Structure Functions and Hard Processes in the k_T Factorization Scheme

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LMU, Dec. 93

1. MASS FACTORIZATION AND AP EVOLUTION
2. LIPATOV EQU.: NO STRONG ORDERING
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6. HO CORRECTIONS TO F_L
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1. MASS FACTORIZATION AND ALTARELLI-PARISI EVOLUTION

∴ NS-evolution:

q 

$$dq_{NS}(x,k^2) = \frac{1}{8\pi^2}\, g_s(k^2)\, P_{NS}(x)\, \frac{dk^2}{k^2} \otimes q_{NS}(x,k^2)$$

STRONG $k_\perp$ ORDERING:

$$Q_0^2 \ll k_{\perp 1}^2 \ll \ldots \ll k_{N\perp}^2 \ll Q^2$$

$k_\perp$ & k_L integrations factorize.

$$M_n(Q^2) = M_n(Q_0^2)\exp\left\{-\frac{\gamma_{NS}^{(0,n)}}{8\pi\beta_0}\ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]\right\}$$

$$M_n(Q^2) = \int_0^1 dx\, x^{n-1}\, F_{NS}(x,Q^2)$$

$$\boxed{\curvearrowright\ \forall M^2: \quad M_n(Q^2) = M_n(M^2)\cdot\exp\left\{-\frac{\gamma_{NS}^{(0,n)}}{8\pi\beta_0}\ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(M^2/\Lambda^2)}\right]\right\}}$$

$$F_{NS}(x,Q^2) = F_{NS}(x,M^2) \otimes C_{NS}\left(x,\frac{Q^2}{M^2}\right)$$

$$[A \otimes B](x) = \int_0^1 dx_1 \int_0^1 dx_2 \; \delta(x - x_1 x_2)\, A(x_1) B(x_2).$$

AN <u>INDEPENDENT</u> INTEGRATION IS CERTAINLY NOT POSSIBLE, IF $x \ll 1$.

$$k^2_{\perp max} = Q^2\left(\frac{1-x}{x}\right) \underset{\text{assumption}}{\approx} Q^2 \quad , \text{ i.e. } \frac{1-x}{x} \approx 1$$

$$1 \approx 2x$$

$$x \sim 0.5$$

$\longrightarrow$ IF $x \lll 1$ (e.g.) $k^2_{\perp max}$ becomes x dependent

- WHAT MEANS THE NEGLECTION OF $k_\perp^2$-DEP. CONTRIBUTIONS IN $\hat{\sigma}$?

ALLOWED, IF $\langle k_\perp^2 \rangle / Q^2 \ll 1$ HOLDS.

$\longrightarrow$ NOT NECESSARILY FULFILLED AT SMALL x.

2. LIPATOV – EQUATION : NO STRONG $k_\perp$-ORDERING

LIPATOV et al. 1975, 77, 78

~ RANGE: $\alpha_s \cdot \log \frac{Q^2}{Q_0^2} \ll 1$

$\alpha_s \cdot \log \frac{1}{x} \sim 1$ (STRONG ORDERING IN X).

DERIVATION: FOR α_s = CONST. ONLY. : Df: $f(n, k^2) = \frac{\partial M_n(x, t)}{\partial \log k^2}$

$$\boxed{\begin{aligned} f(n,k^2) &= \frac{1}{n-1} f_0(n,k^2) + \frac{3\alpha_s}{\pi(n-1)} \mathbb{L}_1 \otimes f \\ \mathbb{L}_1 \otimes f &= \int_0^\infty \frac{dk'^2}{k'^2} \left\{ \frac{k^2}{|k'^2 - k^2|} \left[f(n,k'^2) - f(n,k^2) \right] + \frac{f(n,k^2)}{\sqrt{k^4 + 4k'^4}} \right\} \end{aligned}}$$

$\mathbb{L}_1$ is homogeneous in k^2 & k'^2 $\curvearrowright$ EIGENFUNCTIONS ARE SIMPLE POWERS.

$$\mathbb{L}_1 \otimes f_\omega = K(\omega) \otimes f_\omega$$

$$f_\omega = (k^2)^{\omega + \frac{1}{2}}$$

$$K(\omega) = -2\gamma_E - \psi\left(\tfrac{1}{2}+\omega\right) - \psi\left(\tfrac{1}{2}-\omega\right).$$

$$\psi(x) = \frac{d}{dx} \ln \Gamma(x).$$

SOLUTION OF THE EQUATION:

$$f_0(n, k^2) = \int \frac{d\omega}{2\pi i} (k^2)^{\omega+\frac{1}{2}} e_0(n,\omega)$$

$$f(n, k^2) = \int \frac{d\omega}{2\pi i} (k^2)^{\omega+\frac{1}{2}} e(n,\omega)$$

$$\boxed{e(n,\omega) = \frac{e^0(n,\omega)}{n-1 - \frac{3\alpha_s}{\pi} K(\omega)}}$$

VALIDITY : $n \approx 1$ $\quad \ln\frac{1}{x} \gg 1;$ $\quad x \ll 1.$

BRANCH POINT SINGULARITY AT

$$\boxed{n_0 = 1 + \frac{3\alpha_s}{\pi} K(0) = 1 + 2.64\,\alpha_s}$$

$$\frac{1}{n-\omega} = \int_0^1 dx\, x^{n-1} \underline{\underline{x^{-\omega}}} \;!$$

$$\frac{\partial\, xG}{\partial \log Q^2} \sim x^{-n_0}.$$

WHAT is n_0 ?

$\alpha_s = $ const.

e.g.: $\alpha_s \sim 0.25$ ('educated guess' - toy number!)

$$\frac{\partial G}{\partial \log Q^2} \sim \frac{1}{x^{1.66}}$$

$$\alpha_s \sim 0.18939$$

$$\frac{\partial G}{\partial \log Q^2} \sim \frac{1}{x^{1.5}}$$

ATTEMPTS TO LET α_s RUN:

COLLINS, KWIECINSKI 1989

PRIZE: IF α_s RUNS ONE HAS TO 1°: FREEZE IT FOR LOW Q^2

2°: ONE HAS TO CUT SOMEHOW.

(PROBLEMATIC!)

$$f(n, k^2) = \frac{f_0(n, k^2, k_0^2)}{n-1} + \frac{1}{n-1} \mathbb{L}_2 (k_0^2, k^2) \otimes f$$

$$\mathbb{L}_2 \otimes f = \frac{3\alpha_s(k^2) k^2}{\pi} \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \left[\frac{f(n, k'^2) - f(n, k^2)}{|k'^2 - k^2|} + \frac{f(n, k^2)}{\sqrt{k^4 + 4k'^4}} \right]$$

DISCRETE SPECTRUM, BOUNDED FROM BELOW,

$$1 + \frac{3.6}{\pi}\alpha_s(k_0^2) \leq n_0 \leq 1 + 4\ln 2\left(\frac{3}{\pi}\right)\alpha_s(k_0^2)$$

EXAMPLE:

$Q^2 = 10\,\mathrm{GeV}^2$
$\Lambda = 200\,\mathrm{MeV}$
$N_f = 4$

$\curvearrowright$ $1.31 \leq n_0 \leq 1.72.$

CAUTION:

- LIPATOV EQUATION & RUNNING OF α_s NOT SETTLED YET. $\longrightarrow$ NO RGE FOR $\frac{s}{t} \gg 1$.

  2 DIM. EFFECT. FIELD THEORY
  WHAT IS THE EFFECTIVE COUPLING CONSTANT THERE?

$\Longrightarrow$ LIPATOV LIKE SUMMATIONS, ALTHOUGH STUDIED FIRST FOR p-p SCATTERING ADVOCATE A MORE GENERAL FACTORIZATION THAN MASS FACTORIZATION.

3. $k_\perp$ - FACTORIZATION - 1ST ATTEMPTS

SMALL x & LEADING MOMENTS ?

FOLKLORE: 'DOMINANCE OF THE 0^{th} MOMENT'

($\rightarrow$ WHERE ?!)

$$\int_0^1 dx\, x^{\alpha-1} F_{const} \sim \frac{A}{\alpha} \gg 1 \; ; \; \alpha \to 0$$

HOWEVER:

$$\mathbb{F}(x) = \int_x^1 \frac{dy}{y} \hat{\sigma}_F\left(\frac{x}{y}\right) \mathbb{G}(y)$$

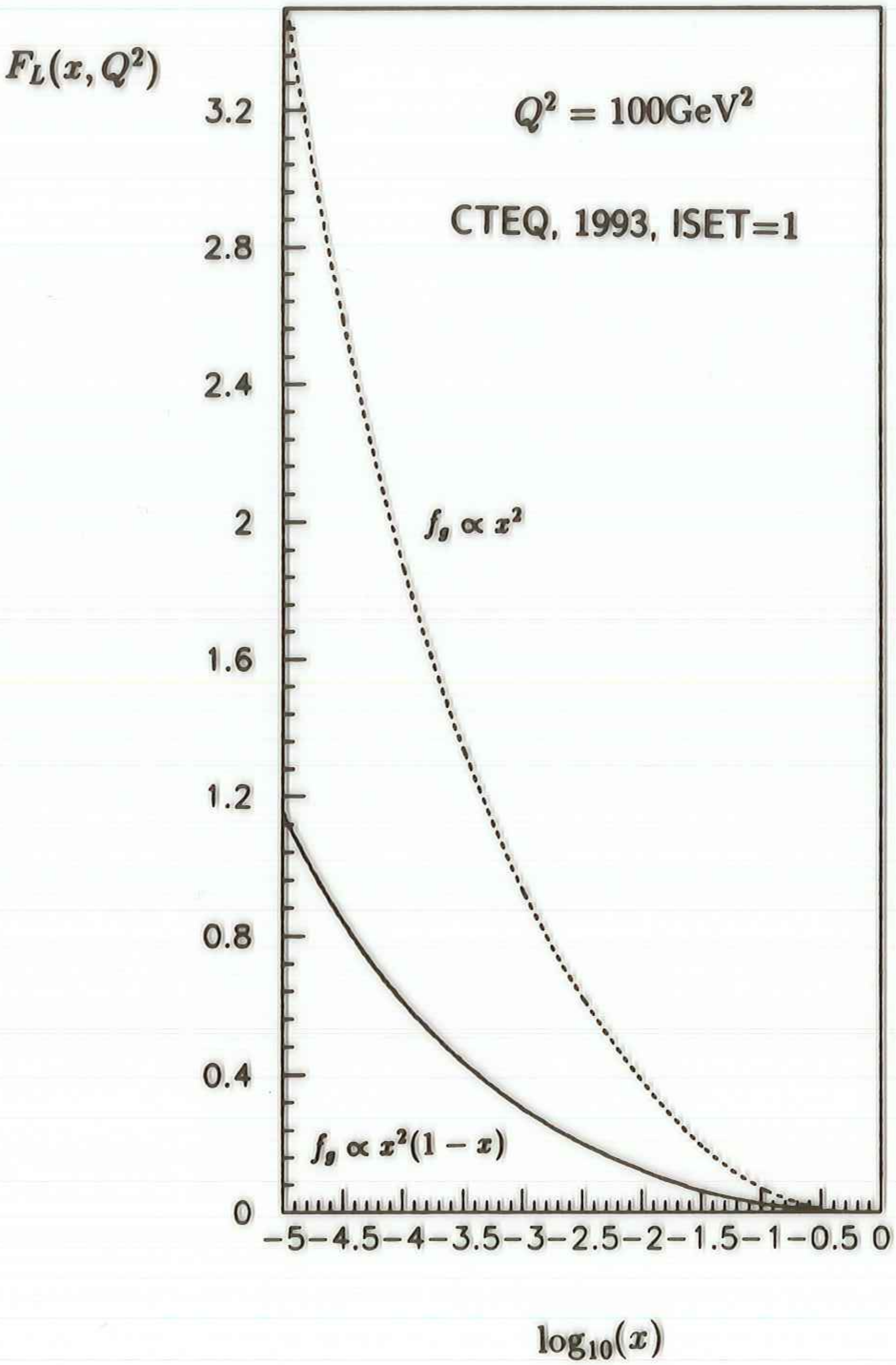
- $\mathbb{G}(y)$ FALLS WITH $y \to 1$ RAPIDLY
- MEDIUM! x/y VALUES ARE IMPORTANT

$\hat{\sigma}_F(\xi \to 0)$ MAY BE INSUFFICIENT.

