

VI Rencontres de Blois
June 1994

Small x Contributions to Structure Functions

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1 Introduction

- In the small x range a novel behaviour of nucleon structure functions is expected.

Among the possible dynamical effects are: Screening and effects due to non strong k_{\perp} ordering.

- This requires to consider rather:

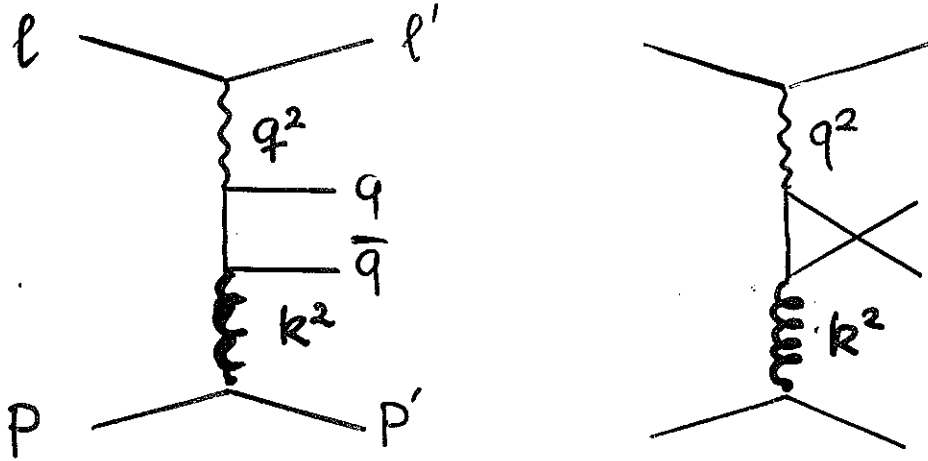
$$\int^{\mu^2} dK^2 \frac{\partial xG(x, K^2)}{\partial K^2} \quad (1)$$

than

$$xG(x, \mu^2) \quad (2)$$

- To obtain results which are consistent with perturbative QCD a consistent factorization relation has to be derived.
- Furthermore, approximations of the x behaviour of the k_{\perp} dependent coefficient functions or (additional) low Q^2 'cuts' bear the danger to obtain partly unphysical and misleading results (e.g. *negative* structure functions).
- The goal of the present paper is to find a consistent solution of the above problems and to extend earlier investigations (LEVIN AND RYSKIN 1991, ASKEW ET AL. 1993).
- We aim on a **general** formulation of the gluon contribution to the structure functions which offers the possibility to **unfold** the k_{\perp} dependence of the gluon at small x . Theoretical predictions of its small x behaviour can thus be confronted with the data with respect to the k_{\perp} dependence directly.

2 Kinematics



$$q + k = p_1 + p_2 \quad (3)$$

The $2 \rightarrow 2$ process is described by the invariants:

$$\begin{aligned} \hat{s} &= (q + k)^2 = (p_1 + p_2)^2 \\ \hat{t} &= (k - p_1)^2 = (q - p_2)^2 \\ \hat{u} &= (q - p_1)^2 = (k - p_2)^2 \end{aligned} \quad (4)$$

with

$$\hat{s} + \hat{t} + \hat{u} = -Q^2 - K^2 + m_1^2 + m_2^2 \quad (5)$$

and $Q^2 = -q^2$, $K^2 = -k^2$, $q' = q + xP$.

$$k_\mu = \xi q'_\mu + \eta P_\mu + k_{\perp\mu} \quad (6)$$

with $k_\perp \cdot q' = k_\perp \cdot P = 0$ and

$$x = \frac{Q^2}{2P \cdot q} \quad (7)$$

$$\xi = \frac{2k \cdot P}{2q' \cdot P} \quad (8)$$

$$\eta = \frac{2k \cdot q'}{2q' \cdot P} \quad (9)$$

and

$$K^2 = k^2 - \xi\eta\frac{Q^2}{x} \quad (10)$$

with $k_{\perp}^2 = -k^2$. between P and k is needed. We will assume that

$$\xi \ll \eta \quad (11)$$

which implies

$$\xi \approx 0 \quad P \cdot k \approx 0 \quad k_{\mu} \approx \eta P_{\mu} + k_{\perp\mu} \quad K^2 \approx k^2 \quad (12)$$

In the scaling limit, $\xi = 0$ and $k_{\perp} = 0$, the gluon momentum is

$$\underline{k_{\mu} = \eta P_{\mu}} \quad (13)$$

and the relations given below simplify accordingly. The general representation of the particle momenta is thus given by:

$$\begin{aligned} k &= (K_0, 0, 0, |\vec{k}|) \\ q &= (Q_0, 0, 0, -|\vec{k}|) \\ P &= E_p(1, \sin \beta, 0, \cos \beta) \\ p_1 &= (E_1, q_1 \sin \theta \cos \varphi, q_1 \sin \theta \sin \varphi, q_1 \cos \theta) \\ p_2 &= (E_2, -q_1 \sin \theta \cos \varphi, -q_1 \sin \theta \sin \varphi, -q_1 \cos \theta) \end{aligned} \quad (14)$$

