Large x Higher Twist Contributions in DIS

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in collaboration with H. Böttcher

- Non-Singlet QCD Analysis of $l^{\pm}N$ Data
- Extraction of Non-leading Twist Contribution

[1] J.B. and H. Böttcher, arXiv:0802.0408 [hep-ph]; Phys. Lett. **B** (2008) in print.

[2] J.B., A. Guffanti, and H. Böttcher, Nucl.Phys.**B774** (2007) 182.

Introduction

- Higher Twist Contributions are expected to contribute to DIS Structure Functions both

  - at large x

    e.g. S. Gottlieb, Nucl. Phys. **B139** (1978) 125; H.D. Politzer, Nucl. Phys. **B172** (1980) 349; R.K. Ellis, W. Furmanski, R. Petronzio, Nucl. Phys. **B207** (1982) 1; **B212** (1983) 29

  - at small x

    e.g. L.V. Gribov, E.M. Ryskin, and M.G. Ryskin, Nucl. Phys. **B188** (1981) 555; A.H. Mueller and J.W. Qiu, Nucl. Phys. **B268** (1986) 427; J. C. Collins and J. Kwiecinski, Nucl. Phys. **B335** (1990) 89; J. Bartels, J. Blümlein, and G.A. Schuler, Z. Phys. **C50** (1991) 91

- In this talk we will investigate the Large x Region.

- How to separate Twist-2 and Higher Twist contributions ?
  - Systematic investigation of $\ln(Q^2)$-slopes of $F_i(x, Q^2)$ cutting from large Q^2
  - A cut of $W^2 > 12.5\ \mathrm{GeV}^2$ allows the separation (see below).
- No sufficient Theoretical Description of Higher Twist Contributions is available for :
  - Anomalous Dimensions
  - Wilson Coefficients
  - HT Correlation Functions
- Fits starting with a **phenomenological Ansatz** are therefore not possible.

Fits under Phenomenological Assumptions :

e.g. K. Varvewll et al. [BEBC] Z. Phys. **C36** (1987) 1; S.I. Alekhin, A.L. Kataev, Phys. Lett. **B452** (1999) 402; M. Botje, Eur. Phys. J. **C14** (2000) 285; S. Simula, Phys. Lett. **B493** (2000) 325; A.L. Kataev, Nucl. Phys. B (Suppl. Proc.) **116** (2003) 105; A. Martin et al., Eur. Phys. J. C35 (2004) 325; S.I. Alekhin, S.A. Kulagin, S. Liuti, Phys. Rev. **D69** (2004) 114009; M. Osipenko et al., Nucl. Phys. **A766** (2006) 142;

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Extraction of HT Terms :

- Twist-2 QCD Fit in the large W^2 region
- Application of Target Mass Corrections
- Extrapolation of the Leading Twist Results into the lower W^2 region:

$W^2, Q^2 \geq 4$ GeV^2.

Twist-2 Non-Singlet Analysis [2]

- Separate the kinematic region $x < 0.4, \;\; x > 0.4$
- $x < 0.4$: $F_{2\mathrm{NS}}$ from p and d data and $\bar{d} - \bar{u}$ (Drell-Yan process.)
- $x > 0.4$: Valence approximation for $F_2^{p,d}$ from p
- 3-Loop QCD Analysis

  3L Anom. Dim. S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B688** (2004) 101
- Known Heavy Flavor contributions are taken into account (NLO)

  1% and smaller
- Effective 4-Loop QCD Analysis possible

  3L WC: J. Vermaseren, A. Vogt., S. Moch Nucl. Phys. **B724** (2005) 3

Finally:

Account for the dominant large x Terms in the Wilson Coefficient to $O(\alpha_s^4)$

