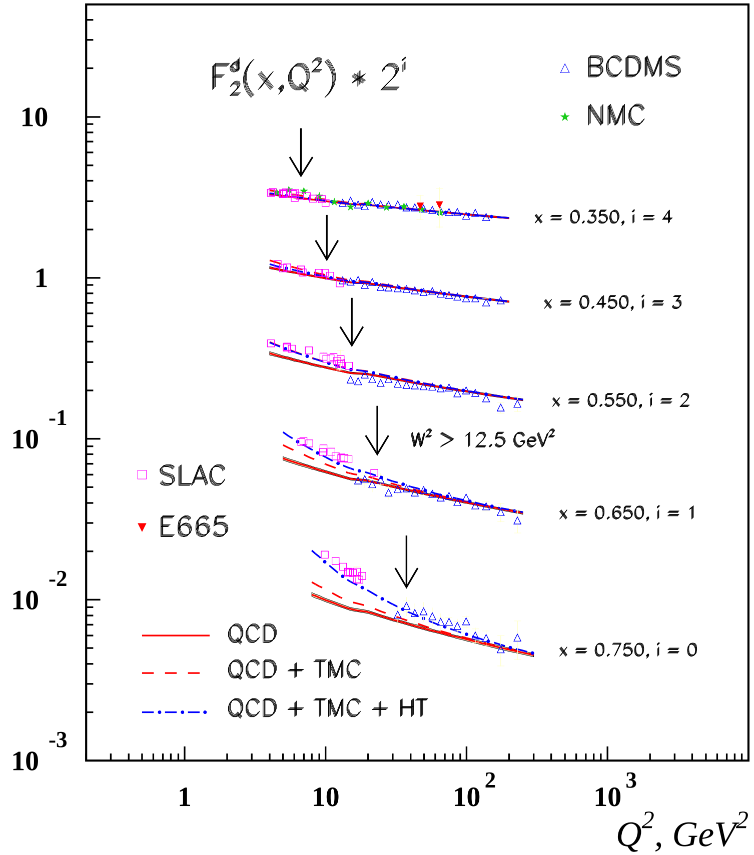
P. Baikov, K. Chetyrkin Nucl.Phys.Proc.Suppl. **160** (2006) 76.

$$\begin{aligned}\gamma_2^{3;NS} \quad &= \quad \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 \\ &+ \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4\end{aligned}$$

The results agree better than 20%.

# $F_2(x, Q^2)$ in the Valence Region




No validity of the twist-2 approximation left from the arrow.

## $\alpha_s(M_Z^2)$

$$\frac{da(\mu^2)}{d\ln(\mu^2)} = -\sum_{k=0}^{\infty} \beta_k a^{k+2}(\mu^2) \ .$$

$\alpha_s(M_Z^2)$ is to be determined together with the valence distributions in the same analysis.

## Overview of the analyzes :

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses $\nu N$ world data
- NS-N$^3$LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

# $\alpha_s(M_Z^2)$

| **NNLO** | $\alpha_s(M_Z^2)$ | expt | theory | Ref. |
|---|---|---|---|---|
| MRST03 | 0.1153 | $\pm 0.0020$ | $\pm 0.0030$ | [2] |
| A02 | 0.1143 | $\pm 0.0014$ | $\pm 0.0009$ | [3] |
| SY01(ep) | 0.1166 | $\pm 0.0013$ | | [8] |
| SY01($\nu$N) | 0.1153 | $\pm 0.0063$ | | [8] |
| GRS | 0.111 | | | [10] |
| A06 | 0.1128 | $\pm 0.0015$ | | [11] |
| BBG | 0.1134 | $+0.0019/-0.0021$ | | [9] |
| **$N^3$LO** | $\alpha_s(M_Z^2)$ | expt | theory | Ref. |
| BBG | 0.1141 | $+0.0020/-0.0022$ | | [9] |