with

$$\begin{aligned} K_0 &= \mathcal{E}(\hat{s}, -K^2, -Q^2) \\ Q_0 &= \mathcal{E}(\hat{s}, -Q^2, -K^2) \\ E_1 &= \mathcal{E}(\hat{s}, m_1^2, m_2^2) \\ E_2 &= \mathcal{E}(\hat{s}, m_2^2, m_1^2) \\ |\vec{k}| &= \mathcal{P}(\hat{s}, -Q^2, -K^2) \\ q_1 &= \mathcal{P}(\hat{s}, m_1^2, m_2^2) \\ \cos \theta &= \frac{2K_0 E_1 + K^2 - m_1^2 + \hat{t}}{2|\vec{k}|q_1} \end{aligned}$$

$$E_p = \mathcal{E}(\hat{s}, 0, t_{qP'}) = \frac{Q^2}{2x\sqrt{\hat{s}}}$$

$$\cos \beta = \frac{K_0}{|\vec{k}|} = \frac{1 - \zeta/2}{\sqrt{1 - x\zeta}} \quad (15)$$

where

$$\zeta = \frac{4K^2x}{Q^2} \quad (16)$$

$$\mathcal{E}(a, b, c) = \frac{a + b - c}{2\sqrt{a}}$$

$$\mathcal{P}(a, b, c) = \sqrt{\frac{\lambda(a, b, c)}{4a}} \quad (17)$$

$$\lambda(a, b, c) = (a - b - c)^2 - 4bc \quad (18)$$

$$t_{qP'} = (q - P')^2 = \hat{s} - Q^2/x \quad (19)$$

φ is defined by

$$\cos \varphi = \frac{(q \times P) \cdot (q \times p_1)}{|q \times P| |q \times p_1|} \Big|_{p_1 = -p_2} \quad (20)$$

In the limit $m_{1,2} \rightarrow 0$ the 3-particle phase space is described by

$$dPS^{(3)} = \frac{1}{128\pi^3} \frac{d\varphi_P}{2\pi} \frac{d\hat{s}d\hat{t}dK^2}{\lambda^{1/2}(\hat{s}, -K^2, -Q^2)\lambda^{1/2}(s_\gamma, 0, -Q^2)} \frac{d\varphi}{2\pi}$$

$$\times \Theta\{-\mathcal{G}(s_\gamma, -K^2, \hat{s}, 0, -Q^2, 0)\}$$

$$\times \Theta\{-\mathcal{G}(\hat{s}, \hat{t}, 0, -K^2, -Q^2, 0)\}\Theta\{\hat{s}\} \quad (21)$$

with $\mathcal{G}(x, y, z, u, v, w)$ the Caley determinants.

$$\int dPS^{(3)} = \frac{1}{128\pi^3} \int_{\eta_{\min}}^{\eta_{\max}} d\eta \int_{K_{\min}^2(\eta)}^{K_{\max}^2(\eta)} dK^2 \int_0^{2\pi} \frac{d\varphi_P}{2\pi} \frac{1}{2} \int_{-1}^1 d\cos \theta \int_0^{2\pi} \frac{d\varphi}{2\pi} \quad (22)$$

The variables in (22) are bounded by:

$$\begin{aligned}K_{min}^2 &= 0 \\K_{max}^2 &= Q^2 \frac{\eta - x}{x} \\ \eta_{min} &= x \\ \eta_{max} &= 1\end{aligned}\tag{23}$$

3 Factorization

- The k_{\perp} integration starts at $K^2 = 0$.
However, a physical definition of $\partial_x G(x, K^2)/\partial K^2$ is ONLY possible for $K^2 \geq Q_0^2$!

• The naive k_{\perp} factorization fails.

$$F_i^g(x, Q^2) = \int_0^1 \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) \int d^2 k f_i(x_1, K^2/Q^2) \frac{\partial x_2 G(x_2, K^2)}{\partial K^2} \quad (24)$$

One observes:

for: $Q_0^2 \ll Q^2$

one has for $K^2 \leq Q_0^2$: $f_i^{gG}(K^2/Q^2, z) \Rightarrow f_i^{gG}(K^2 \rightarrow 0, z)$

The k_{\perp} Factorization Relation is thus:

$$\begin{aligned} F_L^g(x, Q^2) &= \int_x^1 \frac{dz}{z} f_L^{g,0}(z) \frac{x}{z} G(z, Q_0^2) \\ &= \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K_{max}^2} dK^2 f_L^g(z, \frac{K^2}{Q^2}) \frac{x}{z} \frac{\partial G(x/z, K^2)}{\partial K^2} \theta(K_{max}^2 - Q_0^2) \\ F_2^g(x, Q^2) &= F_2^{g, coll}(x, Q^2) \\ &+ \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{K_{max}^2} dK^2 \left\{ f_2^g(z, \frac{K^2}{Q^2}) - f_2^g(z, \frac{\Lambda^2}{Q^2}) \theta(Q^2 - K^2) \right\} \\ &\times \frac{x}{z} \frac{\partial G(x/z, K^2)}{\partial K^2} \theta(K_{max}^2 - Q_0^2) \end{aligned} \quad (25)$$

The Scale of α_s :

For the present leading order calculation we have chosen: Q^2 .
Other characteristic scales would do as well.