S. Moch, J. Vermaseren, A. Vogt. Nucl. Phys. **B688** (2004) 101;

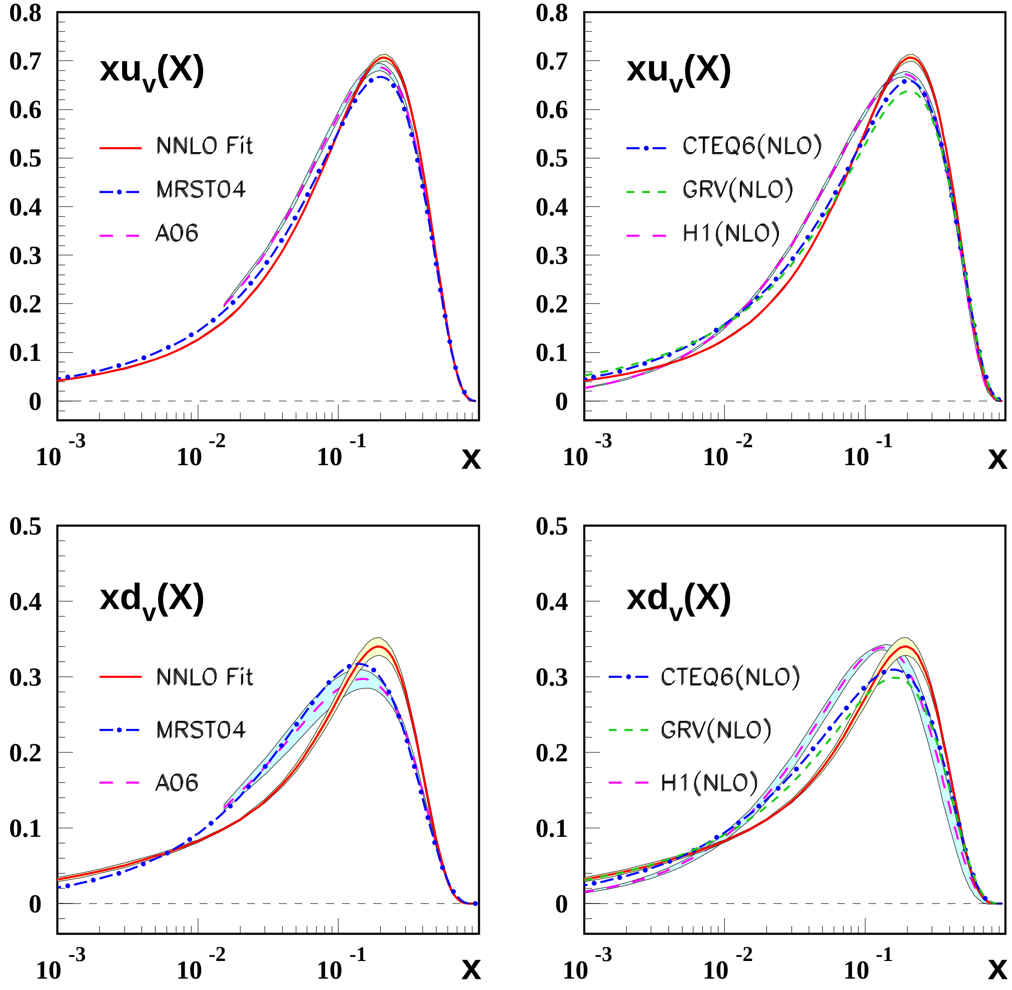
V. Ravindran, Nucl. Phys. **B752** (2006) 173

Formalism

$$F_2^{p,d;\mathrm{NS}}(N,Q^2) = \sum_{k=0}^{\infty} a_s^{k-1}(Q^2) C_{k-1}^{\mathrm{NS}}(N) f_2^{p,d;\mathrm{NS}}(N,Q^2) \ ,$$

$$\begin{aligned}
f_2^{p,d;\mathrm{NS}}(N,Q^2) \quad = \quad & f_2^{p,d;\mathrm{NS}}(N,Q_0^2) \left(\frac{a}{a_0}\right)^{-\hat{P}_0(N)/\beta_0} \Bigg\{ 1 - \frac{1}{\beta_0}(a-a_0)\left[\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right] \\
& - \frac{1}{2\beta_0}\left(a^2-a_0^2\right)\left[\hat{P}_2^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_1^+(N) + \left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right] \\
& + \frac{1}{2\beta_0^2}(a-a_0)^2\left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^2 \\
& - \frac{1}{3\beta_0}\left(a^3-a_0^3\right)\Bigg[\hat{P}_3^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_2^+(N) + \left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_1^+(N) \\
& + \left(\frac{\beta_1^3}{\beta_0^3} - 2\frac{\beta_1\beta_2}{\beta_0^2} + \frac{\beta_3}{\beta_0}\right)\hat{P}_0(N)\Bigg] \frac{(a-a_0)\left(a_0^2-a^2\right)}{2\beta_0^2}\left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right) \\
& \times \left[\hat{P}_2(N) - \frac{\beta_1}{\beta_0}\hat{P}_1(N) - \left(\frac{\beta_1^2-\beta_0\beta_2}{\beta_0^2}\right)\hat{P}_0(N)\right] \\
& - \frac{(a-a_0)^3}{6\beta_0^3}\left(\hat{P}_1^+(N) - \frac{\beta_1}{\beta_0}\hat{P}_0(N)\right)^3 \Bigg\} \ .
\end{aligned}$$