An Example:

COLLINEAR APPROACH: $O(\alpha_s)$

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} x^2 \int_x^1 \frac{dy}{y^3} \left\{ \frac{8}{3} F_2(y,Q^2) + 2 \left(\sum_{i=1}^{2f} e_i^2 \right) \left(1 - \frac{x}{y} \right) y G(y,Q^2) \right\}$$



4. $k_\perp$ Factorization in the DIS Scheme

- The $k_\perp$ integration starts at $K^2 = 0$.
  However, a physical definition of $\partial xG(x, K^2)/\partial K^2$ is only possible for $K^2 \leq Q_0^2$!

$\boxed{\Longrightarrow \textbf{ Naive } k_\perp \textbf{ Factorization fails.}}$

(SEE DISCUSSION ABOVE)

Consider:

$$Q_0^2 \ll Q^2$$

For $K^2 \leq Q_0^2$: $f_i^{qG}\left(\frac{K^2}{Q^2}, \frac{x}{\eta}\right) \Longrightarrow f_i^{qG}\left(\frac{K^2}{Q^2} \to 0, \frac{x}{\eta}\right)$, $i = 2, L$

- The 'naive' factorization relation:

$$F_i(x, Q^2) = \int_0^1 \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) \int d^2\ k f_i(x_1, K^2/Q^2) \frac{\partial x_2 G(x_2, K^2)}{\partial K^2}$$

has to be replaced by:

$$\begin{aligned}
F_L(x, Q^2) &= \int_x^1 \frac{d\eta}{\eta} f_L\left(\frac{x}{\eta}\right) \eta G(\eta, Q_0^2) + \int_x^1 \frac{d\eta}{\eta} \int_{Q_0^2}^{K^2_{max}} dK^2 f_L\left(\frac{x}{\eta}, \frac{K^2}{Q^2}\right) \frac{\partial \eta G(\eta, K^2)}{\partial K^2} \\
F_2(x, Q^2) &= F_2^{coll}(x, Q^2) \\
&+ \int_x^1 \frac{d\eta}{\eta} \int_{Q_0^2}^{K^2_{max}} dK^2 \left\{ f_2\left(\frac{x}{\eta}, \frac{K^2}{Q^2}\right) - f_2\left(\frac{x}{\eta}, \frac{\Lambda^2}{Q^2}\right) \theta(Q^2 - K^2) \right\} \frac{\partial \eta G(\eta, K^2)}{\partial K^2}
\end{aligned}$$

$F_2^{coll}(x, Q^2)$ denotes the structure function F_2 calculated for collinear initial state gluons in the DIS scheme.

$$\sum_\lambda \varepsilon_\mu^*(\lambda) \varepsilon_\nu(\lambda) = \frac{P_\mu P_\nu}{k^2}$$

The scale of α_s:

For the present leading order calculation one has to choose a typical hard scale, characterizing the process: Q^2, W^2.

AKMS

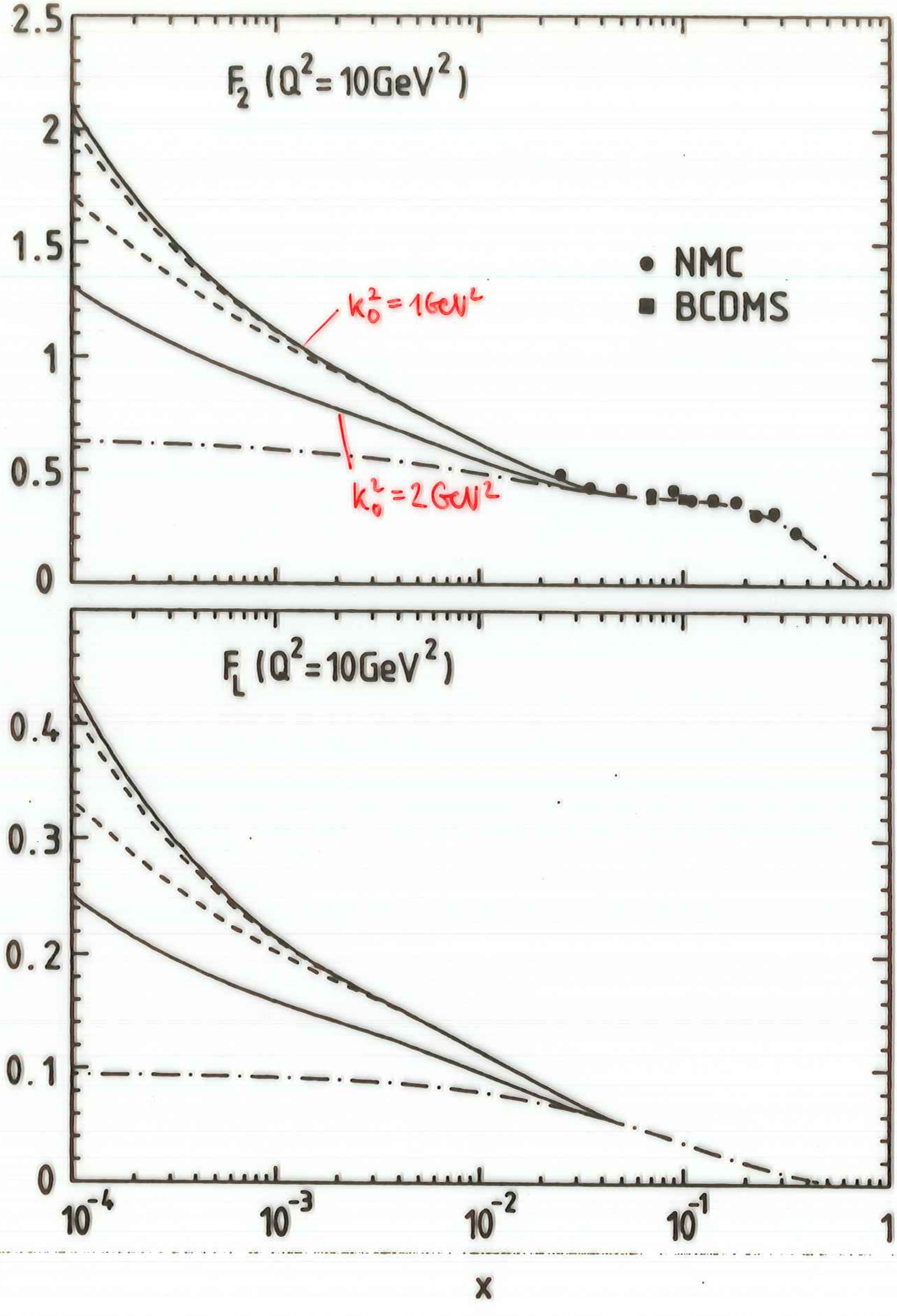
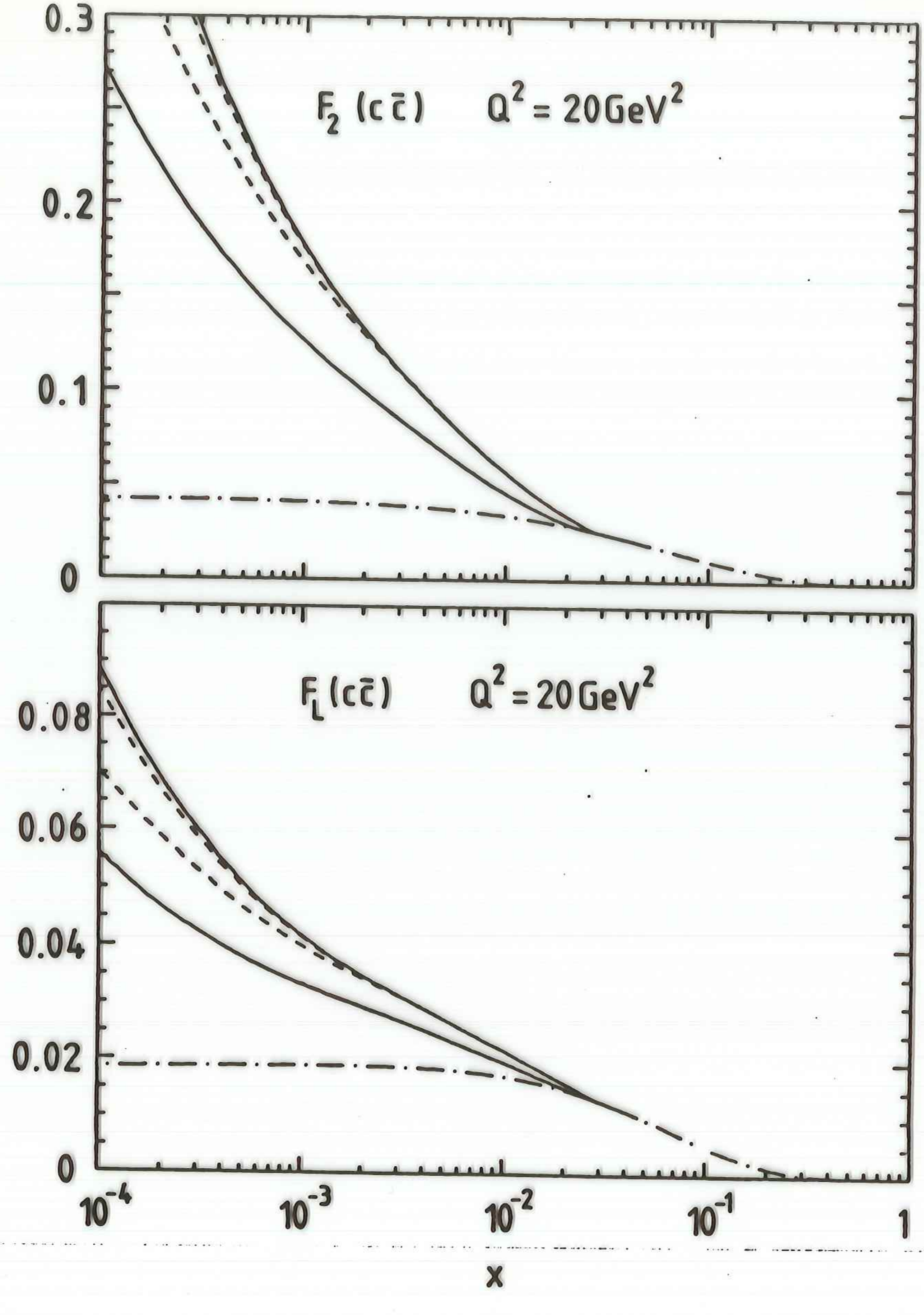

F2 (Q2=10GeV2)
NMC
BCDMS
k0^2 = 1 GeV^2
k0^2 = 2 GeV^2
FL (Q2=10GeV2)
2.5
2
1.5
1
0.5
0
0.4
0.3
0.2
0.1
10^-4
10^-3
10^-2
10^-1
1
x

Fig. 5

AKMS



0.3
0.2
0.1
0
$F_2\,(c\bar{c})$ $Q^2 = 20\,GeV^2$
0.08
0.06
0.04
0.02
0
$F_L\,(c\bar{c})$ $Q^2 = 20\,GeV^2$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Fig. 7

PROBLEMS TO BE SOLVED:

1) FACTORIZATION OF THE INCOMING PARTON DENSITY

2) $\boxed{\text{CONVOLUTION}}$ IN X-SPACE

$$F(x) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x - x_1 x_2) \, f(x_1) \, D(x_2)$$

f AT $x_1 \sim 1$ SAMPLES D AT $x_2 \sim x$
AND VICE VERSA!