NNLO and $N^3$LO

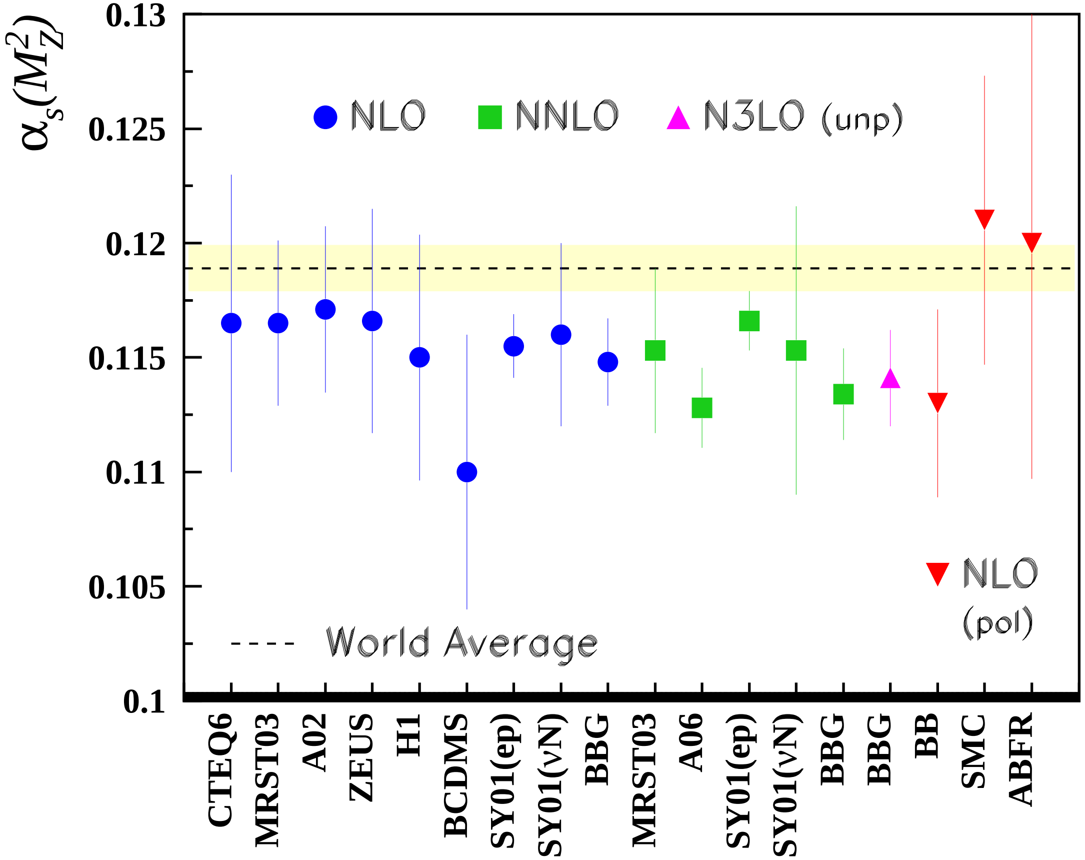
BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26\text{MeV}$

Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16\text{MeV}$

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26\text{MeV}$

# $\alpha_s(M_Z^2)$




J.B., H. Böttcher, A. Guffanti, 2006 [2]

# Higher Twist at Large x

- Non-leading twist terms :

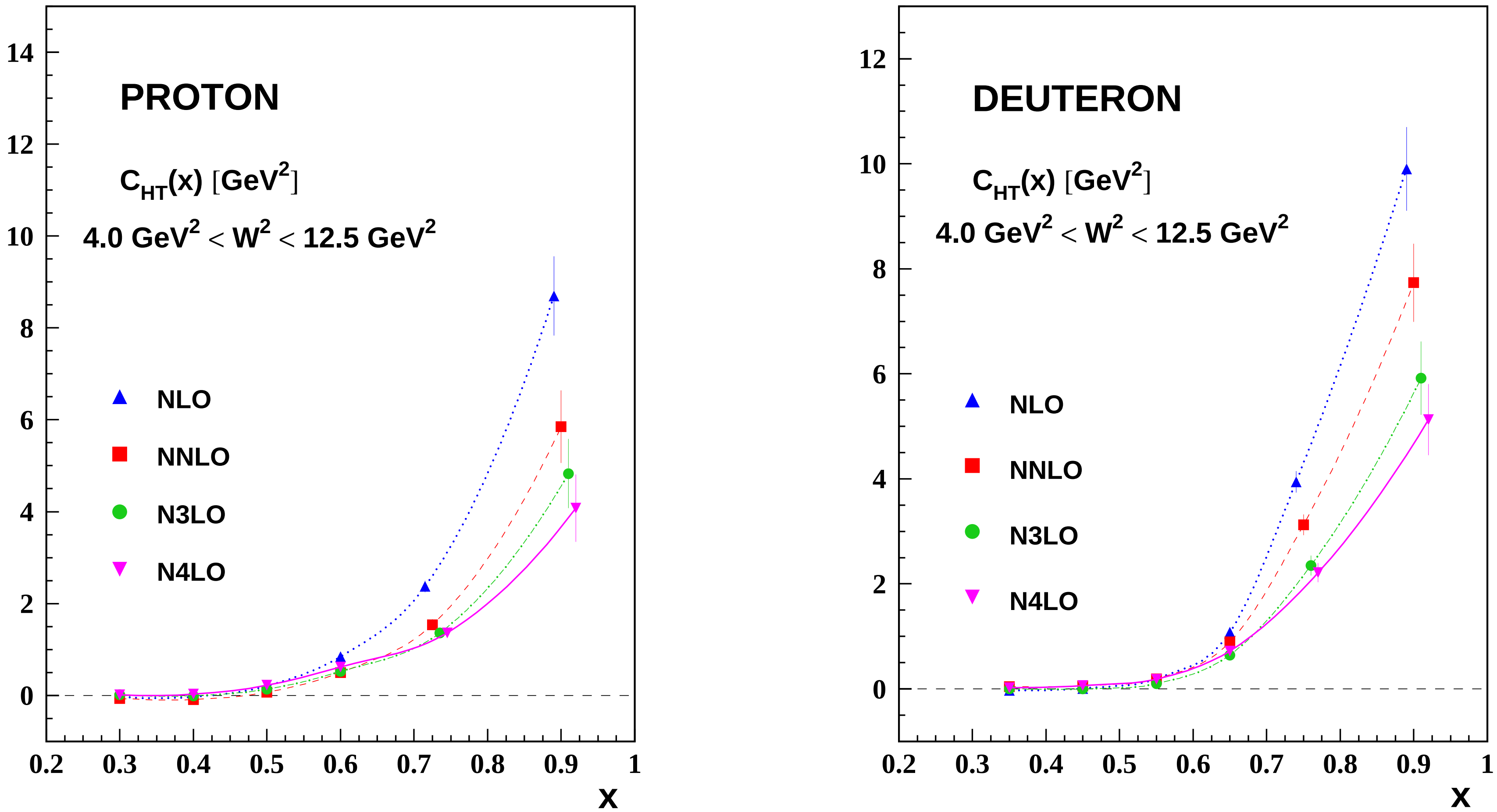
$$F_2^{\rm exp}(x,Q^2) = F_2^{\rm tw2}(x,Q^2) \cdot \left[\frac{O_{\rm TMC}\left[F_2^{\rm tw2}(x,Q^2)\right]}{F_2^{\rm tw2}(x,Q^2)} + \frac{C_{\rm HT}(x,Q^2)}{Q^2[1\,{\rm GeV}^2]}\right] .$$