4 The Structure Functions in the k_{\perp} Factorization Scheme in $\mathcal{O}(\alpha_s)$

$$\frac{d^2\sigma}{dQ^2 dy} = 2\pi\alpha_s^2 \frac{Ms}{(s-M^2)^2} \frac{1}{Q^4} L_{\mu\nu} W^{\mu\nu} \quad (26)$$

$$L_{\mu\nu} = 2[l_{\mu}l'_{\nu} + l'_{\mu}l_{\nu} - g_{\mu\nu}l.l']$$

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_n \langle P | J_{\mu}^{em\dagger}(0) | n \rangle \langle n | J_{\nu}^{em}(0) | P \rangle (2\pi)^4 \delta^{(4)}(P+q-p_n) \quad (27)$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right) W_1(x, Q^2) + \frac{1}{M^2} \left[\left(P_{\mu} - \frac{P \cdot q}{q^2} q_{\mu}\right) \left(P_{\nu} - \frac{P \cdot q}{q^2} q_{\nu}\right) \right] W_2(x, Q^2) \quad (28)$$

$$F_2(x, Q^2) = x T_{\mu\nu}^1 W^{\mu\nu} = x \left(-g_{\mu\nu} + \frac{12x^2}{Q^2} P_{\mu}P_{\nu}\right) W^{\mu\nu}$$

$$F_L(x, Q^2) = x T_{\mu\nu}^2 W^{\mu\nu} = \frac{8x^3}{Q^2} P_{\mu}P_{\nu} W^{\mu\nu} \quad (29)$$

$$-g^{\mu\nu} \widehat{W}_{\mu\nu} = 32\pi\alpha_s e_q^2 \left\{ \frac{(p_1 \cdot P)^2 + (p_2 \cdot P)^2}{\hat{t}\hat{u}} - \frac{Q^2}{K^2} \left[\frac{p_1 \cdot P}{\hat{t}} - \frac{p_2 \cdot P}{\hat{u}} \right]^2 \right\} \quad (30)$$

$$P^{\mu}P^{\nu} \widehat{W}_{\mu\nu} = 64\pi\alpha_s e_q^2 \frac{1}{K^2} \left\{ -2 \frac{(p_1 \cdot P)^2 (p_2 \cdot P)^2}{\hat{t}\hat{u}} + \frac{(p_1 \cdot P)^3 (p_2 \cdot P)}{\hat{t}^2} + \frac{(p_1 \cdot P)(p_2 \cdot P)^3}{\hat{u}^2} \right\} \quad (31)$$

$$f_L^g(z, \frac{K^2}{Q^2}) = \frac{2}{\pi} \alpha_s(Q^2) \sum_{q=1}^{N_f/2} (e_{q_u}^2 + e_{q_d}^2) \left\{ \frac{1}{64z} \left(\frac{Q^2}{K^2}\right)^2 G_{1L}(\omega, \beta) \right.$$

$$\left. + z \frac{1}{16K^2} \left[G_{2L}(\omega, \beta) + G_{3L}(\omega, \beta) \log \left| \frac{1-\omega}{1+\omega} \right| \right] \right\} \quad (32)$$

with

$$\omega = \sqrt{1 - 4K^2 z / Q^2} \quad (33)$$

$$G_{1L}(\omega, \beta) = \sum_{k=0}^4 \frac{1}{\omega^k} g_{k1}^{(L)}(\beta) \quad (34)$$

$$G_{2L}(\omega, \beta) = \sum_{k=2}^4 \frac{1}{\omega^k} g_{k2}^{(L)}(\beta) \quad (35)$$

$$G_{3L}(\omega, \beta) = \sum_{k=1}^5 \frac{1}{\omega^k} g_{k3}^{(L)}(\beta) \quad (36)$$

and

$$\begin{aligned} g_{01}^{(L)}(\beta) &= \frac{5}{2} + 3 \cos^2 \beta - \frac{3}{2} \cos^4 \beta \\ g_{11}^{(L)}(\beta) &= 4 \cos \beta - 12 \cos^3 \beta \\ g_{21}^{(L)}(\beta) &= 3 - 18 \cos^2 \beta + 15 \cos^4 \beta \\ g_{31}^{(L)}(\beta) &= -12 \cos \beta + 20 \cos^3 \beta \\ g_{41}^{(L)}(\beta) &= -\frac{3}{2} + 15 \cos^2 \beta - \frac{35}{2} \cos^4 \beta \\ g_{22}^{(L)}(\beta) &= 4 - 12 \cos^2 \beta \\ g_{23}^{(L)}(\beta) &= -24 \cos \beta + 40 \cos^3 \beta \\ g_{24}^{(L)}(\beta) &= -3 + 30 \cos^2 \beta - 35 \cos^4 \beta \\ g_{31}^{(L)}(\beta) &= \frac{3}{4} + \frac{1}{2} \cos^2 \beta + \frac{3}{4} \cos^4 \beta \\ g_{32}^{(L)}(\beta) &= 2 \cos \beta - 6 \cos^3 \beta \\ g_{33}^{(L)}(\beta) &= \frac{7}{2} - 15 \cos^2 \beta + \frac{15}{2} \cos^4 \beta \\ g_{34}^{(L)}(\beta) &= -18 \cos \beta + 30 \cos^3 \beta \\ g_{35}^{(L)}(\beta) &= -\frac{9}{4} + \frac{45}{2} \cos^2 \beta - \frac{105}{4} \cos^4 \beta \end{aligned} \quad (37)$$