$\longrightarrow$ CALCULATE $f(x_1)$ FOR ALL x_1.

3) HOW TO MAKE CONTACT WITH THE REAL GLUON & QUARK DENSITIES ?

$\longrightarrow$ SCHEME DEPENDENCE

$\longrightarrow$ TREATMENT OF COLINEAR SINGULARITIES

$\longrightarrow$ THIS OPENS THE WAY TO MORE GENERAL GLUON DENSITIES ; NOT ONLY $\alpha_s = $ CONST.
LIPATOV SOLUTION CAN BE IMPLEMENTED!

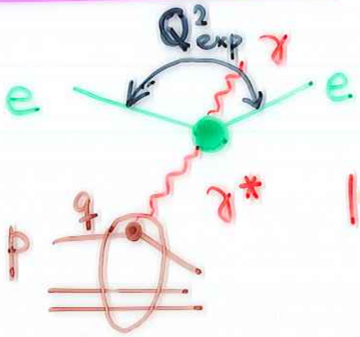
$\longrightarrow$ GLR-LIKE SOLUT. etc. ALSO.

4) TECHNICAL QUESTION: LOOK FOR A SUITABLE REF. FRAME: JB $\leftrightarrow$ CIAFALONI et al.

A STEP ASIDE:

ACCESS TO STRUCTUREFUNCTIONS AT SMALL x & SMALL Q^2

COMPTON - RC:

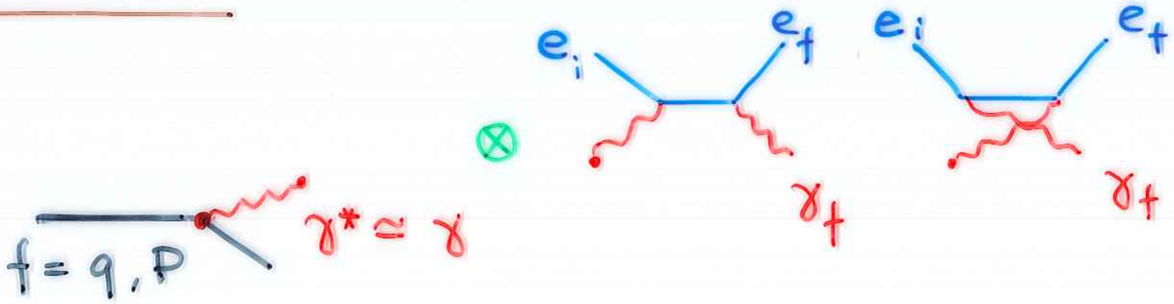


$|q|^2 \ll M_p^2$

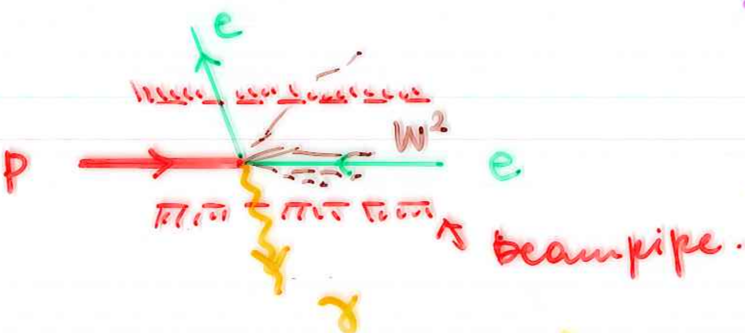
⇒ POTENTIAL CONTRIBUTION TO RC's

→ LEPTONMEASUREMENT!

DIAGRAMS:



SIGNATURE:



- φ nearly balanced γ-e pair
- little W^2_{vis}, if any

⇒ NO DIS CLASSIFICATION!

PRIZE TO PAY:

- TERMS (SOME OF THEM)

$$\sim \left(\frac{Q_0^2}{Q^2}\right)^n \quad \text{ARE NEGLECTED.}$$

$\longrightarrow$ 'HIGHER TWISTS' ARE NEGLECTED IN OTHER PLACES TOO!

$$\longmapsto \quad Q_0^2/Q^2 \ll 1 \quad \& \quad x \leq \frac{1}{1+\frac{Q_0^2}{Q^2}} .$$

CAN ONE MAKE USE OUT OF THIS 'BACKGROUND' ?

· JB, 1989
· JB, SPIESBERGER, LEVMAN 1990; 93

$$\frac{d\sigma^c}{dx_l\, dy_l} = \frac{\alpha^3}{x_\ell S} \frac{1+(1-y_l)^2}{1-y_\ell} \int_{x_\ell}^{1} dz \int_{Q^2_{h\,min}}^{Q^2_l} \frac{dQ^2_h}{Q^2_h} \frac{z}{x_\ell}$$

$$\times \left[\frac{1+(1-z)^2}{z} F_2\left(\frac{x_\ell}{z}, Q^2_h\right) - F_L\left(\frac{x_l}{z}, Q^2_h\right) \right]$$

⟶ find $F_2 - F_L$ combination at small x & —"— Q^2_h.

COMPLETELY NONPERTURBATIVE RANGE !

⟶ USE OF THAT ?

⟼ • PHOTONSTRUCTURE $x \ll 1$

• INPUT FOR INITIAL STATE RADIATION (BORN CROSS SECTION $x \ll 1$, $Q^2 \ll M_p^2$);

⟶ IF WE DO NOT HAVE IT, THE SMALL x UNFOLDING OF $F_2(x, Q^2)$ AT HIGHER Q^2! BECOMES UNRELIABLE.

ONE CAN UNFOLD:

$$D_{\gamma/p}(x_\ell, Q_\ell^2) = \frac{\alpha}{2\pi} \int_{x_\ell}^{1} dz \int_{Q^2_{h,min}}^{Q_\ell^2} dQ_h^2/Q_h^2 \; \frac{z}{x_\ell} \left[\frac{1+(1-z)^2}{z^2} f_2\left(\frac{x_\ell}{z}, Q_h^2\right) - f_L\left(\frac{x_\ell}{z}, Q_h^2\right) \right]$$

USING:

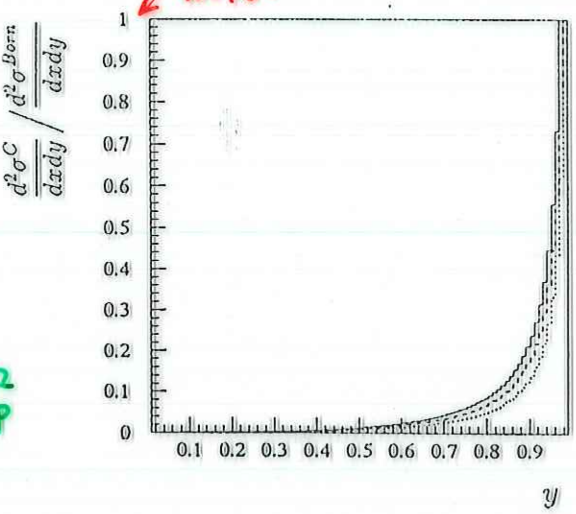
$$\frac{d^2\sigma^c}{dx_\ell dy_\ell} = \int_0^1 \frac{dz}{z} \underline{D_{\gamma/p}(z, Q_\ell^2)} \left. \frac{d^2\hat{\sigma}(e\gamma \to e\gamma)}{d\hat{x}\, dy_\ell} \right|_{\substack{\hat{s} = zs \\ \hat{x} = x_\ell/z}}$$

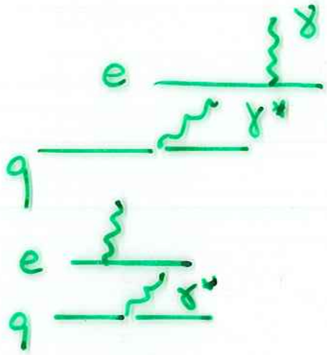
$$\frac{d^2\hat{\sigma}}{d\hat{x}\, dy_\ell}(e\gamma \to e\gamma) = \frac{2\pi\alpha^2}{\hat{s}} \frac{1+(1-y_\ell)^2}{1-y_\ell} \delta(1-\hat{x}).$$

'COMPTON' PROCESS

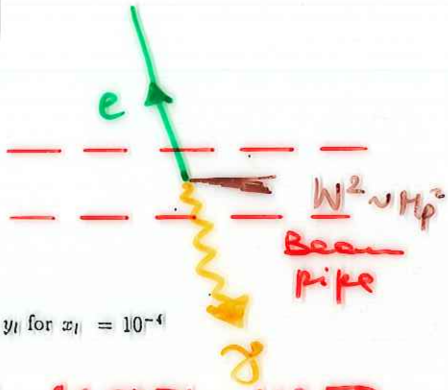
JB, LEVMAN, SPIESBERGER

100%.





$Q^2 \ll M_p^2$



Final state

Figure 1: Differential Compton cross section Eq. (8) as a function of y_l for $x_l = 10^{-4}$ (dotted line), 10^{-3} (dashed line), and 10^{-2} (full line).

(SOME MODEL USED

$F_i \sim \left(\frac{Q^2}{Q^2+\mu^2}\right)^{\alpha} \cdots$)

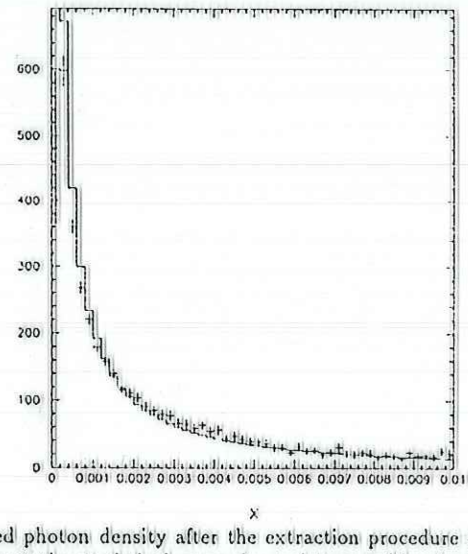


Figure 5: The expected photon density after the extraction procedure described in the text. The errors represent the statistical errors for an integrated luminosity of 100 pb^{-1}. The solid histogram is the prediction of Eq. (13) for HMRSB [21]. The units of the vertical scale are arbitrary.

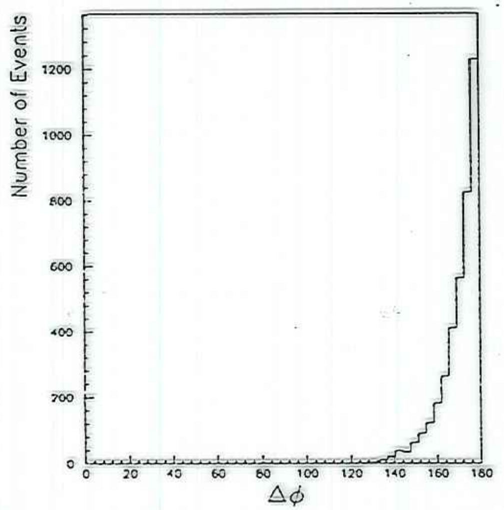


Figure 2: The difference in azimuth of the photon and electron for accepted Compton events.