World Data Analysis: Valence Distributions



World data:

NS-analysis

$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$

N^3LO :

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

J.B., H. Böttcher, A. Guffanti,

Nucl. Phys. **B774** (2007) 182

Why an $O(\alpha_s^4)$ analysis can be performed?

Assume an $\pm 100\%$ error on the Padé approximant $\longrightarrow \pm 2$ MeV in Λ_{QCD}

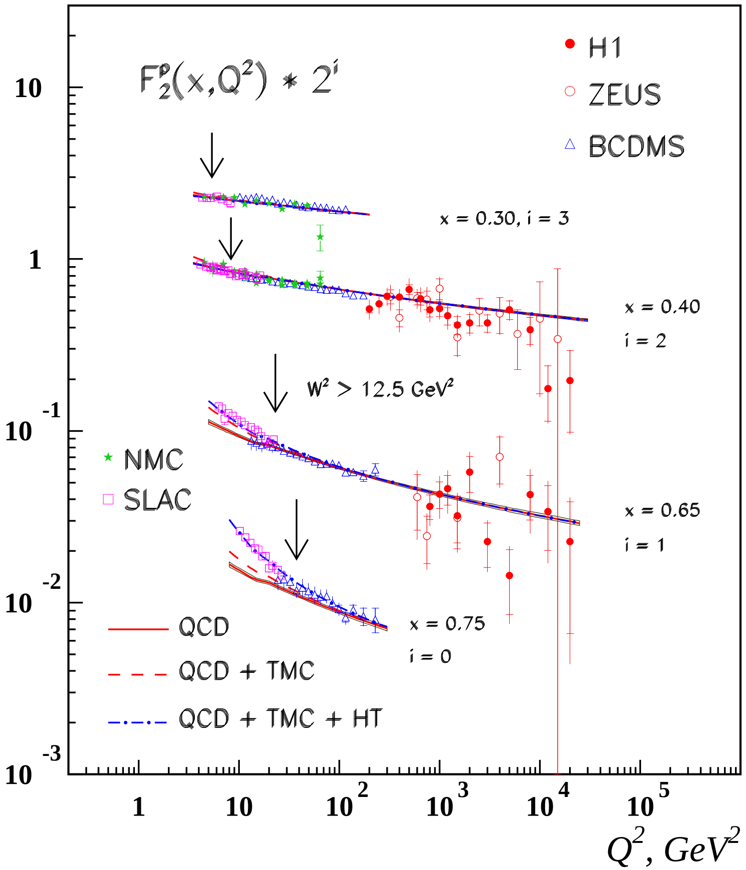
$$\gamma_n^{approx:3} = \frac{{\gamma_n^{(2)}}^2}{\gamma_n^{(1)}}$$