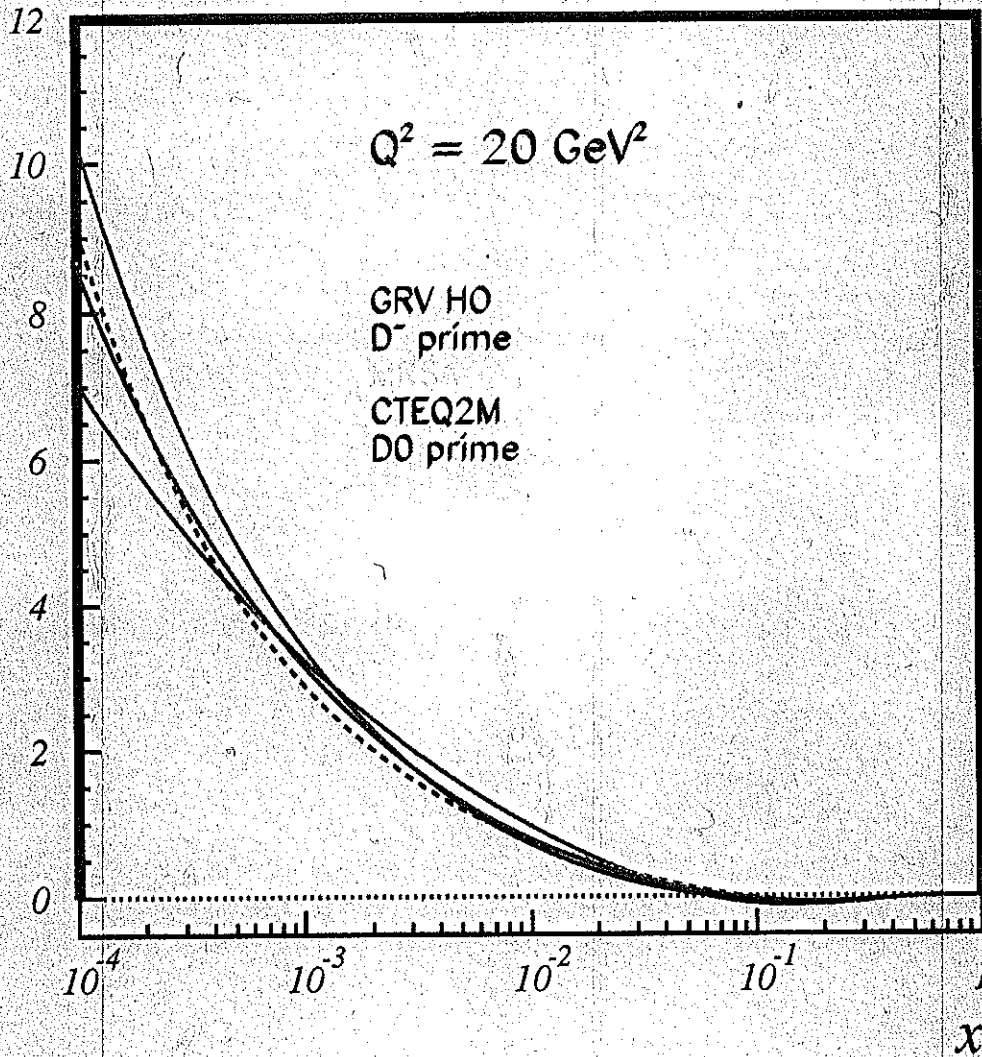
5 Numerical Results

- CONSIDER $F_L(x, Q^2)$ AS AN EXAMPLE.

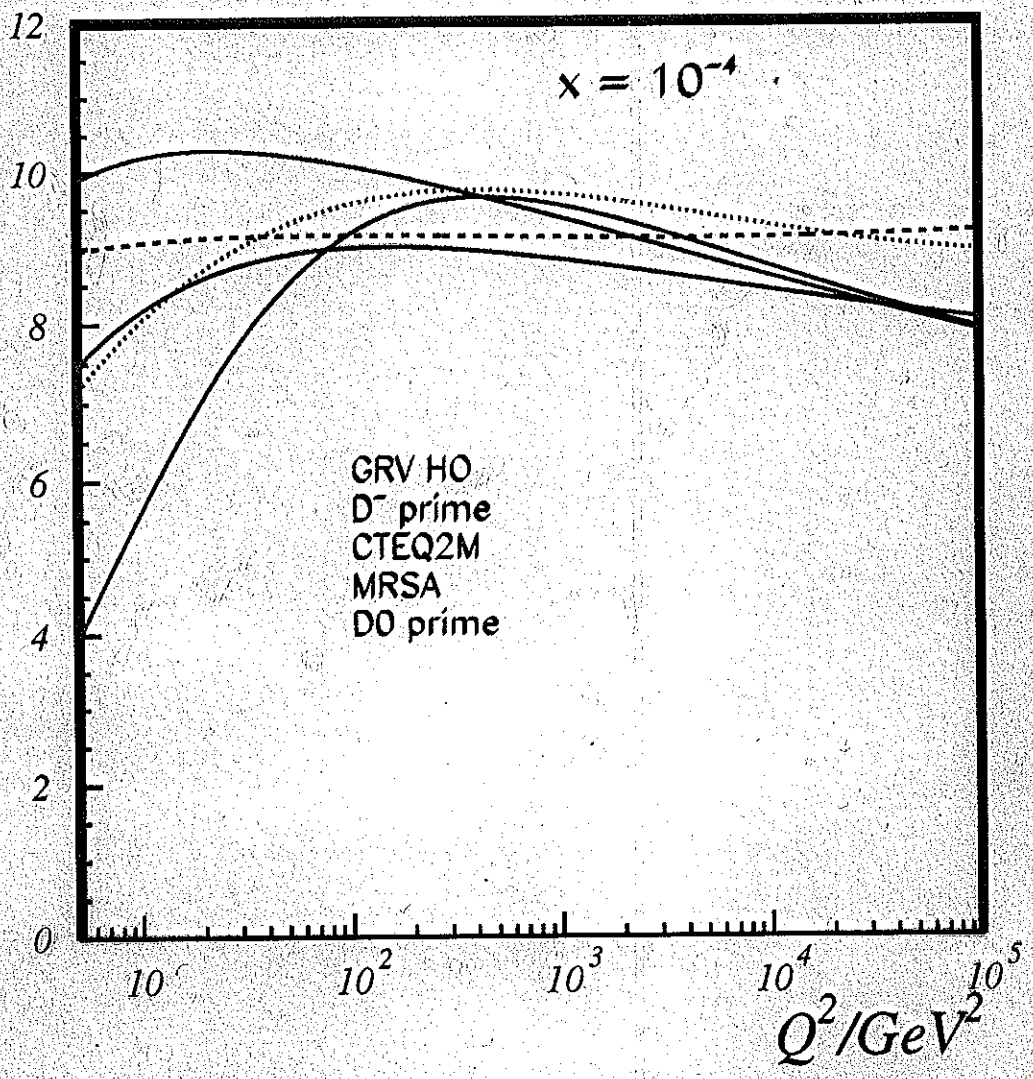
- $\frac{\partial x G(x, K^2)}{\partial \log K^2}$:
 - DO'
 - D''
 - MRSA
 - CTEQ2M
 - GRV HO } SMOOTHED BY R. ROBERTS. NEW FIT!