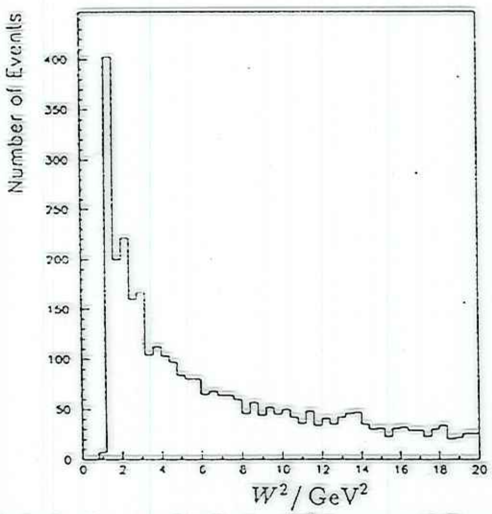


Figure 3: The hadronic mass distribution W^2 for accepted Compton events.

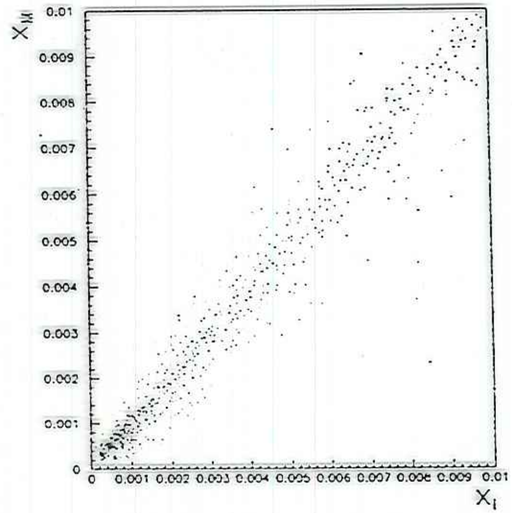


Figure 4: A scatter plot of $x_l = Q_l^2/2p \cdot (l - l')$ and $x_M = M_{e\gamma}^2/s$.

OTHER APPROACHES:

E.M. LEVIN, M.G. RYSKIN, Sov. J. Nucl. Phys. 53 (1991) 653

A.J. ASKEW, J. KWIECINSKI, A. MARTIN, P. SUTTON, Phys. Rev. D47 (1993) 3775.

$\longrightarrow$ FACTORIZATION UNCLEAR

- LR : $\hat{\sigma}(x, k_\perp^2)$ ONLY FOR $x \to 0$

  $\frac{\partial xG}{\partial \ln Q^2} \propto$ LIP. EQ ONLY

- AKMS : 'INFRARED' PROBLEM

  α_s FREEZING $\leftrightarrow$ NON PERT. CONTR.: LATTICE RESULTS LÜSCHER et al. '93.

  - CUT IN $k_\perp \geq k_{\perp min} > 0$ $\sim 1 GeV^2$

    NOT POSSIBLE IN INCLUSIVE QUANTITIES!

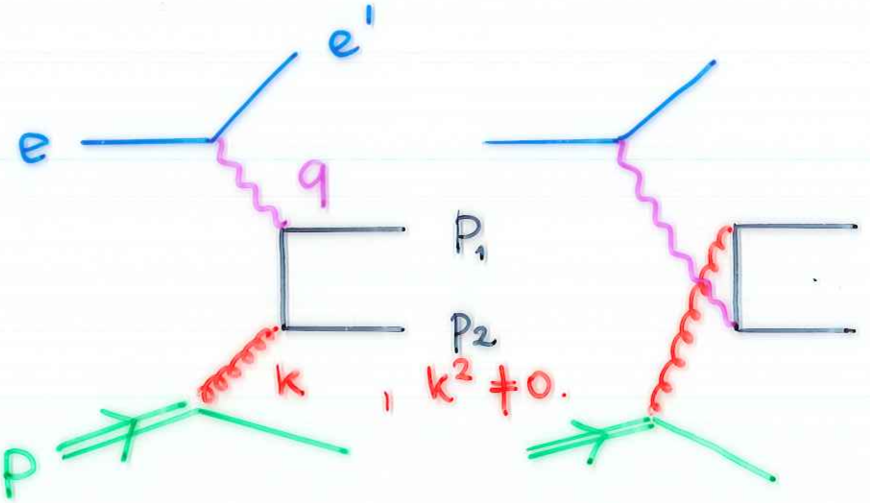
$\Longrightarrow$ DYING OUT OF $xG(x, Q^2 \to 0) \propto \frac{Q^2}{Q^2+\mu^2}$?

$\longleftrightarrow$ PERTURBATION THEORY

WHAT DO WE LEARN UNDER THESE ASSUMPTIONS ?

5. CALCULATION OF $\hat{\sigma}_{2,L}(x, k^2, Q^2)$

2 Phase space



$$q + k = p_1 + p_2$$

TECHNICALLY IMPORTANT !

(CIAFALONI et al. SUDAKOV VAR. BASED ON P & ℓ → PROBLEM TO OBTAIN $d\sigma_{diff}$)

$$\begin{aligned}\hat{s} &= (q+k)^2 = (p_1+p_2)^2 \\ \hat{t} &= (k-p_1)^2 = (q-p_2)^2 \\ \hat{u} &= (q-p_1)^2 = (k-p_2)^2\end{aligned}$$

$$\hat{s} + \hat{t} + \hat{u} = -Q^2 - K^2 + m_1^2 + m_2^2$$

SUDAKOV NOTATION:

$$k_\mu = \xi q' + \eta P_\mu + k_{\perp\mu}$$

$$\xi = \frac{2k.P}{2q'.P}$$

$$\eta = \frac{2k.q'}{2q'.P}$$

$$x = \frac{Q^2}{2P.q}$$

$$K^2 = \boldsymbol{k}^2 - \xi\eta\frac{Q^2}{x}$$

$$y = \frac{P.q}{P.l_e} = \frac{Q^2}{sx}$$

$$\xi \ll \eta$$

$$\xi \approx 0 \qquad P.k \approx 0 \qquad k_\mu \approx \eta P_\mu + k_{\perp\mu} \qquad \underline{K^2 \approx \boldsymbol{k}^2}$$

The following relations may be derived with these definitions

$$\begin{aligned}
\hat{s} &= Q^2\left(\frac{\eta}{x}-1\right)-K^2 \\
\eta &= x\frac{\hat{s}+K^2+Q^2}{Q^2} \\
\hat{t} &= m_1^2-K^2-z(\hat{s}+K^2+Q^2) \\
s_\gamma &\equiv W^2 \approx Q^2\frac{1-x}{x}
\end{aligned}$$

The representation of the particle momenta is thus given by:

$$\begin{aligned}
k &= (K_0, 0, 0, |\vec{k}|) \\
q &= (Q_0, 0, 0, -|\vec{k}|) \\
P &= E_p(1, \sin\beta, 0, \cos\beta) \\
p_1 &= (E_1, q_1\sin\theta\cos\varphi, q_1\sin\theta\sin\varphi, q_1\cos\theta) \\
p_2 &= (E_2, -q_1\sin\theta\cos\varphi, -q_1\sin\theta\sin\varphi, -q_1\cos\theta)
\end{aligned}$$

with

$$\begin{aligned}
K_0 &= \mathcal{E}(\hat{s}, -K^2, -Q^2) \\
Q_0 &= \mathcal{E}(\hat{s}, -Q^2, -K^2) \\
E_1 &= \mathcal{E}(\hat{s}, m_1^2, m_2^2) \\
E_2 &= \mathcal{E}(\hat{s}, m_2^2, m_1^2) \\
|\vec{k}| &= \mathcal{P}(\hat{s}, -Q^2, -K^2) \\
q_1 &= \mathcal{P}(\hat{s}, m_1^2, m_2^2) \\
\cos\theta &= \frac{2K_0E_1+K^2-m_1^2+\hat{t}}{2|\vec{k}|q_1} \\
E_p &= \mathcal{E}(\hat{s}, 0, t_{qP'}) = \frac{Q^2}{2x\sqrt{\hat{s}}} \\
\cos\beta &= \frac{K_0}{|\vec{k}|}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{E}(a,b,c) &= \frac{a+b-c}{2\sqrt{a}} \\
\mathcal{P}(a,b,c) &= \sqrt{\frac{\lambda(a,b,c)}{4a}}
\end{aligned}$$

$$\cos\varphi = \left.\frac{(\boldsymbol{q}\times\boldsymbol{P}).(\boldsymbol{q}\times\boldsymbol{p}_1)}{|\boldsymbol{q}\times\boldsymbol{P}||\boldsymbol{q}\times\boldsymbol{p}_1|}\right|_{\boldsymbol{p}_1=-\boldsymbol{p}_2}$$

$$\cos\beta = \frac{\hat{s}-K^2+Q^2}{[(\hat{s}-K^2+Q^2)^2+4K^2\hat{s}]^{1/2}} = \frac{1-\rho}{\sqrt{1-2\rho x/\eta}}$$

$$\rho = \frac{2K^2x}{Q^2\eta}$$

ONE OBTAINS:

FROM: $-1 \leq \cos\beta \leq +1$

$$\rho\left[\rho - 2\left(1-\frac{x}{\eta}\right)\right] = 0 \qquad \text{(BOUNDARIES)}$$

$$0 \leq k^2 \leq Q^2\left(\frac{\eta - x}{x}\right)$$

or:

$$x\left(1+\frac{k^2}{Q^2}\right) \leq \eta \leq 1$$

COLLINEAR LIMIT:

$k^2 \to 0$

$$x \leq \eta \leq 1$$

$$\cos\beta \equiv 1.$$

i.e.: THE PROTON IN THE (q, k) CMS IS ALONG k.

(OTHER AUTHORS USE $k_\perp$ FOR THE GLUON IN THE $(\mathbb{P}, q)$ OR $(\mathbb{P}, \ell)$ CMS

$\longrightarrow$ LEADS TO TECHNICAL DIFFICULTIES SOMETIMES!).

$$
\begin{aligned}
dPS^{(3)} &= \frac{1}{128\pi^3}\frac{d\varphi_P}{2\pi}\frac{d\hat{s}d\hat{t}dK^2}{\lambda^{1/2}(\hat{s},-K^2,-Q^2)\lambda^{1/2}(s_\gamma,0,-Q^2)}\frac{d\varphi}{2\pi} \\
&\times\ \Theta\{-\mathcal{G}(s_\gamma,-K^2,\hat{s},0,-Q^2,0)\}\Theta\{-\mathcal{G}(\hat{s},\hat{t},0,-K^2,-Q^2,0)\}\Theta\{\hat{s}\}
\end{aligned}
$$

$$
\int dPS^{(3)} = \frac{1}{128\pi^3}\int_{\eta_{min}}^{\eta_{max}} d\eta \int_{K^2_{min}(\eta)}^{K^2_{max}(\eta)} dK^2 \int_0^{2\pi}\frac{d\varphi_P}{2\pi}\frac{1}{2}\int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\varphi}{2\pi}
$$

$$
\begin{aligned}
K^2_{min} &= 0 \\
K^2_{max} &= Q^2\frac{\eta-x}{x} \\
\eta_{min} &= x \\
\eta_{max} &= 1
\end{aligned}
$$

$$
dPS^{(2)} = \frac{1}{8\pi}\frac{1}{2}\int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\varphi}{2\pi}
$$