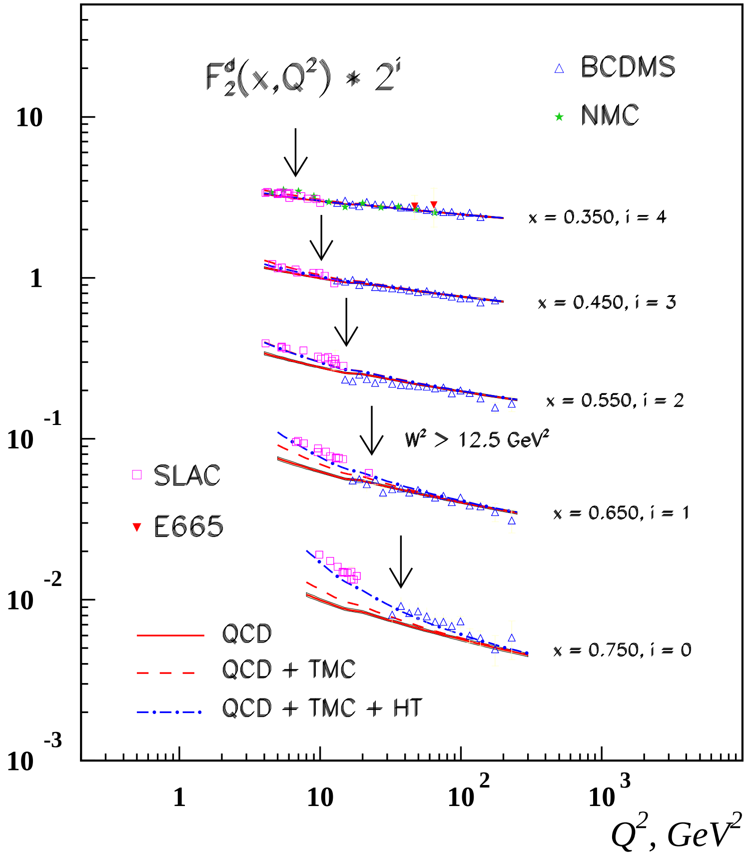
P. Baikov, K. Chetyrkin Nucl.Phys.Proc.Suppl. **160** (2006) 76.

$$\begin{aligned}\gamma_2^{3;NS} \quad = \quad & \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[\frac{3936832}{6561} - \frac{10240}{81}\zeta_3\right] a_s^3 \\ & + \left[\frac{1680283336}{1777147} - \frac{24873952}{6561}\zeta_3 + \frac{5120}{3}\zeta_4 - \frac{56969}{243}\zeta_5\right] a_s^4\end{aligned}$$

The results agree better than 20%.

$F_2(x, Q^2)$ in the Valence Region





No validity of the twist-2 approximation left from the arrow.

$\alpha_s(M_Z^2)$

$$\frac{da(\mu^2)}{d\ln(\mu^2)} = -\sum_{k=0}^{\infty} \beta_k a^{k+2}(\mu^2) \ .$$

$\alpha_s(M_Z^2)$ is to be determined together with the valence distributions in the same analysis.

Overview of the analyzes :

- Various NLO analyses; $\Rightarrow$ Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses $e(\mu)N$ world data
- S- and NS-NNLO moment analyses νN world data
- NS-N^{3}LO analysis $e(\mu)N$ world data
- NLO analyses polarized $e(\mu)N$ world data
- Lattice measurements

$\alpha_s(M_Z^2)$

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	± 0.0020	± 0.0030	[2]
A02	0.1143	± 0.0014	± 0.0009	[3]
SY01(ep)	0.1166	± 0.0013		[8]
SY01(νN)	0.1153	± 0.0063		[8]
GRS	0.111			[10]
A06	0.1128	± 0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]
N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