- X DEPENDENCE
 - Q^2 DEPENDENCE
 - Q_0^2 DEPENDENCE (?)
- OF THE K_{\perp} EFFECTS

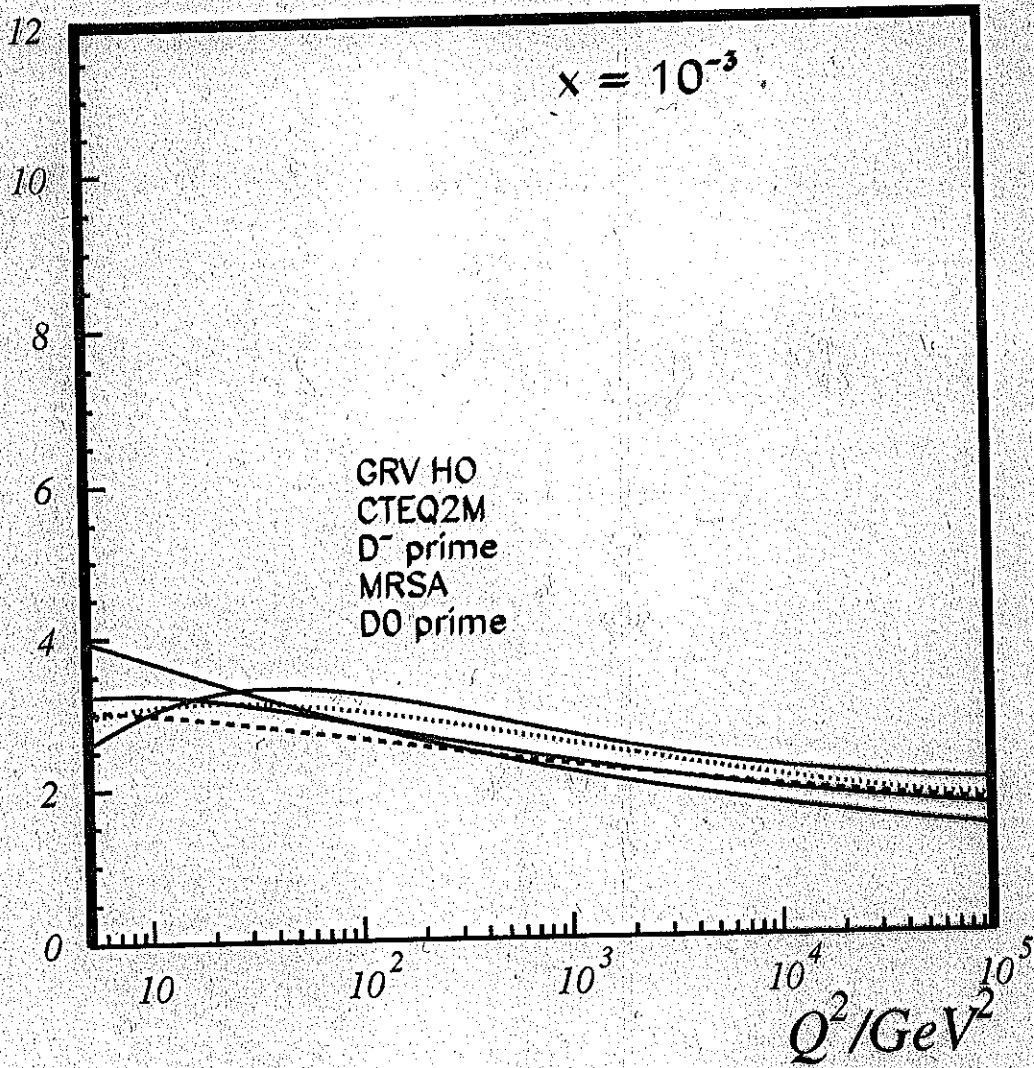
$dxG(x, Q^2)/d\log Q^2$

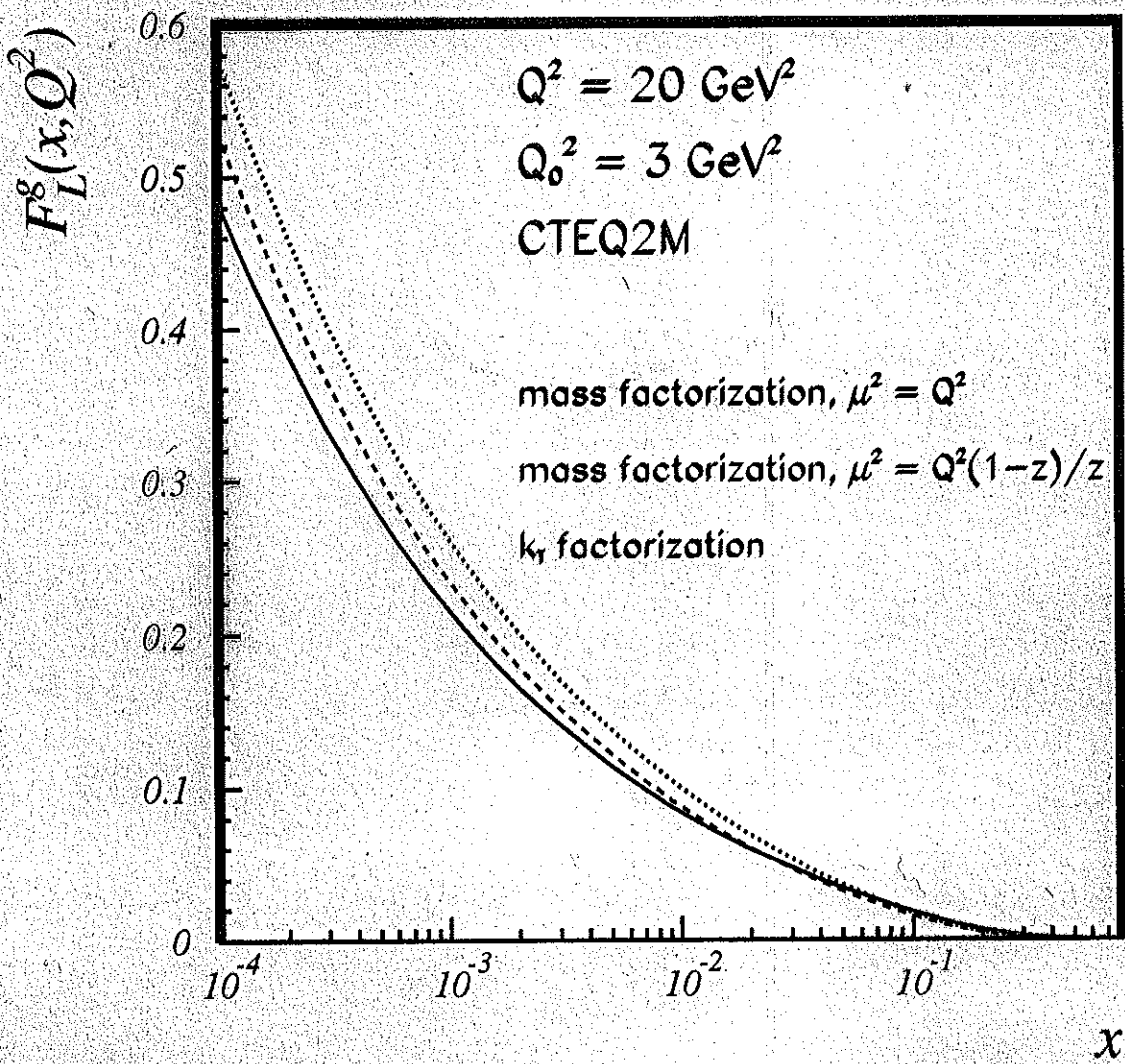


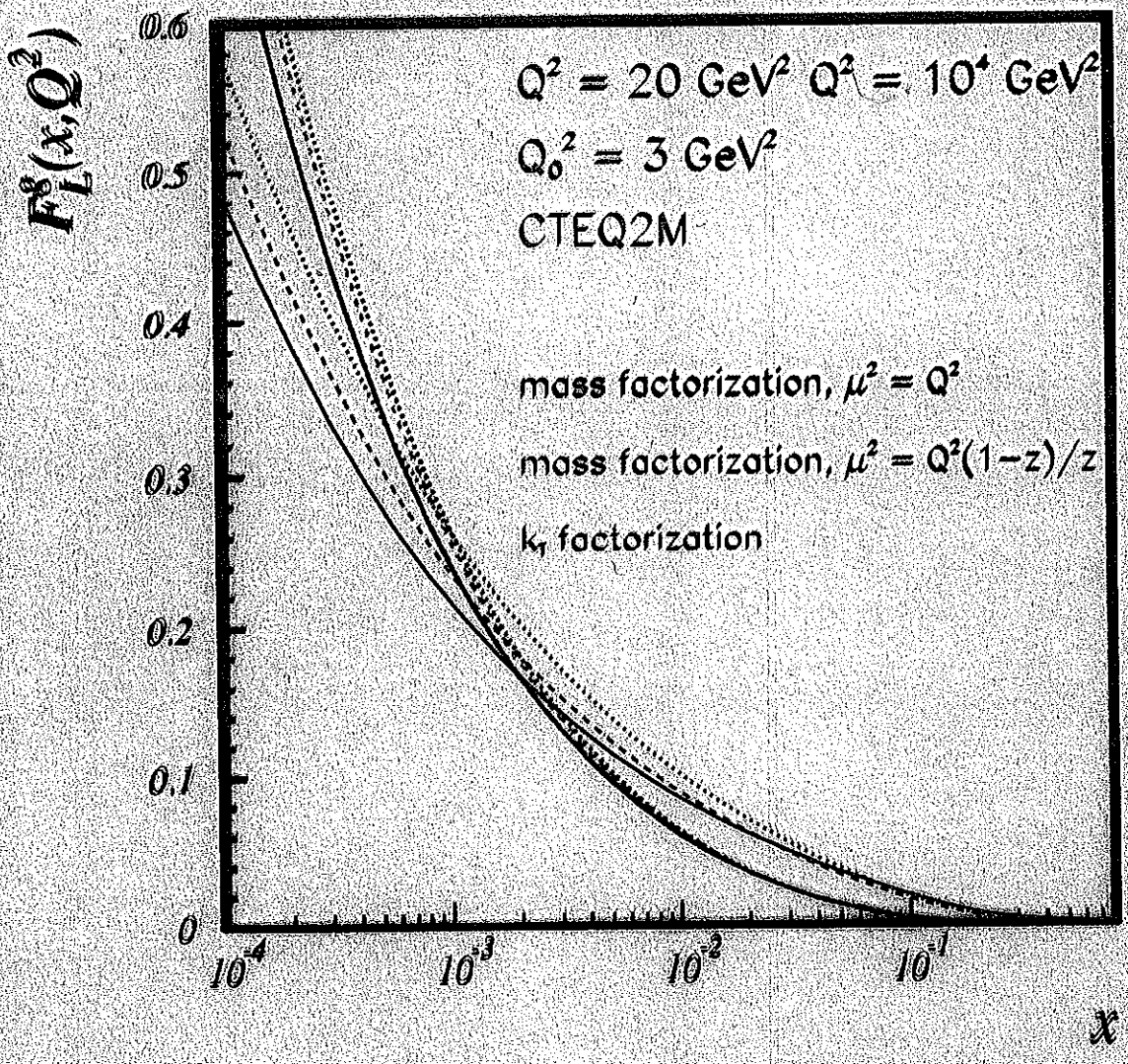
$dxG(x, Q^2)/d\log Q^2$



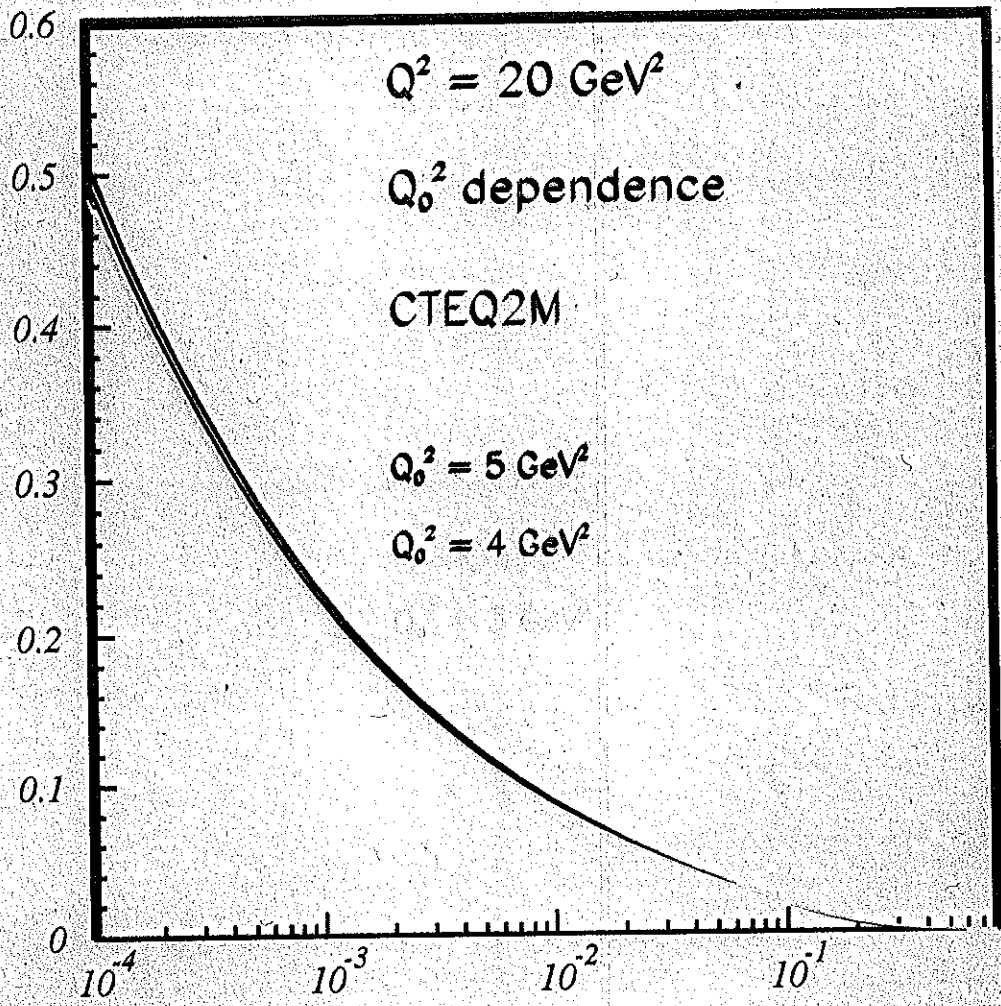
$dxG(x, Q^2)/d\log Q^2$



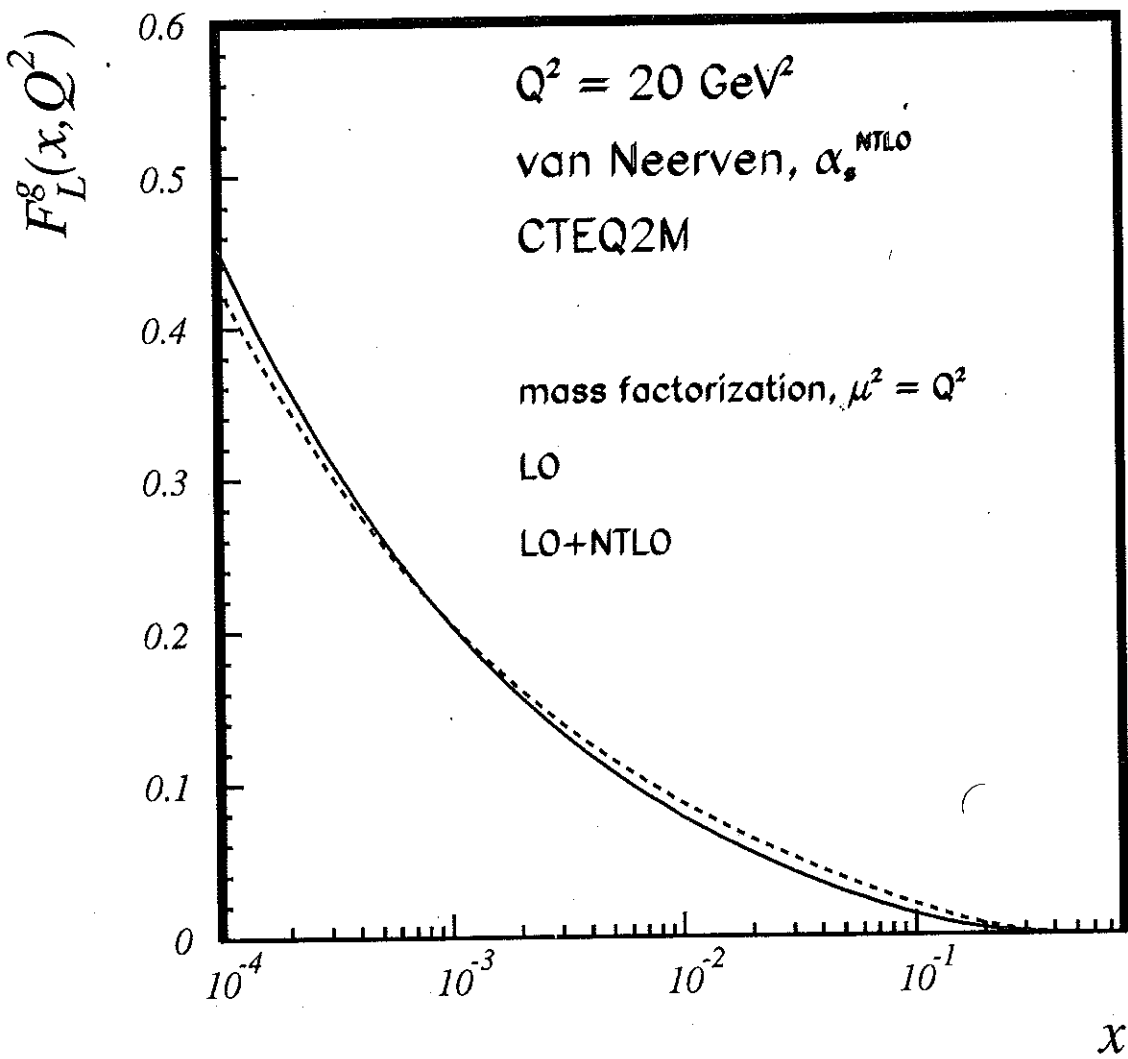




$F_L^g(x, Q^2)$



x



6 Conclusions

1. A derivation of k_{\perp} factorization which is consistent with perturbative QCD has been given.