4 The Structure Functions

$$\frac{d^2\sigma}{dQ^2dy} = 2\pi\alpha^2\frac{Ms}{(s-M^2)^2}\frac{1}{Q^4}L_{\mu\nu}W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left[l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu} l.l'\right]$$

$$W_{\mu\nu} = \frac{1}{4\pi}\sum_n \langle P|J_\mu^{em\dagger}(0)|n\rangle\langle n|J_\nu^{em}(0)|P\rangle(2\pi)^4\delta^{(4)}(P+q-p_n)$$

$$W_{\mu\nu} = \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)W_1(x,Q^2) + \frac{1}{M^2}\left[\left(P_\mu - \frac{P.q}{q^2}q_\mu\right)\left(P_\nu - \frac{P.q}{q^2}q_\nu\right)\right]W_2(x,Q^2)$$

$$F_2(x,Q^2) = xT^1_{\mu\nu}W^{\mu\nu} = x\left(-g_{\mu\nu}+\frac{12x^2}{Q^2}P_\mu P_\nu\right)W^{\mu\nu}$$

$$F_L(x,Q^2) = xT^2_{\mu\nu}W^{\mu\nu} = \frac{8x^3}{Q^2}P_\mu P_\nu W^{\mu\nu}$$

The projections onto $-g_{\mu\nu}$ and $P_\mu P_\nu$ are

$$-g^{\mu\nu}\widehat{W}_{\mu\nu} = 32\pi\alpha_s e_q^2\left\{\frac{(p_1.P)^2+(p_2.P)^2}{\hat{t}\hat{u}} - \frac{Q^2}{K^2}\left[\frac{p_1.P}{\hat{t}} - \frac{p_2.P}{\hat{u}}\right]^2\right\}$$

$$P^\mu P^\nu \widehat{W}_{\mu\nu} = 64\pi\alpha_s e_q^2\frac{1}{K^2}\left\{-2\frac{(p_1.P)^2(p_2.P)^2}{\hat{t}\hat{u}} + \frac{(p_1.P)^3(p_2.P)}{\hat{t}^2} + \frac{(p_1.P)(p_2.P)^3}{\hat{u}^2}\right\}$$

$K^2 \to 0$:

$$-g^{\mu\nu}\widehat{W}_{\mu\nu} = 8\pi\alpha_s e_q^2\left\{\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{s}Q^2}{\hat{t}\hat{u}}\right\}$$

$$P_\mu P_\nu \widehat{W}_{\mu\nu} = 8\pi\alpha_s e_q^2\hat{s}$$

$$\boxed{F_L(x, Q^2),\ F_2(x, Q^2)}$$

F_L :

$$
\begin{aligned}
f_L^{qG}(K^2, x, Q^2) &= -\frac{1}{4\pi}\int dPS^{(2)} x T^2_{\mu\nu} \widehat{W}^{\mu\nu} \\
&= \frac{\alpha_s e_q^2}{4\pi}\left\{\frac{4Q^4}{K^4 x} G_{1L}(\beta,\zeta) + \frac{xQ^2}{K^2}\frac{1}{\sqrt{1-\zeta}}\log\left|\frac{1+\sqrt{1-\zeta}}{1-\sqrt{1-\zeta}}\right| G_{2L}(\beta,\zeta) \right. \\
&\qquad \left. + \frac{2xQ^2}{K^2} G_{3L}(\beta,\zeta)\right\}
\end{aligned}
$$

with

$$\zeta = \frac{4K^2x^2}{Q^2}$$

and

$$G_{iL}(\beta,\zeta) = -\sum_{j=0}^{4} g_{ji}^{L}(\beta)\left(\frac{\zeta}{W(\zeta)}\right)^j$$

where

$$W(\zeta) = 1 - \zeta + \sqrt{1-\zeta}$$

$$f_L^{qG(0)}(x, Q^2) = \frac{8x^3}{Q^2}\frac{1}{4\pi}\int dPS^{(2,0)} P_\mu P_\nu \widehat{W}^{\mu\nu} = \frac{2}{\pi} e_q^2 \alpha_s x^2 (1-x)$$

COEFFICIENTS:

$$
\begin{aligned}
g_{01}^{(L)}(\beta) &= -\frac{1}{8}+\frac{1}{4}\cos\beta-\frac{1}{4}\cos^3\beta+\frac{1}{8}\cos^4\beta\\
g_{02}^{(L)}(\beta) &= -\frac{1}{4}+2\cos\beta-\cos^2\beta-3\cos^3\beta+\frac{9}{4}\cos^4\beta\\
g_{03}^{(L)}(\beta) &= -\frac{1}{4}+6\cos\beta-\frac{9}{2}\cos^2\beta-10\cos^3\beta+\frac{35}{4}\cos^4\beta\\
g_{11}^{(L)}(\beta) &= \cos\beta-\frac{3}{4}\cos^2\beta-\frac{3}{2}\cos^3\beta+\frac{5}{4}\cos^4\beta\\
g_{12}^{(L)}(\beta) &= \frac{1}{4}+\frac{13}{2}\cos\beta-\frac{15}{2}\cos^2\beta-\frac{21}{2}\cos^3\beta+\frac{45}{4}\cos^4\beta\\
g_{13}^{(L)}(\beta) &= 1+18\cos\beta-24\cos^2\beta-30\cos^3\beta+35\cos^4\beta\\
g_{21}^{(L)}(\beta) &= \frac{3}{16}+\frac{9}{8}\cos\beta-\frac{9}{4}\cos^2\beta-\frac{15}{8}\cos^3\beta+\frac{45}{16}\cos^4\beta\\
g_{22}^{(L)}(\beta) &= \frac{5}{4}+\frac{27}{4}\cos\beta-15\cos^2\beta-\frac{45}{4}\cos^3\beta+\frac{75}{4}\cos^4\beta\\
g_{23}^{(L)}(\beta) &= \frac{7}{2}+18\cos\beta-42\cos^2\beta-30\cos^3\beta+\frac{105}{2}\cos^4\beta\\
g_{31}^{(L)}(\beta) &= \frac{3}{16}+\frac{6}{16}\cos\beta-\frac{15}{8}\cos^2\beta-\frac{5}{2}\cos^3\beta+\frac{35}{4}\cos^4\beta\\
g_{32}^{(L)}(\beta) &= \frac{9}{8}+\frac{9}{4}\cos\beta-\frac{45}{4}\cos^2\beta-\frac{15}{4}\cos^3\beta+\frac{105}{8}\cos^4\beta\\
g_{33}^{(L)}(\beta) &= 3+6\cos\beta-30\cos^2\beta-10\cos^3\beta+35\cos^4\beta\\
g_{41}^{(L)}(\beta) &= \frac{3}{64}-\frac{15}{32}\cos^2\beta+\frac{35}{64}\cos^4\beta\\
g_{42}^{(L)}(\beta) &= \frac{9}{32}-\frac{45}{16}\cos^2\beta+\frac{105}{32}\cos^4\beta\\
g_{43}^{(L)}(\beta) &= \frac{3}{4}-\frac{15}{2}\cos^2\beta+\frac{35}{4}\cos^4\beta
\end{aligned}
$$

$$G_{3L}\cdot\frac{1}{k^2} \rightarrow \frac{1}{k^2}\left(g_{03}(k^2)+ ck^2 g_{13}(k^2)+\ldots\right)$$