NNLO and N^{3}LO

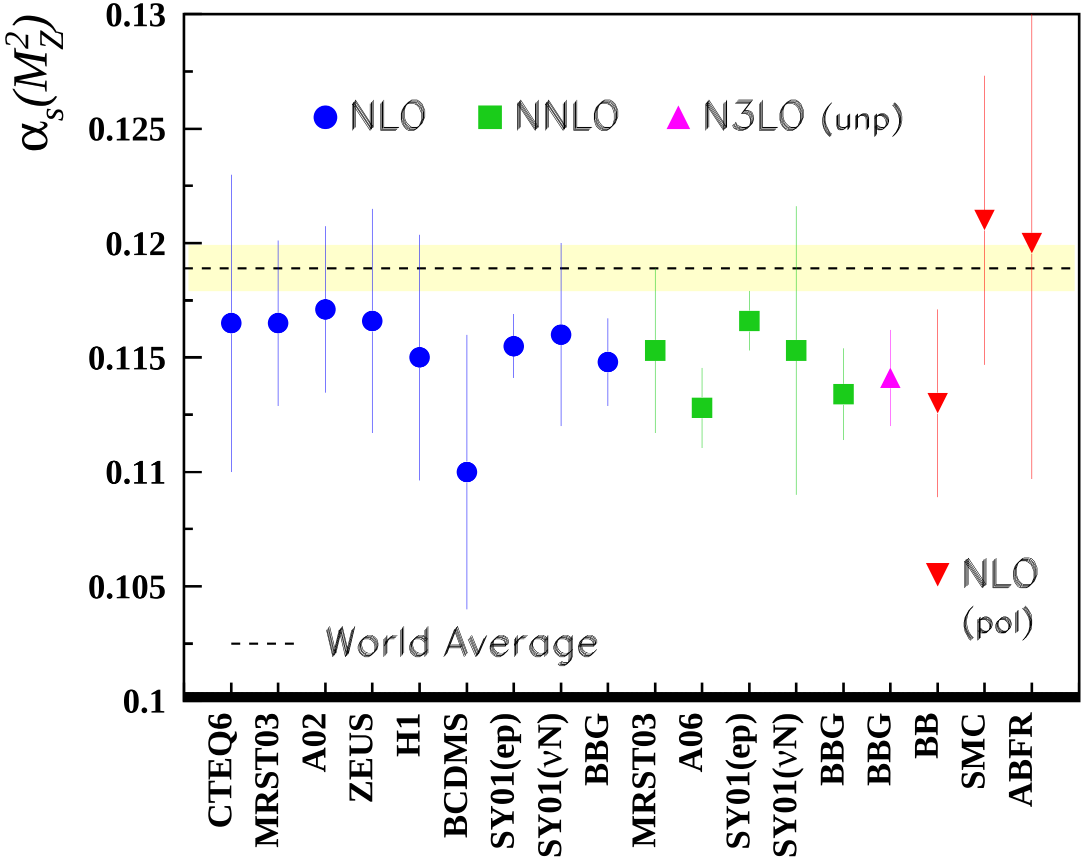
BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26\text{MeV}$

Lattice results :

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16\text{MeV}$

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26\text{MeV}$

$\alpha_s(M_Z^2)$



J.B., H. Böttcher, A. Guffanti, 2006 [2]

Higher Twist at Large x

- Non-leading twist terms :

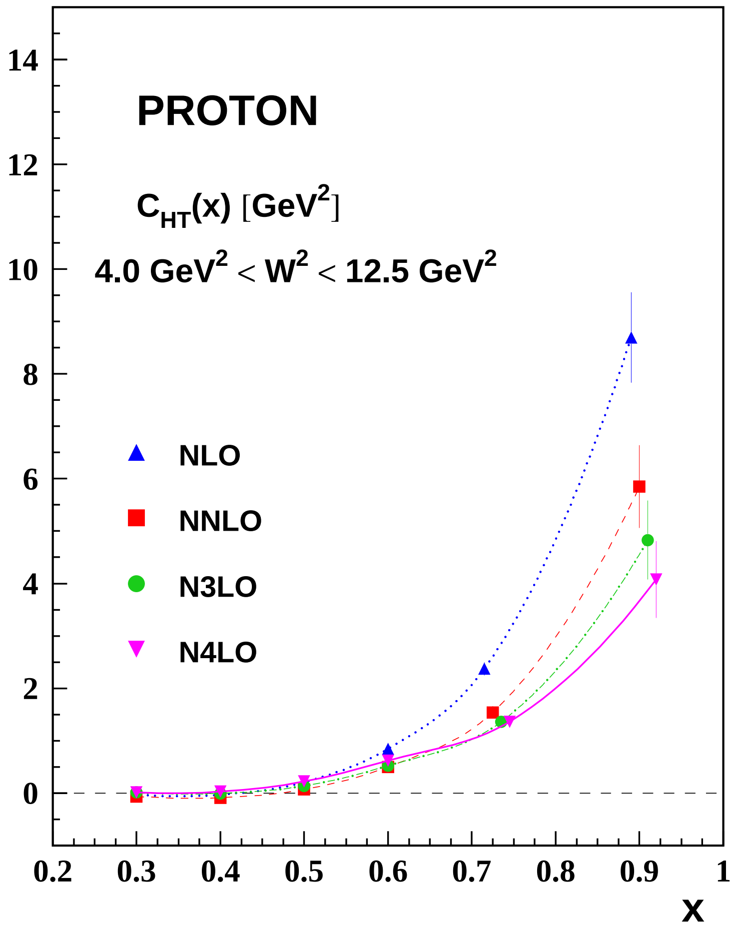
$$F_2^{\rm exp}(x,Q^2) = F_2^{\rm tw2}(x,Q^2) \cdot \left[\frac{O_{\rm TMC}\left[F_2^{\rm tw2}(x,Q^2)\right]}{F_2^{\rm tw2}(x,Q^2)} + \frac{C_{\rm HT}(x,Q^2)}{Q^2[1\,{\rm GeV}^2]}\right] .$$