2. The gluon contribution to the structure functions $F_L(x, Q^2)$ and $F_2(x, Q^2)$ has been calculated using k_{\perp} factorization without approximations of the Mellin convolution and the x dependence of the coefficient functions contrasting earlier investigations. The obtained contributions to the structure functions are *positive* in the whole x range.
3. The derived coefficient functions approach those found using mass factorization in the limit $K^2 \rightarrow 0$.
4. The numerical result obtained for $F_i(x, Q^2)$ in k_{\perp} factorization for suitably 'large' values of x approaches the result obtained ignoring the k_{\perp} dependence of the coefficient functions. (This is an expectation in the parton model.)
5. The numerical results behave *very* stable against the choice of the scale Q_0^2 . There is essentially no change for *all* x , as long as the condition $Q^2 \gg Q_0^2$ is met. (As we do not provide a thorough inclusion of higher twist effects this is a natural choice.)
6. There is *no fixed onset* (e.g. $x \sim 10^{-2}$) of small x effects observed. Deviations from the $k_{\perp} \rightarrow 0$ results become smaller with rising Q^2 at $x = \text{const}$.
7. The k_{\perp} dependence of the coefficient functions and gluon distribution results into SMALLER VALUES of the structure functions in the small x range. A similar behaviour has been observed recently by VAN NEERVEN ET AL. in a higher order calculation using mass factorization.