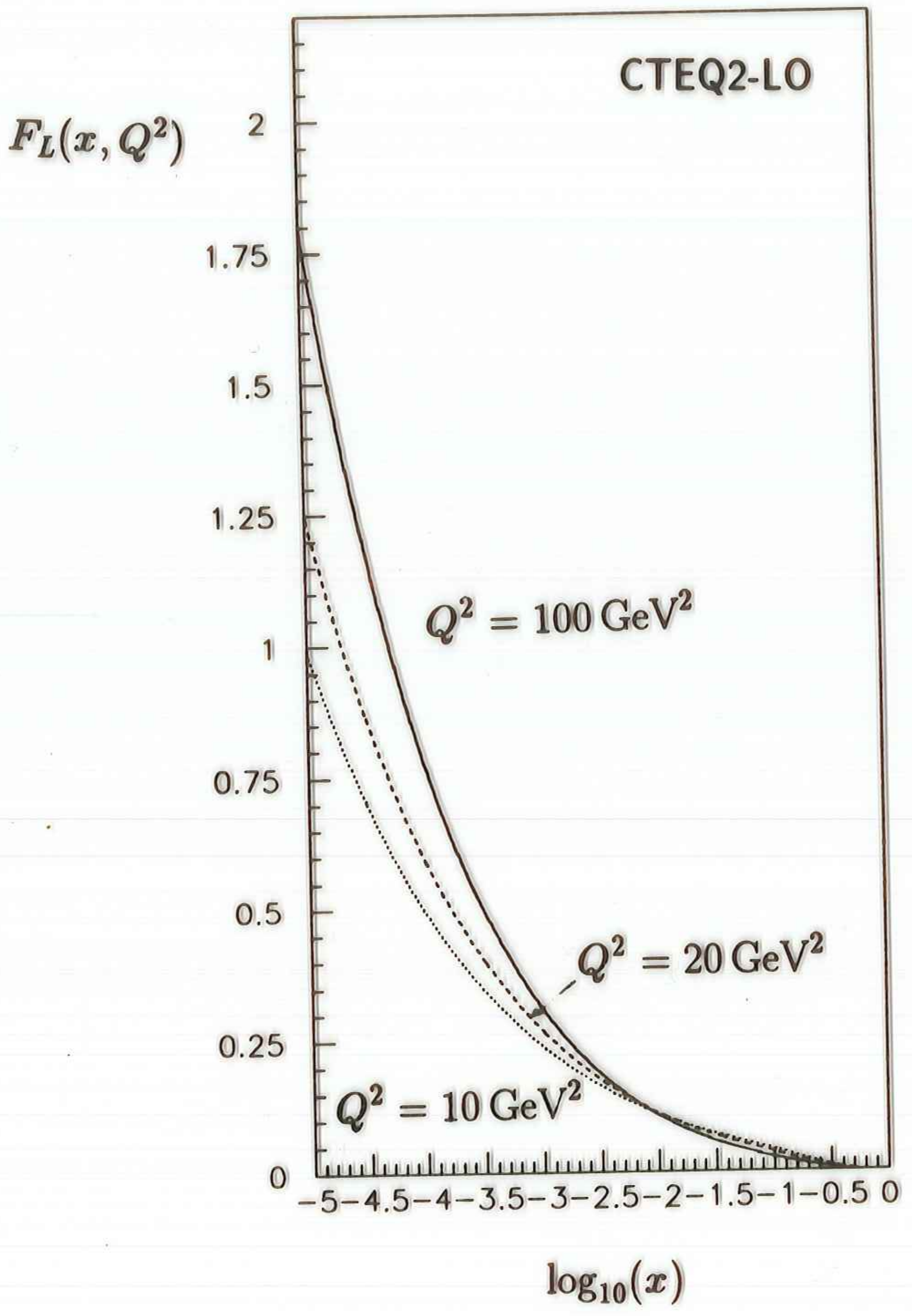
$$\uparrow$$

$$F_L(k^2\to 0) \qquad + \qquad 0$$

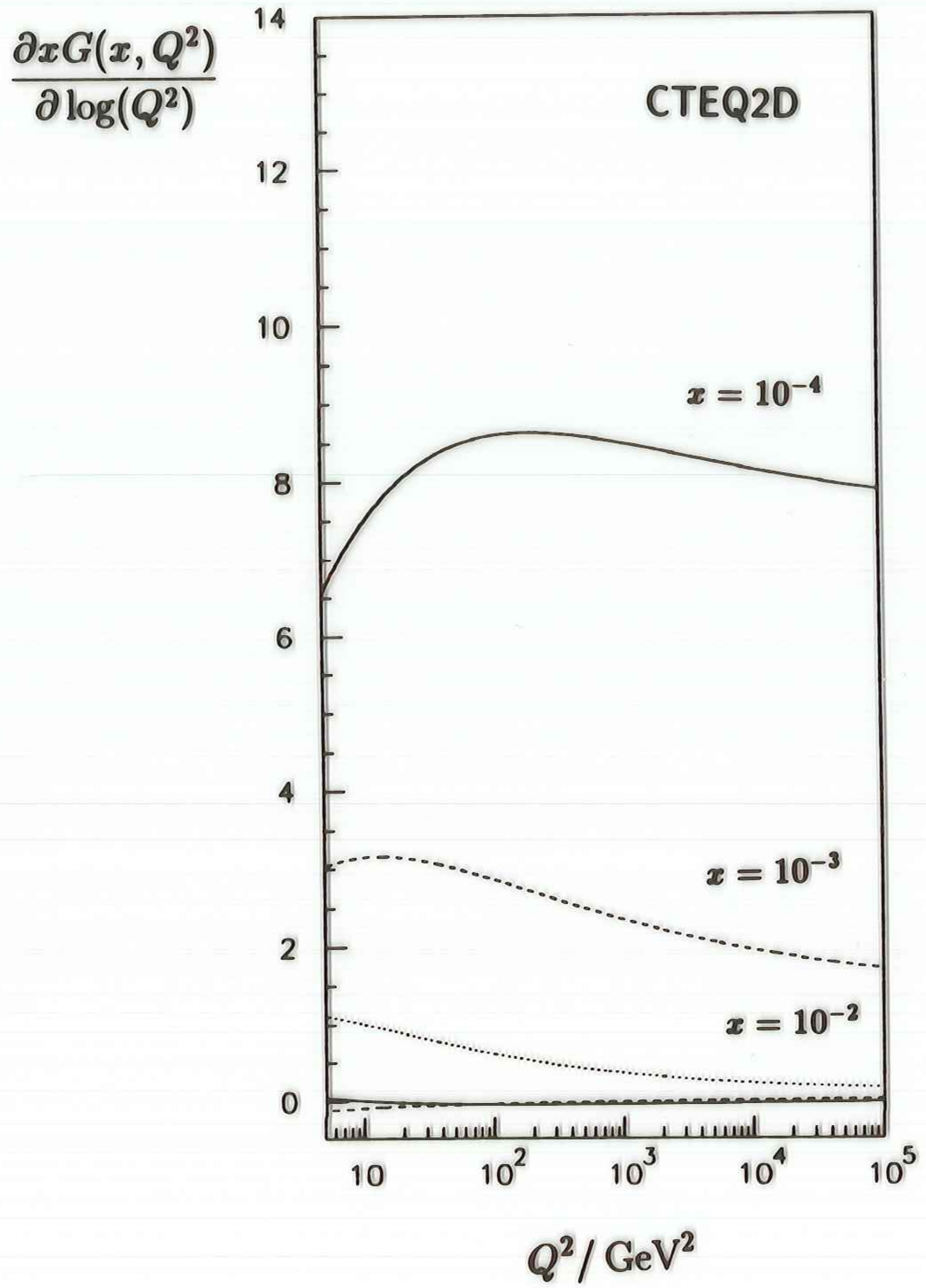
OTHER TERMS DISAPPEAR FOR $k^2\to 0$.

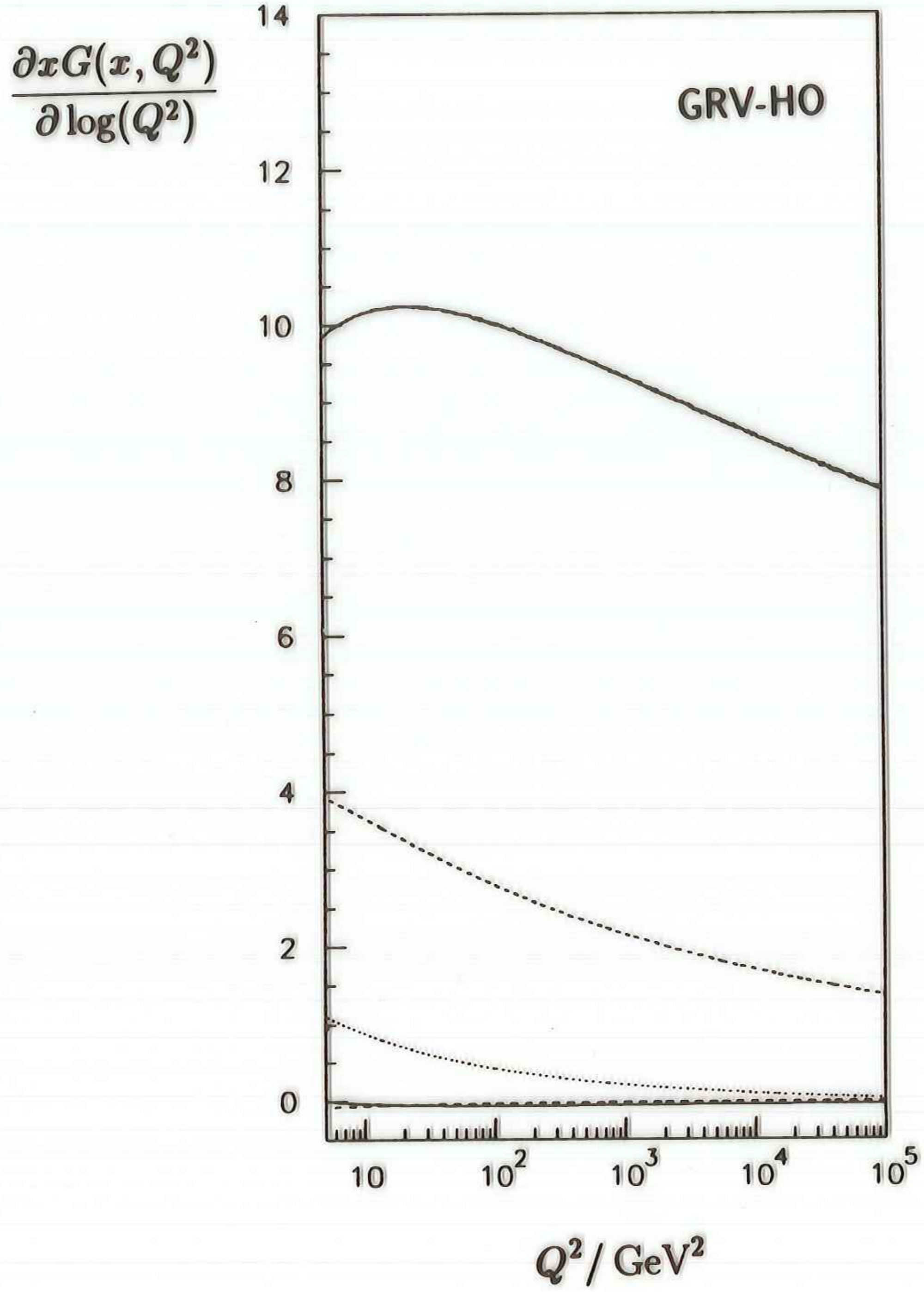
F_2 :

$$\begin{aligned}
g_{01}^{(2)}(\beta) &= -\frac{3}{2}+\frac{1}{2}\cos^2\beta \\
g_{11}^{(2)}(\beta) &= 2\cos\beta \\
g_{21}^{(2)}(\beta) &= \frac{1}{2}-\frac{3}{2}\cos^2\beta \\
g_{02}^{(2)}(\beta) &= -1-\cos^2\beta \\
g_{12}^{(2)}(\beta) &= 4\cos\beta \\
g_{22}^{(2)}(\beta) &= 1-3\cos^2\beta \\
g_{03}^{(2)}(\beta) &= -3+\cos^2\beta \\
g_{13}^{(2)}(\beta) &= 0 \\
g_{23}^{(2)}(\beta) &= 1-3\cos^2\beta \\
g_{04}^{(2)}(\beta) &= 0 \\
g_{14}^{(2)}(\beta) &= 0 \\
g_{24}^{(2)}(\beta) &= 4-12\cos^2\beta
\end{aligned}$$



CTEQ2-LO
$F_L(x, Q^2)$
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
$Q^2 = 100\,\mathrm{GeV}^2$
$Q^2 = 20\,\mathrm{GeV}^2$
$Q^2 = 10\,\mathrm{GeV}^2$
−5 −4.5 −4 −3.5 −3 −2.5 −2 −1.5 −1 −0.5 0
$\log_{10}(x)$


$\frac{\partial xG(x,Q^2)}{\partial \log(Q^2)}$
CTEQ2D
$x = 10^{-4}$
$x = 10^{-3}$
$x = 10^{-2}$
14
12
10
8
6
4
2
0
10
10^2
10^3
10^4
10^5
Q^2 / GeV^2



$\frac{\partial xG(x,Q^2)}{\partial \log(Q^2)}$
GRV-HO
14
12
10
8
6
4
2
0
10
10^2
10^3
10^4
10^5
Q^2 / GeV^2

6. HO CORRECTIONS TO F_L

$k_{\perp i}^2 \equiv 0.$

• J. SANCHEZ GUILLÉN et al. 1991

• ZIJLSTRA VAN NEERVEN 1991/92

$$F_L(x,Q^2) = \int_x^1 \frac{dy}{y} K^{NS}(y,Q^2)\mathcal{F}(x/y,Q^2) + \int_x^1 \frac{dy}{y} K^S(y,Q^2)\mathcal{F}^S(x/y,Q^2) + \int_x^1 \frac{dy}{y} K^G(y,Q^2)\mathcal{G}(x/y,Q^2), \tag{1}$$

where

$$\mathcal{F}(x,Q^2) = \sum_{i=1}^{n_f} e_i^2 x(q_i(x,Q^2) + \bar{q}_i(x,Q^2)),$$
$$\mathcal{F}^S(x,Q^2) = \delta_\psi^2 \sum_{i=1}^{n_f} x(q_i(x,Q^2) + \bar{q}_i(x,Q^2)),$$
$$\mathcal{G}(x,Q^2) = xG(x,Q^2), \tag{2}$$

$$K^{NS}(x,Q^2) = \frac{\alpha_s}{4\pi} f_{L,q}^{(1)}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{NS(2)}(x),$$
$$K^S(x,Q^2) = 0 + \left(\frac{\alpha_s}{4\pi}\right)^2 f_{L,q}^{S(2)}(x),$$
$$K^G(x,Q^2) = \frac{\alpha_s}{4\pi}\delta_\psi^2 f_{L,G}^{(1)}(x) + \left(\frac{\alpha_s}{4\pi}\right)^2 \delta_\psi^2 f_{L,G}^{(2)}(x),$$

$$\left.\begin{aligned} f_{L,q}^{(1)}(x) &= 4C_F x^2, \\ f_{L,G}^{(1)}(x) &= 8n_f x^2(1-x), \end{aligned}\right\} \text{LO}$$

$$\begin{aligned} f_{L,q}^{NS(2)}(x) = 4C_F(C_A - 2C_F)x^2 &\times \Bigg[4\frac{6-3x+47x^2-9x^3}{15x^2}\ln x - 4\,\mathrm{Li}_2(-x)(\ln x - 2\ln(1+x)) - 8\zeta(3) \\ &- 2\ln^2 x \ln(1-x^2) + 4\ln x \ln^2(1+x) - 4\ln x\,\mathrm{Li}_2(x) \\ &+ \tfrac{2}{5}(5-3x^2)\ln^2 x - 4\frac{2+10x^2+5x^3-3x^5}{5x^3} \\ &\times(\mathrm{Li}_2(-x) + \ln x\ln(1+x)) + 4\zeta(2)\left(\ln(1-x^2) - \frac{5-3x^2}{5}\right) \\ &+ 8S_{1,2}(-x) + 4\,\mathrm{Li}_3(x) + 4\,\mathrm{Li}_3(-x) - \tfrac{23}{3}\ln(1-x) \\ &- \frac{144+294x-1729x^2+216x^3}{90x^2}\Bigg] \\ &+ 8C_F^2 x^2\Bigg[\mathrm{Li}_2(x) + \ln^2\left(\frac{x}{1-x}\right) - 3\zeta(2) - \frac{3-22x}{3x}\ln x \\ &+ \frac{6-25x}{6x}\ln(1-x) - \frac{78-355x}{36x}\Bigg] - \tfrac{8}{3}C_F n_f x^2\left[\ln\left(\frac{x^2}{1-x}\right) - \frac{6-25x}{6x}\right], \end{aligned}$$