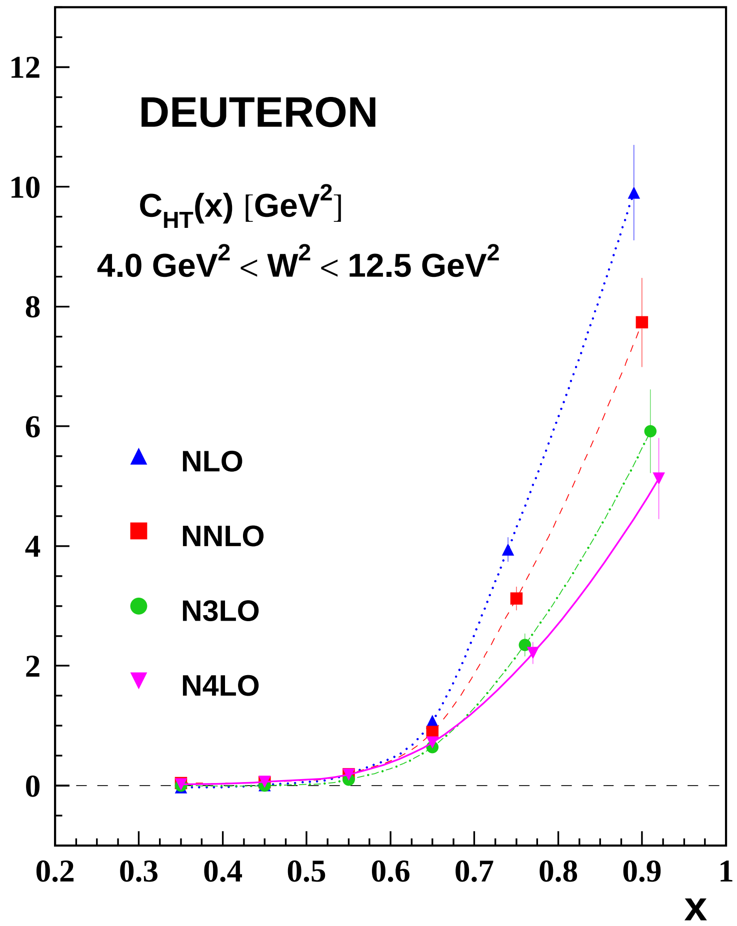
- Extraction for all (x, Q^2) bins
- No HT-assumptions, evolution etc. needed
- Complete Analysis : 2-loop, 3-loop
- 4-Loop analysis Padé for the 4-loop anomalous dimension with 100% error assumed
- Beyond that level: Large x contributions to the Wilson coefficient at $O(\alpha_s^4)$; neglecting anomalous dimension effects. The latter term is used for HT extraction only.
- Mathematical Structure of the large x terms in Mellin Space

$$S_{1,\ldots,1}(N) = \frac{1}{n}\left[S_n(N) + S_1(N)S_{n-1}(N) + S_{1,1}(N)S_{n-2}(N) + \ldots\right] .$$

$$\mathrm{S}_1(N) \propto \ln(N) + \gamma_E; \qquad\qquad S_l(N) \propto \zeta_l, \quad l \geq 2 ,$$

Higher Twist at Large x





Each higher order leads to a depletion. N4LO converged for $x \leq 0.8$

- Agreement between p and d analysis
- LGT simultations of these terms are of interest

Conclusions

- We performed a NS QCD Analysis to Three Loop Order, and under a weak assumption, to Four Loop Order determining the valence quark distributions and $\Lambda_{\rm QCD}$.
- Within this analysis we determined, free of assumptions, the higher twist contributions in the large x region. We also considered the large x contributions to the NS Wilson coefficient in $O(\alpha_s^4)$.
- HT extractions based on a Phenomenological Ansatz have the problem of yet unknown HT anomalous dimensions, Wilson Coefficients, and correlation functions.
- We observe gradual depletion of the higher twist contributions with increasing order of α_s. The present description converges for values $x \leq 0.8$