- Extraction for all $(x, Q^2)$ bins
- No HT-assumptions, evolution etc. needed
- Complete Analysis : 2-loop, 3-loop
- 4-Loop analysis Padé for the 4-loop anomalous dimension with 100% error assumed
- Beyond that level: Large $x$ contributions to the Wilson coefficient at $O(\alpha_s^4)$; neglecting anomalous dimension effects. The latter term is used for HT extraction only.
- Mathematical Structure of the large $x$ terms in Mellin Space

$$S_{1,\ldots,1}(N) = \frac{1}{n}\left[S_n(N) + S_1(N)S_{n-1}(N) + S_{1,1}(N)S_{n-2}(N) + \ldots\right] .$$

$$\mathrm{S}_1(N) \propto \ln(N) + \gamma_E; \qquad S_l(N) \propto \zeta_l, \quad l \geq 2 ,$$

# Higher Twist at Large $x$




Each higher order leads to a depletion. N4LO converged for $x \leq 0.8$

- Agreement between $p$ and $d$ analysis
- LGT simulations of these terms are of interest

# Twist-2 Integral Relations

$$g_2^{\tau=2}(x,Q^2) = -g_1^{\tau=2}(x,Q^2) + \int_x^1 \frac{dy}{y} g_1^{\tau=2}(y,Q^2)$$

$$g_3^{\tau=2}(x,Q^2) = 2x \int_x^1 \frac{dy}{y^2} g_4^{\tau=2}(y,Q^2)$$

$$g_5^{\tau=2}(x,Q^2) = \frac{1}{2x}\left[g_4^{\tau=2}(x,Q^2) + \Delta_{CG}\left(x, M^2/Q^2\right)\right]$$


[1 ] S. Wandzura and F. Wilczek, Phys. Lett. B **72** (1977) 195.

[2 ] J.B., and N. Kochelev, Nucl.Phys.**B498** (1997) 285.

[3 ] J.B., and A. Tkabladze, Nucl.Phys.**B553** (1999) 427.

[4 ] for the gluonic case (pure photon exchange): J.B., V. Ravindran and W. L. van Neerven, Phys. Rev. D **68** (2003) 114004

# Twist-3 Integral Relations

$$g_1^{\tau=3}(x,Q^2) = \frac{4M^2x^2}{Q^2}\left[g_2^{\tau=3}(x,Q^2) - 2\int_x^1 \frac{dy}{y} g_2^{\tau=3}(y,Q^2)\right]$$

$$\frac{4M^2x^2}{Q^2} g_3^{\tau=3}(x,Q^2) = g_4^{\tau=3}(x,Q^2)\left(1+\frac{4M^2x^2}{Q^2}\right) + 3\int_x^1 \frac{dy}{y} g_4^{\tau=3}(y,Q^2)$$

$$g_5^{\tau=3}(x,Q^2) = -\frac{1}{2x}\int_x^1 \frac{dy}{y} g_4^{\tau=3}(y,Q^2)$$

One may therefore unfold both the twist-2 and -3 contributions to all the five polarized structure functions using the above relations.

# Conclusions

- We performed a NS QCD Analysis to Three Loop Order, and under a weak assumption, to Four Loop Order determining the valence quark distributions and $\Lambda_{\rm QCD}$.
- Within this analysis we determined, free of assumptions, the higher twist contributions in the large $x$ region. We also considered the large $x$ contributions to the NS Wilson coefficient in $O(\alpha_s^4)$.
- HT extractions based on a Phenomenological Ansatz have the problem of yet unknown HT anomalous dimensions, Wilson Coefficients, and correlation functions.
- We observe gradual depletion of the higher twist contributions with increasing order of $\alpha_s$. The present description converges for values $x \leq 0.8$
- The twist-2 and twist-3 contributions to the polarized electro-weak structure functions can be unfolded using the respective Integral Relations.