✓

$$f_{\mathrm{L},\mathrm{q}}^{\mathrm{S}(2)}(x) = \tfrac{16}{9} C_{\mathrm{F}} n_{\mathrm{f}} \Big[3(1-2x-2x^2)(1-x)\ln(1-x)$$

$$+ 9x^2\big(\mathrm{Li}_2(x) + \ln^2(x) - \zeta(2)\big) + 9x(1-x-2x^2)\ln x$$

$$-9x^2(1-x) - (1-x)^3\Big], \tag{9}$$

√

$$f_{\mathrm{L},\mathrm{G}}^{(2)}(x) = 16 C_{\mathrm{A}} n_{\mathrm{f}} x^2 \Bigg[\frac{1-3x-27x^2+29x^3}{3x^2} \ln(1-x) - 2(1-x)\ln x \ln(1-x)$$

$$+ 2(1+x)\mathrm{Li}_2(-x) + 4\,\mathrm{Li}_2(x) + 3\ln^2 x + 2(x-2)\zeta(2)$$

$$+ (1-x)\ln^2(1-x) + 2(1+x)\ln x \ln(1+x)$$

$$+ \frac{24+192x-317x^2}{24x}\ln x + \frac{-8+24x+501x^2-517x^3}{72x^2}\Bigg]$$

$$-16 C_{\mathrm{F}} n_{\mathrm{f}} x^2 \Bigg[\mathrm{Li}_2(x) + 2\frac{5+3x^2}{15}\ln^2(x) - \frac{1+3x-4x^2}{2x}\ln(1-x)$$

$$+ \frac{-2+10x^3-12x^5}{15x^3}\big(\mathrm{Li}_2(-x) + \ln x \ln(1+x)\big)$$

$$- \frac{5+12x^2}{15}\zeta(2) + \frac{4+13x+78x^2-36x^3}{30x^2}\ln x$$

$$- \frac{4-16x-213x^2+225x^3}{30x^2}\Bigg]. \tag{10}$$

J. SANCHEZ
GUILLEN

$$C_{\mathrm{L}}^{(2),\mathrm{G}}(x,1) = n_{\mathrm{f}} C_{\mathrm{F}} \Bigg[16x[\mathrm{Li}_2(1-x) + \ln x \ln(1-x)] + \left(-\tfrac{32}{3}x + \tfrac{64}{5}x^3 + \frac{32}{15x^2}\right)[\mathrm{Li}_2(-x) + \ln x \ln(1+x)]$$

$$+ (8+24x-32x^2)\ln(1-x) - (\tfrac{32}{3}x + \tfrac{32}{5}x^3)\ln^2 x + \frac{1}{15}\left(-104-624x+288x^2-\frac{32}{x}\right)\ln x$$

$$+ \left(-\tfrac{32}{3}x + \tfrac{64}{5}x^3\right)\zeta(2) - \tfrac{128}{15} - \tfrac{304}{5}x + \tfrac{336}{5}x^2 + \frac{32}{15x}\Bigg]$$

$$+ n_{\mathrm{f}} C_{\mathrm{A}} \Bigg[-64x\,\mathrm{Li}_2(1-x) + (32x+32x^2)[\mathrm{Li}_2(-x) + \ln x \ln(1+x)] + (16x-16x^2)\ln^2(1-x)$$

$$+ (-96x+32x^2)\ln x \ln(1-x) + \left(-16-144x+\tfrac{464}{3}x^2 + \frac{16}{3x}\right)\ln(1-x) + 48x\ln^2 x$$

$$+ (16+128x-208x^2)\ln x + 32x^2\zeta(2) + \tfrac{16}{3} + \tfrac{272}{3}x - \tfrac{848}{9}x^2 - \frac{16}{9x}\Bigg].$$

ZIJLSTRA
& VAN
NEERVEN

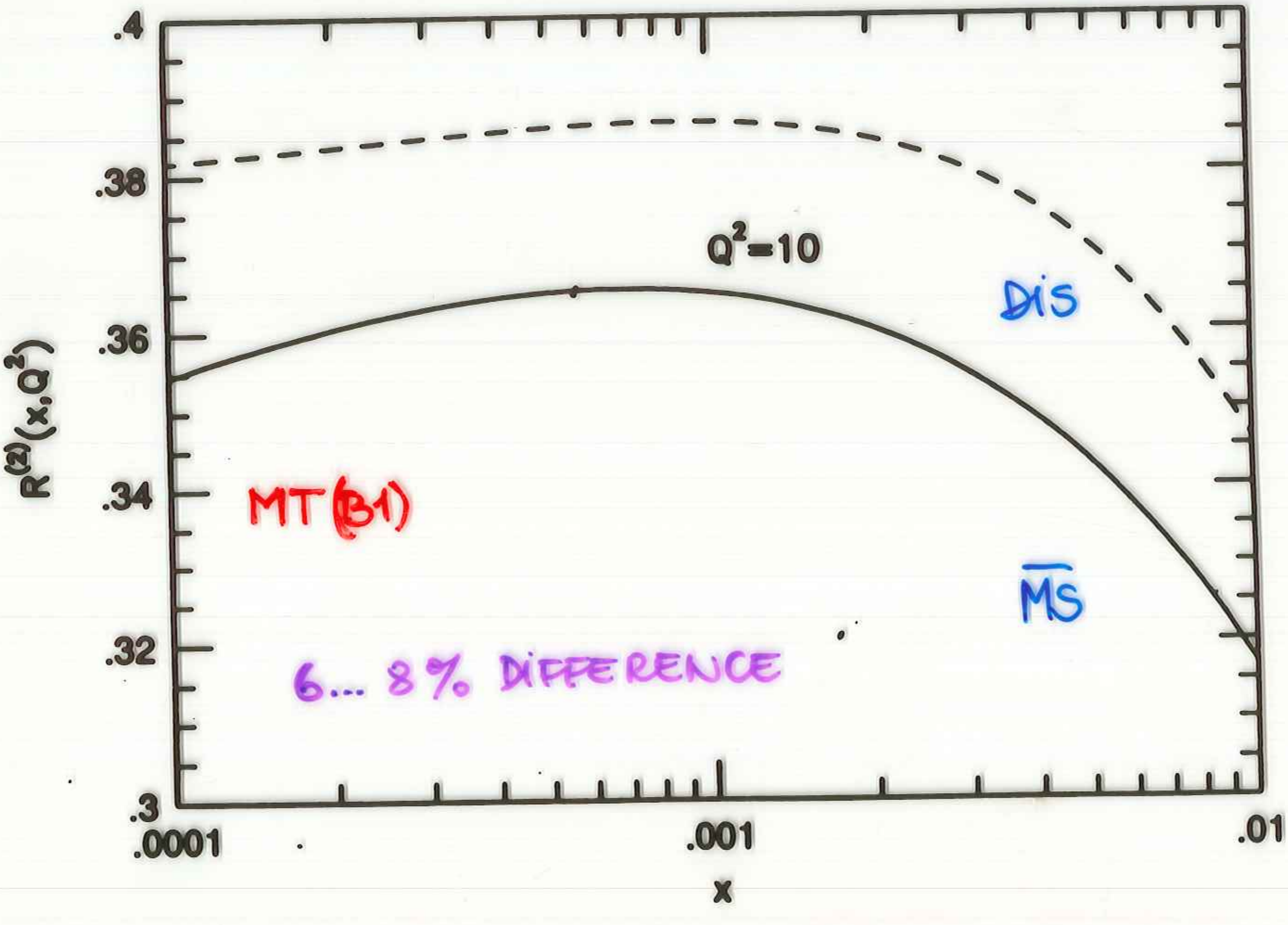
agree with
MOMENT
CALCULATION:

LARIN &
VERMASEREN.

$$R^{(2)}(x,Q^2) = \frac{F_L^{(2)}(x,Q^2)}{\left(1+\frac{4M_p^2x^2}{Q^2}\right)F_2^{(1)}(x,Q^2) - F_L^{(2)}(x,Q^2)}$$

$O(\alpha_s^2)$

ZIJLSTRA, VAN NEERVEN



$F_L(x, Q^2)$ & $xG(x, Q^2)$

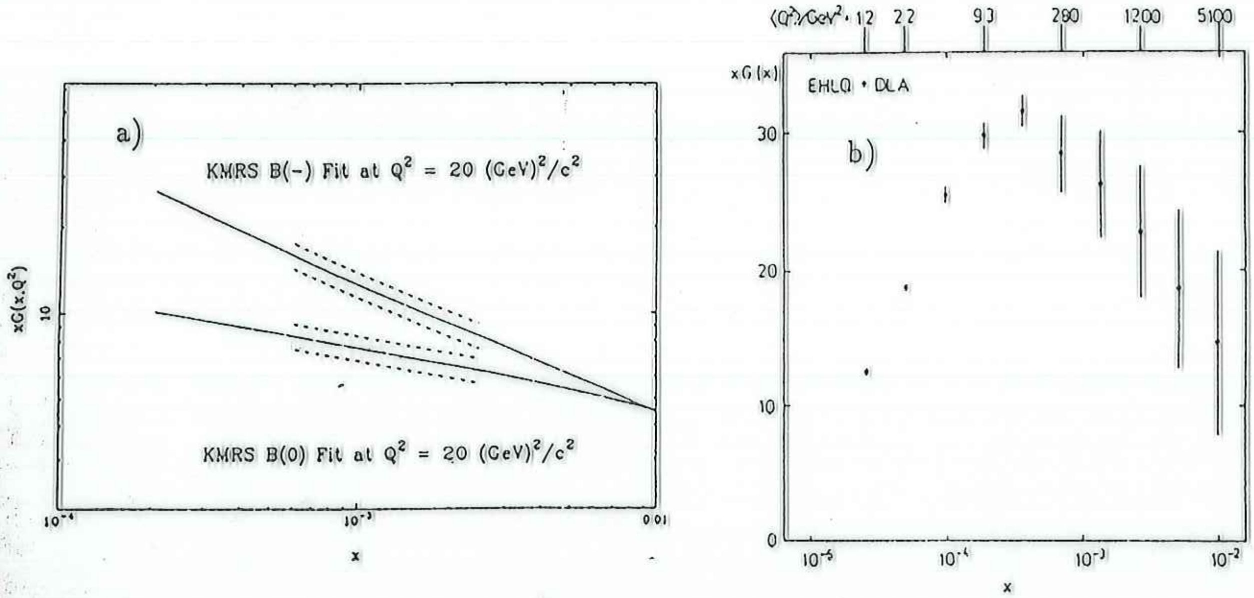
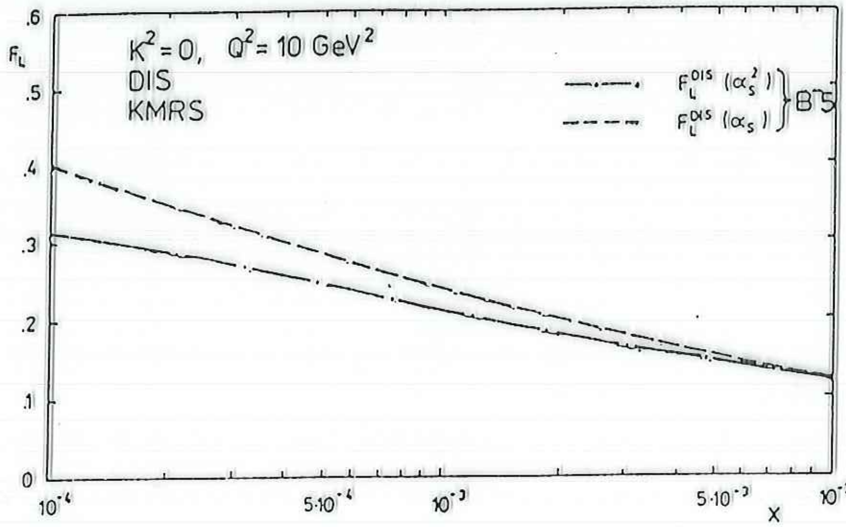


Figure 33: (a) Gluon distribution at low x and measureable domain with errors at HERA, [57]; (b) statistical precision of a possible measurement of $xG(x)$ at LEP $\otimes$ LHC using (62). Here, F_L was determined in the overlap range of a combined measurement at $\sqrt{s} = 1265$ GeV, $\mathcal{L} = 1 fb^{-1}$, and $\sqrt{s} = 1789$ GeV, $\mathcal{L} = 100 pb^{-1}$, and the average over Q^2 was taken, from [36].

25%!



$\mathcal{O}(\alpha_s^2)$

Figure 34: Comparison of 1st and 2nd order contributions to $F_L(x, Q^2)$ [59] for $Q^2 = 10\,\mathrm{GeV}^2$ using the parton distributions KMRS B^- [32].

7. OTHER CROSS SECTIONS

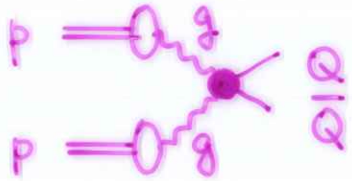
- HEAVY FLAVOUR: PHOTOPRODUCTION $\gamma p \to Q\bar{Q}$

$ep \to Q\bar{Q}$

$p\bar{p} \to Q\bar{Q}$

CIAFALONI, CATANI, HAUTMANN

K. ELLIS, COLLINS (LEVIN et al.)



$+ \ldots$

$$\sigma(s, 4m^2) = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \; f_{i/p_1}(x_1,\mu)\, f_{j/p_2}(x_2,\mu)\, \hat{\sigma}_{ij}(\hat{s}, m, \mu, \alpha_s(\mu))$$

(COLLINEAR PARTONS)

$$\sigma_{gg}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty d\vec{k}_1^2 \int_0^\infty d\vec{k}_2^2 \; \mathcal{f}(x_1, \vec{k}_1, \mu)\, \mathcal{f}(x_2, \vec{k}_2, \mu)$$
$$\times\, I_2(x_1 x_2 s, \vec{k}_1, \vec{k}_2).$$

$$f_g(x,\mu) = \int_0^{\mu^2} d\vec{k}^2 \; \mathcal{f}(x, \vec{k}, \mu).$$

$\longrightarrow$ SIMILAR TECHNIQUE TO OURS TO RENDER THE INTEGRAL FINITE FOR $k_1^2, k_2^2 \to 0$.

$$I_2(\ldots, k_i^2 \to 0) \equiv I_2(\ldots, 0)$$
$$I_2(\ldots, k_1^2, k_2^2 \to 0) \to \hat{\sigma}_{gg}.$$

μ^2 TAKES THE RÔLE OF Q^2: DIS.

$\mu_0^2 \ll \mu^2$. $\quad (\mu_0^2/\mu^2)^n$ TERMS ARE NEGL.

f' IS NOW CONSTRUCTED FROM f TO CONTAIN THE SOLUTION OF THE LIPATOV EQ.

IN MOMENT SPACE ONE HAS:

$$\boxed{j-1-\bar{\alpha}_s\,\chi\left(\gamma_c(j,\bar{\alpha}_s)\right)=0}$$

$$\chi(g)=2\psi(1)-\psi(g)-\psi(1-g).$$

$$\tilde{f}'(j,\vec{k},\mu)=\gamma_c(j,\bar{\alpha}_s)\,\frac{1}{k^2}\left(\frac{k^2}{\mu^2}\right)^{\gamma_c(j,\bar{\alpha}_s)}\cdot\tilde{f}_g(j,\mu)\qquad(*)$$

WITH THE MELLIN MOMENTS:

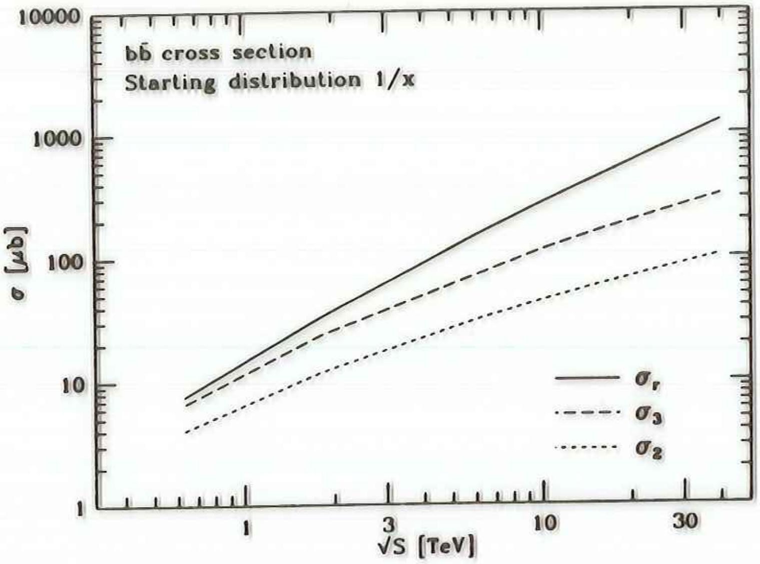
$$\tilde{P}(j)=\int_0^1 dx\,x^{j-1}\,P(x).$$

(ONE HAS $\int_0^{\mu^2} dk^2\,\tilde{f}'(j,\vec{k},\mu)=\tilde{f}_g(j,\mu)$.)

THE 1^{ST} ORDER TERM EXPANDING $(*)$ IN $\bar{\alpha}_s$ IS:

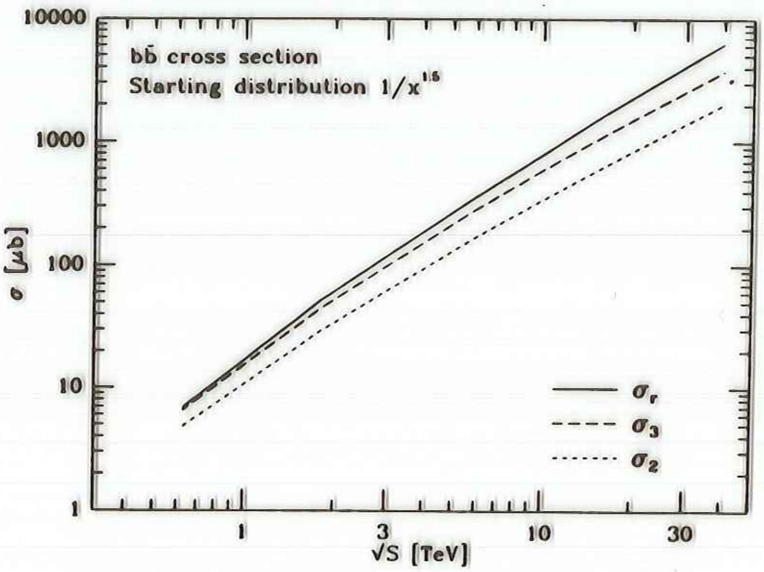
$$\tilde{f}'(j,\vec{k},\mu)=\frac{\bar{\alpha}_s}{j-1}\,\frac{1}{k^2}\,\tilde{f}(j,\mu).$$

$$\gamma_c=\frac{\bar{\alpha}_s}{j-1}+2\zeta(3)\,\frac{\bar{\alpha}_s^4}{(j-1)^4}+O(\bar{\alpha}_s^6)\quad:\quad\bar{\alpha}_s\ll1.$$



$f_g \sim \frac{1}{x}$

Fig. 9. Bottom cross section starting with $1/x$ behavior at small x. σ_2, σ_3, and σ_r are the cross section calculated with respectively the lowest-order approximations everywhere, with the order-α_s^3 approximation to our resummation, and with the full resummation.



$f_g \sim \frac{1}{x^{3/2}}$

Fig. 10. Bottom cross section starting with $1/x^{1.5}$ behavior at small x.

σ_2 : COLLINEAR $gg \to Q\bar{Q}$ $\sim (f_g \otimes f_g)$

σ_3 : „$O(\alpha_s^3)$" $\sim (\mathcal{F}_g \otimes f_g + f_g \otimes \mathcal{F}_g + f_g \otimes f_g)$

σ_R : $\sim \mathcal{F}_g \otimes \mathcal{F}_g$

8. CONCLUSIONS

1) MASS FACTORIZATION IS A GOOD CONCEPT WHEN INITIAL STATE $k_\perp$ EFFECTS ARE UNIMPORTANT (MEDIUM VALUES OF x).

2) $k_\perp$ FACTORIZATION GENERALIZES MASS FACTORIZATION:

$$\hat{\sigma}(k_\perp^2=0, x) \otimes G(x,\mu^2) \Leftarrow \int_0^{\mu^2} dk_\perp^2 \frac{dG(x,k_\perp^2)}{dk_\perp^2} \otimes \hat{\sigma}(k_\perp^2, x)$$

3) MORE INVOLVED EXPRESSIONS FOR $\hat{\sigma}(k_\perp \neq 0)$ ARE OBTAINED.

4) RELATIONS OF THE TYPE:

$$F(x,Q^2) = \int dk_\perp^2 \; \hat{\sigma}_F(x,k_\perp^2) \otimes \frac{dG(x,k_\perp^2)}{dk_\perp^2}$$

OR

$$\sigma_H(s) = \int dk_{\perp 1}^2 dk_{\perp 2}^2 \; \hat{\sigma}_H(k_{\perp 1}^2, k_{\perp 2}^2) \otimes_1 \frac{dG(x_1,k_{\perp 1}^2)}{dk_{\perp 1}^2} \otimes_2 \frac{dG(x_2,k_{\perp 2}^2)}{dk_{\perp 2}^2}$$

MAY BE USED AS MAPPINGS TO EXTRACT

$$\frac{dG(x,k_\perp^2)}{dk_\perp^2}.$$

5)
$\hat{\sigma}_F, \hat{\sigma}_H$ ARE OBTAINED IN PERTURBATIVE QCD WHILE $dG(x,k_\perp^2)/dk_\perp^2$ BEARS NON-PERTURBATIVE CONTRIBUTIONS — ALSO? DYNAMICAL ONES.

(THIS IS ALREADY ANOTHER STORY....)